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2019

Technical mathematics

Revision

BOOK

PAPER 1

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| **GENERAL ALGEBRA** |

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| **QUESTION 1** | | | | | |
|  |  | |  |  |  |
| 1.1 | Given: | | | | |
|  |  |  | | |  |
|  | 1.1.1 | For what value of *x* is *E* undefined? | | | (1) |
|  |  | | | | (2) |
|  | 1.1.2 | Determine the value(s) of *x* for which *E* = 0. | | |  |
|  |  | | | |  |
| 1.2 | Solve the following simultaneous equations for *x* and *y*.  and | | | | (9) |
|  |  |  | |  |  |
| 1.3 | The formula is used to calculate the sag *S* at the centre of a wire. | | | | |
|  |  |  | |  |  |
|  | 1.3.1 | Show that | | | (3) |
|  |  |  | |  |  |
|  | 1.3.2 | Determine the value of *L* , if *d* = 1,65 and *d* = 0,82. | | | (2) |
|  |  |  | |  |  |
|  | 1.3.3 | Write your answer in QUESTION 1.3.2 in scientific notation. | | | (2) |
|  |  |  | |  |  |
| 1.4 | Given the following binary numbers : 11012 ; 10012 and 111012 | | | |  |
|  |  |  | |  |  |
|  | 1.4.1 | Without using a calculator, determine the **sum** of the given binary numbers. | | | (3) |
|  |  |  | |  |  |
|  | 1.4.2 | If the answer to QUESTION 1.4.1 is 1100112, express this number as a positive integer. | | | (3) |
|  |  |  | |  |  |
|  |  |  | |  | **[25]** |

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| **QUESTION 2** | | | | |
|  |  |  |  |  |
| Consider the equation: .  Use the given equation to answer the following questions. | | | | |
|  |  |  |  |  |
| 2.1 | Solve for *x* , if *k* = 3 | | | (4) |
|  |  |  |  |  |
| 2.2 | Without solving the equation, discuss the nature of the roots of the equation, | | | (4) |
|  |  |  |  |  |
| 2.3 | Calculate the value(s) of *k* for which the equation has equal roots. | | | (4) |
|  |  |  |  |  |
|  |  |  |  | **[12]** |

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| **QUESTION 3** | | | | |
|  |  |  |  |  |
| 3.1 | **Without the use of a calculator** simplify (showing all calculations) the following expressions, completely. | | | |
|  |  |  |  |  |
|  | 3.1.1 |  | | (4) |
|  |  |  |  |  |
|  | 3.1.2 |  | | (4) |
|  |  |  |  |  |
|  | 3.1.3 |  | | (3) |
|  |  |  |  |  |
| 3.2 | Solve for ***x*** in each of the following: | | | |
|  |  |  |  |  |
|  | 3.2.1 |  | | (4) |
|  |  |  |  |  |
|  | 3.2.2 |  | | (5) |
|  |  |  |  |  |
|  | 3.2.3 |  | | (4) |
|  |  |  |  |  |
| 3.3 | Express the electric field intensity of a light wave  z , in rectangular form, i.e. | | | (6) |
|  |  |  |  |  |
| 3.4 | Solve for *k* and *p* if : | | | (4) |
|  |  |  |  | **[34]** |

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| **QUESTION 4** | | | | |
|  | | | | |
| 4.1 | Solve for *x* in each of the following, and give the answers correct to TWO decimal places where necessary. | | | |
|  |  |  |  |  |
|  | 4.1.1 |  | | (6) |
|  |  |  | |  |
|  | 4.1.2 |  | | (8) |
|  |  |  | |  |
|  | 4.1.3 | , and illustrate your answer on a number line. | | (5) |
|  |  |  | |  |
| 4.2 | Given : | | | |
|  |  |  | |  |
|  | 4.2.1 | Show that | | (3) |
|  |  |  | |  |
|  | 4.2.2 | Hence, determine the value of *g*, if T = 1,56 ; *l* = 243 and , correct to the nearest integer value. | | (2) |
|  |  |  | |  |
| 4.3 | In the given diagram, ABCD represents a rectangular piece of land. The area of the land is 52 m2. AB is *x* metres and BC is *y* metres.  B  A  *x*  *y*  C  D | | | |
|  | 4.3.1 | If BC is 1,5 m less than AB, express BC in terms of *x*. | | (1) |
|  |  |  | |  |
|  | 4.3.2 | If the area of the rectangle is given by: , determine the length of AB. | | (6) |
|  |  |  | |  |
|  | 4.3.3 | Hence, write down the value of BC. | | (1) |
|  |  |  | | **[32]** |

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| **QUESTION 5** | | |
|  | | |
| 5.1 | Discuss the nature of the roots of , without solving the equation. | (4) |
|  |  |  |
| 5.2 | Find the value(s) of *k* if has equal roots. | (5) |
|  |  |  |
| 5.3 | Show that the equation has real and unequal roots for all real values of *m.* | (5) |
|  |  |  |
|  |  | **[14]** |

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| **QUESTION 6** | | | |
|  |  | |  |
| 6.1 | Simplify the following **without the use of a calculator**. | | |
|  |  |  |  |
|  | 6.1.1 |  | (3) |
|  |  |  |  |
|  | 6.1.2 |  | (3) |
|  |  |  |  |
|  | 6.1.3 |  | (4) |
|  |  |  |  |
|  | 6.1.4 |  | (5) |
|  |  |  |  |
|  | 6.1.5 |  | (5) |
|  |  |  | **[20]** |

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| **QUESTION 7** | | | |
|  | | | |
| 7.1 | Solve for *x* in each of the following. | | |
|  |  |  |  |
|  | 7.1.1 |  | (5) |
|  |  |  |  |
|  | 7.1.2 |  | (5) |
|  |  |  |  |
|  | 7.1.3 |  | (5) |
|  |  |  |  |
| 7.2 | If , determine the value(s) of *x* and *y*. | | (6) |
|  |  |  |  |
|  |  |  | **[21]** |

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| **QUESTION 8** | | |
|  | | |
| 8.1 | Simplify fully: | (3) |
|  |  |  |
| 8.2 | Determine the value(s) of *x* for which: | (3) |
|  |  |  |
| 8.3 | Solve for *x* and *y* in the following simultaneous equations:  And ; | (6) |
|  |  |  |
| 8.4 | Without the use of a calculator, determine the value of: | (3) |
|  |  |  |

|  |  |  |  |
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| 8.5 | Given: | |  |
|  | | | |
|  | 8.5.1 | Show that can be written as | (2) |
|  |  |  |  |
|  | 8.5.2 | Hence, solve the equation: | (5) |
|  |  |  |  |
|  |  |  | **[22]** |

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| **QUESTION 9** | | | |
|  | | | |
| 9.1 | Determine the values of *k* for which the equation has real roots. | | (5) |
|  |  |  |  |
| 9.2 | The numerical value of the perimeter of a rectangle with integral sides is equal to one-fifth the numerical value of its area. Show that there is more than one rectangle which satisfies these conditions, and calculate the dimensions of such a rectangle with the smallest breadth. | | (11) |
|  |  |  |  |
|  |  |  | **[16]** |

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| **Nature of roots** |

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| 1.1 | Determine the nature of the roots of a quadratic equation if: | | | | |  |
|  |  | | |  |  |  |
|  | 1.1.1 | | |  | | (2) |
|  |  | | |  | |  |
|  | 1.1.2 | | |  | | (2) |
|  |  | | |  | |  |
| 1.2 | Prove that the roots of are rational if *n* is an integer. | | | | | (4) |
|  |  |  | | | |  |
| 2.1 | What deductions can you make about the roots of the equation  , if: | | | | |  |
|  |  | | |  | |  |
|  | 2.1.1 | | | is a perfect square | | (1) |
|  |  | | |  | |  |
|  | 2.1.2 | | |  | | (1) |
|  |  | | |  | |  |
|  | 2.1.3 | | |  | | (2) |
|  |  | | |  | |  |
| 2.2 | Determine the value(s) of *m* for which the quadratic equation  has equal roots. | | | | | (6) |
|  |  | | |  | |  |
| 3.1 | Determine the nature of the roots of the following equations: | | | | |  |
|  |  | |  | | |  |
|  | 3.1.1 | |  | | | (3) |
|  |  | |  | | |  |
|  | 3.1.2 | |  | | | (4) |
|  |  | |  | | |  |
|  | 3.1.3 | |  | | | (3) |
|  |  | |  | | |  |
| 3.2 |  | | | | |  |
|  | 3.2.1 | | Determine the value(s) of *k* if the roots of  are real and unequal | | | (5) |
|  |  | |  | | |  |
|  | 3.2.2 | | Hence, determine the nature of the roots of the equation in QUESTION 3.2.1, if *k* = 8. | | | (3) |
|  |  | |  | | |  |
| 3.3 | Determine the value(s) of *p* for which the roots of will be non-real. | | | | | (6) |
|  |  | |  | | |  |
| 3.4 | Prove that the equation has rational roots for all values of *p*. | | | | | (6) |

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| **Complex Numbers** |

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| --- | --- | --- | --- | --- |
| 1. | Represent each of the following complex numbers on an Argand diagram. | | | |
|  |  |  |  |  |
|  | 1.1 |  | | (2) |
|  |  |  | |  |
|  | 1.2 |  | | (2) |
|  |  |  | |  |
|  | 1.3 |  | | (2) |
|  |  |  | |  |
|  | 1.4 |  | | (2) |
|  |  |  | |  |
|  | 1.5 |  | | (2) |
|  |  |  | |  |
| 2. | Determine the modulus and argument of each of the complex numbers in QUESTION 1 above. Show ALL working details clearly. | | | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 3. | In each of the following you are given the modulus and the argument of a complex number *z*. Use the given information to express *z* in the form  *z* = *a* + *bi,* where *a* and *b* are real numbers. | | | |
|  | |  |  |  |  | | --- | --- | --- | --- | | **No.** | **Modulus** | **Argument** | ***z* = *a + bi*** | | 3.1 | 2 |  |  | | 3.2 | 4 | 45o |  | | 3.3 | 1 |  |  | | 3.4 | 4 |  |  | | 3.5 | 6 |  |  | | | | |
|  |  |  |  |  |
| 4. | Express the following complex numbers in polar form | | |  |
|  |  |  |  |  |
|  | 4.1 |  | |  |
|  |  |  | |  |
|  | 4.2 |  | |  |
|  |  |  | |  |
|  | 4.3 |  | |  |
|  |  |  | |  |
|  | 4.4 |  | |  |
|  |  |  | |  |
|  | 4.5 |  | |  |
|  |  |  | |  |
|  | 4.6 |  | |  |
|  |  |  | |  |
|  | 4.7 |  | |  |
|  |  |  | |  |
| 5. | The complex numbers and are defined by :  and | | |  |
|  |  |  | |  |
|  | 5.1 | Represent the complex numbers and on an Argand diagram. | | (2) |
|  |  |  | |  |
|  | 5.2 | Determine | | (2) |
|  |  |  | |  |
|  | 5.3 | Hence, determine the modulus of | | (2) |
|  |  |  | |  |
|  | 5.4 | Determine the complex number in the form *a* + *bi*, where *a* and *b* are rational numbers. | | (4) |
|  |  |  | |  |
|  | 5.5 | Hence, determine the argument of , in radians correct to TWO decimal places | | (3) |
|  |  |  | |  |
|  | 5.6 | Determine the values of the real constants *x* and *y*, such that: | | (6) |
|  |  |  | |  |
| 6. | Given the complex number | | |  |
|  |  |  | |  |
|  | 6.1 | Determine the modulus and argument of *z* | | (3) |
|  |  |  | |  |
|  | 6.2 | Determine and write your answer in the form *a* + *bi* | | (3) |
|  |  |  | |  |
|  | 6.3 | Represent on an Argand diagram the points A, B and C, representing the complex numbers and respectively. | | (5) |
|  |  |  | |  |
|  | 6.4 | Hence, determine the magnitude of | | (4) |
|  |  |  | |  |
| 7. | The points O , P and Q in the Complex plane represent the complex numbers 0 + 0*i* ; 4 + 2*i* and 3– *i*  respectively. | | |  |
|  |  |  | |  |
|  | 7.1 | Represent the points, O, P and Q on an Argand diagram. | | (3) |
|  |  |  | |  |
|  | 7.2 | Determine the length of PQ in simplified surd form | | (3) |
|  |  |  | |  |
|  | 7.3 | Hence, show that is a right-angled triangle. | | (3) |
|  |  |  | |  |
| 8. | Solve for *x* and *y* in each of the following. | | | |
|  |  |  | |  |
|  | 8.1 |  | | (4) |
|  |  |  | |  |
|  | 8.2 |  | | (4) |
|  |  |  | |  |
|  | 8.3 |  | | (6) |
|  |  |  | |  |
|  | 8.4 |  | | (4) |
|  |  |  | |  |
|  | 8.5 |  | | (4) |
|  |  |  | |  |
|  | 8.6 |  | | (6) |
|  |  |  | |  |
|  | 8.7 |  | | (3) |
|  |  |  | |  |
|  | 8.8 |  | | (5) |

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| **FINANCIAL MATHEMATICS** |

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| **Nominal & Effective Interest Rates** |

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| 1. | Calculate the **effective rate** , correct to TWO decimal places, if an investment offers a nominal rate of 14% p.a. compounded: | | | | |
|  |  | |  |  |  |
|  | 1.1 | | Quarterly | | (3) |
|  |  | |  | |  |
|  | 1.2 | | Daily | | (3) |
|  |  | |  |  |  |
| 2. | Calculate the **nominal rate** , correct to TWO decimal places, if the effective rate is : | | | | |
|  |  | |  |  |  |
|  | 2.1 | | 13 % p.a. compounded monthly | | (3) |
|  |  | |  |  |  |
|  | 2.2 | | 16% p.a. compounded semi-annually. | | (3) |
|  |  | |  |  |  |
| 3. | Your local bank offers you the following options, if you invest an amount of R 10 000,00 or more with them for a year or longer. | | | |  |
|  |  | |  |  |  |
|  | 3.1 | | **Option 1**: 10,55% p.a. compounded daily | | (2) |
|  |  | |  |  |  |
|  | 3.2 | | **Option 2:** 10,55% p.a. compounded monthly | | (2) |
|  |  | |  |  |  |
|  | 3.3 | | Which option will you select? Provide a reason for your choice. | | (2) |
|  |  | |  |  |  |
| 4. | An investment earns interest at a rate of 10% p.a., compounded monthly. Calculate the effective annual interest rate, correct to TWO decimal places, on the investment. | | | | (3) |
|  |  | |  |  |  |
| 5. | Calculate the nominal interest rate if the effective rate is 18% p.a. compounded quarterly. | | | | (3) |
|  |  | |  |  |  |
| 6. | Yasmeen would like to save her pocket money she saved to date. The bank offers her a fixed interest rate of 7,5% p.a. compounded monthly. Calculate the annual effective interest rate that she will receive. Give your answer to the nearest integer. | | | | (3) |
|  |  | |  |  |  |
| 7. | Malume wants to purchase a house that costs R 950 000. The bank requires that he pays a deposit of 12% of the cost of the home and borrow the rest from them. | | | | |
|  |  |  | |  |  |
|  | 7.1 | How much in rands, must Malume pay as the deposit. | | | (2) |
|  |  |  | | |  |
|  | 7.2 | Hence, determine the amount in rands that he needs to borrow from the bank. | | | (2) |
|  |  |  | | |  |
|  | 7.3 | The bank charges interest of 9% p.a., compounded monthly on the loan amount. Malume, works out that the loan will carry an effective interest rate of 9,6% p.a. Is her calculation correct? Provide a reason for your response. | | | (4) |
|  |  |  | | |  |
| 8. | Frankel invests an amount of R 120 000,00 with a local financial institution. The bank quotes him a nominal interest rate of 7,2% p.a. compounded monthly. | | | |  |
|  |  |  | |  |  |
|  | 8.1 | Calculate the effective interest rate, per annum, correct to THREE decimal places. | | | (3) |
|  |  |  | |  |  |
|  | 8.2 | If the effective interest rate calculated in QUESTION 8.1, is 7,442% p.a., calculate the value of Frankel investment if the money is invested for three years. | | | (4) |
|  |  |  | |  |  |
|  | 8.3 | Suppose that Frankel invests the money for four years, but after 18 months withdraws an amount of R 20 000,00. Determine the amount which he will receive at the end of four years. | | | (6) |
|  |  |  | |  |  |
| 9. | Portia invests a certain sum of money for five years. She receives interest of 12% p.a. compounded monthly for the first two years. For the remaining term, the interest rate changes to 14% p.a. compounded semi-annually. At the end of the five years the investment is worth  R 75 000,00. | | | |  |
|  |  |  | |  |  |
|  | 9.1 | Calculate the effective interest rate per annum during the first year. | | | (3) |
|  |  |  | |  |  |
|  | 9.2 | Calculate the amount of money which Portia initially invested. | | | (4) |
|  |  |  | |  |  |

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| **Simple & Compound Growth & Decay** |

|  |  |  |  |  |
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| 1. | Determine the accumulated amount if an amount of R 60 000,00 is invested at 10% p.a for five years. | | | |
|  |  |  |  |  |
|  | 1.1 | Using simple interest | | (3) |
|  |  |  | |  |
|  | 1.2 | Using compounded interest, compounded annually. | | (3) |
|  |  |  | |  |
| 2. | Calculate the reduced value of a R 10 000,00 item which depreciates by 8% p.a over a five year period. (Hint: Use A = P(1 – *in*)) | | | (3) |
|  |  |  |  |  |
| 3. | An item costing R 25 000,00 depreciates at 18% p.a. over a four year period. Determine the reduced amount if the depreciation is calculated: | | | |
|  |  |  |  |  |
|  | 3.1 | On the straight-line method | | (4) |
|  |  |  | |  |
|  | 3.2 | On the reducing-balance method. | | (4) |
|  |  |  |  |  |
| 4. | A new depreciates in value by 18% p.a. on the reducing balance method. What was the initial cos t of a car 4 years ago, if the current book value is R 100 000,00. Give your answer correct to the nearest rand. | | | (5) |
|  |  |  |  |  |
| 5. | The value of a piece of equipment depreciates from R 100 000,00 to  R 50 000,00 in four years. What is the rate of depreciation (correct to TWO decimal places) if calculated on the: | | | |
|  |  |  |  |  |
|  | 5.1 | Straight-line method | | (4) |
|  |  |  | |  |
|  | 5.2 | Reducing-balance method. | | (6) |
|  |  |  |  |  |
| 6. | Fatima invests R 110 000,00 at 12,5% p.a. compound interest for six years, compounded annually. What is the value of her investment at the end of the period. | | | (4) |
|  |  |  |  |  |
| 7. | A car costing R 100 000,00 depreciates at 18% p.a, compounded annually. What is the car worth after four years? | | | (3) |
|  |  | | |  |
| 8. | Given the formula: A = P(1 + *i)*n. Use the formula to determine the value of A (correct to TWO decimal places), if P = R 70 000,00 ;  *i* = 3% and *n* = 8. | | | (3) |
|  |  | | |  |
| 9. | Thabang bought a new car for R 355 000. It depreciates in value, according to the reducing balance method by 12,5% p.a. Determine the value of the car after three years. | | | (3) |
|  |  | | |  |
| 10. | A computer is purchased for R 26 000,00. It depreciates at 15% p.a. | | |  |
|  |  |  |  |  |
|  | 10.1 | Determine the value of the computer after three years, if depreciation is calculated according to the straight line method. | | (3) |
|  |  |  |  |  |
|  | 10.2 | If the book value of the computer is R 14300,00, determine the depreciation rate of the computer after three years, if the computer is depreciated according to the reducing-balance method. | | (3) |
|  |  |  |  |  |
| 11. | Uwais purchased a cellphone for R 17 200,00. Determine the depreciated value of the cellphone after 3 years, if it depreciates at 25% p.a. according to the reducing-balance method. | | | (3) |
|  |  |  |  |  |
| 12. | The population of a certain town decreases at a rate of 9,5% p.a. due to migration. Calculate the decrease in population over a period of six years if the initial population was 1 850 000. | | | (3) |
|  |  |  |  |  |
| 13. | A new car depreciates in value by 18% p.a. compounded annually. Determine the initial cost of the vehicle, if it is worth R 283 680,00 after one year. | | | (3) |
|  |  |  |  |  |
| 14. | The present value of a car is R 48 000,00. It was worth R 90 000,00 five years ago. Calculate the annual depreciation rate, if the vehicle is depreciated according to the reducing- balance method. | | | (4) |
|  |  |  |  |  |
| 15. | Thabiso, recently bought a new computer for R 15800,00. The computer will be worth R 8 642,73 in four years’ time. The price of a computer increases by 14% p.a compounded annually. | | |  |
|  |  |  |  |  |
|  | 15.1 | Determine the cost a similar computer in four years’ time | | (3) |
|  |  |  |  |  |
|  | 15.2 | Determine the amount of money Thabiso needs to save in order to purchase a replacement computer in four years’ time. | | (3) |

