MATHEMATICS

MATERIAL FOR GRADE 12

FUNCTIONS

MEMORANDA

COMPILED BY EXPERTS: K. NCUBE & T. MJIKWA

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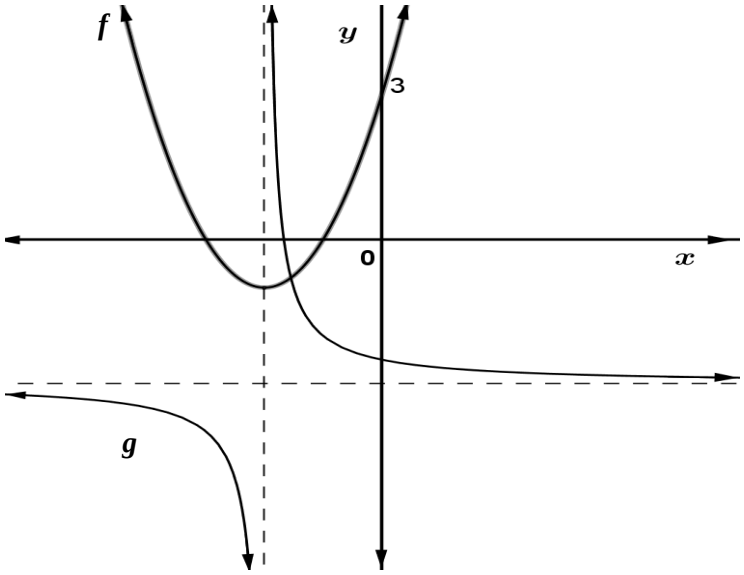
QUESTION 1

1.1 Sketched are the functions $f(x) = (x + p)^2 + q$, and $g(x)$: $(x + 2)(y + 3) = t$ If $g(0) = -\frac{5}{2}$ and g is a rectangular hyperbola with one of its asymptotes an axis of symmetry for f as shown. Answer the following:



1.1.1 Write down the equations of the asymptotes of g. (2)

1.1.2 Determine the values of:

1.1.2.1 t (2)

1.1.2.2 p and q (3)

1.2 Write g in the form $y = \frac{a}{x + p} + q$ (3)

1.3 If $h(x) = x - 1$ is the line of symmetry to g, determine the co-ordinates of the points of intersection h and g. (6)

1.4 If $k = x^2 + 4x + 3$. Determine the values of k if its roots are:

1.4.1 non-real (2)

1.4.2 negative and unequal (2)

1.5 Write down the:

1.5.1 domain of g (2)

1.5.2 range of f (1)

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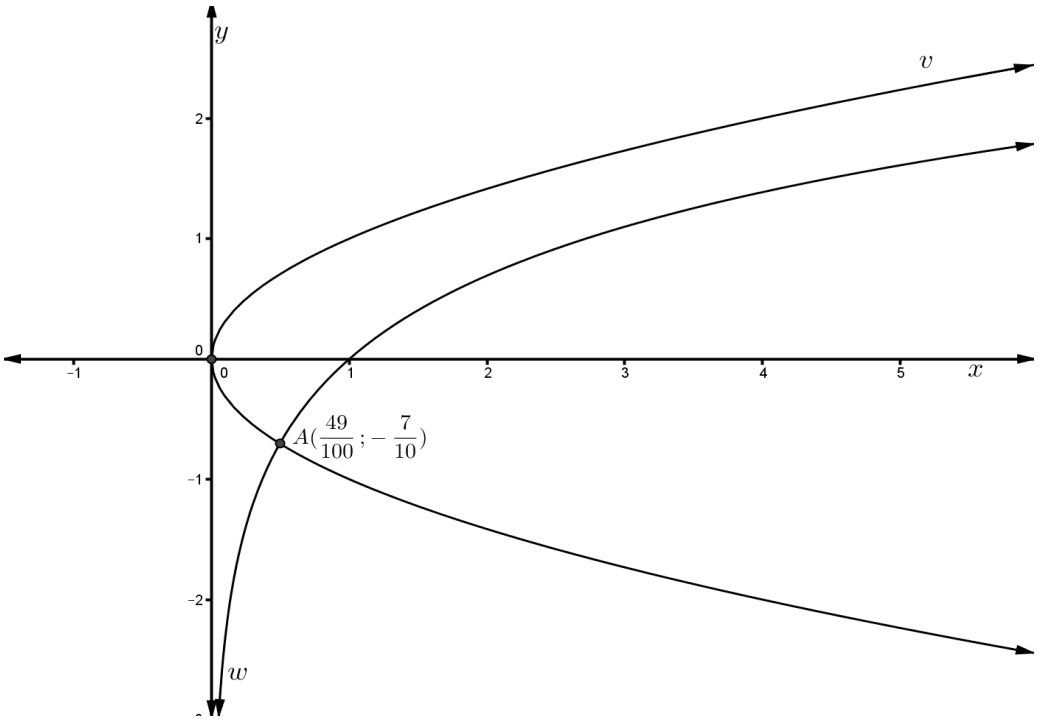
QUESTION 2

2.1 Study the diagram which shows the sketch graphs of $v(x) = \pm\sqrt{x}$ and $w(x) = logx$ then answer the questions that follow:



