ANSWERS: GR 11 NATURE OF THE ROOTS

Exercise 7 (page 45)

1.1 $\Delta = b^2 - 4ac$
$= 25 - 24$
$= 1$
$\therefore$ Roots are real $(\Delta > 0)$
rational (Δ is a perfect square)
unequal $(\Delta \neq 0)$

1.2 $2x^2 + 3x + 7 = 0$
$\therefore \Delta = 9 - 56$
$= -47$
$\therefore$ Roots are non-real $(\Delta < 0)$

1.3 $7x^2 - 6x = 0$
$\therefore \Delta = 36 - 0$
$= 36$
$\therefore$ Roots are real $(\Delta > 0)$
rational (Δ is a perfect square)
unequal $(\Delta \neq 0)$

1.4 $3x(x - 8) = 1$
$\therefore 3x^2 - 24x - 1 = 0$
$\therefore \Delta = 576 + 12$
$= 588$
$\therefore$ Roots are real $(\Delta > 0)$
irrational (Δ is not a perfect square)
unequal $(\Delta \neq 0)$

1.5 $x^2 - 3x(4x - 3) = (x - 5)^2$
$\therefore x^2 - 12x^2 + 9x = x^2 - 10x + 25$
$\therefore -12x^2 + 19x - 25 = 0$
$\Delta = 361 - 1\,200$
$= -839$
$\therefore$ Roots are non-real $(\Delta < 0)$

2.1 $2x^2 + 3x - 2 = k$
$\therefore 2x^2 + 3x - 2 - k = 0$
$\Delta = (3)^2 - 4(2)(-2 - k)$
$= 9 + 16 + 8k$
$= 25 + 8k$
$\Delta = 0$ (equal roots)
$\therefore 25 + 8k = 0$
$\therefore k = -3\frac{1}{8}$

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2.2 $x^2 - 3x + 4k^2 = 0$

$\Delta = (-3)^2 - 4(1)(4k^2)$

$= 9 - 16k^2$

$\Delta = 0$ (equal roots)

$\therefore 9 - 16k^2 = 0$

$(3 + 4k)(3 - 4k) = 0$

$\therefore k = -\frac{3}{4}$ or $k = \frac{3}{4}$

3. $x^2 - 2x = 4 - k$

$x^2 - 2x - 4 + k = 0$

$\therefore \Delta = (-2)^2 - 4(1)(-4 + k)$

$= 4 + 16 - 4k$

$= 20 - 4k$

$\Delta \geq 0$ (roots are real)

$\therefore 20 - 4k \geq 0$

$-4k \geq -20$

$\therefore k \leq 5$

4. $(a - 1)x^2 + 2ax - x + 2 = 0$

$\therefore (a - 1)x^2 + (2a - 1)x + 2 = 0$

$\Delta = (2a - 1)^2 - 4(a - 1)(2)$

$= 4a^2 - 4a + 1 - 8a + 8$

$= 4a^2 - 12a + 9$

$= (2a - 3)^2$

$\therefore$ Roots are real ($\Delta \geq 0$)

rational (Δ a perfect square)

5. $x^2 + kx + 1 = 0$

$\Delta = k^2 - 4$

$\Delta = 0$ (roots are equal)

$\therefore k^2 - 4 = 0$

$\therefore (k + 2)(k - 2) = 0$

$\therefore k = -2$ or $k = 2$

$x^2 + kx + 6 = 0$

$\therefore x^2 \pm 2x + 6 = 0$

$\Delta = (\pm 2)^2 - 4(1)(6)$

$= 4 - 24$

$= -20$

$\therefore$ Roots are non-real

7. $(x - m)(x - n) = a^2$

$x^2 - mx - nx + mn - a^2 = 0$

$x^2 + (-m - n)x + (mn - a^2) = 0$

$\Delta = (-m - n)^2 - 4(mn - a^2)$

$= m^2 + 2mn + n^2 - 4mn + 4a^2$

$= m^2 - 2mn + n^2 + 4a^2$

$= (m - n)^2 + 4a^2$

$(m - n)^2 \geq 0$ and $4a^2 \geq 0$

$\therefore \Delta \geq 0$

$\therefore$ Roots are real

8. $px^2 + (p + 1)x = -1$

$px^2 + (p + 1)x + 1 = 0$

$\Delta = (p + 1)^2 - 4p$

$= p^2 + 2p + 1 - 4p$

$= p^2 - 2p + 1$

$= (p - 1)^2$

$\Delta = 0$ (roots equal)