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| **Different periods of Compound Growth and Decay** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1. | Complete the following table, if **P rands** is invested at **24% p.a**. compounded annually for 5 years.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | No. | If the interest is compounded…… | The interest rate (***i***) per time period is…… | The number (***n***) of time periods is…. | The final amount (**A**) is …….. | | 1. | **Annually**(Per year ; per annum) |  |  | **A =** | | 2. | **Half-yearly** (semi-annually; twice a year; bi-annually) |  |  | **A =** | | 3. | **Quarterly** (every three months) |  |  | **A =** | | 4. | **Monthly** |  |  | **A =** | | 5. | **Daily (assuming a non-leap year)** |  |  | **A =** | | | | |
|  |  |  |  |  |
| 2. | Phillip bought a new car for R 290 000,00. It depreciates at 18% p.a. compounded semi-annually. Determine its value, to the nearest rand after five years. | | | (3) |
|  |  |  |  |  |
| 3. | Sontho invested an amount of R 120 000,00 for three years at 16% p.a. compounded interest. Determine the final value if interest is added semi-annually. | | | (3) |
|  |  |  |  |  |
| 4. | A new house costs R 1 500 000,00. If inflation is calculated at 18% p.a. compounded half-yearly, what would the same house cost in ten years’ time? | | | (3) |
|  |  |  |  |  |
| 4. | Sahal wishes to invest R 100 000,00 for five years. His local bank offers him the following options:   * **OPTION 1**: An interest rate of 16% p.a. compounded quarterly * **OPTION 2**: An interest rate of 15% p.a. compounded monthly. | | | |
|  | If you were the investment advisor, show by using appropriate calculations which option would you recommend to Sahal that will ensure a maximum return on his investment? | | | (7) |
|  |  |  |  |  |
| 5. | Calculate the interest earned if R 19 500,00 is invested for 5,5 years at an interest of 13% p.a. compounded half-yearly. | | | (3) |
|  |  |  |  |  |
| 6. | Two friends, Thandi and Nalini each receive an amount of  R 600 000,00 to invest for a period of five years. They invest their money as follows: | | | |
|  | * **Thandi:** 8,5% p.a. simple interest. At the end of the five year period Thandi will receive a bonus of exactly 5% of the principal amount. * **Nalini :** 8% p.a. compounded quarterly. | | | |
|  | At the end of the five year period, who will have the better return on their investment? Justify your answer, using appropriate mathematical calculations. | | | (8) |
|  |  |  |  |  |
| 7. | Themba buys a house for R 2 420 000,00 and a car for R 255 000,00. The house appreciates in value at a rate of 11% p.a. compounded bi-annually, while the car depreciates at a rate of 9% p.a. compounded annually. Determine the value of each item at the end of five years. | | | (7) |
|  |  |  |  |  |
| 8. | An amount of R 500,00 is invested at an interest rate of *x*% p.a., compounded half-yearly. After six years the amount has grown to  R 1 126,10. Calculate the value of *x,* correct to TWO decimal places. | | | (6) |
|  |  |  |  |  |
| 9. | The original price of a motor vehicle in 1990 was R 40 000,00. Its value depreciates by a certain percentage (*x*%) every six months. The motor vehicle was worth R 21 544,60 , five years after it was bought. Calculate the value of *x,* the annual interest rate, correct to TWO decimal places. | | | (6) |

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| **Timelines** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1. | Heidi invested an amount of R 10 000,00 at 9% p.a. compounded half-yearly. Two years after her initial investment, the interest rate changed to 12% p.a. compounded monthly. Determine the value of the investment (correct to the nearest rand) , after five years. | | | (4) |
|  |  |  |  |  |
| 2. | Santha, made an initial deposit of R 7 200,00 into a savings account. Three years later she made a further deposit of R 5 000,00. Fifteen months after the second deposit, she deposited a further R 10 000,00 into the same account. Calculate the accumulated amount she had saved after 8 years, if the interest rate for the entire duration was 10% p.a. compounded monthly. | | | (6) |
|  |  |  |  |  |
| 3. | Waseela bought a car, and paid an initial deposit of R 50 000,00. Two years later she pays a further R 48 500,00; and three years thereafter, a further R 20 000,00 is paid. The interest rate is 18% p.a. compounded monthly.Calculate the original price of the car. | | | (6) |
|  |  |  |  |  |
| 4. | Benjamin invested an amount of R 50 000,00 in a savings account which offers 8% p.a. interest compounded quarterly for the first 18 months; thereafter the interest rate changes to 6% p.a. compounded monthly. Two years after the initial deposit, he withdraws R 10 000,00 to pay for his son’s studies. Calculate the amount of money in his account after four years. | | | (8) |
|  |  |  |  |  |
| 5. | Susan invests R 12 500,00 for five years at 12% p.a. compounded monthly for the first two years and 14% p.a. compounded semi-annually for the next three years. How much will Susan receive in total after five years? | | | (6) |
|  |  |  |  |  |
| 6. | Lebogang invests R 35 000,00 in a savings account. The interest rate for the first four years is 8% p.a. compounded monthly, thereafter the interest rate changes to 9% p.a. compounded semi-annually for the next five years. | | |  |
|  |  |  |  |  |
|  | 6.1 | Determine the amount of money that Lebogang will have in her savings account at the end of this period. | | (4) |
|  |  |  | |  |
|  | 6.2 | Calculate the total interest earned over the nine year period. | | (3) |
|  |  |  |  |  |
| 7. | Hilda invests a sum of R 20 000,00 at 10% p.a. compounded annually. Two years later the interest rate changes to 12% p.a. compounded quarterly, and, three years later to 15% p.a. compounded monthly. Determine the value of the investment after a decade. | | | (8) |
|  |  |  |  |  |
| 8. | On her first birthday, Kgomotso’s grandfather deposited R 20 000,00 into a savings account offering 7,5% p.a. interest compounded quarterly. She will be able to access the money on her 21st birthday, i.e. at the end of 20 years of accumulated interest. | | | |
|  |  |  |  |  |
|  | 8.1 | Calculate the amount which Kgomotso’s grandfather expected her to receive from this investment. | | (3) |
|  |  |  |  |  |
|  | 8.2 | Eight years after the initial deposit, the interest rate changed to 7,75% p.a. compounded monthly. Calculate how her final pay-out will differ from that which was originally expected. | | (6) |
|  |  |  |  |  |
| 9. | Rafia is saving for university and decides to invest her money into a fixed deposit paying 10% p.a. compounded annually. She starts her savings with R 10 000,00. Three years later, she deposits a further  R 4 000,00 and eight years after her initial deposit a further  R 10 000,00 is made. Calculate the amount in her account after 10 years. | | | (8) |

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| **POLYNOMIALS:**  **Remainder and Factor Theorems** |

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| --- | --- | --- | --- | --- |
| **QUESTION 1** | | | | |
|  |  |  |  |  |
| 1.1 | Calculate the remainder, when is divided by *x* + 1 | | | (3) |
|  |  | | |  |
| 1.2 | If *x* *–* 2 is a factor of *x*3 + *px* –108, calculate the numerical value of *p.* | | | (3) |
|  |  | | |  |
| 1.3 | Show that *x*3 –4*x*2 + *x* + 6 is exactly divisible by *x*– 3. Hence, factorise the expression completely and solve the equation *x*3 –4*x*2 + *x* + 6 = 0. | | | (9) |
|  |  | | |  |
|  |  | | | **[15]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 2** | | |
|  |  |  |
| 2.1 | Given that *f(x)* = *ax*3 + *bx*2 + *cx* + d, and *f (*2) = 0. What deduction can you make from the given information? | (1) |
|  |  |  |
| 2.2 | If *x* + 3 is a factor of 2*x*3 + *x*2 – 5*x* – *a*, determine the value of *a.* | (6) |
|  |  |  |
| 2.3 | Solve for *x*  in: *x*3 – 7*x* + 6 = 0 | (11) |
|  |  |  |
|  |  | **[18]** |

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| **QUESTION 3** | | | |
|  |  | |  |
| 3.1 | Given: *f (x)* = *x*3 – 2*x*2 – 9*x* +18 | |  |
|  |  |  |  |
|  | 3.1.1 | Determine *f* (2) | (2) |
|  |  |  |  |
|  | 3.1.2 | Solve for *x* , if *f (x)* = 0 | (5) |
|  |  |  |  |
| 3.2 | If *g (x)* = *x*3 + *ax*2 – 3 , is divided by *x* + 1, then the remainder is –2. Determine the value of *a*. | | (4) |
|  |  |  | **[11]** |
| **QUESTION 4** | | | |
|  |  | |  |
| 4.1 | Determine the remainder when 1 – *x*3 is divided by *x* + 1. | | (3) |
|  |  | |  |
| 4.2 | Determine the value of *k* if *x* + 2 is a factor of *x*3 + *kx*2 – *kx* – 6. | | (4) |
|  |  | |  |
| 4.3 | Given: *f (x)* = 2*x*3 + *ax*2 + *bx* –3; and *f* (-2) = 9 and *f*(1) = 0. Use the given information to determine the values of *a* and *b*. | | (9) |
|  |  | |  |
|  |  | | **[16]** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 5** | | | |
|  |  | |  |
| 5.1 | **Given** : | |  |
|  |  |  |  |
|  | 5.1.1 | Show that *x* + 2 is a factor of *f (x).* | (3) |
|  |  |  |  |
|  | 5.1.2 | Factorise completely. | (4) |
|  |  |  |  |
|  | 5.1.3 | Explain why *f (x)* has only two factors. | (3) |
|  |  |  |  |
| 5.2 | Prove that if *x* + 3 is a factor of 3*x*3 + 8*x*2 + *ax* + *c*, then | | (5) |
|  |  | |  |
|  |  | | **[15]** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 6** | | | |
|  |  | |  |
| Given: *f (x)*  = 2*x*3 – *x*2 – 18*x* + 9 | | |  |
|  |  |  |  |
| 6.1 | Write down one factor of *f (x)* if *f* (– 0,5) = 0. | | (2) |
|  |  |  |  |
| 6.2 | Prove that *x* + 3 is a factor of *f (x).* | | (4) |
|  |  |  |  |
| 6.3 | Hence, solve *f (x)* = 0. | | (6) |
|  |  |  |  |
|  |  |  | **[12]** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **QUESTION 7** | | | | |
|  |  | |  |  |
| 7.1 | Determine the remainder when *x*3 –27 is divided by *x* – 4. | | | (3) |
|  |  | | |  |
| 7.2 | Given the expression: *x*3 – 7*x* – 6. | | |  |
|  |  |  | |  |
|  | 7.2.1 | Show that *x* + 1 is a factor of *x*3 – 7*x* – 6. | | (3) |
|  |  |  | |  |
|  | 7.2.2 | Hence, or otherwise, factorise the expression completely. | | (3) |
|  |  |  | |  |
|  | 7.2.3 | Solve for *x* in : *x*3 – 7*x* – 6 = 0 | | (3) |
|  |  |  | |  |
| 7.3 | If  *x* – 2 is a factor of 2*x*3 + *ax*2 – (*a* + 1)*x* – 5 , show that *a* = – 4,5. | | | (4) |
|  |  |  | |  |
|  |  | | | **[16]** |

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| **QUESTION 8** | | | |
|  |  | |  |
| 8.1 | Without carrying out division, determine the remainder when  2*x*3 – 4*x*2 + *x* is divided by *x* – 3. | | (3) |
|  |  | |  |
| 8.2 | Given: *p(x)* = 2*x*3 + *x*2 –5*x* + 2 | |  |
|  |  |  |  |
|  | 8.2.1 | Determine the value of *p* (–2) | (2) |
|  |  |  |  |
|  | 8.2.2 | Hence, factorize *p (x)* completely. | (4) |
|  |  | |  |
|  |  | | **[9]** |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **QUESTION 9** | | | | | | | | |
|  |  | | | | | | |  |
| 9.1 | When 2*x*2 –*ax* –3 is divided by *x* + 1, the remainder is 5. Determine the value of *a.* | | | | | | | (4) |
|  |  | | | | | | |  |
| 9.2 | Without, carrying out a division, determine whether 3*x* + 1 is a factor of 3*x*3 – 2*x*2 – 7*x* – 2 or not? | | | | | | | (4) |
|  |  | | | | | | |  |
| 9.3 | The polynomials 2x3 –*ax* + *b* and *x*3 – *ax*2 – *bx* – 8 have a common factor of *x*– 2. Determine the values of *a* and *b.* | | | | | | | (6) |
|  |  | | | | | | |  |
|  |  | | |  | | |  | **[14]** |
| **QUESTION 10** | | | | | | | | |
|  | |  | | |  |  | |  |
| 10.1 | | State the remainder theorem without, proof. | | | | | | (2) |
|  | |  | | | | | |  |
| 10.2 | | Without doing long division, determine the remainder when the polynomial 4*x*2 + 1 is divided by 2*x* + 1. | | | | | | (2) |
|  | |  | | | | | |  |
| 10.3 | | If *2x* – 1 is a factor of *px*3 –3*x*2 –3*x* + *p*, determine the value of p. | | | | | | (4) |
|  | |  | | |  |  | |  |
| 10.4 | | **Given:** *f (x)* = 2*x*3 –3*x*2 – 11*x* + 6 | | | | | |  |
|  | |  |  | | |  | |  |
|  | | 10.4.1 | Prove that *x* + 2 is a factor of *f (x).* | | | | | (2) |
|  | |  |  | | | | |  |
|  | | 10.4.2 | Factorize 2*x*3 –3*x*2 – 11*x* + 6 completely. | | | | | (4) |
|  | |  |  | | | | |  |
|  | | 10.4.3 | Hence, determine the value(s) of *x* for which *f (x)* = 0. | | | | | (3) |
|  | |  |  | | | | |  |
|  | |  |  | | | | | **[17]** |
| **QUESTION 11** | | | | | | | | |
|  | |  |  | | |  | |  |
| 11.1 | | Determine the value of *k* if the remainder when the polynomial is divided by is 60. | | | | | | (6) |
|  | |  |  | | |  | |  |
| 11.2 | | Given : | | | | | |  |
|  | |  |  | | |  | |  |
|  | | 11.2.1 | Factorise fully. | | | | | (6) |
|  | |  |  | | |  | |  |
|  | | 11.2.2 | Hence, or otherwise solve for *x* in : | | | | | (8) |
|  | |  |  | | |  | |  |
|  | |  |  | | |  | | **[20]** |

|  |  |  |  |  |
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| **QUESTION 12** | | | | |
|  |  |  |  |  |
| 12.1 | State the Remainder theorem without proof. | | | (2) |
|  |  |  |  |  |
| 12.2 | If *f(x)* is a polynomial of the third degree, and  , determine: | | | |
|  |  |  |  |  |
|  | 12.2.1 | The remainder if *f(x)* is divided by (*x* – 1); and | | (2) |
|  |  |  |  |  |
|  | 12.2.2 | *Q(x)* , if *Q(x)* is a factor of *f(x).* |  | (3) |
|  |  |  |  |  |
| 12.3 | Calculate the value(s) of *a* if is divisible by | | | (6) |
|  |  |  |  |  |
|  |  |  |  | **[13]** |

|  |  |  |  |  |
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| **QUESTION 13** | | | | |
|  |  |  |  |  |
| Given:, and is a factor of | | | | |
|  |  |  |  |  |
| 13.1 | Determine the numerical value(s) of *m.* | | | (4) |
|  |  | | |  |
| 13.2 | Determine the remainder when is divided by | | | (2) |
|  |  | | |  |
| 13.3 | Use your results to QUESTION 13.1 to show that 102 is a factor of  4 000 400. | | | (3) |
|  |  | | |  |
|  |  | | | **[9]** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **QUESTION 14** | | | | |
|  |  | | |  |
| 14.1 | If a polynomial *f(x)* is divided by *x* – 1, it gives a quotient of  and a remainder of *k*, independent of *x;* and *f*(–1) = 9.  Use the given information to determine the polynomial in descending powers of *x.* | | | (5) |
|  |  | | |  |
| 14.2 | What must be added to such that is a factor of the new expression? | | | (3) |
|  |  | | |  |
| 14.3 | If *x* – *a* is a common factor of and , show that | | | (7) |
|  |  |  |  |  |
|  |  |  |  | **[15]** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **QUESTION 15** | | | | |
|  |  |  |  |  |
| 15.1 | State the Remainder Theorem without proof | | | (2) |
|  |  | | |  |
| 15.2 | Determine the remainder when is divided by 2*x* + 1 | | | (3) |
|  |  | | |  |
| 15.3 | Given: *f(x)* = 2*x*3 + *x*2 – 5*x* + 2. Factorise *f(x)* completely by using the Factor theorem. | | | (7) |
|  |  | | |  |
| 15.4 | What must be added to the expression 2*x*3 – *x* + 1 in order to make it exactly divisible by 2*x* – 3? | | | (3) |
|  |  | | |  |
|  |  | | | **[12]** |