2.1.1 State whether $v(x)$ is a function or not, motivate your answer. (2)

2.1.2 Write down conditions that will make v a function. (2)

2.1.3 Determine all values of:

2.1.3.1 y for which $w(x) < 0$ (1)

2.1.3.2 x for which $w(x) \leq -\frac{7}{10}$ (2)

2.1.4 If a function $v(x)$ is as determined in 5.1.2 write down the equation(s) of $v^{-1}(x)$. (3)

2.1.5 If $h(x) = w(x) - \sqrt{x}$ where the range of $\sqrt{x}$ is $(0;\infty)$, calculate the range of $h(1)$. (1)

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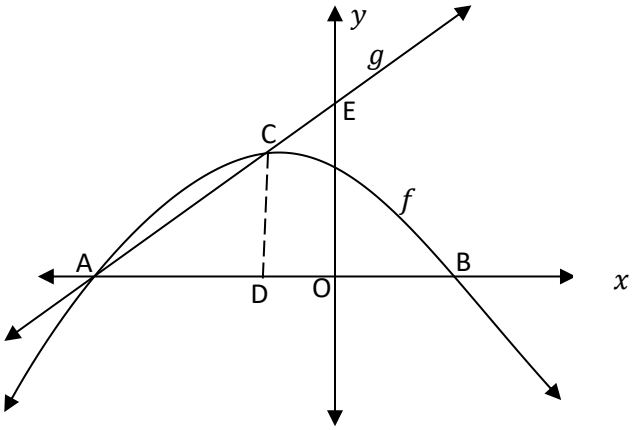
QUESTION 3

The functions $f(x) = -x^2 - 2x + 3$ and $g(x) = mx + c$ are drawn below, with g passing through E, C and A.

A and B are the $x-$ intercepts of , and CD is the axis of symmetry of f.

E is the $y-$ intercept of g



3.1 Determine the coordinates of C, the turning point of the graph of f. (3)

3.2 Determine the coordinates of A and B. (3)

3.3 Write down the coordinates of the $y-$ intercept of the graph of f. (1)

3.4 Calculate the length of CE. (6)

3.5 Determine the equation of $g^{-1}(x)$ in the form y = …. (2)

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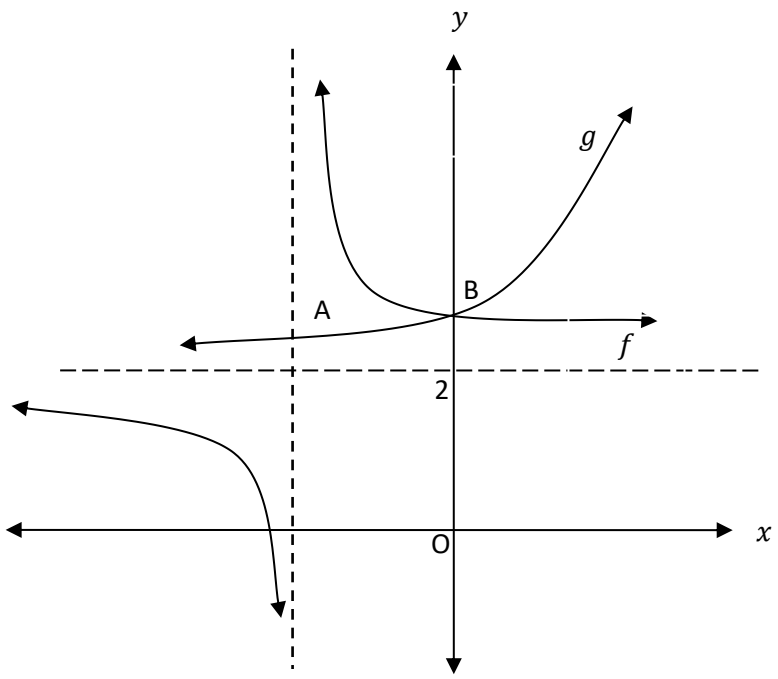
QUESTION 4

Below are graphs of the functions $f(x) = \frac{3}{x-p} + q$ and $g(x) = 2^x + e$.

Both f and g have the same horizontal asymptote and y −intercept B.

A is the point of intersection of g and the vertical asymptote of f.



