Exercise 11.2 (P. 244)

1. b) $(6;1)$ $(-6;-6)$

for distance → $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$d^2 = (-6-6)^2 + (6-1)^2$

$d^2 = 144 + 25$

$d = \sqrt{169}$

$d = 13$ units

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1. d) $(-4;3)$ $(0;0)$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$d^2 = (0-(-4))^2 + (0-3)^2$

$d^2 = 16 + 9$

$d = \sqrt{25}$

$d = 5$ units

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1. e) $(-2;1)$ $(-4;-1)$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

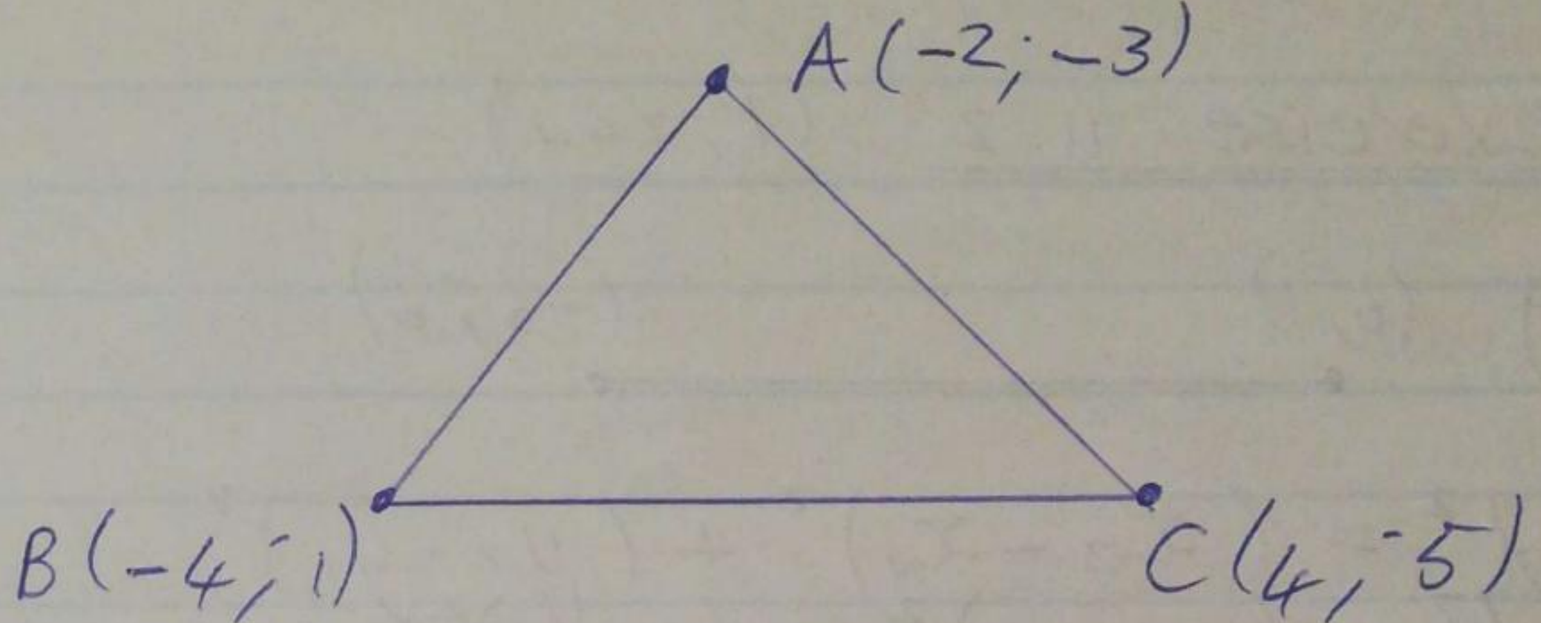
$d^2 = (-4-(-2))^2 + (-1-1)^2$

$d^2 = 4 + 4$

$d = \sqrt{8}$ $(= 2\sqrt{2})$

$d = 2,83$ units

2.C



i) Perimeter = AB + BC + AC

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$AB^2 = (-4 - (-2))^2 + (1 - (-3))^2$
$AB = \sqrt{20}$
$AB = 2\sqrt{5}$ units

$BC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$BC^2 = (4 - (-4))^2 + (5 - 1)^2$
$BC = \sqrt{80}$
$BC = 4\sqrt{5}$ units

$AC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$AC^2 = (4 - (-2))^2 + (5 - (-3))^2$
$AC = \sqrt{100}$
$AC = 10$ units

$\therefore$ Perimeter $= 2\sqrt{5} + 4\sqrt{5} + 10$
$= 23,42$ units

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ii) Scalene - no equal sides

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iii) Check if hypotenuse (AC) is 10 units with Pythagoras.