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| **FUNCTIONS** |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **QUESTION 1** | | | | | |
|  |  | |  |  |  |
| **Given:** | | | | | |
|  |  | |  |  |  |
| 1.1 | Write down the coordinates of the turning point of *f.* | | | | (2) |
|  |  | | | |  |
| 1.2 | Write down: | | | |  |
|  |  |  | | |  |
|  | 1.2.1 | The domain of *f* in set-builder notation. | | | (2) |
|  |  |  | | |  |
|  | 1.2.2 | The range of *f* in interval notation. | | | (2) |
|  |  |  | | |  |
| 1.3 | Show that | | | | (3) |
|  |  |  | | |  |
| 1.4 | Write down the *y*-intercept of *f.* | | | | (1) |
|  |  |  | | |  |
| 1.5 | Without solving the equation , discuss the nature of its roots. | | | | (3) |
|  |  | | | |  |
| 1.6 | Sketch the graph of *f*, clearly showing all intercepts with the axes as well the turning point. | | | | (3) |
|  |  | | | |  |
| 1.7 | Determine the numerical value(s) of *k*, correct to two decimal places if | | | | (6) |
|  |  | | | |  |
|  |  | | | | **[22]** |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **QUESTION 2** | | | | | |
|  |  | | | |  |
| Given: | | | | | |
|  |  | |  |  |  |
| 2.1 | Explain why the value of *y* **can never** be zero? | | | | (2) |
|  |  | | | |  |
| 2.2 | Write down: | | | |  |
|  |  |  | | |  |
|  | 2.2.1 | The domain of F in interval notation. | | | (2) |
|  |  |  | | |  |
|  | 2.2.2 | The range of F in set-builder notation | | | (2) |
|  |  |  | | |  |
| 2.3 | Is F a function or a non-function? Provide a reason for your response. | | | | (2) |
|  |  |  | | |  |
| 2.4 | Determine the value of F(-5), leaving your answer in simplified surd form, with a rational denominator. | | | | (3) |
|  |  |  | | | **[11]** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **QUESTION 3** | | | | |
|  |  | |  |  |
| The sketch which is not drawn to scale, represents the graphs of :   * + | | | | |
|  | | | | |
| T (–2 ;3) is a point on the semi-circle and the hyperbola. P(1 ; 3) is a point on the straight line *f*. The point S is point of intersection of the straight line *k* and the semi-circle, *h*. The straight lines *k* and *f* intersect at (0 ; 1). The line *y* = 0,5 is the horizontal asymptote of *g.* | | | | |
|  |  |  | |  |
| 3.1 | Determine the numerical values of: | | |  |
|  |  |  | |  |
|  | 3.1.1 | *m*  and *c* | | (3) |
|  |  |  | |  |
|  | 3.1.2 | *a* and *q* | | (3) |
|  |  |  | |  |
| 3.2 | Determine the equation of the straight line defined by *k* , in the form :  *ax* + *by* + *c* = 0, if the straight lines *f* and *k* intersect at right angles at  (0 ; 1). | | | (4) |
|  |  |  | |  |
| 3.3 | In order to determine the co-ordinates of S, the point of intersection of the straight line *k* and the semi-circle *h*, Musa, started as follows:    ……………(2)  Substitute (2) into (1): +  =    Complete Musa’s work, in order to determine the co-ordinates of S. Give your answers correct to TWO decimal places, if necessary. | | | (6) |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 4** | | | |
|  |  |  |  |
| **Given:** and | | | |
|  |  |  |  |
| 4.1 | Determine the value of in simplified surd form. | | (2) |
|  |  |  |  |
| 4.2 | Write down the horizontal asymptote of *k.* | | (1) |
|  |  |  |  |
| 4.3 | Determine : | |  |
|  |  |  |  |
|  | 4.3.1 | The *x*-intercept(s) of *k.* | (2) |
|  |  |  |  |
|  | 4.3.2 | The *y* – intercepts of *p.* | (2) |
|  |  |  |  |
| 4.4 | On the same set of axes draw the graphs of *p* and *k*, clearly showing all intercepts with the axes as well as the asymptotes. | | (5) |
|  |  |  |  |
| 4.5 | Show, by means of a dotted line and the letter **A**, where on the graph will you find the value of | | (2) |
|  |  |  |  |
| 4.6 | Show, using appropriate algebraic calculations that the *x*-values of the intersection of the circle and the hyperbola are given by the solution to the equation:  **(DO NOT SOLVE THE EQUATION!)** | | (4) |
|  |  |  | **[18]** |
| **QUESTION 5** | | | |
|  |  |  |  |
| **Given:**  ; . | | | |
|  |  |  |  |
| 5.1 | Determine: | |  |
|  |  |  |  |
|  | 5.1.1 | The *x*-intercepts of *g*. | (4) |
|  |  |  |  |
|  | 5.1.2 | The *y*-intercept of *g* | (2) |
|  |  |  |  |
|  | 5.1.3 | The co-ordinates of the turning point of *g.* | (4) |
|  |  |  |  |
|  | 5.1.4 | The gradient of *h* | (1) |
|  |  |  |  |
|  | 5.1.5 | The *y* –intercept of *h.* | (2) |
|  |  |  |  |
| 5.2 | On the same set of axes draw the graphs of *g* and *h*, clearly showing all intercepts with the axes as well as the turning point. | | (6) |
|  |  |  |  |
| 5.3 | Show that | | (3) |
|  |  |  |  |
| 5.4 | For what value(s) of , is . | | (5) |
|  |  |  |  |
| 5.5 | A line defined by is a tangent to the graph of *g*. Determine the numerical values of *k.* | | (4) |
|  |  |  |  |
|  |  |  | **[31]** |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **QUESTION 6** | | | | | | | | |
| In the given diagram, the graphs (not drawn to scale) of and are drawn.  It is further given that and . M is the point of intersection of *p* and *k.* | | | | | | | | |
|  | | | | | | | | |
| 6.1 | Use the given information to determine the numerical values of: | | | | | | | |
|  | 6.1.1 | | | | *a* | | | (4) |
|  |  | | | |  | | |  |
|  | 6.1.2 | | | | *b* | | | (2) |
|  |  | | | |  | | |  |
|  | 6.1.3 | | | | *c* | | | (1) |
|  |  | | | |  | | |  |
|  | 6.1.4 | | | | *m* | | | (2) |
|  |  | | | |  | | |  |
|  | 6.1.5 | | | | *q* | | | (1) |
|  |  | | | |  | | |  |
| 6.2 | If , determine the co-ordinates of the turning point. | | | | | | | (4) |
|  |  | | | |  | | |  |
| 6.3 | Use an algebraic method to determine the co-ordinates of T and M if *k*(*x*) = *x*. | | | | | | | (6) |
|  |  | | | |  | | |  |
| 6.4 | Calculate | | | | | | | (5) |
|  |  | | | |  | | | **[25]** |
| **QUESTION 7** | | | | | | | | |
|  | | |  | | | |  |  |
| 7.1 | | | The given figure shows the graphs (not drawn to scale), of and . The point is a point on and is an asymptote of . | | | | | |
|  | | | | | | | | |
| 7.1.1 | | | Write down the value of *q.* | | | | | (1) |
|  | | |  |  | | | |  |
| 7.1.2 | | | Determine the numeric value of *a*. | | | | | (3) |
|  | | |  | | | |  |  |
| 7.1.3 | | | Write down: | | | | |  |
|  | | |  |  | | | |  |
|  | | | a) | The domain of *g* | | | | (2) |
|  | | |  |  | | | |  |
|  | | | b) | The range of *g* | | | | (2) |
|  | | |  |  | | | |  |
| 7.2 | | | **Given:** . Complete the following table with respect to *h.* | | | | | |
|  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | | 7.2.1 | 7.2.2 | 7.2.3 | 7.2.4 | 7.2.5 | | *x*-intercepts | *y*-intercept | Equation of axis of symmetry | Minimum value | Range | | (4) | (2) | (1) | (1) | (2) | | | | | | |
|  | | |  |  | | | |  |
| 7.3 | | | Use the information from the table in QUESTION 7.2, draw a graph of *h*, clearly showing all intercepts with the axes a well as the turning point. | | | | | (4) |
|  | | |  |  | | | | **[22]** |
| **QUESTION 8** | | | | | | | | |
|  | |  | | | |  | |  |
| **Given:** | | | | | | | | |
|  | |  | | | |  | |  |
| 8.1 | | Is *f* an increasing or a decreasing function? Provide a reason. | | | | | | (2) |
|  | |  | | | |  | |  |
| 8.2 | | Sketch the graph of *f* for the domain . | | | | | | (4) |
|  | |  | | | |  | |  |
| 8.3 | | By using dotted lines and the letters P and Q respectively, indicate on your graph where you would obtain the solution to each of the following equations: | | | | | | |
|  | |  | | | |  | |  |
|  | | 8.3.1 | | | |  | | (2) |
|  | |  | | | |  | |  |
|  | | 8.3.2 | | | |  | | (2) |
|  | |  | | | |  | |  |
| 8.4 | | Write down the equation of the graph which is a reflection of *f* in the *y*-axis. | | | | | | (2) |
|  | |  | | | |  | |  |
| 8.5 | | Use the graph of *f* and another appropriate graph to determine the solution of the equation | | | | | | (3) |
|  | |  | | | |  | |  |
|  | |  | | | |  | | **[15]** |

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| --- | --- | --- | --- |
| **QUESTION 9** | | | |
|  |  |  |  |
| The figure below shows the graphs of and | | | |
|  | | | |
| The graphs of *f* and *g* intersect at O (0 ; 0) and C. The straight line *h* has its *x*-intercept at F and *y*-intercept at E. The quadratic function defined by *f* cuts the *x*-axis at the origin and B ; and has its turning point at D. The quadratic function defined by *g* cuts the *x*-axis at A and O; and turns at C. The straight line *h* and the parabola *g* intersect at K. | | | |
|  |  |  |  |
| 9.1 | Write down the coordinates of: | | |
|  |  |  |  |
|  | 9.1.1 | A | (2) |
|  |  |  |  |
|  | 9.1.2 | B | (2) |
|  |  |  |  |
|  | 9.1.3 | C | (2) |
|  |  |  |  |
|  | 9.1.4 | D | (2) |
|  |  |  |  |
|  | 9.1.5 | E | (1) |
|  |  |  |  |
|  | 9.1.6 | F | (1) |
|  |  |  |  |
| 9.2 | Determine the co-ordinates of K. | | (6) |
|  |  |  |  |
| 9.3 | Show that | | (3) |
|  |  |  |  |
| 9.4 | Determine the value of : | | (6) |
|  |  |  |  |
| 9.5 | If *y* = *kx* – 9 is a tangent to *g*, determine the numerical value of *k*. | | (3) |
|  |  |  |  |
| 9.6 | If *y* = *kx* – 9 is a tangent to *f*, determine the numerical value of *k*. | | (3) |
|  |  |  |  |
| 9.7 | If *y* = *kx* – 9 is a common tangent to *f* and *g*, determine the numerical value of *k*. | | (4) |
|  |  |  |  |
|  |  |  | **[35]** |

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| **QUESTION 10** | | | |
|  |  |  |  |
| 10.1 | In each of the following cases draw sketch graphs of the function  *y*  ; clearly showing the position of the graph in relation to the *x* – and *y* – axis. | | |
|  |  |  |  |
|  | 10.1.1 |  | (3) |
|  |  |  |  |
|  | 10.1.2 |  | (3) |
|  |  |  |  |
|  | 10.1.3 |  | (3) |
|  |  |  |  |
| 10.2 | Given: ; and | | |
|  |  |  |  |
|  | 10.2.1 | Write down the value of | (1) |
|  |  |  |  |
|  | 10.2.2 | Determine the *x*-intercepts of *f* | (4) |
|  |  |  |  |
|  | 10.2.3 | Determine the co-ordinates of the turning point of  *f*. | (5) |
|  |  |  |  |
|  | 10.2.4 | Hence, write down the range of *f*  in interval notation. | (2) |
|  |  |  |  |
|  | 10.2.5 | Write down the *x* – and *y* – intercepts of *h.* | (3) |
|  |  |  |  |
|  | 10.2.6 | On the same set of axes, draw the graphs of *f , g* and *h*, clearly showing all intercepts with the axes as well as the turning point. | (6) |
|  |  |  |  |
|  | 10.2.7 | Determine the value(s) of *x* for which *f(x) = h(x).* | (4) |
|  |  |  |  |
|  | 10.2.8 | For what value(s) of *x* if *g(x) = h(x)*? | (1) |
|  |  |  |  |
|  | 10.2.9 | Write down the value(s) of x for which | (3) |
|  |  |  |  |
|  | 10.2.10 | Determine the value(s) of *k* for which is a tangent to | (6) |
|  |  |  |  |
|  |  |  | **[43]** |

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| **QUESTION 11** | | | | |
|  |  | |  |  |
| In the given diagram, the graphs of a semi-circle with *x*-intercepts A (– 6 ;0) ;  B (6 ;0) and *y*-intercept C (0 ;6); a straight line with equation *y – x* = 0 and a hyperbola defined by are shown. | | | | |
| 11.1 | Write down the equation of the semi-circle in the form: *y* =……… | | | (3) |
|  |  |  | |  |
| 11.2 | Write down: | | |  |
|  |  |  | |  |
|  | 11.2.1 | The radius of the semi-circle. | | (1) |
|  |  |  | |  |
|  | 11.2.2 | The domain of g. | | (2) |
|  |  |  | |  |
|  | 11.2.3 | The range of g. | | (2) |
|  |  |  | |  |
|  | 11.2.4 | The *x*-intercept of g. | | (2) |
|  |  |  | |  |
| 11.3 | Use an algebraic method to determine the co-ordinates of P, the point of intersection of the hyperbola and the straight line *y* = *x.* | | | (6) |
|  |  |  | |  |
|  |  |  | | **[16]** |
|  |  |  | |  |
| **QUESTION 12** | | | | |
|  |  |  | |  |
| The given diagram shows the graphs of :   * ; * ; * ; and * . | | | | |
| The quadratic function *m* , cuts the *x*-axis at A(– 5 ; 0) and at O(0 ; 0). The straight line *y = 2* passes through B, the turning point of *m*. | | | | |
|  | | | | |
| 12.1 | Write down: | | | |
|  |  |  | |  |
|  | 12.1.1 | The co-ordinates of A and C, the *x*-intercepts of the semi-circle , *f*. | | (2) |
|  |  |  | |  |
|  | 12.1.2 | The co-ordinates of F, the *x*-intercept of *h*. | | (2) |
|  |  |  | |  |
|  | 12.1.3 | The equation of the axis of symmetry of the parabola, *m*. | | (2) |
|  |  |  | |  |
|  | 12.1.4 | Hence, write down the maximum value of the turning point of the parabola, *m*. | | (1) |
|  |  |  | |  |
| 12.2 | Use the results obtained in QUESTIONS 12.1.3 and 12.1.4 to determine the equation of the parabola ,*m.* | | | (3) |
| 12.3 | Explain, why if the parabola, *m*, is moved down to touch the *x*-axis, it must pass through E on the circle , *h*. | | | (2) |
|  |  |  | |  |
|  |  |  | | **[12]** |

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| **QUESTION 13** | | | |
|  |  |  |  |
| In the given diagram, the following graphs are drawn (not to scale):   * A circle , ; * An exponential function, and * A hyperbola, | | | |
|  | | | |
| The point A(– 2 ; 4) lies on the circle and the exponential function; while the point C(4 ; 2) is the point of intersection of the circle and the hyperbola.  The point D is the y-intercept of the exponential function. | | | |
|  |  |  |  |
| 13.1 | Write down the radius of the circle in simplified surd form. | | (2) |
|  |  | |  |
| 13.2 | Write down the value of *q.* | | (1) |
|  |  | |  |
| 13.3 | Show that *k* = 4 and *b* = | | (5) |
|  |  | |  |
| 13.4 | Determine the co-ordinates of D, the *y*-intercept of *g.* | | (3) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **QUESTION 14** | | | | | | | |
|  | |  | |  |  |  | |
| The diagram, below shows the graphs of a semi-circle  , and a parabola, *g* defined by ,  The parabola and the semi-circle intersect at A and B. The point A on the *x*-axis is the turning point of the parabola and the point B is the *y*-intercept. The point is a point on the semi-circle. The straight line defined by , is a tangent to *f* at P, intersects the *x*-axis at E and the *y*-axis at D. | | | | | | | |
|  | | | | | | | |
| 14.1 | Determine the equation of the semi-circle, *f.* | | | | | | (2) |
|  |  | |  | |  | |  |
| 14.2 | If the equation of the semi-circle , *f* is given by , write down the co-ordinates of A , B and C. | | | | | | (4) |
|  |  | | |  |  | |  |
| 14.3 | If A (3 ;0) and B (0 ; –3), determine the numerical values of *a* , *p* and *q.* | | | | | |  |
|  |  | | |  |  | |  |
| 14.4 | Write down the gradient of OP. | | | | | | (2) |
|  |  | | |  |  | |  |
| 14.5 | Show, that the equation of *h*, is given by *x* + 9 = 0 | | | | | | (3) |
|  |  | | |  |  | |  |
| 14.6 | If OP = 3 units and PD = units, determine the length of OD. | | | | | | (1) |
|  |  | | |  |  | |  |
| 14.7 | Hence, determine the size of OP | | | | | | (3) |
|  |  | | |  |  | |  |
| 14.8 | If and , determine the co-ordinates of F, the point of intersection of *h* and *g.* | | | | | | (7) |
|  |  | | |  |  | | **[22]** |

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| **QUESTION 15** | | | | | |
|  |  | |  |  |  |
| 15.1 In the given diagram, (not drawn to scale), the following graphs are shown:   * ; * , and * .   The straight line *y* = 1 is a tangent to the parabola at the point B. The radius of the circle is units. The point B is the point of intersection of the parabola and the circle. The point A(0 ; 9) lies on the parabola. | | | | | |
| 15.1.1 | Show that *f*(2) = 1 | | | | (3) |
|  |  | |  |  |  |
| 15.1.2 | Hence, write down the equation of the axis of symmetry and the minimum value of *g.* | | | | (2) |
|  |  | |  |  |  |
| 15.1.3 | If *h* can be written in the form , , determine the value of *a* if *p* = 2 and *q* = 1 | | | | (4) |
|  |  | |  |  |  |
| 15.1.4 | Determine the equation of the circle *f*, if B(2;1) is a point on *f*. | | | | (3) |
|  |  | |  |  |  |
| 15.1.5 | Determine : A = | | | | (6) |
|  |  | |  |  |  |
| 15.2 | Given the functions and. | | | | |
|  |  |  | |  |  |
|  | 15.2.1 | Determine the *y*-intercept of *k*. | | | (2) |
|  |  |  | | |  |
|  | 15.2.2 | Determine the *x* and *y* –intercepts of *p*. | | | (4) |
|  |  |  | | |  |
|  | 15.2.3 | Write down the equation of the asymptote of *k.* | | | (1) |
|  |  |  | | |  |
|  | 15.2.4 | On the same set of axis draw the graphs of *k* and *p*. Clearly show all intercepts with the axes as well as the asymptotes. | | | (6) |
|  |  |  | | |  |
|  | 15.2.5 | Write down the value(s) of *x* for which | | | (2) |
|  |  |  | | |  |
|  |  |  | | | **[33]** |

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| **DIFFERENTIAL CALCULUS** |

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| --- |
| **Average Gradient** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 1** | | |
| 1.1 | Calculate the average gradient of the graph of *y* = *x*3 between the points on the curve where *x* = 3 and *x* = 1 | (6) |
|  |  |  |
| 1.2 | Given: . Determine the average gradient between the points on the curve where *x* = – 1 and *x* = 2. | (5) |
|  |  |  |
| 1.3 | Calculate the average gradient of the curve defined by :  , between *x* = 2 and *x* = 6. | (4) |
|  |  |  |
| 1.4 | Determine the average gradient of between the points where *x* = 1 and *x* = 3. | (6) |
|  |  |  |
|  |  | **[21]** |

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| **Limits** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 1** | | |
|  |  |  |
| Determine the following limits | | |
|  |  |  |
| 1.1 |  | (2) |
|  |  |  |
| 1.2 |  | (2) |
|  |  |  |
| 1.3 |  | (3) |
|  |  |  |
| 1.4 |  | (3) |
|  |  |  |
| 1.5 |  | (3) |
| 1.6 |  | (2) |
|  |  |  |
| 1.7 |  | (4) |
|  |  |  |
|  |  | **[22]** |

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| **Derivatives from First Principles** |

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| --- | --- | --- |
| **QUESTION 1** | | |
|  |  |  |
| Determine the derivative (or gradient function) of the following using **FIRST PRINCIPLES** | | |
|  | | |
| 1.1 |  | (2) |
|  |  |  |
| 1.2 |  | (3) |
|  |  |  |
| 1.3 |  | (5) |
|  |  |  |
| 1.4 |  | (5) |
|  |  |  |
| 1.5 |  | (5) |
|  |  |  |
|  |  | **[20]** |

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| --- | --- | --- | --- |
| **QUESTION 2** | | | |
|  |  | |  |
| 2.1 | Given*: f(x)* = *x*. | |  |
|  |  | |  |
|  | 2.1.1 | Determine from FIRST PRINCIPLES. | (5) |
|  |  |  |  |
|  | 2.1.2 | Hence, determine the equation of the tangent to *f* at the point  (2 ; 12). | (2) |
|  |  |  |  |
| 2.2 | Given: | |  |
|  |  |  |  |
|  | 2.2.1 | Determine : | (3) |
|  |  |  |  |
|  | 2.2.2 | Use your result from QUESTION 2.2.1, to determine the average gradient of *g(x)* = 2*x* + 1, between the points (– 1 ; 2) and (2 ; 8) | (3) |
|  |  |  |  |
|  | 2.2.3 | Determine: | (2) |
|  |  |  |  |
|  | 2.2.4 | Use your result in QUESTION 2.2.3, to determine the gradient of *g(x)* = 2*x* + 1 at the point (– 1 ; 2). | (2) |
|  |  |  |  |
| 2.3 | Given: *f(x)* = 5*x* + 3 | |  |
|  |  |  |  |
|  | 2.3.1 | Determine from FIRST PRINCIPLES. | (5) |
|  |  |  |  |
|  | 2.3.2 | Hence, determine the value of the gradient of the tangent at the point (1 ; 9). | (2) |
|  |  |  |  |
| 2.4 | If *f(x)* = 5*x* + 4 , show using FIRST PRINCIPLES , that = 5 | | (6) |
|  |  |  |  |
| 2.5 | Differentiate *f(x)* = 0,5*x* – 3 using FIRST PRINCIPLES. | | (6) |
|  |  |  |  |
| 2.6 | Determine from FIRST PRINCIPLES, given that *f(x)* = 10*x* -1 | | (5) |
|  |  |  |  |
|  |  |  | **[41]** |