4.1 Write down the values of e and q. (2)

4.2 Determine the value of p. (3)

4.3 Determine the coordinates of A. (3)

4.4 Determine x such that $f(x) - g(x) \geq 0$ (2)

4.5 If the graph of f is shifted two units to the right, determine the equation of the shifted graph of f. (2)

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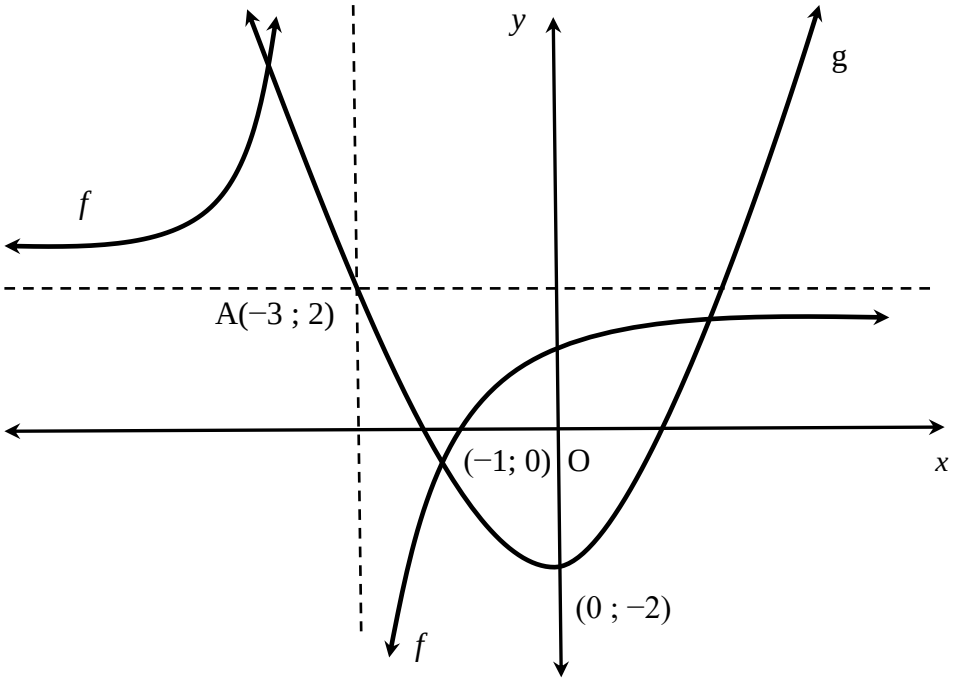
QUESTION 5

The sketch below represents the graphs of: $f(x) = \frac{a}{x+p} + q$ and $g(x) = bx^2 + c$.



The point A (−3; 2) is the point of intersection of the asymptotes of f. The graph of f intersects the x-axis at (−1; 0). The graph of g intersects the y-axis at (0; −2).

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Determine the equation of f. (3)

5.3 Write down the equation of the axes of symmetry of f in the form $y = mx + c$ if $m < 0$. (2)

5.4 Write down the domain of $5f(x - 1)$. (2)

5.5 Write down the equation of k, the reflection of f about the y-axis. Leave your answer in the form $y = \frac{a}{x+p} + q$. (2)

5.6 For which value(s) of x is $f(x).g'(x) \geq 0$? (2)

5.7 Determine the equation of g. (3)

5.8 Determine the equation of $h^{-1}(x)$ if $h(x) = g(x) + 2$. Leave your answer in the form $y = ...$ (3)

5.9 The inverse of h is not a function. Restrict the domain of h such that h^{-1} is a function. Sketch the restricted graph of h and h^{-1} on the same system of axes. (2)

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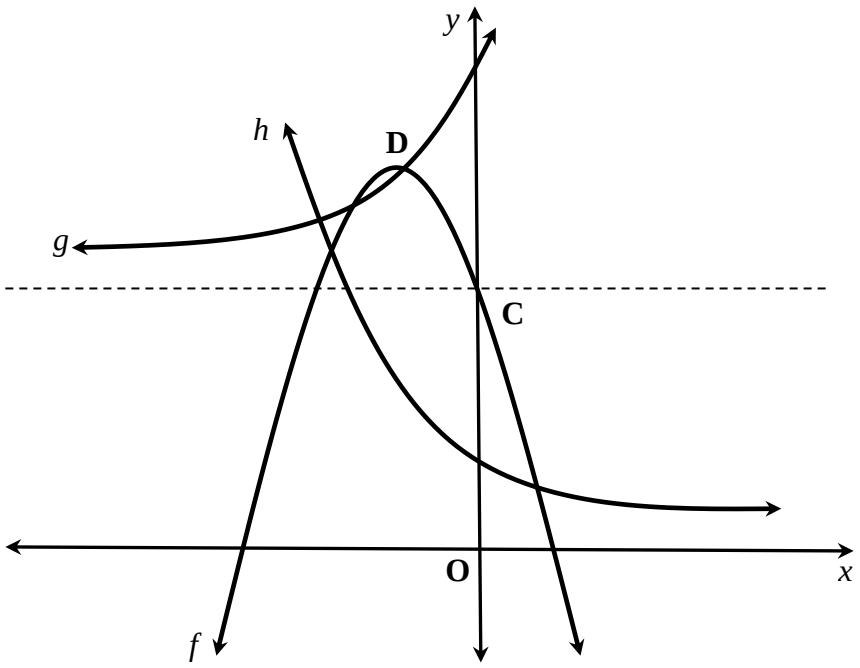
QUESTION 6

The graphs of $h(x) = 3^{-x}$, $f(x) = -(x+1)^2 + 9$ and $g(x) = a.2^x + q$ are represented in the sketch below. D, the turning point of *f*, is also a point of intersection of *g* and *f*. The asymptote of *g* passes through C, the *y*-intercept of *f*.