$AC^2 = AB^2 + BC^2$
$AC^2 = (2\sqrt{5})^2 + (4\sqrt{5})^2$
$AC^2 = 20 + 80$
$AC = \sqrt{100}$
$AC = 10$ units $\therefore \triangle ABC$ is right-angled

3a) $(x, -3)$ — 5 units — $(0, 0)$ ← origin

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$5^2 = (0 - x)^2 + (0 - (-3))^2$$
$$25 = x^2 + 9$$
$$0 = x^2 - 16$$
$$0 = (x + 4)(x - 4)$$
$$x + 4 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -4 \qquad x = 4$$

If no sketch is present, then both answers are correct.

3c) $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$$AB^2 = (5 - (-3))^2 + (-2 - 0)^2$$
$$AB = \sqrt{68} = 2\sqrt{17}$$
$$AB = BC$$

$\therefore$ B(5, -2) — $2\sqrt{17}$ units — C(13, y)

$$BC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$(2\sqrt{17})^2 = (13 - 5)^2 + (y - (-2))^2$$
$$68 = 64 + (y + 4)^2$$
$$68 = 64 + y^2 + 8y + 16$$
$$0 = y^2 + 8y + 12$$
$$0 = (y + 6)(y + 2)$$
$$y + 6 = 0 \quad \text{or} \quad y + 2 = 0$$
$$y = -6 \qquad y = -2$$

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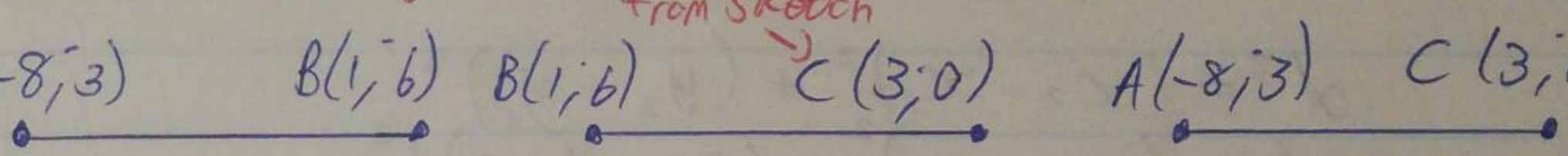
6. Find the length of the longest side:

from sketch

A(-8,3) B(1,6) B(1,6) C(3,0) A(-8,3) C(3,0)



$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$AB^2 = (1 - (-8))^2 + (6 - 3)^2$
$AB = \sqrt{90} = 3\sqrt{10}$ units

$BC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$BC^2 = (3 - 1)^2 + (0 - 6)^2$
$BC = \sqrt{40} = 2\sqrt{10}$ units

$AC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$AC^2 = (3 - (-8))^2 + (0 - 3)^2$
$AC = \sqrt{130}$ units ← Longest side

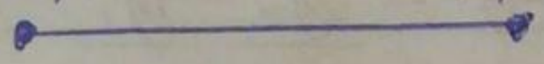
Next, see if Pythagoras gives the same distance for AC

$AC^2 = AB^2 + BC^2$
$AC^2 = (\sqrt{90})^2 + (\sqrt{40})^2$
$AC^2 = 90 + 40$
$AC = \sqrt{130}$

∴ $\triangle ABC$ is a right-angled $\triangle$.

Exercise 11.4 (p. 257)

1.a) $(-3;2)$ $(1;1)$

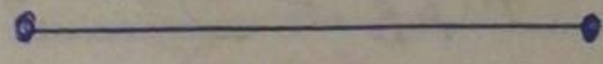


$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{1 - 2}{1 - (-3)}$

$m = -\frac{1}{4}$

1.c) $(-1;2)$ $(1;-1)$

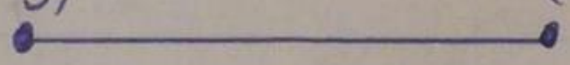


$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-1 - 2}{1 - (-1)}$

$m = -\frac{3}{2}$

1.f) $(a;b)$ $(-2a;-2b)$



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-2b - b}{-2a - a}$

$m = \frac{-3b}{-3a}$

$m = \frac{b}{a}$

2) Remember: $AB \perp BC$; if $m_{AB} \times m_{BC} = -1$

$\therefore$ if $m_{AB} = \frac{a}{b}$, then $m_{BC} = -\frac{b}{a}$.