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| **Differentiation using the rules of differentiation** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 1** | | | |
|  |  |  |  |
| 1.1 | Determine , using the rules of differentiation, in each of the following, and express your answer with a positive index, where necessary. | | |
|  |  |  |  |
|  | 1.1.1 |  | (2) |
|  |  |  |  |
|  | 1.1.2 |  | (3) |
|  |  |  |  |
|  | 1.1.3 |  | (2) |
|  |  |  |  |
|  | 1.1.4 |  | (1) |
|  |  |  |  |
|  | 1.1.5 |  | (3) |
|  |  |  |  |
|  | 1.1.6 |  | (3) |
|  |  |  |  |
|  | 1.1.7 |  | (4) |
|  |  |  |  |
|  | 1.1.8 |  | (4) |
|  |  |  |  |
| 1.2 | Use the rules of differentiation to determine in each of the following: | | |
|  |  |  |  |
|  | 1.2.1 |  | (3) |
|  |  |  |  |
|  | 1.2.2 |  | (3) |
|  |  |  |  |
|  | 1.2.3 |  | (4) |
|  |  |  |  |
|  | 1.2.4 |  | (2) |
|  |  |  |  |
|  | 1.2.5 |  | (2) |
|  |  |  |  |
|  | 1.2.6 |  | (3) |
|  |  |  |  |
|  | 1.2.7 |  | (4) |
|  |  |  |  |
|  | 1.2.8 |  | (2) |
|  |  |  |  |
|  | 1.2.9 |  | (4) |
|  |  |  |  |
|  | 1.2.10 |  | (2) |
|  |  |  |  |

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| **Cubic functions** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 1** | | | |
|  |  |  |  |
| Consider the function | | | |
|  |  |  |  |
| 1.1 | Write down the coordinates of: | |  |
|  |  |  |  |
|  | 1.1.1 | The *x*- intercepts of *f* | (2) |
|  |  |  |  |
|  | 1.1.2 | The *y* –intercepts of *f* | (1) |
|  |  |  |  |
| 1.2 | Determine the derivative of *f(x)* and then calculate the co-ordinates of the points where . Hence, determine whether each of these points is a local maximum or local minimum. | | (8) |
|  |  |  |  |
| 1.3 | Write down the value of *f(x)* when: | |  |
|  |  |  |  |
|  | 1.3.1 |  | (1) |
|  |  |  |  |
|  | 1.3.2 |  | (1) |
|  |  |  |  |
| 1.4 | Use the above information to sketch the graph of *f*. Show all intercepts with the axes as well as the turning points. | | (4) |
|  |  |  | **[17]** |

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| --- | --- | --- | --- |
| **QUESTION 2** | | | |
|  |  |  |  |
| The function *f* defined by , can also be written in the form : | | | |
|  |  |  |  |
| 2.1 | Write down the co-ordinates of the x and y –intercepts of *f.* | | (3) |
|  |  |  |  |
| 2.2 | Determine the derivative of *f (x)* and calculate the co-ordinates of the points where and hence classify each of these points as either a local maximum or local minimum point. | | (9) |
|  |  |  |  |
| 2.3 | Use the above information to sketch the graph of *f.* | | (6) |
|  |  |  |  |
|  |  |  | **[18]** |
| **QUESTION 3** | | | |
|  |  |  |  |
| Given : | | | |
|  |  |  |  |
| 3.1 | Factorize *f(x)* completely. | | (2) |
|  |  | |  |
| 3.2 | Solve for *x* , if *f (x)* = 0 | | (2) |
|  |  | |  |
| 3.3 | Determine the co-ordinates of the stationary points of the curve of *f* and show clearly whether each of these points is a local maximum or a local minimum point. | | (7) |
|  |  | |  |
| 3.4 | Sketch the graph of *f* and clearly indicate the intercepts with the axes as well as the turning points. | | (4) |
|  |  | |  |
| 3.5 | Write down the equation of a straight line that will enable you to solve the equation | | (2) |
|  |  | |  |
|  |  | | **[17]** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 4** | | | |
|  |  | |  |
| The sketch below, not drawn to scale, shows the graphs of and | | | |
|  | | | |
| 4.1 | If , determine the *x* –intercepts of the curve. | | (3) |
|  |  |  |  |
| 4.2 | Solve | | (4) |
|  |  |  |  |
| 4.3 | Hence, write down the co-ordinates of C, the minimum turning point of the curve of *f.* | | (2) |
|  |  |  |  |
| 4.4 | Determine the numerical values of *m* and *n* | | (4) |
|  |  |  |  |
| 4.5 | Write down the length of DC. | | (2) |
|  |  |  |  |
| 4.6 | For what value(s) of *x* is the graph of *f* decreasing? | | (2) |
|  |  |  |  |
|  |  |  | **[17]** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **QUESTION 5** | | | | |
|  |  | |  |  |
| The sketch below, represents the graphs (not drawn to scale), of *g(x)* = 4*x* + 1 and  . B and E are the turning points of *f*. | | | | |
|  |  | |  |  |
|  | | | | |
|  | | | | |
|  |  |  | |  |
| 5.1 | Calculate the lengths of : | | |  |
|  |  |  | |  |
|  | 5.1.1 | OM | | (2) |
|  |  |  | |  |
|  | 5.1.2 | OB | | (2) |
|  |  |  | |  |
|  | 5.1.3 | OL | | (3) |
|  |  |  | |  |
|  | 5.1.4 | KD | | (3) |
|  |  |  | |  |
| 5.2 | Determine the co-ordinates of E | | | (5) |
|  |  |  | |  |
| 5.3 | Determine the co-ordinates of C, the point of intersection of *f* and *g*. | | | (8) |
|  |  |  | |  |
| 5.4 | Determine the gradient of *f* at M(2 ; 0) | | | (3) |
|  |  |  | |  |
|  |  |  | | **[26]** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 6** | | | |
|  | | | |
| The figure below shows the graph (not drawn to scale) of  . The graph cuts the *x*-axis at A, B and C. The point D is the *y*-intercept of *f*. The function has a local maximum at point E. FG is perpendicular to the *x*-axis with G (*x* ; 0). | | | |
|  | | | |
| 6.1 | Show that *f*(1) = 0 | | (2) |
|  |  |  |  |
| 6.2 | If , determine the values of *a* , *b* and *c.* | | (4) |
|  |  |  |  |
| 6.3 | Hence, factorise *f(x)*  completely. | | (2) |
|  |  |  |  |
| 6.4 | Determine the lengths of : | |  |
|  |  |  |  |
|  | 6.4.1 | OA , OB and OC | (3) |
|  |  |  |  |
|  | 6.4.2 | OD | (1) |
|  |  |  |  |
| 6.5 | Determine the derivative , | | (3) |
|  |  |  |  |
| 6.6 | Determine the co-ordinates of E, a turning point of *f*. | | (3) |
|  |  |  |  |
| 6.7 | Determine the co-ordinates of G , if the tangent to the curve at F is parallel to the tangent to the curve at D | | (5) |
|  |  |  |  |
| 6.8 | Determine the area of the shaded region , i.e.: A = | | (8) |
|  |  |  |  |
|  |  |  | **[31]** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 7** | | | |
|  |  |  |  |
| Show, using appropriate calculations and without drawing any graphs that: | | | |
|  |  |  |  |
| 7.1 | , has **two** stationary points. | | (5) |
|  |  |  |  |
| 7.2 | , has **only one** real root. | | (5) |
|  |  |  |  |
| 7.3 | , has **no** stationary points. | | (5) |
|  |  |  |  |
|  |  |  | **[15]** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 8** | | | |
|  | | | |
| 8.1 | Given : | |  |
|  |  |  |  |
| 8.2 | Show that | | (2) |
|  |  | |  |
| 8.3 | Hence, write down the co-ordinates of the *x* – and *y*- intercepts of *f.* | | (4) |
|  |  | |  |
| 8.4 | Determine the derivative of *f (x)* | | (2) |
|  |  | |  |
| 8.5 | Determine the co-ordinates of the turning points of *f.* | | (4) |
|  |  | |  |
| 8.6 | Indicate whether each of the turning points calculated in QUESTION 9.5 is a relative maximum or minimum. | | (2) |
|  |  | |  |
| 8.7 | Use the above information to sketch the graph of *f*. Show all intercepts with the axes as well as the turning points on the graph. | | (5) |
|  |  | |  |
|  |  | | **[13]** |

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| **Maxima and Minima** |

|  |  |  |  |
| --- | --- | --- | --- |
| 1. | Determine the maximum value of , using differential calculus methods. | | (5) |
|  |  | |  |
| 2. | The sum of two numbers is 20. Show that their maximum product cannot exceed 100. | | (7) |
|  |  | |  |
| 3. | Mandla, has 500 metres of fencing available for a perimeter fence for his rectangular plot of land. | |  |
|  |  |  |  |
|  | 3.1 | If the length is *x* metres and the breadth is *y* metres, write down an equation connecting the perimeter, *x* and *y*. | (2) |
|  |  |  |  |
|  | 3.2 | Complete: Area of rectangle = …………….. | (1) |
|  |  |  |  |
|  | 3.3 | Express *y*, the breadth in terms of *x* | (1) |
|  |  |  |  |
|  | 3.4 | Show that the area of the rectangle is given by: | (2) |
|  |  |  |  |
|  | 3.5 | Hence, determine the dimensions of *x* and *y* for which the area will be a maximum. | (3) |
|  |  |  |  |
|  | 3.6 | Write down the maximum area in km2 | (2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 4 | The given figure represents a prism with height (3 – *x*) metres; length (*x* + 3) metres and breadth (*x* + 3) metres .    *x* + 3  3 -*x*  *x* + 3 | | | | | |
|  | 4.1 | | The base of this prism is a square or rectangle (select one). | | | (1) |
|  |  | |  | | |  |
|  | 4.2 | | Write down the volume of the prism in terms of *x.* | | | (2) |
|  |  | |  | | |  |
|  | 4.3 | | Determine | | | (1) |
|  |  | |  | | |  |
|  | 4.4 | | Hence, determine the dimensions of the prism if the volume is to be a maximum. | | | (3) |
|  |  | |  | | |  |
|  | 4.5 | | Calculate the maximum volume. Give your answer in litres | | | (3) |
|  |  | | **A** | | |  |
| 5. | In the diagram alongside, is a right-angled triangle. The lengths AB and BC vary, such that their sum is always 6 cm. | | | | *x*  **C**  **B** | |
|  | 5.1 | If the length of BC is *x* cm, write down, in terms of *x* , the length of AB. | | | | (2) |
|  |  |  | | | |  |
|  | 5.2 | Complete: Area of = | | | | (1) |
|  |  |  | | | |  |
|  | 5.3 | Determine the maximum area of | | | | (6) |
|  |  |  | | | |  |
| 6. | A cylindrical can, with no lid, has a circular base of radius *r* cm, and height of *h* cm. The total surface area of the can is (300)cm2.  The following formula may assist you in answering this question:  Total Surface area of cylinder  = ;  Volume of cylinder = | | | Image result for pictures of cylindrical can no lid | |  |
|  | 6.1 | Express the height , *h*, of the cylinder in terms of and *r.* | | | | (4) |
|  |  |  | | | |  |
|  | 6.2 | Show, that the volume ,V cm3 of the cylindrical can is given by: | | | | (2) |
|  |  |  | | | |  |
|  | 6.3 | Given that the radius *r* may vary, determine the positive value of *r* for which | | | | (3) |
|  |  |  | | | |  |
|  | 6.4 | Show that this value of *r* gives a maximum value of V. | | | | (2) |
|  |  |  | | | |  |
|  | 6.5 | Hence, determine the maximum value of V. | | | | (1) |
|  |  |  | | | |  |
| 7. | The figure alongside represents a window in the shape of a rectangle with a semi-circle on the top. The length of the rectangle is *x* metres and the breadth is *y* metres. The perimeter of the rectangle is 4 metres.  O is the centre of the semi-circle. | | | Image result for pictures of a tray with semicircular regions and a rectangle  **O**  *y* | | |
| 7.1 | Express the radius of the semi-circle in terms of *x*. | | | | | (1) |
|  |  | | | | |  |
| 7.2 | Write down an expression for the perimeter P of the window in terms of *x* and *y*. | | | | | (2) |
|  |  | | | | |  |
| 7.3 | Show that the area , A , of the window is given by : | | | | | (5) |
|  |  | | | | |  |
| 7.4 | Determine the value of *x* for the largest possible area holding the perimeter of the rectangle to 4 metres. Use | | | | | (6) |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 8. | The figure alongside, shows the flat surface of a tray consisting of a rectangular region and a semi-circular region on each end.  **X** | | **Y** |  |
| 2*y*  *x*  **W**  **Z**  In the above diagram, WXYZ is a rectangular region of the tray. XY = *x* cm.  The semi-circular regions are on either end with XW and YZ as diameters. The area of rectangle WXYZ is 200 cm2. Let the diameters XW and YZ be 2*y* cm .  **(IGNORE THE SHADED REGIONS)** | | | | |
|  | | | | |
| 8.1 | If the area of the rectangle XWZY is 200 cm2, express *y* in terms of *x*. | | | (3) |
|  |  | | |  |
| 8.2 | Show that the perimeter, P of the tray is given by : | | | (3) |
|  |  | | |  |
| 8.3 | Determine | | | (2) |
|  |  | | |  |
| 8.4 | Hence, determine the smallest of P as *x* varies. | | | (4) |
|  |  | | |  |
|  |  | | |  |
| 9. | A straight line with a positive gradient *m* , passes through the point  (– 1 ; 2) and cuts the *x* –axis at A and the *y*-axis at B. | | |  |
|  |  |  | |  |
|  | 9.1 | Determine the co-ordinates of A and B in terms of *m*. | | (4) |
|  |  |  | |  |
|  | 9.2 | Show that the area, A , of OXY, where O is the origin, is given by | | (3) |
|  |  |  | |  |
|  | 9.3 | Show that A has a minimum value of 4 . | | (6) |
|  |  |  | |  |
|  |  |  | |  |
| 10. | The figure below shows a right circular cone of base radius 20 cm and height 40 cm standing on a horizontal table. A cylinder of radius *r* units stands inside the cone with its axis coincident with the axis of symmetry of the cone and such that the cylinder touches the curved surface of the cone as shown. The volume of the cylinder is V cm3 | | |  |
|  |  |  | |  |
| Related image  The following formulae can assist you in answering this question.  Volume of cone  =  Volume of cylinder  = | | | | |
| 10.1 | Determine the volume of the cylinder in terms of *r*. | | | (6) |
|  |  |  | |  |
| 10.2 | Given that *r* can vary, determine the maximum value of V and state the value of *r* for which this maximum volume occurs. | | | (6) |

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| **Rates of change** |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1. | A rocket is fired vertically into the sky. After *t* seconds, the rocket reaches a height of *f (t)* metres, where *f (t)* = 100*t* – 5*t*2 | | | |  |
|  |  |  | |  |  |
|  | 1.1 | After how many seconds will the rocket reach a height of 320 metres? | | | (5) |
|  |  |  | | |  |
|  | 1.2 | Determine the speed of the rocket after 2 seconds. | | | (3) |
|  |  |  | | |  |
|  | 1.3 | After how many seconds will the rocket reach its maximum height? | | | (2) |
|  |  |  | | |  |
|  | 1.4 | Calculate the maximum height reached by the rocket. | | | (2) |
|  |  |  | | |  |
|  | 1.5 | If the speed of the rocket is given by *g(t)* = 100 –10*t*, calculate the acceleration of the rocket. Provide an explanation for your answer. | | | (3) |
|  |  |  | | |  |
|  | 1.6 | After how many seconds will the rocket reach the ground? | | | (4) |
|  |  |  | | |  |
| 2. |  | Image result for pictures of spherical balloon | A spherical balloon is inflated at a rate of 3cm3/s. Determine the rate of increase of the radius when the radius is 2 cm.  [**Hint**: ] | |  |
|  |  |  | | |  |
| 3. | The side of a square piece of metal increases at a rate of 0,1 cm/s when it is heated. Determine the rate at which the area of the square surface increases. | | | |  |
|  |  |  | | |  |
| 4. | Air is pumped into a spherical balloon at a rate of 250cm3/s. When the radius of the balloon is 15 cm, determine: | | | |  |
|  |  |  | | |  |
|  | 4.1 | The rate at which the radius is increasing. | | |  |
|  |  |  | | |  |
|  | 4.2 | The rate at which the surface area is increasing. | | |  |
| [**Hint**: and Surface area , S = ] | | | | | |

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| **INTEGRAL CALCULUS** |

|  |
| --- |
| **Basic integration** |

|  |  |
| --- | --- |
| 1. | Use the rules of integration to determine the following indefinite integrals |
|  | |
| 1.1 |  |
|  |  |
| 1.2 |  |
|  |  |
| 1.3 |  |
|  |  |
| 1.4 |  |
|  |  |
| 1.5 |  |
|  |  |
| 1.6 |  |
|  |  |
| 1.7 |  |
|  |  |
| 1.8 |  |
|  |  |
| 1.9 |  |
|  |  |
| 1.10 |  |
|  |  |
| 1.11 |  |
|  |  |
| 1.12 |  |
|  |  |
| 1.13 |  |
|  |  |
| 1.1.4 |  |
| 1.15 |  |
|  |  |
| 2. | Determine the value of the following definite integrals |
|  |  |
| 2.1 |  |
|  |  |
| 2.2 |  |
|  |  |
| 2.3 |  |
|  |  |
| 2.4 |  |
|  |  |
| 2.5 |  |
|  |  |
| 2.6 |  |
|  |  |
| 2.7 |  |
|  |  |
| 2.8 |  |
|  |  |
| 2.9 |  |
|  |  |
| 2.10 |  |
|  |  |
| 2.11 |  |
|  |  |
| 2.12 |  |
|  |  |
| 2.13 |  |

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| **Area under a curve** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | The sketch below (not drawn to scale) shows the function , | | | |
|  |  | | | |
|  | Determine the area under the curve between *x* = 1 and *x* = 3. | | | |
|  |  | | | |
| 2 | The sketch below (not drawn to scale) shows the function , | | | |
|  |  | | | |
|  |  | | | |
|  | Determine the area between the curve , and the *x*-axis from: | | | |
|  | 2.1) *x* = – 2 to *x* = 0 2.2) *x* = – 2 to *x* = 2 | | | |
| 3 | The sketch below shows the graph of the function defined by  . Determine the area between the curve and the *x*-axis from *x* = 0 to *x* = 3. | | | |
|  |  | | | |
|  |  | | | |
| 4 | Given: and | | | |
|  |  | |  | |
|  | 4.1 | | On the same set of axes draw the graphs of *f* and *g* | |
|  |  | |  | |
|  | 4.2 | | Use an algebraic method to determine the point(s) of intersection of *f* and *g.* | |
|  |  | |  | |
|  | 4.3 | | Hence, determine , where | |
|  |  | |  | |
| 5 | Given: and | | | |
|  |  | | |  |
|  | 5.1 | | | On the same set of axes draw the graphs of *k* and *m*. |
|  |  | | |  |
|  | 5.2 | | | Use an algebraic method to determine the point(s) of intersection of *k* and *m*. |
|  |  | | |  |
|  | 5.3 | | | If , determine . Show all working details. |
| 6 | Determine the area of the shaded region for each of the given functions. | | | |
|  |  |  | | |
| 6.1 | |  | | |
|  |  |  | | |
| 6.2 | |  | | |
|  |  |  | | |
| 6.3 | |  | | |
|  |  |  | | |
| 6.4 | |  | | |

PAPER 2

|  |
| --- |
| **ANALYTICAL GEOMETRY** |

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| --- | --- | --- | --- | --- |
| **QUESTION 1** | | | | |
| In the given diagram, ABCD is a parallelogram with vertices A and D on the *y*-axis. The side BC is produced to E so that BC = CE. AD is 2 units long. The co-ordinates of the points B and D are (3 ; 9) and (0 ; 2) respectively. The line segments AE and DC intersect at F.  **B**  ***y***  **C**  **A**  **F**  **E**  **D**  ***x***  **O** | | | | |
| 1.1 | Determine the co-ordinates of: | | |  |
|  |  |  |  |  |
|  | 1.1.1 | A | | (1) |
|  |  |  | |  |
|  | 1.1.2 | C | | (2) |
|  |  |  | |  |
|  | 1.1.3 | E | | (1) |
|  |  |  | |  |
| 1.2 | Why is F the midpoint of AE? | | | (2) |
|  |  | | |  |
| 1.3 | Determine the co-ordinates of F. | | | (2) |
|  |  | | |  |
| 1.4 | Use, Analytical Geometry methods to show that the points O, F and B lie on the same straight line. | | | (5) |
|  |  | | | **[13]** |
| **QUESTION 2** | | | | |
| 2.1 | In the given diagram, A,B and C are the vertices of . The coordinates of A are (1 ; – 1). The sides BC and AC are defined by the equations and respectively. | | |  |
|  | | | | |
|  | 2.1.1 | Determine the co-ordinates of B, the *y*-intercept of line BC. | | (2) |
|  |  |  | |  |
|  | 2.1.2 | Write down the gradient of BC. | | (2) |
|  |  |  | |  |
|  | 2.1.3 | Prove, using Analytical Geometry methods that | | (4) |
|  |  |  | |  |
|  | 2.1.4 | Determine the co-ordinates of C, the point of intersection of BC and AC. | | (4) |
|  |  |  | |  |
|  | 2.1.5 | Determine the length of AC in simplified surd form. | | (2) |
|  |  |  | |  |
| 2.2 | Sketch the ellipse defined by . Show the intercepts with the axes. | | | (4) |
|  |  |  | |  |
|  |  |  | | **[18]** |