6.1 Write down the coordinates of C. (2)

6.2 Calculate the values of *a* and *q*. (3)

6.3 Write down the range of *g*. (2)

6.4 Write down the coordinates of D', if D is reflected about the line $y = 8$. (1)

6.5 If $k(x) = (x+2)^2 + 9$, describe the transformation from f to k. (3)

6.6 Write down the equation of $h^{-1}(x)$ in the form $y = ...$ (1)

6.7 Determine the minimum value of $y = \left(\frac{1}{3}\right)^{f(x)-5}$. (2)

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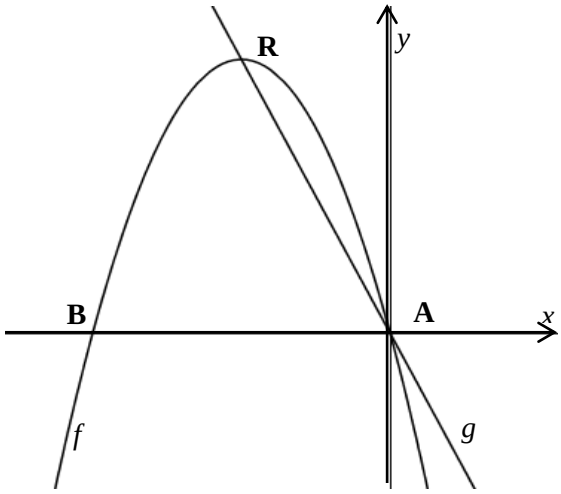
QUESTION 7

Sketched alongside are the graphs of

$f(x) = -(x+2)^2 + 4$ and

$g(x) = ax + q$

R is the turning point of f



7.1 Give the coordinates of R. (2)

7.2 Calculate the length of AB. (2)

7.3 Determine the equation of g. (2)

7.4 For which values of x is $g(x) > f(x)$? (2)

7.5 Write down the equation of the axis of symmetry of h if $h(x) = f(-x)$. (2)

7.6 Give the range of p if $p(x) = -f(x)$. (2)

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QUESTION 8

Given: $h(x) = \frac{2}{x-2} + 1$

8.1 Give the equations of the asymptotes of h (2)

8.2 Determine the x- and y-intercepts of the graph of h. (3)

8.3 Sketch the graph of h using the grid on the DIAGRAM SHEET. (3)

8.4 Give the domain of h. (2)

8.5 Describe the transformation of h to f if :

8.5.1 $f(x) = h(x+3)$ (2)

8.5.1 $f(x) = h(x) - 2$ (2)

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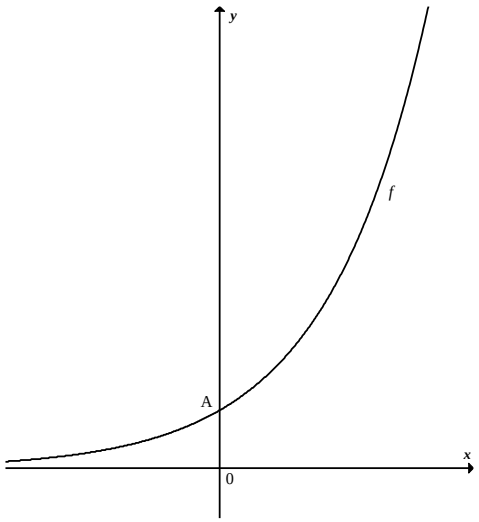
QUESTION 9

9.1 The graph of $f(x) = 3^x$ is sketched alongside.



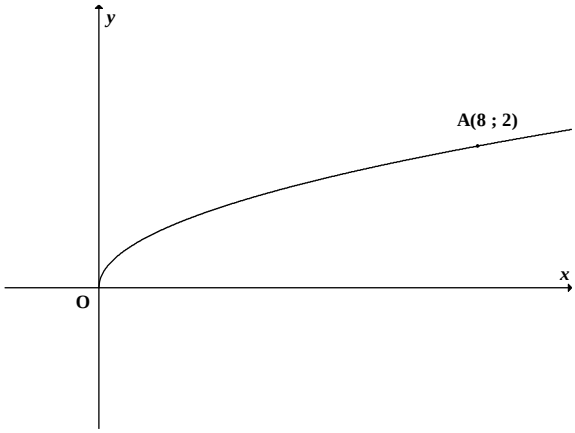
9.1.1 Give the coordinates of A. (1)

9.1.2 Write down the equation of f^{-1} in the form $y = \ldots$ (1)

9.1.3 For which value(s) of x will $f^{-1} \leq 0$? (2)

9.1.4 Write down the equation of the asymptote of $f(x-1)$ (1)

9.2 Sketched is the graph of f, the inverse of a restricted parabola. The point A (8; 2) lies on the graph of f.



9.2.1 Determine the equation of f in the form $y = \ldots$ (2)

9.2.2 Hence, write down the equation of f^{-1} in the form $y = \ldots$ (2)

9.2.3 Give the coordinates of the turning point of $g(x) = f^{-1}(x+3) - 1$. (1)

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QUESTION 10

Given: $f(x) = \frac{3}{x-2} + 1$

10.1 Write down the equations of the asymptotes of f. (2)

10.2 Determine coordinates of B, the *x*-intercept of *f*. (2)

10.3 Write down the domain of *g* if $g(x) = f(x+1)$. (3)

10.4 One of the axes of symmetry of *f* is an increasing function. Write down the equation of this axis of symmetry. (2)

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QUESTION 11

Given $h(x) = 3^{-x}$ and $k^{-1}(x) = 2x^2$ for $x \geq 0$.
A(0,57 ; 0,53) is a point of intersection between *h* and k.