2.a) $m_1 = -\frac{1}{4}$ $\therefore$ $m_2 = \frac{4}{1} = 4$

2.b) $m_1 = -\frac{3}{2}$ $\therefore$ $m_2 = \frac{2}{3}$

2.f) $m_1 = \frac{b}{a}$ $\therefore$ $m_2 = -\frac{a}{b}$

3. a) A(−3; 5) B(5; −1) C(−2; −1) D(1; 3)

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} = \frac{-1 - 5}{5 - (-3)}$

$m_{AB} = -\frac{3}{4}$

$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{CD} = \frac{3 - (-1)}{1 - (-2)}$

$m_{CD} = \frac{4}{3}$

$m_{AB} \neq m_{CD}$ ∴ not parallel

$m_{AB} \times m_{CD} = -\frac{3}{4} \times \frac{4}{3} = -1$

∴ Perpendicular

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3. b) A(−2; −4) B(3; 1) C(5; −1) D(−2; −8)

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{1 - (-4)}{3 - (-2)}$

$= 1$

$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-8 - (-1)}{-2 - 5}$

$= 1$

$m_{AB} = m_{CD}$ ∴ Parallel

3.c) A(-2;1) B(2;4) C(-3;-1) D(0;3)

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4-1}{2-(-2)}$

$= \frac{3}{4}$

$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-(-1)}{0-(-3)}$

$= \frac{4}{3}$

$m_{AB} \neq m_{CD}$ ∴ not parallel

$m_{AB} \times m_{CD} = \frac{3}{4} \times \frac{4}{3} = 1 \neq -1$ ∴ Not perpendicular.

3.d) A(a;b) B(c;d) C(b;a) D(d;c)

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{d-b}{c-a}$

$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{c-a}{d-b}$

$m_{AB} \neq m_{CD}$ ∴ not parallel

$m_{AB} \times m_{CD} = \frac{(d-b)}{(c-a)} \times \frac{(c-a)}{(d-b)} = 1 \neq -1$ ∴ not perpendicular

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3.e) A(a;b) B(ka;kb) C(b;pa) D(pb;a)

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{kb-b}{ka-a}$

$= \frac{b(k-1)}{a(k-1)}$

$= \frac{b}{a}$

$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{a-pa}{pb-b}$

$= \frac{a(1-p)}{b(p-1)}$

$= \frac{-a(p-1)}{b(p-1)} = \frac{-a}{b}$

$m_{AB} \neq m_{CD}$

$m_{AB} \times m_{CD}$

$= \frac{b}{a} \times -\frac{a}{b}$

$= -1$

∴ perpendicular

4. P(1;3) Q(1;-1) R(5;2) S(2;2)

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1 - 3}{1 - 1}$$

$$= \frac{-4}{0}$$

∴ undefined

∴ vertical

$$m_{RS} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 2}{2 - 5}$$

$$= \frac{0}{-3}$$

$$= 0$$

∴ horizontal

∴ PQ ⊥ RS

Exercise 11.4 (P. 257)

5)a) $(-2;-6)$ $(2;-4)$ $(2;-4)$ $(4;-3)$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{-4-(-6)}{2-(-2)}$$

$$m_1 = \frac{1}{2}$$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_2 = \frac{-3-(-4)}{4-2}$$

$$m_2 = \frac{1}{2}$$

$\therefore m_1 = m_2 \therefore$ collinear

5.b) $(-5;5)$ $(1;1)$ $(1;1)$ $(4;-1)$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-5}{1-(-5)}$$

$$= -\frac{2}{3}$$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1-1}{4-1}$$

$$= -\frac{2}{3}$$

$m_1 = m_2 \therefore$ collinear

5.c) $(-5;-6)$ $(-2;0)$ $(-2;0)$ $(-1;2)$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0-(-6)}{-2-(-5)}$$

$$= \frac{6}{3}$$

$$= 2$$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-0}{-1-(-2)}$$

$$= \frac{2}{1}$$

$$= 2$$

$\therefore m_1 = m_2 \therefore$ collinear

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6. A(-2;3) B(1;4) C(-4;1) D(x;4)

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-3}{1-(-2)}$$

$$= \frac{1}{3}$$

6. a) Parallel:

$$m_{CD} = \frac{1}{3}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{3}$$

$$\frac{4-1}{x-(-4)} = \frac{1}{3}$$

$$\frac{3}{x+4} \times 3 = \frac{1}{3} \times (x+4)$$

$$9 = x + 4$$

$$5 = x$$

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6. c) $m_{AB} \times m_{CD} = -1$

$$\frac{1}{3} \times m_{CD} = -1$$

$$m_{CD} = -3$$

$$\frac{3}{x+4} = -3$$

$$3 = -3(x+4)$$

$$3 = -3x - 12$$

$$\frac{3x}{3} = \frac{-15}{3}$$

$$x = -5$$

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6. c) B(-2;3) ——— C(-4;1) C(-4;1) ——— D(x;4)