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| --- | --- | --- | --- |
| **QUESTION 3** | | | |
|  | | | |
| 3.1 | In the given diagram, O is the centre of the circle. The line AB touches the circle at P(- 4 ; 2). POS is a diameter of the circle. TR is a tangent to the circle at R. RT is perpendicular to AB at T. | | |
|  | | | |
|  | | | |
|  | 3.1.1 | Determine the radius of the circle. | (2) |
|  |  |  |  |
|  | 3.1.2 | Write down the equation of the circle in the form | (1) |
|  |  |  |  |
|  | 3.1.3 | Show that the equation of the straight line SOP is given by | (1) |
|  |  |  |  |
|  | 3.1.4 | Hence determine the equation of the straight line AB in the form *ax* + *by* + *c* = 0. | (3) |
|  |  |  |  |
|  | 3.1.5 | Why is TP = TR? | (1) |
|  |  |  |  |
|  | 3.1.6 | Determine the co-ordinates of R if OPTR is a square. | (7) |
|  |  |  |  |
|  | 3.1.7 | Hence, show that the length of RS < 6,5 units if R is the point (2 ; 4) | (4) |
|  |  |  |  |
| 3.2 | A circle with equation intersects the straight line defined by at points A and B, where A is a point in the first quadrant and B is a point in the third quadrant of the Cartesian Plane. | |  |
|  |  |  |  |
|  | 3.2.1 | Determine the co-ordinates of A and B. | (8) |
|  |  |  |  |
|  | 3.2.2 | Determine the equation of a second circle that is a constant 2 units away from the original circle. | (3) |
|  |  |  |  |
|  |  |  | **[30]** |

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| **QUESTION 4** | | |
|  | | |
| In the given diagram, A, B, C and D are the vertices of a square ABCD. The diagonals AC and BD intersect at P. The co-ordinates of B and D are (7 ; 6) and (– 1 ; 2) respectively.  *y*  **A**  **B**  **P**  **D**  **C**  **O**  *x* | |  |
| 4.1 | Write down the co-ordinates of P , the point of intersection of diagonal AC and diagonal BD. | (2) |
|  |  |  |
| 4.2 | Calculate the length of BD. Express your answer in simplified surd form. | (3) |
|  |  |  |
| 4.3 | Complete the following statement: The diagonals of a square intersect each other at …………..angles. | (1) |
|  |  |  |
| 4.4 | Show that the equation of diagonal AC is given by : | (5) |
|  |  |  |
| 4.5 | Write down the length of AP. | (1) |
|  |  |  |
| 4.6 | Calculate the co-ordinates of the points A and C. | (9) |
|  |  |  |
| 4.7 | Hence, determine the area of the square ABCD. | (2) |
|  |  |  |
|  |  | **[23]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 5** | | |
|  |  |  |
| In the given diagram, the points A , B , C and D are the vertices of a rhombus. The co-ordinates of the vertices A , B , C and D are (– 2 ; – 1) ; (2 ; 2) ; (5 ; 6) and  (*x* ; *y*) respectively. The diagonals AC and BD intersect each other at M. | | |
| ***y***  **D**  **C**  **M**  **B**  **O**  ***x***  **A** | | |
| 5.1 | Complete: The diagonals of a rhombus ……. each other at …….. | (2) |
|  |  |  |
| 5.2 | Show, using Analytical Geometry methods that the co-ordinates of the point M are | (2) |
|  |  |  |
| 5.3 | Use the fact that M is the midpoint of BD, to express the co-ordinates of M in terms of *x* and *y*. | (2) |
|  |  |  |
| 5.4 | Hence, determine the co-ordinates of D | (3) |
|  |  |  |
| 5.5 | Determine the length of AC. Leave your answer in simplified surd form. | (3) |
|  |  |  |
| 5.6 | Calculate the area of ABC | (4) |
|  |  |  |
| 5.7 | Hence, determine the area of rhombus ABCD. | (2) |
|  |  |  |
|  |  | **[18]** |

|  |  |  |  |
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| **QUESTION 6** | | | |
|  |  | |  |
| 6.1 | In the figure below the straight line passing through the points D, C and B is defined by . The points C and D are the x and y-intercepts respectively of the straight line.  The circle with centre at the origin cuts the straight line at D and B. The points A and D are the y –intercepts of the circle. | | |
|  |  | | |
|  | 6.1.1 | Write down the co-ordinates of D , the *y*-intercept of the straight line | (2) |
|  |  |  |  |
|  | 6.1.2 | Write down the co-ordinates of C, the *x*-intercept of the straight line. | (2) |
|  |  |  |  |
|  | 6.1.2 | Show that the equation of the circle is given by | (3) |
|  |  |  |  |
|  | 6.1.3 | Hence write down the co-ordinates of A | (1) |
|  |  |  |  |
|  | 6.1.4 | For what value(s) of *k* will be a tangent to the circle at B. | (7) |
|  |  |  |  |
| 6.2 | Sketch the ellipse defined by . Show all intercepts with the axes clearly. | | (4) |
|  |  |  | **[19]** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 7** | | | |
|  | | | |
| 7.1 | Determine the equations of the following circles whose centre’s lie at the origin. | |  |
|  |  |  |  |
|  | 7.1.1 | The radius of the circle is units | (2) |
|  |  |  |  |
|  | 7.1.2 | The circle passes through the point (– 2 ; 4) | (2) |
|  |  |  |  |
| 7.2 | In the figure below, the equation of the circle with centre (0 ; 0) is and the equation of the secant MN is The diameter is defined by the equation | |  |
|  |  |  |  |
|  | | | |
|  | 7.2.1 | Determine the co-ordinates of the points M and N which are the points of intersection of the circle and the secant. Give your answers as integer values. | (7) |
|  |  |  |  |
|  | 7.2.2 | If T is a point on MN such that OT is perpendicular to MN , determine the co-ordinates of T | (2) |
|  |  |  |  |
|  | 7.2.3 | Determine the TH , the height of the minor segment of MN | (5) |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| 7.3 | In the accompanying figure (not drawn to scale), A, B (3; 4), C and  D (–6 ; –3) are the vertices of a rectangle ABCD. The gradient of DC is. | | |
|  | ***y***  **A**  **B**  ***x***  **O**  **D**  **C** | |  |
|  |  |  |  |
|  | 7.3.1 | Write down the gradient of CD. | (1) |
|  |  |  |  |
|  | 7.3.2 | Determine: |  |
|  |  |  |  |
|  |  | 1. The equation of CB | (2) |
|  |  |  |  |
|  |  | 1. The co-ordinates of C ; and | (5) |
|  |  |  |  |
|  |  | 1. The co-ordinates of A , if A lies on the y-axis. | (3) |
|  |  |  |  |
| 7.4 |  | |  |
|  | 7.4.1 | If the centre of an ellipse is at the origin , the major axis is 8 cm and the minor axis on the *x*-axis is 6 cm, determine the equation of the ellipse. | (4) |
|  |  |  |  |
|  | 7.4.2 | Hence, sketch the graph of the ellipse, showing all intercepts with the axes. | (2) |
|  |  |  | **[31]** |
| **TRIGONOMETRY** | | | |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 1** | | | |
|  |  |  |  |
| 1.1 | Without using a calculator , determine the value of : | |  |
|  |  |  |  |
|  | 1.1.1 |  | (2) |
|  |  |  |  |
|  | 1.1.2 |  | (4) |
|  |  |  |  |
|  | 1.1.3 |  | (3) |
|  |  |  |  |
| 1.2 | If and ,determine the value of  1 + sec A, **without using a calculator**. | | (4) |
|  |  |  |  |
| 1.3 | Determine the value of the following expressions: | |  |
|  |  |  |  |
|  | 1.3.1 |  | (4) |
|  |  |  |  |
|  | 1.3.2 |  | (4) |
|  |  |  |  |
|  | 1.3.3 |  | (4) |
|  |  |  |  |
|  |  |  | **[25]** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 2** | | | |
|  |  |  |  |
| 2.1 | Evaluate the following **without using a calculator**: | |  |
|  |  |  |  |
|  | 2.1.1 |  | (2) |
|  |  |  |  |
|  | 2.1.2 |  | (3) |
|  |  |  |  |
|  | 2.1.3 |  | (5) |
|  |  |  |  |
| 2.2 | The following formula is used when checking a taper:  . Determine the value of , when  *H* = 15 ; *C* = 50 ; and *d* = 12. | | (4) |
|  |  |  |  |
| 2.3 | Determine the value of | | (4) |
|  |  |  |  |
| 2.4 | In each of the following, determine the value(s) of where | |  |
|  |  |  |  |
|  | 2.4.1 |  | (3) |
|  |  |  |  |
|  | 2.4.2 |  | (3) |
|  |  |  |  |
|  |  |  | **[24]** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 3** | | | |
|  |  |  |  |
| 3.1 | If , where . Use a suitable diagram, and without the use of a calculator determine the exact numerical value of . | | (7) |
|  |  |  |  |
| 3.2 | If , determine the following in terms of *k*: | |  |
|  |  |  |  |
|  | 3.2.1 |  | (2) |
|  |  |  |  |
|  | 3.2.2 |  | (2) |
|  |  |  |  |
|  | 3.2.3 |  | (2) |
|  |  |  |  |
|  | 3.2.4 |  | (2) |
|  |  |  |  |
| 3.3 | If , determine the value of the following, correct TWO decimal places. | |  |
|  |  |  |  |
|  | 3.3.1 |  | (2) |
|  |  |  |  |
|  | 3.3.2 |  | (3) |
|  |  |  |  |
|  | 3.3.3 |  | (4) |
|  |  |  |  |
| 3.4 | Simplify the following expression using trigonometric identities | | (5) |
|  |  |  |  |
|  |  |  | **[29]** |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 4** | | | |
|  |  |  |  |
| 4.1 | In the given diagram, is a point in the Cartesian plane. OA = 3 units and the obtuse angle formed by OA and the *x* –axis is . | |  |
|  | Determine, without the use of a calculator, the numerical value of: | |  |
|  |  |  |  |
|  | 4.1.1 | *a* | (2) |
|  |  |  |  |
|  | 4.1.2 |  | (2) |
|  |  |  |  |
|  | 4.1.3 |  | (2) |
|  |  |  |  |
| 4.2 | If , determine the value of the following correct to TWO decimal places. | |  |
|  |  |  |  |
|  | 4.2.1 |  | (3) |
|  |  |  |  |
|  | 4.2.2 |  | (2) |
|  |  |  |  |
|  |  |  | **[11]** |

|  |  |  |  |  |
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| **QUESTION 5** | | | | |
|  |  | |  |  |
| 5.1 | Given : . | | |  |
|  |  | |  |  |
|  | 5.1.1 | | In which quadrant(s) does the angle lie | (2) |
|  |  | |  |  |
|  | 5.1.2 | | Hence, determine the numerical values of | (3) |
|  |  | |  |  |
| 5.2 | Solve for , where in the following: | | |  |
|  |  | |  |  |
|  | 5.2.1 | |  | (5) |
|  |  | |  |  |
|  | 5.2.2 | |  | (4) |
|  |  | |  |  |
| 5.3 | Solve : | | | (4) |
|  |  | |  |  |
| 5.4 | Solve: for | | | (5) |
|  |  | |  |  |
| 5.5 | Solve for *x* : and | | | (4) |
|  |  | |  |  |
| 5.6 |  | | |  |
|  | 5.6.1 | | Solve for in , where | (6) |
|  |  | |  |  |
|  | 5.6.2 | | Hence, determine the value of | (3) |
|  |  | |  |  |
| 5.7 | Given: and , determine the exact value of | | | (6) |
|  |  | |  |  |
| 5.8 | Solve for , if and | | | (6) |
|  |  | |  |  |
| 5.9 | If and is an acute angle, determine the value of | | | (5) |
|  |  | |  |  |
| 5.10 | Solve for *x* if and | | | (6) |
|  |  | | |  |
| 5.11 | If , solve the following equations without the use of a calculator: | | |  |
|  |  |  | |  |
|  | 5.11.1 |  | | (1) |
|  |  |  | |  |
|  | 5.11.2 |  | | (2) |
|  |  |  | |  |
|  | 5.11.3 |  | | (4) |
|  |  |  | |  |
| 5.12 | If , solve for *x* in . Give your answer(s) correct to the nearest degree. | | | (5) |

|  |  |  |  |
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| **QUESTION 6** | | | |
|  |  |  |  |
| 6.1 | Simplify the following expressions without the use of a calculator | |  |
|  |  |  |  |
|  | 6.1.1 |  | (7) |
|  |  |  |  |
|  | 6.1.2 |  | (5) |
|  |  |  |  |
|  | 6.1.3 |  | (7) |
|  |  |  |  |
|  | 6.1.4 |  | (8) |
|  |  |  |  |
| 6.2 | Simplify the following expressions fully. | |  |
|  |  |  |  |
|  | 6.2.1 |  | (5) |
|  |  |  |  |
|  | 6.2.2 |  | (6) |
|  |  |  |  |
|  | 6.2.3 |  | (3) |
|  |  |  |  |
|  | 6.2.4 |  | (5) |
|  |  |  |  |
| 6.3 | Simplify the following expressions fully: | |  |
|  |  |  |  |
|  | 6.3.1 |  | (3) |
|  |  |  |  |
|  | 6.3.2 |  | (2) |
|  |  |  |  |
|  | 6.3.3 |  | (3) |
|  |  |  |  |
|  | 6.3.4 |  | (3) |
|  |  |  |  |
|  | 6.3.5 |  | (3) |
|  |  |  |  |
|  | 6.3.6 |  | (3) |
|  |  |  |  |
|  | 6.3.7 |  | (2) |
|  |  |  |  |
|  | 6.3.8 |  | (3) |
|  |  |  |  |
|  | 6.3.9 |  | (3) |

|  |  |  |  |
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| **QUESTION 7** | | | |
|  |  |  |  |
| 7.1 | Given: | |  |
|  |  |  |  |
|  | 7.1.1 | Prove | (4) |
|  |  |  |  |
|  | 7.1.2 | If , for what value(s) of *x* is the given identity not defined. | (2) |
|  |  |  |  |
|  | 7.1.3 | Hence, write down the range of the function which is defined by : | (2) |
|  |  |  |  |
| 7.2 | If , without using a calculator, show that | | (4) |
|  |  |  |  |
| 7.3 | Prove the following identities and in each case give the value(s) of *x* in the interval for which the identity is not defined. | | |
|  |  |  |  |
|  | 7.3.1 |  | (4) |
|  |  |  |  |
|  | 7.3.2 |  | (6) |
|  |  |  |  |
|  | 7.3.3 |  | (7) |
|  |  |  |  |
|  | 7.3.4 |  | (6) |
|  |  |  |  |
|  | 7.3.5 |  | (5) |
|  |  |  |  |
|  | 7.3.6 |  | (6) |
|  |  |  |  |
|  | 7.3.7 |  | (8) |
|  |  |  |  |
|  | 7.3.8 |  | (6) |

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| **QUESTION 8** | | | |
|  |  |  |  |
| 8.1 | Given : | |  |
|  |  |  |  |
|  | 8.1.1 | Sketch the graph of for , clearly showing intercepts with the axes as well as the asymptotes. |  |
|  |  |  |  |
|  | 8.1.2 | Write down the period of *f* |  |
|  |  |  |  |
|  | 8.1.3 | Write down an equation of an asymptote of *f*. |  |
|  | | | |
| 8.2 | The sketch below shows the graphs of and for the interval | |  |
|  |  |  |  |
|  | | | |
| Use the given graphs to answer the following questions. | | | |
|  |  |  |  |
|  | 8.2.1 | Write down the co-ordinates of A, the *y*-intercept of *f.* | (1) |
|  |  |  |  |
|  | 8.2.2 | Write down the co-ordinates of B and F the *x* –intercepts of *f*. | (2) |
|  |  |  |  |
|  | 8.2.3 | Write down the co-ordinates of H , a turning point of *f.* | (1) |
|  |  |  |  |
|  | 8.2.4 | Write down the co-ordinates of C and G the turning points of g. | (2) |
|  |  |  |  |
|  | 8.2.5 | Write down the period of *f* | (1) |
|  |  |  |  |
|  | 8.2.6 | Write down the range of g. | (1) |
|  |  |  |  |
|  | 8.2.7 | Write down the value(s) of *x* for which | (3) |
|  |  |  |  |
|  | 8.2.8 | If the graph of g undergoes a positive vertical shift of 1 unit, explain the influence of this shift on the range of g. | (2) |
|  |  |  |  |
|  | 8.2.9 | Write down the new equation of *f* if it is shifted 30o horizontally to the right. | (2) |

|  |  |  |  |
| --- | --- | --- | --- |
| 8.3 |  | |  |
|  | 8.3.1 | On the same set of axes, draw the graphs of  and for the domain | (8) |
|  |  |  |  |
|  | 8.3.2 | Use your graphs to answer the following questions: |  |
|  |  |  |  |
|  |  | 1. Write down the range of g | (2) |
|  |  |  |  |
|  |  | 1. Use any of the capital letters A , B , C and D to indicate the point(s) on the appropriate axis where you would read off the solution(s) of the equation | (4) |

|  |  |  |  |
| --- | --- | --- | --- |
| 8.4 | The diagram below illustrates the graph of for the domain . The point B lies at the point of intersection of the curve with the x-axis, and both C and D are turning points of the curve. | |  |
|  |  |  |  |
|  | Use the given graph to answer the questions that follow. | |  |
|  | | | |
|  | 8.4.1 | Write down the co-ordinates of B,C and D in that order. | (3) |
|  |  |  |  |
|  | 8.4.2 | If A (*m* ; 0,5) is a point on the curve of *f,* determine the value of *m.* | (2) |
|  |  |  |  |
|  | 8.4.3 | Write down the range of *f*. | (2) |
|  |  |  |  |
|  | 8.4.4 | Determine the equation of *h* in the form *h(x)* = ………..  after *f* has moved 1 unit vertically upwards. | (1) |

|  |  |  |  |
| --- | --- | --- | --- |
| 8.5 |  | |  |
|  | 8.5.1 | Given the functions and  , for . On the same set of axes draw the graphs of *f* and *g* clearly showing the intercepts with the axes as well as the turning points. | (6) |
|  |  |  |  |
|  | 8.5.2 | Write down the amplitude of *g*. | (1) |
|  |  |  |  |
|  | 8.5.3 | Write down the period of *f.* | (1) |
|  |  |  |  |
|  | 8.5.4 | Use your graph to determine the value(s) of *x* for which  in the interval . | (4) |
|  |  |  |  |
|  | 8.5.5 | Use the given functions to show that.  [Hint: Use the fact that. | (3) |
|  |  |  |  |
|  | 8.5.6 | Hence sketch the graph of *h* on the same axes as *f* and *g* for  , clearly showing all asymptotes and intercepts with the axes. | (4) |