11.1 Write down the equation of *k* in the form *y* = … (2)

11.2 Sketch the graphs of *h* and *k* on the same set of axes, clearly indicating the intercepts with the axes. (5)

11.3 Write down the range of *h*. (1)

11.4 For which values of x is $k(x) \leq h(x)$? (2)

11.5 For which values of t will $k(x) + t = h(x)$ have no real roots? (2)

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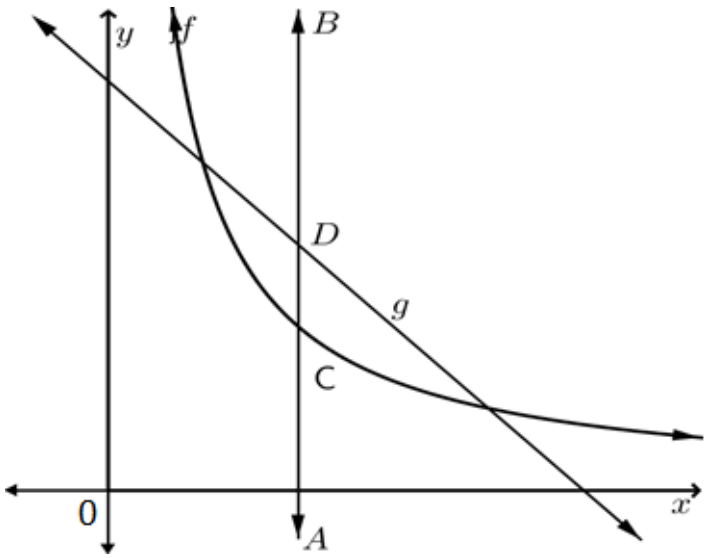
QUESTION 12

The diagram below shows the graphs of $f(x)=\frac{4}{x};\ x>0$ and $g(x)=6-x$. Line BA intersects f and g respectively at points C and D. BA is perpendicular to the *x*-axis.



12.1 Write down the *y*-intercept of *g*. (1)

12.2 Write down the equation of *h* if *h* is a translation of *f* one unit to the right. (1)

12.3 Calculate the values of *x* for which $h(x)=g(x)$. (3)

12.4 Write down the length of CD in terms of *x*. (2)

12.5 Determine the value of x for which CD has a maximum length. (4)

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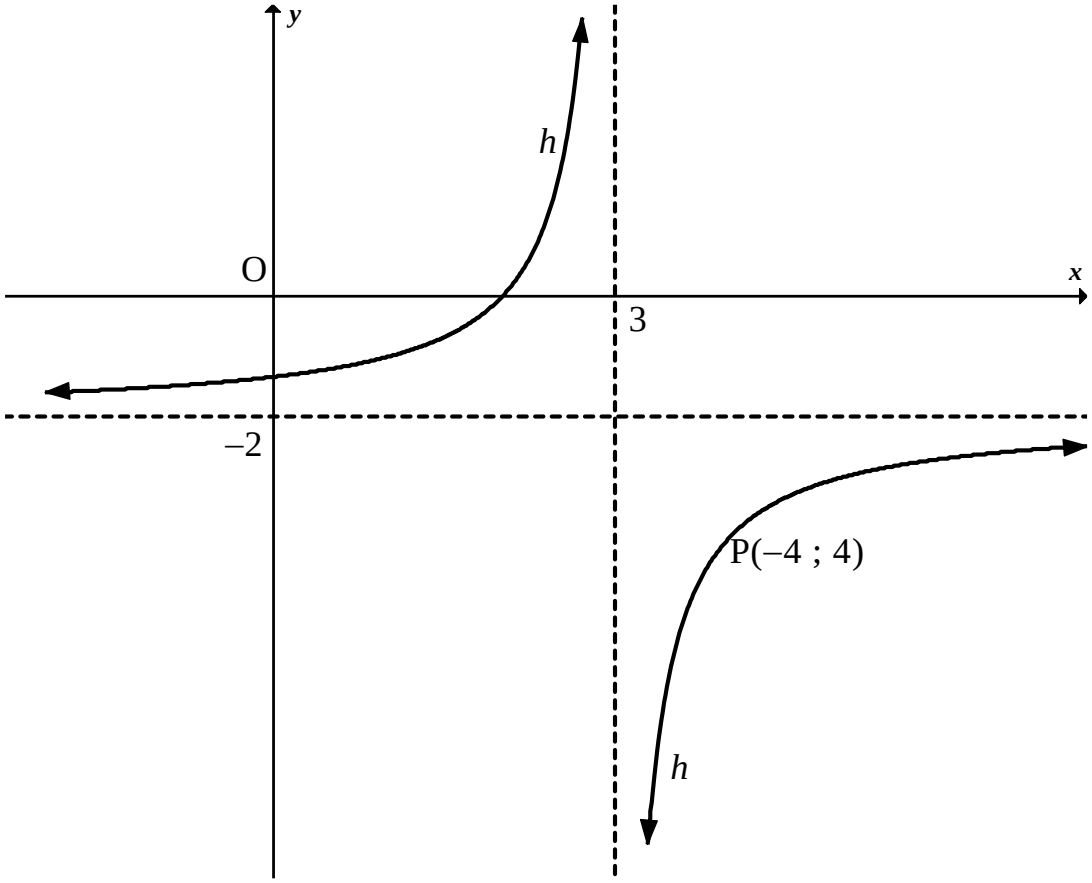
QUESTION 13

The diagram below shows the graph of $h(x) = \frac{a}{x+p} + q$. The lines $x = 3$ and $y = -2$ are asymptotes of h. $P(4;-4)$ is a point on h.



