## SOLVING TRIANGLES IN TWO AND THREE DIMENSIONS

Any triangle can be solved, if **THREE** properties of the triangle are given/known, by using:

- ✓ The trig ratios in RIGHT-ANGLED triangles
- ✓ The area, sine or cosine rule

**When to use what?**

**If the triangle is RIGHT-ANGLED**

↓

*Use the trig RATIOS*

**If the triangle is NOT right-angled**

↓

COSINE RULE if ……

- *3 sides of the triangle are given*
- *2 sides and an included angle of the triangle is given*

SINE RULE if ……

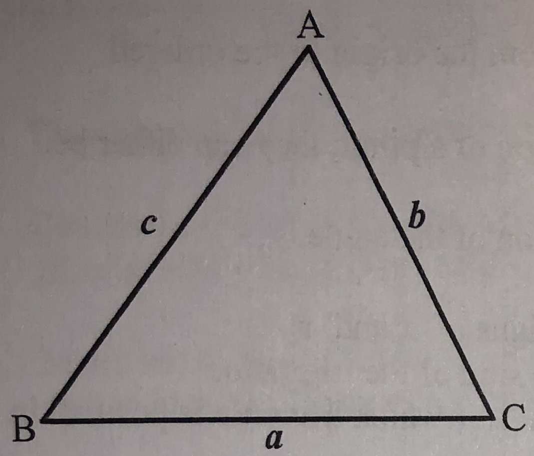
- *Any condition that does NOT satisfy the cosine rule*

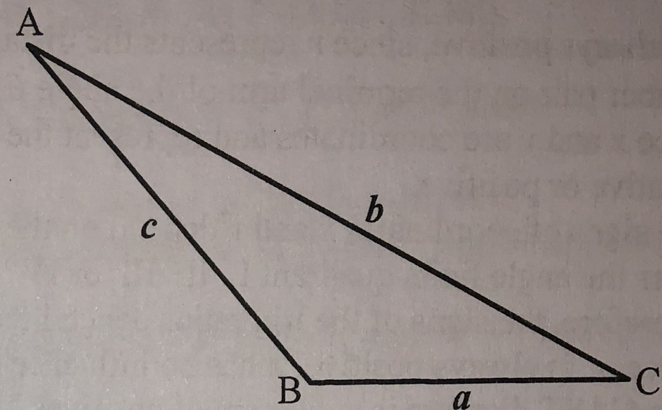
AREA RULE if ……

- *Only if "area" is mentioned*

**Remember:**
*3 properties of a triangle must be given in a triangle in order to work in that triangle*
*(**NOT** angle, angle, angle)*

**IN ANY ΔABC THE RULES ARE APPLIED AS FOLLOW:**




## Sine rule

*If an ANGLE is asked*

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

*If a SIDE is asked*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

## Cosine rule

*If an ANGLE is asked*

- $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
- $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
- $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

*If a SIDE is asked*

- $a^2 = b^2 + c^2 - 2bc \,.\, \cos A$
- $b^2 = a^2 + c^2 - 2ac \,.\, \cos B$
- $c^2 = a^2 + b^2 - 2ab \,.\, \cos C$

**Area rule** ⟶ **Area of ΔABC** $= \frac{1}{2} ab . \sin C$ or

$= \frac{1}{2} bc . \sin A$ or

$= \frac{1}{2} ac . \sin B$

## TIPS FOR SOLVING 2D & 3D PROBLEMS

1. The diagram usually consists of 2 or more triangles with COMMON sides.

2. One of the triangles is often right-angled, so use the trig ratios to solve it. (In triangles without right angles, the Sine, Cosine and Area rules must be applied.)

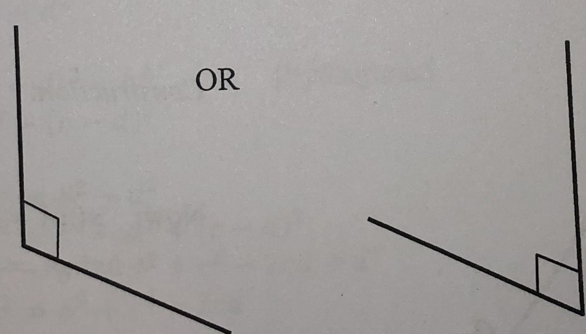
3. * Make use of basic Geometry to obtain additional information, such as vertical opposite angles, interior angles of a triangle, etc.

4. In Grade 12, be on the lookout for Compound and Double angles when simplifying a problem.

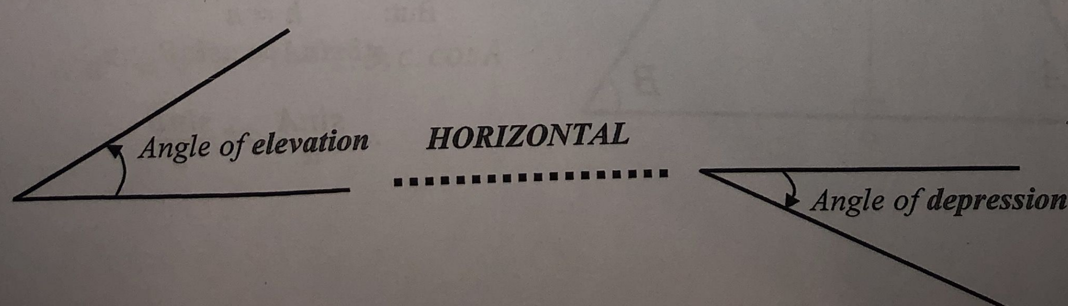
5. Start in the triangle that contains the most information, then move along to the triangle in which the required line/angle is.

move from have → want Δ. = via common side

6. When solving problems in three dimensions:
   - It may help to shade the horizontal area
   - In the diagram, right angles may not look like right angles, e.g. * 3-d perspective




7. In applications, we often use angles of DEPRESSION and ELEVATION. Both are measured from the horizontal.

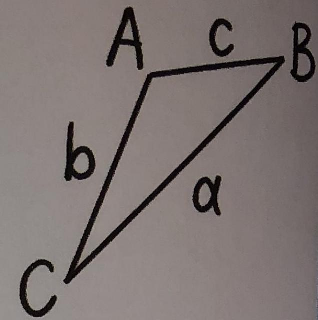


# Three-dimensional Trigonometry

## In any triangle:

Sin rule • $\dfrac{\sin\hat{A}}{a} = \dfrac{\sin\hat{B}}{b} = \dfrac{\sin\hat{C}}{c}$

• $\dfrac{a}{\sin\hat{A}} = \dfrac{b}{\sin\hat{B}} = \dfrac{c}{\sin\hat{C}}$




Cos rule • $a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

Area rule • area $\Delta ABC = \frac{1}{2}ab\sin\hat{C}$

Must be an included angle.

## NB

• In a 90° (right-angled) triangle:
→ use "normal" trig, eg. $\sin x = \frac{o}{h}$, $\cos x = \frac{a}{h}$ and $\tan x = \frac{o}{a}$

• When you need to prove something in a non-right-angled triangle:
→ always use sin rule
→ use cos rule when you need a $\sqrt{\ }$ or $(\ )^2$