|  |  |  |  |
| --- | --- | --- | --- |
| 8.6 | In the given diagram, the graph of for is shown. | | |
|  |  |  |  |
|  | | | |
|  | 8.6.1 | On the same set of axes draw the graph of for . Clearly show the intercepts with the axes and any turning points. | (3) |
|  |  |  |  |
|  | 8.6.2 | Write down the range of *f* in interval notation. | (2) |
|  |  |  |  |
|  | 8.6.3 | Write down the minimum value of *y* = – cos *x* | (1) |
|  |  |  |  |
|  | 8.6.4 | Show by using dotted lines and the letter “**K**” on your graph, where the approximate value of sin 110o can be read off. | (2) |
|  |  |  |  |
|  | 8.6.5 | Determine the value(s) of *x* for which sin 2*x* + cos *x* = 0. | (1) |
|  |  |  |  |
|  | 8.6.6 | For which value(s) of *x* will sin 2*x* decrease as *x* increases for the given interval? | (2) |
|  |  |  |  |
|  | 8.6.7 | For which value(s) of *x* is | (2) |
|  |  |  |  |
|  | 8.6.8 | For which value(s) of *x*  is ? | (2) |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **QUESTION 9** | | | | |
|  |  |  | |  |
| 9.1 | Complete the following: | | |  |
|  |  |  | |  |
|  | 9.1.1 |  | | (1) |
|  |  |  | |  |
|  | 9.1.2 |  | | (2) |
|  |  |  | |  |
| 9.2 | In the accompanying diagram the points S , H, P and K represent the vertices of a piece of land. Isaac divides the land amongst his three sons as shown in the diagram.  HTP is a straight line, with HP.    **H**  HT = KP | | **S**  **K**  **P**  **T** | |
|  | 9.2.1 | Express in terms of . | | (1) |
|  |  |  | |  |
|  | 9.2.2 | Show that in | | (3) |
|  |  |  | |  |
|  | 9.2.3 | Express ST in terms of HT and a trigonometric ratio of | | (2) |
|  |  |  | |  |
|  | 9.2.4 | Hence, determine the area of , if ST = | | (3) |
|  |  |  | |  |
|  | 9.2.5 | Determine the numerical value of the area of , if  HT = 28 m; . Give your answer correct to the nearest m2. | | (4) |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 9.3 | In the diagram alongside, PT is a  2 metre high screen which is 1 metre above eye level, QR , of a learner at R.  QR = *x* metres.  The angle of elevation of P, the top of the screen, from R is , i.e  . | | **P**  **2**  **T**  **1**  **R**  **Q**  ***x*** | |
|  | 9.3.1 | Express in terms of | | (2) |
|  |  |  | |  |
|  | 9.3.2 | Show that | | (2) |
|  |  |  | |  |
|  | 9.3.3 | Express in terms of | | (1) |
|  |  |  | |  |
|  | 9.3.4 | In , show that [Hint: Use the fact that ] | | (3) |
|  |  |  | |  |
|  | 9.3.5 | Hence, prove that | | (2) |
|  |  |  | |  |
|  | 9.3.6 | If , calculate, rounded off to ONE decimal place, how far the learner is sitting from the screen. | | (2) |

|  |  |  |  |
| --- | --- | --- | --- |
| 9.4 | Two sides of a triangular plot of land are 52 metres and 34 metres respectively. If the area of plot of land is 620 m2, determine: | |  |
|  |  |  |  |
|  | 9.4.1 | The size of the included angle. | (3) |
|  |  |  |  |
|  | 9.4.2 | The amount of fencing needed to enclose the plot of land. Give your answer correct to the nearest metre. | (6) |
|  |  |  |  |
|  | 9.4.3 | Hence, determine size of the interior angles of the plot of land. | (4) |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 9.4 | In the diagram alongside, an engine has a seven –inch connecting rod fastened to crank. | |  | |
|  | A simplified version of the engine is shown below.  **A**  1,5  7  *x*  **C**  **B** | | |  |
|  | 9.4.1 | Express the lengths of AB, BC and AC in cm.  (Hint: Use 1 ich = 2,54cm) | | (2) |
|  |  |  | |  |
|  | 9.4.2 | Complete the cosine rule for any  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | (1) |
|  |  |  | |  |
|  | 9.4.3 | Use the cosine rule to show that | | (3) |
|  |  |  | |  |

|  |  |  |
| --- | --- | --- |
| 9.5 | ABCD is a parking lot has the shape of a parallelogram, as indicated in the given diagram. The lengths of two adjacent sides, AB and BC are 70 metres and 100 metres respectively. Calculate the area of the parking lot ABCD, correct to TWO decimal places. |  |
|  | 100 m  D  C  B  A |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 9.6 | | In a process with continuous paper, the paper passes across three rollers of radii three inches, four inches, and six inches, as indicated in the given diagram. The centres of the three-inch and six-inch rollers are *d* inches apart, and the length of the arc in contact with the paper on the four-inch roller is *s* inches. | | | | |  |
|  | | | | | ***d***  **C**  **B**  **A** | | |
|  | | 9.6.1 | | Write down the lengths of AB and BC. | | | (2) |
|  | |  | |  | | |  |
|  | | 9.6.2 | | If AB = 7 inches ; BC = 10 inches and AC = *d* inches, express these lengths in mm. [Hint: Use 1 inch = 2,54 cm]. Give your answers as integer values. | | | (5) |
|  | |  | |  | | |  |
|  | | 9.6.3 | | Show that | | | (3) |
|  |  | | | | | | |
| 9.7 | An idler gear, 30 mm in diameter, has to be fitted between a 70 mm diameter driving gear and a 90 mm diameter driving gear. | | | | | | |
|  | | | | | | | |
| The diameter of the circle with centre A is 90 mm ; the diameter of the circle with centre B is 30 mm, and the diameter of circle with centre C is 70 mm. AC = 99,78mm. | The diagram below represents the given information.  **A**  **B**  **99,78**  Use the given information to determine the size of , correct to TWO decimal places.  **C** | | | | | (5) | |
| 9.8 | A bridge is to be built across a small lake from a gazebo, G to a dock, D. The bearing from the gazebo to the dock is S 41o W. From a tree,T, 100 metres from the gazebo, the bearings to the gazebo and the dock are S74oE and S28oE, respectively. | | | | |  | |
|  | | | | | | | |
|  |  | |  | | |  | |
| ; ; . TG = 100 metres  **T**  **100**  74o  28o  **K**  **G**  41o  **L**  **D** | | | | | | | |
|  | 9.8.1 | | Write down, with reason(s), the size of the interior angles of | | | (5) | |
|  |  | |  | | |  | |
|  | 9.8.2 | | Hence, determine the distance from D to G, correct to TWO decimal places. | | | (4) | |
| **EUCLIDEAN GEOMETRY** | | | | | | | |

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| **QUESTION 1** | | | | |
|  | | | | |
| 1.1 | In the accompanying diagram, BD is a diameter of the circle with centre K. If | | **B**  1  2  **A**  **C**  3  **K**  5  4  **D** | |
|  | 1.1.1 | Name four radii in the given diagram | | (4) |
|  |  |  | |  |
|  | 1.1.2 | Why is ? | | (1) |
|  |  |  | |  |
|  | 1.1.3 | What type of triangle is ? Provide a reason for your answer. | | (2) |
|  |  |  | |  |
|  | 1.1.4 | Determine, with reasons the sizes of the following angles | |  |
|  |  |  | |  |
|  |  |  | | (3) |
|  |  |  | |  |
|  |  |  | | (3) |
|  |  |  | |  |
|  |  |  | | (2) |

|  |  |  |  |  |  |
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| 1.2 |  | | | | |
|  | 1.2.1 | Complete the following statement: If a line segment is drawn from the centre of the circle, perpendicular to a chord, then it……………………………………… | | | (1) |
|  | | | | | |
|  | 1.2.2 | In the diagram alongside, O is the centre of the circle with  AB = 6*x* units and  OD = 2*x* units. | | **B**  **O**  **D**  **A** | |
|  |  |  | |  |  |
|  |  | 1.2.2.1 | Express , with reason, AD in terms of *x* | | (2) |
|  |  |  |  | |  |
|  |  | 1.2.2.2 | Hence, calculate the radius, OA in terms of *x*. | | (4) |

|  |  |  |  |  |
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| 1.3 | In the accompanying diagram, A, B and C are the vertices of ABC.  The straight line through C, parallel to BA, meets the bisector of at D.  2  AB = AC and | | 3  4  1  **D**  **C**  **B**  **A** | |
|  |  |  |  |  |
|  | 1.3.1 | What type of triangle is ? Give a reason for your answer. | | (2) |
|  |  |  | |  |
|  | 1.3.2 | Calculate, with reasons, the size of | | (4) |
|  |  |  | |  |
|  | 1.3.3 | Are the points A, B, C and D concyclic? Explain. | | (4) |

|  |  |  |  |  |  |
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| 1.4 |  |  | | |  |
|  | 1.4.1 | List four ways to prove a quadrilateral is cyclic. | | | (4) |
|  |  |  | | |  |
|  | 1.4.2 | List two ways to show that a line is a tangent to a circle at a given point. | | | (2) |
|  |  |  | |  |  |
|  | 1.4.3.1 | In the given diagram, ABCD is a cyclic quadrilateral. The tangent to the circle at A meets BD produced at E.  and | | **A**  4  3  5  1  **B**  **E**  6  2  **D**  **C** | |
|  |  | Determine with reasons, the magnitude of the following angles. | | | |
|  | | | | | |
|  |  | a) |  | | (2) |
|  |  |  |  | |  |
|  |  | b) |  | | (2) |
|  |  |  |  | |  |
|  |  | c) |  | | (2) |
|  |  |  |  | |  |
|  |  | d) | The interior angles of | | (2) |
|  |  |  |  | |  |
|  | 1.4.3.2 | Is BD a diameter of the circle? Explain. | | | (2) |
|  |  |  |  | |  |
|  | 1.4.3.3 | Show, using appropriate calculations that ABCE is **not** a cyclic quadrilateral. | | | (3) |

|  |  |  |  |  |  |  |
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| 1.5 |  | |  |  |  |  |
|  | 1.5.1 | | Complete the statement of the following theorem in your answer book: The angle which an arc of a circle subtends at the centre ………………………………. | | | (1) |
|  |  | |  |  |  |  |
|  | 1.5.2 | | In the given diagram, R and K are centres of two unequal circles, which intersect at B and D. CBA and EDA are double chords such that CD = DA. | | | |
|  | | | | | | |
| **C**  2  **B**  5  **K**  **A**  3  1  **R**  4  **D**  **E** | | | | | | |
|  | Calculate, with reasons, the sizes of the following angles: | | | | | |
|  |  |  | |  |  |  |
|  |  | i) | |  | | (3) |
|  |  |  | |  | |  |
|  |  | ii) | |  | | (3) |
|  |  |  | |  | |  |
|  |  | iii) | |  | | (2) |
|  |  |  | |  | |  |
|  |  | iv) | |  | | (3) |

|  |  |  |  |  |  |
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| 1.6 |  | | | | |
|  | 1.6.1 | Complete the statement of the following theorem in your answer book: The angle between a tangent to a circle and the chord drawn from the point of contact is equal to………….. | | | (1) |
|  |  |  |  | |  |
|  | 1.6.2 | In the accompanying figure, the points K, N , M and L lie on the circle. The tangent KT to the circle at K is parallel to the chord NM. The chord NL is produced to T.  and . | | | |
| **K**  4  5  1  6  **N**  **L**  **T**  3  2  **M** | | | | | |
|  | Calculate, with reasons, the sizes of the following angles: | | | |  |
|  |  |  |  |  |  |
|  |  | 1.6.2.1 |  | | (2) |
|  |  |  |  | |  |
|  |  | 1.6.2.2 |  | | (3) |
|  |  |  |  | |  |
|  |  | 1.6.2.3 |  | | (2) |

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| 1.7 | Complete the following statements by filling in the missing word(s) so that the statements are correct. | | | |
|  |  |  |  |  |
|  | 1.7.1 | The angle subtended by a chord at the centre of a circle is…… | | (1) |
|  |  |  | |  |
|  | 1.7.2 | The opposite angles of a cyclic quadrilateral are…………….. | | (1) |
|  |  |  | |  |
|  | 1.7.3 | Angles subtended by a chord of a circle in the same………..are equal. | | (1) |
|  |  |  | |  |
|  | 1.7.4 | The exterior angle of a cyclic quadrilateral is……………….. | | (1) |
|  |  |  | |  |
|  | 1.7.5 | The angle between the tangent to a circle and a chord at the point of contact is………………………………………… | | (1) |
|  |  |  | |  |
|  | 1.7.6 | Tangents drawn to a circle from a common point are…………. | | (1) |
|  |  |  | |  |
|  | 1.7.7 | If a line is drawn through the end point of a chord making with the chord an angle equal to an angle in the alternate segment, then the line is a………………..to the circle. | | (1) |

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| 1.8 | In the given diagram, the points A, B, C and D lie on the circle with centre O. RDS is a tangent to circle at D.  BC = DC; . | | | |
|  | **A**  **B**  3  2  1  **O**  1  **C**  4  3  2  5  1  **R**  **D**  **S** | | |  |
|  | 1.8.1 | Provide reason(s) to make the following statements TRUE. | |  |
|  |  |  |  |  |
|  |  | 1.8.1.1 | ABCD is a cyclic quadrilateral. | (1) |
|  |  |  |  |  |
|  |  | 1.8.1.2 | is an isosceles triangle. | (1) |
|  |  |  |  |  |
|  |  | 1.8.1.3 |  | (1) |
|  |  |  |  |  |
|  |  | 1.8.1.4 | OB = OD | (1) |
|  |  |  |  |  |
|  | 1.8.2 | Calculate, with reasons the sizes of the following: | |  |
|  |  |  |  |  |
|  |  | 1.8.2.1 |  | (3) |
|  |  |  |  |  |
|  |  | 1.8.2.2 |  | (2) |
|  |  |  |  |  |
|  |  | 1.8.2.3 |  | (2) |
|  |  |  |  |  |
|  |  | 1.8.2.4 |  | (2) |

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| 1.9 | In the given diagram, O is the centre of the circle. The points S, P, Q and R lie on the circle. TQP is a straight line.  QR = TR.  ;  TR is a tangent to the circle at R. | | | |
| **P** | | | | |
| **Q**  *x*  *y*  **O**  **S**  **T**  **R** | | | | |
|  | 1.9.1 | Name with reason(s) three other angles equal to *x* | | (5) |
|  |  |  |  |  |
|  | 1.9.2 | Express in terms of *x*. | | (2) |
|  |  |  |  |  |
|  | 1.9.3 | Determine the value(s) of *x* for which PTRO **will not** be a cyclic quadrilateral. | | (4) |

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| 1.10 | In the accompanying diagram, M is the centre of the circle. AB is a chord with length 8 units. .  PA = 12 units and PT = 8 units, with T a point on the circle.  MQ = 3 units.  AB = 8 units | | | | |
| **P**  **8**  **12**  **T**  **M**  **3**  **A**  **B**  **Q** | | | | | |
|  | 1.10.1 | Write down, with reason the length of AQ. | | | (2) |
|  |  |  |  | |  |
|  | 1.10.2 | Complete: In a right-angled triangle, the square on the hypotenuse is………………………………………….. | | | (1) |
|  |  |  |  | |  |
|  | 1.10.3 | Determine the length AM | | | (3) |
|  |  |  |  |  |  |
|  | 1.10.4 | Hence, use appropriate calculations to show that PA is a tangent to the circle at A. | | | (4) |

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| 1.11 | In the given diagram, O is the centre of the circle. PT is a tangent to the circle at T. Chord RQ is produced to meet the tangent at P.  QT = QP. . The diameter SOT = 25 cm. | | | | |
| **S** | | | | | |
| 2  **R**  1  **O**  1  **Q**  2  2  1  *x*  3  **T**  **P** | | | | | |
|  | 1.11.1 | Write down the radius of the circle. | | | (1) |
|  |  |  |  |  |  |
|  | 1.11.2 | If , state with reasons two other angles that are equal to *x.* | | | (3) |
|  |  |  |  |  |  |
|  | 1.11.3 | Why is ? | | | (1) |
|  |  |  | | |  |
|  | 1.11.4 | Why is ? | | | (1) |
|  |  |  | | |  |
|  | 1.11.5 | What type of triangle is | | | (1) |
|  |  |  | | |  |
|  | 1.11.5 | Hence, determine the length of TP with reasons , if  RS = 7 cm and SO = 12,5 cm. | | | (6) |

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| 1.12 | In the given diagram, AOBC is a cyclic quadrilateral. A second unequal circle with centre O passes through A and B. CO produced meets the second circle at D.  and AC = OB. Let | | |
| **C** | | | |
| **6**  **5**  **4**  **3**  **2**  **1**  ***x***  **B**  **D**  **O**  **A** | | | |
|  | | | |
|  | 1.12.1 | Why is | (1) |
|  |  |  |  |
|  | 1.12.2 | Prove that | (4) |
|  |  |  |  |
|  | 1.12.3 | Determine the size of in terms of *x* | (3) |
|  |  |  |  |
|  | 1.12.4 | If *x* = 20o , show using appropriate calculations that is an equilateral triangle. | (5) |

|  |  |  |  |
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| 1.13 | In the given diagram, the points B, C, D and E lie on the circle with centre O. BOE is the diameter of the circle. The tangent to the circle at C, meets OD produced at T and the chord EC cuts OD at A. OD is parallel to BC. | | |
|  |  |  |  |
| **D**  **E**  4  **A**  8  7  6  5  4  3  2  1  **M**  **O**  **B**  **F**  **C**  **T** | | | |
|  | 1.13.1 | Why is EDCB a cyclic quadrilateral? | (1) |
|  |  |  |  |
|  | 1.13.2 | Write down with reason the size of | (2) |
|  |  |  |  |
|  | 1.13.3 | Why is EO = OB = OD? | (1) |
|  |  |  |  |
|  | 1.13.4 | Hence, deduce that EA = AC. | (2) |
|  |  |  |  |
|  | 1.13.5 | If = 50o, determine the size of | (2) |
|  |  |  |  |
|  | 1.13.6 | Show that = 25o | (2) |
|  |  |  |  |
|  | 1.13.7 | Hence show that | (3) |
|  |  |  |  |
|  | 1.13.8 | Show, using relevant calculations and providing reasons, that EOCT is a cyclic quadrilateral. | (3) |

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| **Ratio, Proportion and Similarity** |

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| **QUESTION 2** | | | | |
|  |  |  | |  |
| 2.1 |  |  | |  |
|  | 2.1.1 | Complete the following statement: A line drawn parallel to one side of triangle, divides ………………………………… | | (1) |
|  |  |  | |  |
|  | 2.1.2 | In the given diagram, ABCD is a quadrilateral with and | |  |
|  |  | **A**  **B**  **G**  **E**  **C**  **F**  **D** | |  |
|  |  | a. | Write down, with reasons, TWO ratios each equal to  AG:GD | (3) |
|  |  |  |  |  |
|  |  | b. | Hence, prove that | (2) |
|  |  |  |  |  |
|  |  | c. | Prove ||| | (3) |
|  |  |  |  |  |
|  |  | d. | If ; BC = 16 units and DG = 9 units, calculate the lengths of EF and AG (in this order). | (6) |

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| 2.2 |  |  | |  | |  |
|  | 2.2.1 | Complete the following: Two triangles are similar if:  a)……………………………….  b)………………………………. | | | | (2) |
|  |  |  | |  | |  |
|  | 2.2.2 | In the accompanying diagram, ABC is a right –angled triangle with . The point D lies on AB such that CD is perpendicular to AB.  Let:  **A** | | | **C**  1  2  **B**  2  11  **D** | |
|  |  | a) | Express in terms of *x* and *y* | | | (1) |
|  |  |  |  | | |  |
|  |  | b) | In , express in terms of *x* | | | (1) |
|  |  |  |  | | |  |
|  |  | c) | In , express in terms of *y* | | | (1) |
|  |  |  |  | | |  |
|  |  | d) | Hence, prove ||| | | | (4) |
|  |  |  |  | | |  |
|  |  | e) | Name, in order of corresponding letters, another triangle in the figure which is similar to | | | (2) |
|  |  |  |  | | |  |
|  |  | f) | If ||| complete the following ratios: | | | (2) |
|  |  |  |  | | |  |
|  |  | g) | Hence complete: | | | (1) |
|  |  |  |  | | |  |
|  |  | h) | If ||| , show that | | | (2) |
|  |  |  |  | | |  |
|  |  | i) | Prove ||| | | | (3) |
|  |  |  |  | | |  |
|  |  | j) | Hence, deduce that | | | (2) |
|  |  |  |  | | |  |
|  |  | k) | If BC = 4 units, and BD = 2 units, determine the lengths of , in simplified surd form:   1. CD 2. AB 3. BC | | | (8) |

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| 2.3. |  |  |  |  |
|  | 2.3.1 | Complete the following statement: If a line segment joining two points subtends equal angles at two other points on the same side of the line segment,……………………………….. | | (2) |
|  | | | | |
|  | 2.3.2 | In the given diagram, AB is a diameter of the circle with centre O. DO is perpendicular to AB at O. The chord BC meets OE produced at D. OD and AC intersects at E. | | |
|  |  |  |  |  |
| **E**  1  2  1  1  2  3  3  2  2  1  **D**  **C**  **B**  **O**  **A** | | | | |
|  |  | a) | If , name, with reasons THREE other angles equal to 90o | (4) |
|  |  |  |  |  |
|  |  | b) | Hence, deduce that OADC is a cyclic quadrilateral. | (2) |
|  |  |  |  |  |
|  |  | c) | Why is ? | (1) |
|  |  |  |  |  |
|  |  | d) | Hence, show that ? | (2) |
|  |  |  |  |  |
|  |  | e) | Show ||| | (3) |
|  |  |  |  |  |
|  |  | f) | Hence, complete, | (1) |
|  |  |  |  |  |
|  |  | g) | If  *r* is the radius of the circle, determine in terms of *r* | (2) |
|  |  |  |  |  |
|  |  | h) | If , show that OE = *r*.tan *x* | (2) |
|  |  |  |  |  |
|  |  | i) | If , and *r*  = 2 units, determine the area of , in simplified surd form. | (3) |