13.1 Write down the values of p and q. (2)

13.2 Calculate the value of a. (2)

13.3 Calculate the coordinates of the y – intercept of h. (2)

13.4 If $g(x) = h(x+2)$, write down the equation of the vertical asymptote of g. (2)

13.5 If the graph of h is symmetrical about the line $y = -x + c$, determine the value of c. (2)

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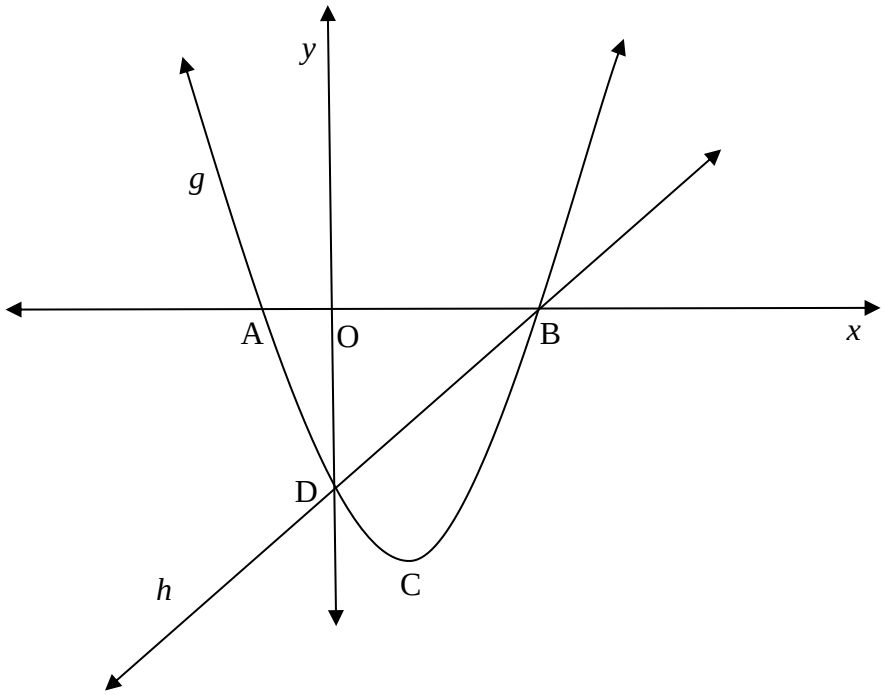
QUESTION 14

The sketch below shows the graphs of $g(x) = x^2 - 3x - 10$ and $h(x) = ax + q$. The graphs intersect at B and D. The graph of g intersects the x – axis at A and B and has a turning point at C. The graph of h intersects the y – axis at D and the x – axis at B.



14.1 Write down the coordinates of D. (1)

14.2 Determine the coordinates of A and B. (4)

14.3 Write down the values of a and q. (2)

14.4 Calculate the coordinates of C, the turning point of g. (3)

14.5 Write down the turning point of t, if $t(x) = g(-x) + 3$. (2)

14.6 For which values of x will $g'(x).h'(x) \geq 0$? (2)

[14]

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QUESTION 15

Given $p(x) = 3^x$.

15.1 Write down the equation of p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

15.2 Sketch in your ANSWER BOOK the graphs of p and p^{-1} on the same system of axes. Show clearly all the intercepts with the axes and at least one other point on each graph. (4)

15.3 Determine the values of x for which $p^{-1}(x) \leq 3$ (4)

[10]

QUESTION 16

Two functions are defined by $f(x) = (x - 4)(x + 2)$ and $g(x) = 2x - 12$.

A is a point on f such that g is a tangent to f at A.

