Number Patterns

Linear number patterns are patterns that have a constand difference,
$\therefore T_2 - T_1 = T_3 - T_2$.

When investigating a linear number pattern, we obtain a general rule:

$$T_n = bn + c$$

where:
- T_n is the value at position n.
- b is the constant difference
- n is the position of a term
- $b + c = T_1$

Examples

1) Consider the number pattern: 6; 10; 14; 18; ...
a) Determine the n^{th} term (General rule)
b) Detemine the 1000^{th} term
c) Which term will be equal to 202?

a) $b+c=\boxed{6}$; 10; 14; ...

$b=\boxed{4}$ $\quad$ 4

$T_n = bn + c$

$b = 4$ $\qquad$ $b + c = 6$

$4 + c = 6$

$c = 2$

$\therefore T_n = 4n + 2$

b) $T_n = 4n + 2$

$T_{1000} = 4(1000) + 2$

$T_{1000} = 4002$

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c) $T_n = 4n + 2$

$202 = 4n + 2$

$\frac{200}{4} = \frac{4n}{4}$

$50 = n$

$\therefore T_{50} = 202$

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2) Consider the pattern: $-6; -11; -16; -21; \ldots$

a) Determine the n^{th} term

b) Determine the 600^{th} term

c) Which term will be equal to -401?

a) $b+c = \boxed{-6}; -11; -16$

$b = \boxed{-5} \quad -5$

$T_n = bn + c$

$b = -5 \qquad b + c = -6$

$-5 + c = -6$

$c = -1$

$\therefore T_n = -5n - 1$

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b) $T_n = -5n - 1$

$T_{600} = -5(600) - 1$

$T_{600} = -3001$

c) $T_n = -5n - 1$

$-401 = -5n - 1$

$\frac{-400}{-5} = \frac{-5n}{-5}$

$80 = n$

$\therefore T_{80} = -401$

Example - November 2019

Given the linear pattern: $2x+1; 3x+3; 4x+5; \ldots$

1) Write down the next term in the pattern.
2) Write in terms of x, the formula for T_n.
3) If the value of the 13^{th} term is 95, calculate the value of x.
4) If $x=5$, determine the largest value of n for which $T_n < 158$.

1) $5x + 7$

2) $2x; 3x; 4x \ldots$ with differences x, x

$1; 3; 5; \ldots$ with differences 2, 2

$T_n = bn + c$

$b = x \quad b + c = 2x$

$x + c = 2x$

$c = x$

$\therefore T_n = xn + x$

$T_n = bn + c$

$b = 2 \quad b + c = 1$

$2 + c = 1$

$c = -1$

$\therefore T_n = 2n - 1$

But these are being added, so:

$$T_n = xn + x + 2n - 1$$
$$T_n = xn + 2n + x - 1$$
$$T_n = \underbrace{(x+2)}_{b}n + \underbrace{x - 1}_{c}$$

3) $T_{13} = 95$

$$T_n = (x+2)n + x - 1$$
$$95 = (x+2)13 + x - 1$$
$$95 = 13x + 26 + x - 1$$
$$95 = 14x + 25$$
$$\frac{70}{14} = \frac{14x}{14}$$
$$5 = x$$

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4) $x = 5 \therefore T_n = (5+2)n + 5 - 1$

$$T_n = 7n + 4$$

$$T_n < 158$$
$$7n + 4 < 158$$
$$\frac{7n}{7} < \frac{154}{7}$$
$$n < 22$$

$\therefore n = 21$ since $n \in \mathbb{Z}$

Exercise 3.5 P. 69

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