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| 2.4 |  |  |  |  |
|  | 2.4.1 | Complete the following statement: If two tangents are drawn to a circle from a common point,………………………. | | (1) |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | 2.4.2 | In the given figure, O is the centre of the circle. AB is a diameter of the circle. The tangent to the circle at B meets the AC produced at E. The tangent to the circle at C meets EB at D.  Let | | |
| **E** | | | | |
| **2**  **1**  **4**  **2**  **3**  **1**  **1**  **2**  **3**  **2**  **1**  **C**  **B**  **O**  **D**  **A** | | | | |
|  |  | a) | Name with reasons, THREE angles equal to *x* | (5) |
|  |  |  |  |  |
|  |  | b) | Name with reason, ONE angle equal to *y.* | (1) |
|  |  |  |  |  |
|  |  | c) | In the given diagram, name with reasons, FIVE angles equal to 90o | (7) |
|  |  |  |  |  |
|  |  | d) | Why is CD = DB? | (1) |
|  |  |  |  |  |
|  |  | e) | Show that BOCD is a cyclic quadrilateral. | (3) |
|  |  |  |  |  |
|  |  | f) | Prove that | (2) |
|  |  |  |  |  |
|  |  | g) | Hence, deduce that DB = DE | (2) |
|  |  |  |  |  |
|  |  | h) | Prove ||| | (3) |
|  |  |  |  |  |
|  |  | i) | Complete: | (1) |
|  |  |  |  |  |
|  |  | j) | If BD = CE = 2 units, determine the length of CA. | (7) |

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| 2.5 | In the given diagram, CEDB represents a rectangular shed 2metres wide and 3 metres high. The shed stands against a perpendicular building of height 5,5 metres. A ladder , FA is used to gain access to the roof of the building. AF is the minimum length of the ladder.  The diagram below shows a side view of the shed and a ladder leaning against a building AC. EF is the minimum distance of the ladder to the shed. | | |
|  | | | |
| **A**  **D**  **B**  5,5  2  3  **F**  **C**  **E** | | | |
|  | 2.5.1 | Why is EC? | (1) |
|  |  |  |  |
|  | 2.5.2 | Write down, with reason, the size of | (1) |
|  |  |  |  |
|  | 2.5.3 | Prove ||| | (4) |
|  |  |  |  |
|  | 2.5.4 | Complete: | (2) |
|  |  |  |  |
|  | 2.5.5 | Determine the length of AB | (1) |
|  |  |  |  |
|  | 2.5.6 | Show, that the minimum distance from the bottom of the ladder to the shed cannot be more than 3 m | (4) |

|  |  |  |  |
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| 2.6 | In the given diagram, A, E and C are the vertices of . EA is drawn parallel to DB, with D a point on EC.  AE = 10 cm; AB = 6 cm and BC = 9 cm. (**Note** : ) | | |
|  | | | |
|  | **A**  **6**  **B**  **10**  **9**  **C**  **E**  **D**  **12** | |  |
|  | 2.6.1 | Complete the following statement: A line drawn parallel to one side of a triangle…………………………………….. | (1) |
|  |  |  |  |
|  | 2.6.2 | Prove ||| | (4) |
|  |  |  |  |
|  | 2.6.3 | Complete: | (1) |
|  |  |  |  |
|  | 2.6.4 | Hence, show that CD = 7,2 cm and BD = 6 cm. | (3) |
|  |  |  |  |
|  | 2.6.5 | Hence, determine the size of , correct to TWO decimal places. | (3) |
|  |  |  |  |
|  | 2.6.6 | Determine the numerical value of . Give your answer as an integer value. | (4) |

|  |  |  |  |
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| 2.7 | In the given diagram, two triangles are shown, namely and  ; AB = *x* mm ; AC = 14,58 mm and  CB = 25,69 mm.  ; DE = 4,74mm ; EF = *y* mm and  FD = 7,36 mm | | |
|  | | | |
| **E**  *y*  **C**  4,74  14,58  25,69  7,36  **F**  **D**  **A**  ***x***  **B** | | | |
|  | 2.7.1 | Determine the size of the remaining interior angles of the given triangles. | (2) |
|  |  |  |  |
|  | 2.7.2 | Show that ||| | (3) |
|  |  |  |  |
|  | 2.7.3 | Hence, determine the numerical values of *x* and *y*. | (4) |

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| 2.8 | In the given diagram, PQR is an equilateral triangle with sides 4 cm. PQ and PR are produced to meet in S and T respectively. ST is parallel to QR. PS = 9 cm. | |  |
|  | | | |
| **P**  **2**  **1**  **R**  **Q**  **1**  **2**  **T**  **S** | | | |
|  | 2.8.1 | Write down, with reason, the sizes of the interior angles of | (2 |
|  |  |  |  |
|  | 2.8.2 | Write down, with reason the size of | (2) |
|  |  |  |  |
|  | 2.8.3 | Write down the length of QS | (1) |
|  |  |  |  |
|  | 2.8.4 | Prove ||| | (3) |
|  |  |  |  |
|  | 2.8.5 | Hence, determine the length of ST. | (3) |
|  |  |  |  |
|  | 2.8.6 | X is an arbitrary point on ST, such that PX is the bisector of . Determine the length of PX, correct to TWO decimal places. | (3) |

|  |  |  |  |
| --- | --- | --- | --- |
| 2.9 | In the given diagram DE . AE and DB intersect at C. | |  |
|  | | | |
|  | **A**  **A**  **21**  **1**  **3**  **C**  **4**  **6**  **5**  **E**  **D** | |  |
|  |  |  |  |
|  | 2.9.1 | Determine the length of BC if AB = 6 cm ; DE = 8 cm and DC = 3 cm | (4) |
|  |  |  |  |
|  | 2.9.2 | Determine the length of DE when EC = 2 cm ; AC = 5 cm and AB = 10 cm | (4) |

|  |  |  |  |
| --- | --- | --- | --- |
| 2.10 | In the given diagram CF ; CE and, . | | |
|  |  |  |  |
|  | **A**  **F**  **E**  **D**  **C**  **B** | |  |
|  | Determine the following ratios with reasons. | |  |
|  |  |  |  |
|  | 2.10.1 | DC: CB | (2) |
|  |  |  |  |
|  | 2.10.2 |  | (3) |
|  |  |  |  |
|  | 2.10.3 |  | (5) |

|  |  |  |  |
| --- | --- | --- | --- |
| 2.11 | In the given diagram ABCD is a parallelogram with the diagonals intersecting at T. The circle passing through the points A, B and C cuts DB at O. [Note: T is **not** the centre of the circle.] | |  |
|  |  |  |  |
| 7  **T**  4  2  2  6  5  4  3  2  1  8  1  **O**  **D**  **C**  **B**  **A** | | | |
|  | 2.11.1 | Why is AT = TC? | (1) |
|  |  |  |  |
|  | 2.11.2 | Complete the following statement: Vertically opposite angles are……………………. | (1) |
|  |  |  |  |
|  | 2.11.3 | Why is |  |
|  |  |  |  |
|  | 2.11.4 | Prove ||| | (3) |
|  |  |  |  |
|  | 2.11.5 | Complete: | (2) |
|  |  |  |  |
|  | 2.11.6 | If AC = 6 units and BT = 4 units |  |
|  |  |  |  |
|  |  | 1. Show that OT = | (2) |
|  |  |  |  |
|  |  | 1. Hence, determine the length of DO. | (4) |

|  |  |  |  |
| --- | --- | --- | --- |
| 2.12 | In the given diagram, K, L and M are the vertices of , with O on KL and P on KM. OP is parallel to LM.  KP = 3 cm ; area of | | |
|  |  |  |  |
|  | **K**  **3**  **P**  **O**  **M**  **L** | |  |
|  | 2.12.1 | Complete the following statement: A line drawn parallel to one side of a triangle………………………………………… | (1) |
|  |  |  |  |
|  | 2.12.2 | Provide the geometrical name for the quadrilateral LOPM and provide a reason for your answer. | (2) |
|  |  |  |  |
|  | 2.12.3 | Prove ||| | (4) |
|  |  |  |  |
|  | 2.12.4 | Write down the area of | (1) |
|  |  |  |  |
|  | 2.12.5 | Hence, determine the length of PM | (4) |
|  |  |  |  |

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| CIRCLES, ANGLES AND ANGULAR MOVEMENT |

|  |  |
| --- | --- |
| **QUESTION 1** | |
|  | |
| 1. Determine the **number of revolutions** in the following: | |
|  | |
| 1. 330o | 1. 88o51’ |
| 1. 55o | 1. 348o15’ |
| 1. 34,12o | 1. 58o6’ |
| 1. 33,55o | 1. 60o |
| 1. 19o31’ | 1. 360o |
|  |  |
| 1. Express the following in **degrees** only: | |
|  | |
| 1. 0,5 revolutions | 1. 48o33’ |
| 1. 67o60’ | 1. 0,35 revolutions |
| 1. 20o8’ | 1. 0,17 revolutions |
| 1. 25o33’ | 1. 0,53 revolutions |
| 1. 333o33’33’’ | 1. 15o58’60’’ |
|  |  |
| 1. Write the following in **degrees, minutes and seconds** only. | |
|  | |
| 1. 0,16 revolutions | 1. 35,9o |
| 1. 225,25o | 1. 0,12 revolutions |
| 1. 48,3o | 1. 0,34 revolutions |
| 1. 50,65o | 1. 72,51o |
| 1. 0,15 revolutions | 1. 20,33o |

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| **QUESTION 2** | |
|  | |
| 1. Convert the following to **radians.** | |
|  | |
| 1. 214o | 1. 134,3o |
| 1. 300o12’ | 1. 112o24’6’’ |
| 1. 0,25 revolutions | 1. 360o |
| 1. 1o10’ | 1. 0,66 revolutions |
|  | |
| 1. Convert the following to **degrees and minutes.** | |
|  | |
| 1. 0,434 revolutions | 1. 2,976 radians |
| 1. 1 radian | 1. radians |
| 1. 11,4 radians | 1. 1,571 radians |
|  | 1. 0, 3 revolutions |
|  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 3** | | | |
|  | | | |
| 3.1. | A wheel rotates at 15 rps (revolutions per second). Calculate the angular velocity of the wheel. | | |
|  |  | | |
| 3.2 | The angular velocity of a wheel is 180 rad/s (radians per second). Determine the number of revolutions the wheel completes in one second. | | |
|  |  | | |
| 3.3 | The diameter of a wheel is 80 mm. Calculate the peripheral and angular velocity of the wheel if it rotates at 21 rps. | | |
|  |  | | |
| 3.4 | Image result for picture of a driver wheel moving a belt | | A point on the circumference of a driver wheel driving a belt moves through 2m every second. The diameter of the driver wheel is 0,4 m. Calculate:  4.1) The distance that will be covered by the point in 10 s. (Hint: Use the formula ***s = vt***)  4.2) The rotational frequency of the driver wheel. (Hint: Use the formula )  4.3) The time taken in minutes for the driver wheel to make 12 complete revolutions. |
|  |  | | |
| 3.5 | The Ferris Wheel at the V&A Waterfront in Cape Town has a diameter of 50 m. The distance from the ground to the bottom of the wheel is 3 m. The Ferris Wheel completes one complete revolution in 3,75 minutes.  3.5.1 Calculate the linear velocity in km/h of an individual riding the Ferris Wheel.  3.5.2 The height of a rider in terms of time, *t* , where *t* is measured in minutes. | Image result for pictures of ferris wheel clip art with a person riding | |
|  |  | | |
| 3.6 | A train moving on a circular track with a diameter of 14,642 km, takes 50 minutes to complete one revolution. Calculate the peripheral velocity of the train in km/h. | | |
|  |  | | |

|  |  |  |
| --- | --- | --- |
| 3.7 | Related image | A spoke of a bicycle wheel turns through 8,42 radians in one second. How long, in minutes, will it take the wheel to complete four complete revolutions. |

|  |  |  |  |
| --- | --- | --- | --- |
| 3.8 |  | Tebogo is a regular participant in the Cape Argus cycling tour. His bicycle has a 150 mm-diameter chainring and a 95 mm-diameter sprocket. The sprocket and the rear wheel rotate at the same rate. The diameter of the rear wheel is 700 mm.  Calculate his speed in km/h, if he was pedaling at 90 rpm (revolutions per minute).  [Hint: Use 1 km = 106 mm] | |
|  |  | | |
| 3.9. |  | | A fire truck is parked on the shoulder of a highway next to a long block wall. The red light on the top of the truck is 10 feet from the wall and rotates through a complete revolution every 2 seconds.  3.9.1 Convert 10 ft to metres. Use the fact that 1 foot = 0,3048 m.  3.9.2 Determine the angular velocity of the red light in rad/s.  3.9.3 Express *d* and *l* in metres, in terms of trigonometric ratios of .  3.9.4 Hence, show that |

|  |  |  |
| --- | --- | --- |
| 3.10. |  | The diagram alongside shows a larger pulley with radius 15 cm and a smaller pulley with radius of 8 cm. The larger pulley rotates at 25 times per 36 sec.  3.10.1 Calculate the angular velocity of the larger pulley in rad/s.  3.10.2 Calculate the linear velocity of the larger pulley in cm/s.  3.10.3 If the linear velocity of the larger pulley is 65,45 cm/s, show, using appropriate calculations that the angular velocity of the smaller pulley is less than rad/s. |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 4** | | | |
|  | | | |
| * 1. In a circle sector, *s* = arc length; *r* = radius and = central angle. Calculate the value of the missing variable in each of the following. **NB :** | | | |
|  | | | |
| |  |  |  |  | | --- | --- | --- | --- | | **No** | **Arc length , s** | **Radius, r** | **Measure of central angle,** | | 1.1 | 0,6 m | 1,3 m |  | | 1.2 |  | 0,9 m | 3 rad | | 1.3 | 4,6 m |  | 2,5 rad | | 1.4 |  | 600 mm |  | | 1.5 | 800 mm | 600 mm |  | | | | |
|  | | | |
| 4.2 | The length of the arc of a circle sector is 2,2m and the radius is 0,9 m. Calculate, correct to TWO decimal places: | | |
|  | | | |
|  | 4.2.1 | The size of the central angle | |
|  | | | |
|  | 4.2.2 | The area of the sector. | |
|  | | | |
| 4.3. | The radius of a circle from which a circle sector has been cut is 400 mm. The angle subtended at the centre by the arc is 63o36’. Calculate correct to TWO decimal places: | | |
|  |  |  |  |
|  | 4.3.1 | The length of the arc | |
|  |  |  |  |
|  | 4.3.2 | The area of the circle sector. | |
|  | | | |
| 4.4 | The arc length of a sector of a circle is 1,6m and it subtends an angle of 3 radians at the centre. Calculate correct to TWO decimal places: | | |
|  |  |  |  |
|  | 4.4.1 | The radius ; | |
|  |  |  |  |
|  | 4.4.2 | The area of the sector. | |
|  | | | |
| 4.5 | The area of a circle sector is 218 m2 and the length of the arc is 8 m. Calculate correct to TWO decimal places: | | |
|  |  |  |  |
|  | 4.5.1 | The length of the radius | |
|  |  |  | |
|  | 4.5.2 | The magnitude of the central angle in degrees, minutes and seconds. | |
|  |  |  |  |
| 4.6 | The area of a circle is 128m2. The radius is 4 m long. Calculate , correct to TWO decimal places: | | |
|  |  |  |  |
|  | 4.6.1 | The size of the central angle in radians. | |
|  |  |  | |
|  | 4.6.2 | The arc length. | |

|  |  |  |
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| 4.7 |  | A cardboard name tag is in the shape of a sector of a circle. The radii OA =OB = 6 cm. The arc AB subtends an angle of 40o at the centre of the sector.  Calculate correct to TWO decimal places:  4.7.1. The area of the name tag.  4.7.2. Calculate the perimeter of the name tag. |
|  | | |
| 4.8 |  | ABC is a sector of a circle with centre A.  AB = 10 cm ; arc length BC = 8 cm.  Calculate , correct to TWO decimal places:  4.8.1. The size of ;  4.8.2. The area of sector ABC. |

|  |  |  |
| --- | --- | --- |
| 4.9 | A chord of a circle subtends an angle of radians at the centre of the circle, cuts off a segment equal to of the area of the whole circle. | |
|  | | |
|  | 4.9.1. | Write down the area of the full circle in terms of |
|  |  |  |
|  | 4.9.2 | Show that |
|  |  |  |
|  | 4.9.3 | Verify that . |
|  | | |
| 4.10 | A chord of a circle subtends an angle of radians at the centre of the circle, cuts off a segment equal to of the area of the whole circle. | |
|  | | |
|  | 4.10.1 | Show that |
|  |  |  |
|  | 4.10.2 | Verify that . |

|  |
| --- |
| **QUESTION 5** |
|  |
| In each of the following , calculate the value of the missing variable. In each case use the formula: 4*h*2 – 4*Dh* + *x*2 = 0. In each case give answer correct to TWO decimal places, where necessary. |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | | **No.** | **Height(s) of segment, *h*** | **Diameter of circle, *D*** | **Length of chord, *x*** | **Radius of circle , *r*** | | 1. | 120 mm |  | 250 mm |  | | 2. | 2 m |  | 3,2 m |  | | 3. | 0,28 m |  | 150 mm |  | | 4. | 80 mm | 35 cm |  |  | | 5. | 150 mm | 400 mm |  |  | | 6. | 200 mm |  |  | 205 mm | | 7. | H = 400 mm ; h = 300 mm |  |  |  | | 8. | 1,5 m | 3 m |  |  | | 9. |  | 3 m | 200 cm |  | | 10. |  |  | 220 mm | 18 cm | | 11. |  | 450 mm | 250 mm |  | | 12. |  | 320 mm | 150 mm |  | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **QUESTION 6** | | | | |
|  | | | | |
| 6.1 | In the accompanying diagram BC is a diameter of the circle with centre O.  The chord AB = 114 cm , AO is perpendicular to BC at O, and  = 45o | | **B**  **A**  **O**  **C** | |
|  | 6.1.1 | Why is OC = OB? | | (1) |
|  |  |  | |  |
|  | 6.1.2 | Show OB = AB | | (2) |
|  |  |  | |  |
|  | 6.1.3 | Hence, determine the length of BC, correct to TWO decimal places. | | (2) |

|  |  |  |  |
| --- | --- | --- | --- |
| 6.2 | In the accompanying figure O is the centre of the circle. The point A is on the circle. OB is perpendicular to chord AC. OA = 10 cm; OB = 6cm. | | |
|  |  | **D** |  |
|  | **C**  **B**  **O**  **A** | | |
|  | 6.2.1 | Why is CB = BA? | (1) |
|  |  |  |  |
|  | 6.2.2 | Complete: In a right-angled triangle, the square on the hypotenuse is………………………………… | (1) |
|  |  |  |  |
|  | 6.2.3 | Determine the length of AB | (3) |
|  |  |  |  |
|  | 6.2.4 | Hence, write down the length of AC | (1) |
|  |  |  |  |
|  | 6.2.5 | Determine the height of the minor segment and hence write down the length of the height of the major segment. | (4) |
|  |  |  |  |
|  | 6.2.6 | Determine the area of ,in mm2 | (4) |
|  |  |  |  |
|  | 6.2.7 | Determine the circumference of the circle. | (2) |

# Formulae for you to know and apply:

|  |  |  |
| --- | --- | --- |
| **Angular Velocity** |  |  |
|  |  |
|  |  |
| **Peripheral Velocity** |  | *D* = Diameter of circle  *n* = rotational frequency |
| **Linear Velocity** |  |  |
| **Arc length** |  |  |
| **Area of sector** |  |  |
| **Area of circle** | Area = |  |
| **4*h*2 – 4*Dh* + *x*2 = 0** | | *D* = Diameter of circle  *h* = Height of segment  *x* = Length of chord |