16.1 Write down the gradient of g. (1)

16.2 Calculate the coordinates of A. (5)

16.3 Determine the equation of the graph h which is the reflection of f about the $y-$ axis. (2)

16.4 Determine value(s) of x for which $f(x).g(x) < 0$, given that $x > 0$. (2)

16.5 Determine g^{-1}, the inverse of g, in the form $y = \ldots$. (2)

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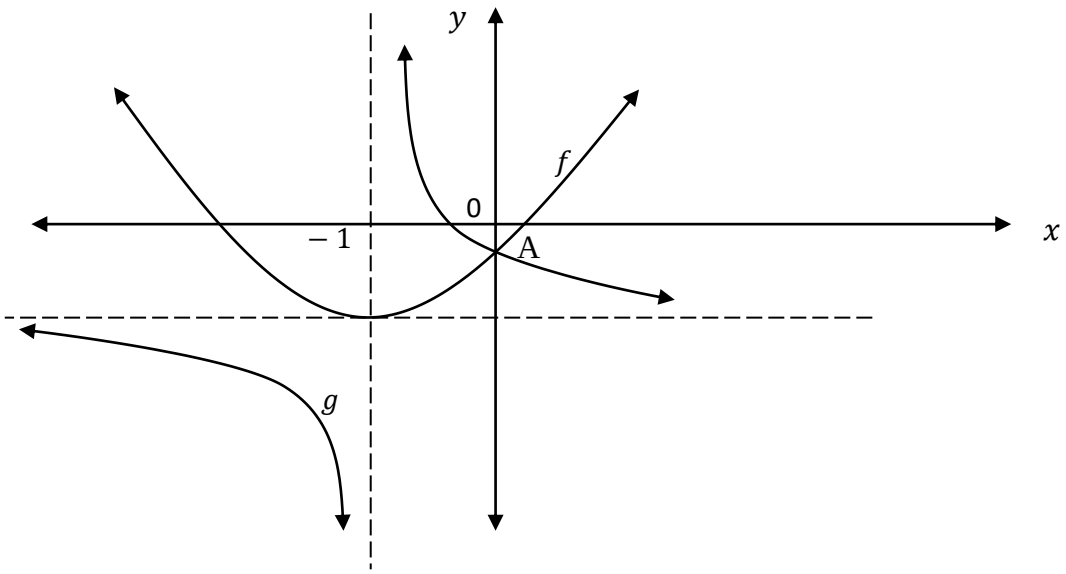
QUESTION 17

The graphs of $f(x) = x^2 + 2x - 3$ and $g(x) = \frac{a}{x+p} + q$ are drawn below. A is the $y-$ intercept of both f and g. The horizontal asymptote of g is also a tangent to f at the turning point of f. The equation of the vertical asymptote of g is $x = -1$.



17.1 Determine the equations of the asymptotes of g. (3)

17.2 Write down the coordinates of A. (1)

17.3 Determine the equation of g. (4)

17.4 Determine points of intersection of g with its axis of symmetry that has a positive gradient. (4)

17.5 Write down the range of $-f(x)$. (2)

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QUESTION 18

Given: $f(x) = 3^{-x+1} - 3$

18.1 Draw the graph of f showing all asymptotes and intercepts with the axes. (3)

18.2 Calculate the $x-$ value when $y = 5$. (3)

18.3 If $h(x) = 3^x$; explain what transformations f has undergone to become h. (3)

[9]

QUESTION 19

Given: $f(x) = \frac{-3}{x-2} + 1$

19.1 Calculate the coordinates of the y-intercept of f. (2)

19.2 Calculate the coordinates of the x-intercept of f. (2)

19.3 Sketch the graph of f in your ANSWER BOOK, clearly showing the asymptotes and the intercepts with the axes. (3)

19.4 Write down the range of f. (2)

19.5 Another function h, is formed by translating f 3 units to the right and 4 units down. Write down the equation of h. (2)

19.6 For which value(s) of x is $h(x) \leq -4$? (3)

19.7 Determine the equations of the asymptotes of $k(x) = \frac{3x-5}{x-1}$. (3)

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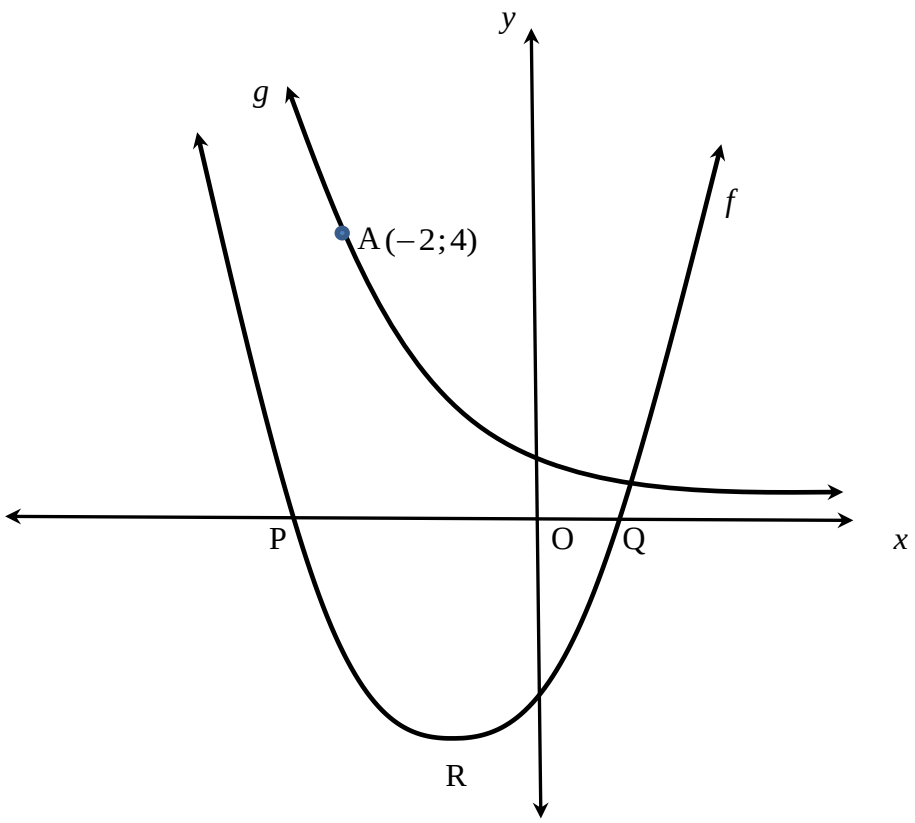
QUESTION 20

The graphs of $f(x) = 2(x+1)^2 - 8$ and $g(x) = \left(\frac{1}{2}\right)^x$ are represented in the sketch below. P and Q are the x-intercepts of f and R is the turning point of f. The point $A(-2; 4)$ is a point on the graph of g.