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-3}{-4-(-2)}$$

$$= 1$$

$$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-1}{x-(-4)}$$

$$= \frac{3}{x+4}$$

collinear

$\therefore m_{BC} = m_{CD}$

$(x+4) \times 1 = \frac{3}{(x+4)} \times (x+4)$

$x+4 = 3$

$x = -1$

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7. b) (x;y) ——— (3;-5) (3;-5) ——— (4;-4)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5-y}{3-x}$$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-4-(-5)}{4-3} = 1$$

$\therefore \frac{-5-y}{3-x} = 1$

$-5-y = 3-x$

$-5-3+x = y$

$x-8 = y$

Exercise 11.3 (P. 261)

2. a) $A(-2;-1)$ $M(x;y)$ $B(-1;9)$

$$x = \frac{x_1 + x_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

$$= \frac{-2 + (-1)}{2} \qquad = \frac{-1 + 9}{2}$$

$$= -\frac{3}{2} \qquad = 4$$

$$\therefore M\left(-\frac{3}{2}; 4\right)$$

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2. b) $A(4;5)$ $M(1;3)$ $(x;y)$

$$x_M = \frac{x_1 + x_2}{2} \qquad y_M = \frac{y_1 + y_2}{2}$$

$$1 = \frac{4 + x}{2} \qquad 3 = \frac{5 + y}{2}$$

$$2 = 4 + x \qquad 6 = 5 + y$$

$$-2 = x \qquad 1 = y$$

$$\therefore (-2;1)$$

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2. c) $(0;-2)$ $M(-1;y)$ $(x;8)$

$$x_M = \frac{x_1 + x_2}{2} \qquad y_M = \frac{y_1 + y_2}{2}$$

$$-1 = \frac{0 + x}{2} \qquad y = \frac{-2 + 8}{2}$$

$$-2 = 0 + x \qquad y = 3$$

$$-2 = x$$

$$\therefore M(-1;y) = M(-1;3)$$

$$(x;8) = (-2;8)$$

2. d) $M(5;-2) \quad C(x;3) \quad N(-7;y)$

$$x_c = \frac{x_1 + x_2}{2}$$

$$x = \frac{5 + (-7)}{2}$$

$$x = -1$$

$$y_c = \frac{y_1 + y_2}{2}$$

$$3 = \frac{-2 + y}{2}$$

$$6 = -2 + y$$

$$8 = y$$

$\therefore (x;3) = (-1;3)$

$N(-7;y) = N(-7;8)$

~~2. e)~~

~~3. d)~~

3. a) $A(-3;1)$ $R(x;y)$ $B(-5;-3)$

$$x_R = \frac{x_1 + x_2}{2} \qquad y_R = \frac{y_1 + y_2}{2}$$

$$x_R = \frac{-3 + (-5)}{2} \qquad = \frac{1 + (-3)}{2}$$

$$x_R = -4 \qquad = -1$$

$$\therefore R(-4;-1)$$

$B(-5;-3)$ $S(x;y)$ $C(1;-5)$

$$x_S = \frac{x_1 + x_2}{2} \qquad y_S = \frac{y_1 + y_2}{2}$$

$$= \frac{-5 + 1}{2} \qquad = \frac{-3 + (-5)}{2}$$

$$= -2 \qquad = -4$$

$$\therefore S(-2;-4)$$

$A(-3;1)$ $T(x;y)$ $C(1;-5)$

$$x_T = \frac{x_1 + x_2}{2} \qquad y_T = \frac{y_1 + y_2}{2}$$

$$= \frac{-3 + 1}{2} \qquad = \frac{1 + (-5)}{2}$$

$$= -1 \qquad = -2$$

$$\therefore T(-1;-2)$$

3.b) C(1;-5) R(-4;-1)

$CR^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$CR^2 = (-4 - 1)^2 + (-1 - (-5))^2$
$CR = \sqrt{41}$
$CR = 6,40$ units

A(-3;1) S(-2;-4)

$AS^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$AS^2 = (-2 - (-3))^2 + (-4 - 1)^2$
$AS = \sqrt{26}$
$AS = 5,10$ units

B(-5;-3) T(-1;-2)

$BT^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$BT^2 = (-1 - (-5))^2 + (-2 - (-3))^2$
$BT = \sqrt{17}$
$BT = 4,12$ units