**MENSURATION**

| **Name of shape / object** | **Figure** | **Area formula** |
| --- | --- | --- |
| **Triangle** | ***h***  ***base*** | A =  **A =** |
| If the lengths of all three sides of a triangle are known, its area, A, can be calculated using **Heron’s formula**  ***l*** | **Heron’s formula**:  **A =**  , where ***a***, ***b*** and ***c*** are the lengths of the three sides and s is the semi-perimeter, i.e.:  ***s*** = |
| **Rectangle** | ***b*** | A = *length*  **A = *l*** |
| **Square** | ***l***  ***l*** | A = *length*  **A = *l*2** |
| **Parallelogram** | ***b***  ***h*** | A =  **A =** |
| **Rhombus** |  | A = |
| **Kite** |  | A = |
| **Trapezium** | **>**  **>** | A = sum of parallel sides height |

|  |  |  |
| --- | --- | --- |
| **Circle** | ***r*** | **A =** , where *r* is the radius of the circle  **A =** ; where *D* is the diameter of the circle |
| **Sector** |  | **A =**  , where is measured in *radians*  **A =** , where is measured in *degrees*.  A = , where *s* is the arc length of the sector |
| **Ellipse** |  | A = , where ***a*** and ***b*** are the lengths of the *semi-major* and *semi-minor* axes respectively. |

|  |  |  |
| --- | --- | --- |
|  | **QUESTION 1** | |
|  |  | |
| **Find the areas of the following plane figures, correct to TWO decimal places.** | | |
|  | | |
| a) | In ABC, AB = 3 cm; BC = 6 cm and AC = 5 cm.  **A**  **C**  **B** |  |
| b) | In the figure alongside. O is the intersection of the major and minor axis of an ellipse. OB is the semi-major axis and OA is the semi-minor axis. OA = 2cm and OB = 5 cm. | *5 cm*  *2 cm*  **B**  **O**  **A** |
| c) | In the accompanying diagram O is the centre of a sector of a circle. Angle DOB = 40o, and the radius of the circle is 15 cm.  Use = 3,14 | **D**  **O**  **B** |
| d)  **A** | In the accompanying figure ABCD is a quadrilateral, with AC = 8 cm ; BE = 6 cm and DF = 2 cm. BE AC and DF AC.  **B**  **F**  **C**  **E**  **D** | |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 2** | | | |
|  | | | |
| 2.1 | Determine the areas of the following geometric shapes, correct to one decimal place. | | |
| a) | In the given diagram ABCD is a square, with length 4cm.  **C**  **B**  **A**  **D** | b) | EFGH is a rectangle, with FG = 4 cm and HG = 12 cm.  **F**  **E**  **G**  **H** |
| c) | Points K,L and N are vertices of a triangle with KM perpendicular to LN. KM = 15 cm and LN = 10 cm.  **K**  **L**  **N**  **M** | d) | In the given diagram PQRS is a trapezium, with PQ = 18 cm; SR = 12 cm ; PT = 8 cm and ST PQ.  **T**  **Q**  **P**  **S**  **R** |
| e) | AOB is the diametre of a circle with centre O. AOB = 15 cm  **A**  **O**  **B** | f) | ABCD is a rhombus with diagonals AC and BD.  AC = 15 cm and BD = 10 cm. |
| g) | Determine the area of ABC, if  AB = 5 cm; BC = 12 cm and  AC = 16 cm. | h) | In the accompanying figure, O is the centre of the circle, with A and B points on the circumference. = 70o and OB is 18 cm.  **70o**  **B**  **A**  **18**  **O** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 2.2\* | Thabo, an excavator operator has 100 metres of barricade mesh and needs to enclose an area to work in safely. He decides to make a rectangular region with dimensions ***x*** and ***y.***   1. Write an equation for the enclosed perimeter in terms of ***x*** and ***y***. 2. Hence, make ***y*** the subject of the formula. 3. Write an equation for the area of the enclosed region in terms of ***x.*** 4. Complete the following table for different values of ***x***  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | ***x*** | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | | **Area** |  |  |  |  |  |  |  |  |  |  |  |  1. Can *x* have a value greater than 50? Explain. 2. Sketch a graph of the area against *x.* 3. Determine the value of *x* that ensures a maximum area. 4. What is the value of *y* for the maximum area? 5. What shape encloses the maximum area? 6. Calculate the maximum area.   Before, Thabo constructs the rectangular region, his colleague Lebogang, suggests that he considers constructing a circular region instead of a rectangular region with the same 100 metres of barricade mesh.   1. Determine the radius of this circular region. 2. What area is enclosed in this circular region? 3. Which of the two shapes encloses the greater area? What is the difference between the two areas? 4. Which shape should Thabo use for the construction of the enclosure? |
| 2.3\* | In the accompanying figure, ABC is a scalene triangle with a base length of 80 cm and a perpendicular height of 40 cm. A right-angled triangle, AED, is nestled within ABC such that DE is 10 cm to the left of the perpendicular height, as shown. Determine the numerical values of ***x*** and ***y***, if the shorter side of the two is 20 cm less than the longer side and the areas of the two shaded regions are equal.  **C**  **NB: Angle C 90o**  **D**  **B**  **A**  **E**  **F**  AE = ***x*** cm ; DE = *y* cm ; EF = 10 cm ; AB = 80 cm and CF = 40 cm.  CF AB ; DA = 90o |

|  | **FORMULAE FOR** | |
| --- | --- | --- |
| **Name of Shape** | **TOTAL SURFACE AREA (TSA)** | **VOLUME (V)** |
| Prism | TSA = | V = |
|  | | |
| Pyramid | TSA = |  |
|  | | |
| Cylinder | TSA = | V = |
|  | | |
| Cone | TSA = | V = |
|  | | |
| Sphere | TSA = | V = |
|  | | |
| Hemisphere | TSA = | V = |
|  | | |

|  |  |  |  |
| --- | --- | --- | --- |
| **QUESTION 3** | | | |
|  | | | |
| 3.1 | A rectangular block of alloy has dimensions of 60 mm by 30 mm by 12 mm. Calculate the volume of the alloy in cubic centimetres. | | (3) |
|  |  | |  |
| 3.2 | Determine how many cubic metres of concrete are required for a  120 m long path, 400 mm wide and 10 cm deep. | | (3) |
|  |  | |  |
| 3.3 | Determine the volume of a cone with height 12,5 cm and base diameter 6 cm, correct to ONE decimal place. | | (3) |
|  |  | |  |
| 3.4 | A piece of alloy with dimensions 25 mm by 60 mm by 1,60 m is melted down and re-cast into a cylinder whose diameter is 150 mm. Assume no wastage, determine the height of the cylinder in centimetres, correct to ONE decimal place. | | (4) |
|  |  | |  |
| 3.5 | Determine the maximum capacity in litres of a fish tank measuring 50 cm by 40 cm by 2,5 m. [Note: 1 litre = 1000 cm3] | | (4) |
|  |  | |  |
| 3.6 | The diagram below shows a right-angled triangular prism.  AB = 3 cm ; AC = 5 cm ; BC = 4 cm and CD = 11 cm | |  |
|  | **F** | |  |
|  | Image result for Picture of right angled triangular prism  **E**  **B**  **C**  **D**  **A** | |  |
|  | 3.6.1 | Determine the volume of the prism in m3 | (3) |
|  |  |  |  |
|  | 3.6.2 | Determine the Total surface area of the prism in mm2 | (5) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 3.8 | A pyramid has a rectangular base 3,60 cm 5,40 cm is shown below on the right. The slant length is 15 cm. | | | | |
|  |  | **F** | |  | |
| Image result for pictures of pyramid with rectangular base  The following formulae may assist you in answering this question:  Volume of Pyramid = =  TSA Pyramid = Sum of areas of triangles forming sides + Area of base  Picture of a Pyramid in Cairo,Egypt. | | | **D**  **C**  **E**  **G**  **B**  **A** | | |
|  | 3.8.1 | Why is DE = EB? | | | (1) |
|  |  |  | | |  |
|  | 3.8.2 | Use the Theorem of Pythagoras to calculate the length of DB, correct to TWO decimal places. | | | (2) |
|  |  |  | | |  |
|  | 3.8.3 | Hence, write down the length of EB. | | | (1) |
|  |  |  | | |  |
|  | 3.8.4 | Calculate the length of EF, correct to TWO decimal places. | | | (3) |
|  |  |  | | |  |
|  | 3.8.5 | Determine the volume of the pyramid, correct to TWO decimal places. | | | (2) |
|  |  |  | | |  |
|  | 3.8.6 | If, H is the midpoint of DA, determine FH and hence calculate the area of , correct to THREE decimal places. | | | (5) |
|  |  |  | | |  |
|  | 3.8.7 | Calculate the , if FG = 14,75 cm. | | | (2) |
|  |  |  | | |  |
|  | 3.8.8 | Hence, determine the Total Surface Area of the pyramid, in cm2. | | | (4) |
|  |  |  | | |  |

|  |  |  |  |
| --- | --- | --- | --- |
| 3.9 | Determine the volume and total surface area of a hemisphere with diameter 5 cm. | | (5) |
|  | | | |
| 3.10 | A rectangular piece of metal having dimensions  is melted down and recast into a pyramid having a rectangular base measuring . Calculate the perpendicular height of the pyramid. Assume no waste of metal happened. | | (6) |
|  |  |  |  |
| 3.11 | A rivet consists of a cylindrical head, of diameter 1 cm and depth 2mm, and a shaft of diameter 2 mm and length 1,5 cm. | |  |
|  | | | |
|  | Image result for pictures of rivet with a cylindrical head and shaft | | |
|  | 3.11.1 | Determine the volume of the cylindrical head [Use . | (3) |
|  |  |  |  |
|  | 3.11.2 | If the total volume of one rivet is 0,2042 cm3, determine correct to TWO decimal places, the volume of the cylindrical shaft of the rivet. | (2) |
|  |  |  |  |
|  | 3.11.3 | Hence, determine the volume of metal in 2000 rivets of the same type. | (2) |

|  |  |  |  |
| --- | --- | --- | --- |
| 3.12 | A solid metal cylinder of radius 6 cm and height 15 cm is melted down and recast into a shape comprising a hemisphere surmounted by a cone. Suppose that 8% of the metal is wasted in the process. | |  |
|  | | | |
|  | *The following formulae can be used to assist you in answering this question:*  *Volume of Sphere = ; Volume of Cone =*  *Curved surface area of Sphere = ; TSA of Cone =* | | |
| radius  C  B  Image result for pictures of a solid metal cylinder  D  **After**  **Before** | | |  |
|  | 3.12.1 | Determine the volume of the cylinder in cm3. Use the following formula : Volume = Area of base X Height ; | (3) |
|  |  |  |  |
|  | 3.12.2 | If 8% of the metal is wasted, what percentage of the original material does the volume in QUESTION 3.12.1 represent? | (1) |
|  |  |  |  |
|  | 3.12.3 | If the volume of the hemisphere and the cone is given by,  Volume = , determine the height, AD, of the conical portion, if its diameter , BC is 12 cm. | (6) |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 3.13 | A block of copper having a mass of 50 kg is drawn out to make 500 m of wire of uniform cross-section. The density of copper is 8,91 g/cm3  [**Note:** A density of 8,91 g/cm3 means that 8,91 g of copper has a volume of 1 cm3, or 1 g of copper has a volume of (1 8,91) cm3] | | | |
|  |  |  | |  |
| Image result for pictures of a block of copper  Copper block | | | Image result for pictures of copper wire of uniform cross section | |
|  | 3.13.1 | The formula used to calculate the density of an object is given by : .  Make Volume, the subject of the formula. | | (2) |
|  |  |  | |  |
|  |  |  | |  |
|  | 3.13.2 | Express 50 kg in grams | | (2) |
|  |  |  | |  |
|  | 3.13.2 | Hence, determine the Volume of the 50 kg block of copper. Express your answer in cm3. | | (3) |
|  |  |  | |  |
|  | 3.13.3 | Determine the cross-sectional area of the wire, if the volume of the wire is 5612 cm3  [Hint : Use the formula: Volume of wire = Area of cross-section X Length of wire] | | (3) |
|  |  |  | |  |
|  | 3.13.4 | Determine the diameter of the cross-section of the wire, if the cross-sectional area of the wire is 0,1122 cm2. Express your answer in mm. [Use ]. | | (4) |

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| --- | --- | --- | --- |
| 3.14 | In a frustum of cone the diameter of the ends are 6 cm and 4 cm and its perpendicular height is 3,6 cm. | | |
|  | | | |
| Image result for pictures of a frustum of a cone | | | |
| The diagram below shows a section through the vertex of a complete cone. | | | |
| **A**  In the diagram alongside; DE = 4 cm ; DP = 2 cm ; BR = 1 cm ; BQ = 3 cm ; BC = 6cm and DR = 3,6 cm  **P**  **E**  **D**  **B**  **C**  **Q**  **R** | | | |
|  | 3.14.1 | Show that ||| | (3) |
|  |  |  |  |
|  | 3.14.2 | Hence, determine the length of AP | (3) |
|  |  |  |  |
|  | 3.14.3 | Write down, with reason the height of AQ | (2) |
|  |  |  |  |
|  | 3.14.4 | Determine the volume of the large cone, correct to TWO decimal places. | (3) |
|  |  |  |  |
|  | 3.14.5 | Determine the volume of the frustum of the cone, if the volume of the smaller cone is 30,16 cm3 | (3) |
|  |  |  |  |
|  | 3.14.6 | Determine the Total Surface Area of the frustum | (7) |

|  |  |
| --- | --- |
| 3.15 | A lampshade is in the shape of a frustum of a cone. The vertical height of the shade is 25 cm and the diameters of the ends are 20 cm and 10 cm respectively. Determine the area of the material needed to form the lampshade, correct to TWO decimal places.  [Note: The curved surface area of frustum of cone = ] |
|  | Image result for pictures of a lampshade in the shape of a frustum of a cone |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 3.1.6 | A cooling tower is in the form of a cylinder surmounted by a frustum of a cone, as shown in the figure below. | | | |
|  |  |  | |  |
|  |  | | |  |
|  | 3.16.1 | | Determine, the volume of the cylindrical portion of the tower. | (3) |
|  |  | |  |  |
|  | 3.16.2 | | Calculate the volume of the frustum of the cone [ Use the following formula : Vol.of frustum of cone = | (5) |
|  |  | |  |  |
|  | 3.16.3 | | Hence write down the total volume of the cooling tower. | (2) |
|  |  | |  |  |
|  | 3.16.4 | | If the total volume of the cooling tower is 10 928 m3, calculate the volume of air space, in m3 in the tower if 40% of the space is used for pipes and other structures. | (3) |

**AREA OF AN IRREGULAR FIGURE USING MID-ORDINATE RULE**

|  |
| --- |
| **Question 1***:* The area of an irregular metal plate needs to be calculated. The ordinates are drawn 5 cm apart across the surface of the metal plate. The lengths of the ordinates in cm’s are: 29; 32; 33; 32,5; 32; 31; 31; 32; 33; 35; 37; 39; 40. |
| 1.1 The following table is provided to you. Determine the missing values, A, B and C in the given table.   |  |  |  | | --- | --- | --- | | **Ordinate** | **Calculation** | **Mid-ordinate** | | **29** |  | **30,5** | | **32** |  | **32,5** | | **33** |  | **A** | | **32,5** |  | **32,25** | | **32** |  | **31,5** | | **31** | **B** | **31** | | **31** |  | **31,5** | | **32** |  | **32,5** | | **33** |  | **34** | | **35** |  | **36** | | **37** |  | **38** | | **39** |  | **39,5** | | **40** |  |  | | **Sum of mid-ordinates** | | **C** |   1.2 Hence determine the area of the sheet metal using the mid-ordinate rule. |

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| --- | --- |
| **QUESTION 2** | |
|  |  |
| 2.1 | Calculate the area of an irregular shape, with the following information:   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | **Ordinates** | 6 *m* | 8 *m* | 10 *m* | 9 *m* | 8 *m* | 6 *m* |   The common interval is 5 *m*. |
|  |  |
| 2.2 | The ordinates, in mm, of an irregular sheet of metal are as follows: 116; 121; 132; 137; 141; 152; 140; 132; 124 and 114 respectively. The common distance is 14 mm. Use the **mid-ordinate rule** to calculate the area of the sheet. |
|  |  |
| 2.3 | Calculate the area of an irregular metal plate. The lengths of the ordinates in *mm* are: 0; 4; 8; 6; 2; 0. The common interval is 5 *mm*. |
|  |  |
| 2.4 | |  |  |  | | --- | --- | --- | | **Ordinates** | **Calculation** | **Mid-Ordinate** | | 40 *mm* |  | 50 | | 60 *mm* |  | 62 | | 64 *mm* |  | 65 | | 66 *mm* |  |  | | 64 *mm* |  |  | | 61 *mm* |  |  | | 60 *mm* |  |  | | 59 *mm* |  |  | | 57 *mm* |  |  | | 48 *mm* |  |  | | 38 *mm* |  |  | | **Sum of mid-ordinates** | |  |   Complete the following table, to calculate the area of an irregular shape. The common interval is 10*mm* |
|  |  |
| 2.5 | An irregular figure has the following ordinates: 60 *cm*; 100 *cm*; ***x*** *cm*; 40 *cm*.  The common interval is 200 *mm*. Calculate the numerical value of ***x*** if the area is 6280 *cm*2. |
| 2.6. | The graph represents part of a circle with diameter 1*metre*.  2.6.1. Write down the equation of the circle in the form:  ***y*** =…………….  2.6.2 Use the graph to read off the ordinates(*y*-values) for the indicated ***x*** -values   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | ***x*- value** | 0 | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | | ***y*- value** | 0,5 |  |  |  | 0,3 |  |   2.6.3 Use the **mid-ordinate rule** to calculate the area of the shown area.  2.6.4 Verify your response by using a known formula. |
|  |  |
| 2.7 | The ordinates of an irregular shape are: 36; 44; 54; 40; 32; 40; 47; 68 and 53 *mm.* Calculate the area of the figure, using the **mid-ordinate rule** if the common distance is 18 *mm*. |
|  |  |
| 2.8. 1 | Draw a semi-circle with radius 8 cm. Use the diameter as the base line and draw six mid ordinates. Calculate the area of the semi-circle by using the **mid-ordinate rule**. |
| 2.8.2 | Verify the accuracy of your answer by calculating the area of the semi-circle by using a known formula for the area of a circle.(**Area of circle =** ) |
|  |  |
| 2.9. | An irregular sheet of metal has a straight edge measuring 100 cm. Ordinates are drawn from the straight edge to the irregular edge at intervals of 0,1 m and are found to be 32; 41; 52; 76; 101; 110; 95; 74; 65; 41 and 28 cm. Calculate the area of the sheet by using the **mid-ordinate rule**. Give your answer in m2. |
|  |  |
| 2.10. | ***P*** is the load on the piston of a steam engine at a distance ***d*** from the beginning of the stroke. The following readings were taken under test conditions:   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | ***d*** (in cm) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | ***P*** (in Newtons) | 500 | 540 | 530 | 446 | 364 | 286 | 216 | 180 | 0 |   Determine, with the aid of the mid-ordinate rule, the work done in joules, in one forward movement of the piston over the given distance of 8 cm.  (**NB**: 1 joule = 1 Newton) |
| 2.11. | The width of a field varies along its length as shown in the table below. Without drawing the graph, calculate the area of the field using the **mid-ordinate rule.**   |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Length (in ***m***) | 0 | 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 | | Width (in ***m***) | 61 | 69 | 75 | 65 | 60 | 70 | 78 | 82 | 85 | 92 | 99 | |
|  |  |
| 2.12. | The following table gives the corresponding ***x*** and ***y*** values which lie on a curve. Plot these points on the cartesian plane and draw the graph. Calculate the aea between the curve, the ***x***-axis and ***x*** = 5 and ***x*** = 50. Draw nine mid ordinates.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | ***x*** | 5 | 16 | 26 | 35 | 50 | | ***y*** | 13 | 20 | 24 | 27 | 31 | |
|  |  |
| \*2.13. | A tree branch, sawn off at each end, has across sectional area of **Y** cm2 at a distance of **X** cm from one end. The following table gives the corresponding **X** and **Y** values.   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | **X** | 0 | 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | | **Y** | 105 | 106 | 108 | 113 | 118 | 124 | 131 | 136 | 144 |   If the total length of the branch is 160 cm, calculate the volume of the branch in dm3 (cubic decimetres) using the **mid-ordinate rule**. |