20.1 Write down the equation of the axis of symmetry of f. (1)

20.2 Write down the coordinates of the turning point of f. (1)

20.3 Determine the length of PQ. (4)

20.4 Write down the equation of k, if k is the reflection of f in the y-axis. Give your answer in the form $y = ax^2 + bx + c$. (3)

20.5 Write down the equation of g^{-1}, the inverse of g, in the form $y = \ldots$ (1)

20.6 Sketch the graph of g^{-1} in your ANSWER BOOK, clearly showing the intercept with the axis as well as ONE other point on the graph of g^{-1}. (3)

20.7 For which value(s) of x will:

20.7.1 $g^{-1}(x) \geq -2$ (2)

20.7.2 $x.f(x) < 0$ (4)

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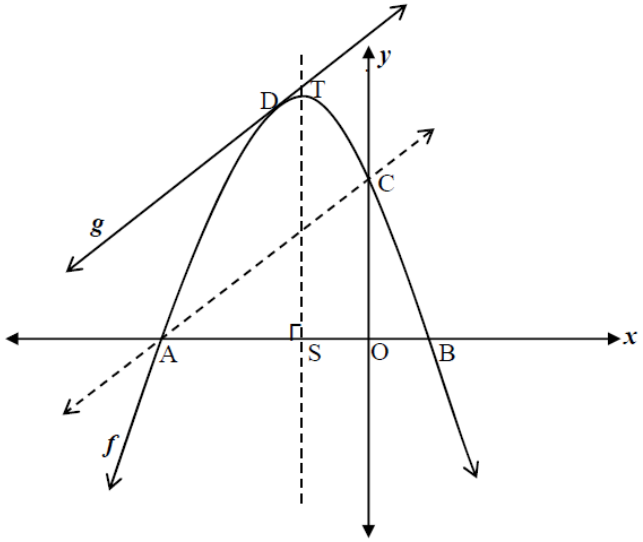
QUESTION 21

The diagram shows the graphs of $f(x) = -2x^2 - 4x + 6$ and $g(x) = mx + c$. A, B and C are the intercepts of f with the axes. T is the turning point of the graph of f. The graph of g is a straight line parallel to AC, and is a tangent to the graph of f at D.



21.1 Determine the lengths of OC and AB. (5)

21.2 Determine the equation of the axis of symmetry of the graph of f. (2)

21.3 Show that the length of ST = 8 units. (2)

21.4 Calculate the gradient of AC. (2)

21.5 Hence, or otherwise, calculate the coordinates of D. (5)

21.6 For which value(s) of a will $f(a + t) = f(a - t)$ for all values of t? (2)

[18]

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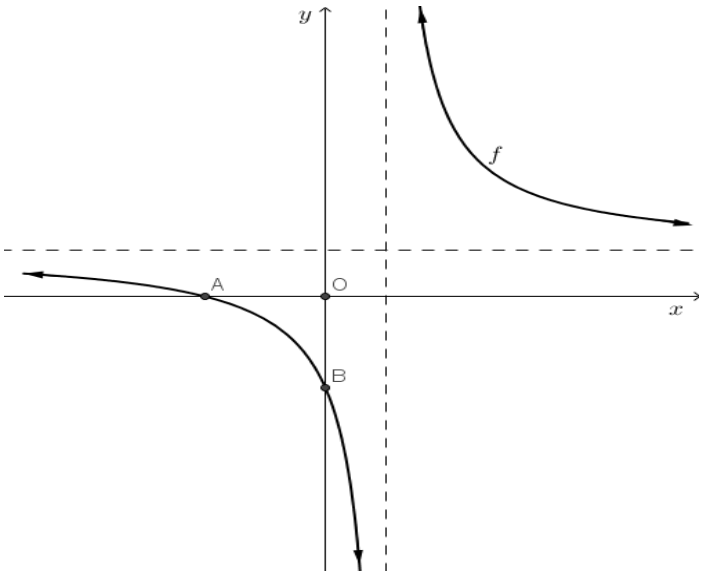
QUESTION 22

The sketch of $f(x) = \frac{2+x}{x-1}$ is drawn below.



22.1 Write down the equation of the vertical asymtote of f.

22.2 Determine the coordinates of A, the x-intercept of the hyperbola.

22.3 Calculate the area of ΔAOB.

22.4 Show that $f(x)$ can be rewritten as $f(x) = \frac{3}{x-1} + 1$

22.5 The graph of f is shifted such that point A lies on the origin. What are the coordinates of the point of intersection of the asymptotes of the new graph?

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QUESTION 23

23.1 Given: $f(x) = 2.2^x - 1$

23.1.1 Write down the range of f.

23.1.2 $g(x) = f(x-1) + 1$. Write down the equation of g^{-1}, the inverse of g in the form y =...

23.2 Given: $h(x) = -\sqrt{\frac{x}{3}}$; $x \geq 0$

23.2.1 If $k(x)$ is the inverse of h, give the equation of $k(x)$

23.2.2 Give the coordinates of the point of intersection of $h(x)$ and $k(x)$

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