$\therefore (p - 1)^2 = 0$

$\therefore p = 1$

9. $(c + d)x^2 - (c + 2d)x - 2c = 0$

$(c + d)x^2 + (-c - 2d)x - 2c = 0$

$\Delta = (-c - 2d)^2 - 4(c + d)(-2c)$

$= c^2 + 4cd + 4d^2 + 8c^2 + 8cd$

$= 9c^2 + 12cd + 4d^2$

$= (3c + 2d)^2$

$\therefore$ Roots are real ($\Delta \geq 0$)

rational (Δ a perfect square)

Exercise 8 (page 47)

1. $ax^2 + bx = bx^2 + a$

$(a - b)x^2 + bx - a = 0$

$\Delta = b^2 + 4a(a - b)$

$= b^2 + 4a^2 - 4ab$

$= (b - 2a)^2$

Roots are real ($\Delta \geq 0$)

rational (Δ is a perfect square)

equal if $b = 2a$ ($\Delta = 0$)

unequal if $b \neq 2a$ ($\Delta \neq 0$)

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6. $kx^2 - 2x - kx = 4$

$\therefore kx^2 + (-2 - k)x - 4 = 0$

$\Delta = (-2 - k)^2 - 4(k)(-4)$

$= 4 + 4k + k^2 + 16k$

$= k^2 + 20k + 4$

Let $k = 1$:

$\Delta = (1)2 + 20(1) + 4 = 25$

$\therefore$ Roots are rational (Δ is a perfect square)

$\therefore$ smallest value of $k = 1$

2.1 $ax^2 + bx + c = 0$

$\Delta = b^2 - 4ac$

$= (+)^2 - 4(-)(0)$

$= (+)^2$

= perfect square

$\therefore$ Roots are real, rational and unequal $(\Delta > 0)$

2.2 $b^2 = 4ac + 8$

$b^2 - 4ac = 8$

$\therefore \Delta = 8$

$\therefore$ Roots are real, irrational and unequal

2.3 $b^2 = ac$:

$\Delta = b^2 - 4ac$

$= b^2 - 4(b^2)$

$= -3b^2$

Roots are non-real $(b \neq 0)$

3. $5x^2 - kx = 2$

$5x^2 - kx - 2 = 0$

$\Delta = k^2 + 40$

For $k = 1$: $\Delta = 41$

For $k = 2$: $\Delta = 44$

For $k = 3$: $\Delta = 49$

$\therefore$ For $k = 3$ roots are rational

(Δ is a perfect square)

4. $x^2 - kx + k + x = 3$

$x^2 - kx + x + k - 3 = 0$

$x^2 - (k - 1)x + k - 3 = 0$ (standard form)

$x^2 + (-k + 1)x + k - 3 = 0$

$\Delta = b^2 - 4ac$

$= (-k + 1)^2 - 4(k - 3)$

$= k^2 - 2k + 1 - 4k + 12$

$= k^2 - 6k + 13$

$= k^2 - 6k + 9 - 9 + 13$ (completing the square)

$= (k - 3)^2 + 4$

$(k - 3)^2 \geq 0$

$\therefore (k - 3)^2 + 4 \geq 4$

$\therefore \Delta > 0$

$\therefore$ Roots are real

5. $3x^2 - 5x + \frac{k}{4} = 0$

6. $\frac{3x^2 + x - 2}{x^2 - 3x + 1} = k$

$3x^2 + x - 2 = k(x^2 - 3x + 1)$

$3x^2 + x - 2 = kx^2 - 3kx + k$

$3x^2 - kx^2 + x + 3kx - 2 - k = 0$

$(3 - k)x^2 + (1 + 3k)x + (-2 - k) = 0$

$\therefore \Delta = (1 + 3k)^2 - 4(3 - k)(-2 - k)$

$= 1 + 6k + 9k^2 - 4(-6 - k + k^2)$

$= 1 + 6k + 9k^2 + 24 + 4k - 4k^2$

$= 5k^2 + 10k + 25$

$= 5(k^2 + 2k + 5)$

$= 5(k^2 + 2k + 1 - 1 + 5)$

$= 5((k + 1)^2 + 4)$

$= 5(k + 1)^2 + 20$

$\therefore \Delta \geq 20$

$\therefore$ Roots real $(\Delta > 0)$

7. $k = \frac{x^2 + x + 1}{x^2 - x + 1}$

$k(x^2 - x + 1) = x^2 + x + 1$

$kx^2 - kx + k = x^2 + x + 1$

$kx^2 - x^2 - kx - x + k - 1 = 0$

$\therefore (k - 1)x^2 + (-k - 1)x + k - 1 = 0$

$\Delta = (-k - 1)^2 - 4(k - 1)(k - 1)$

$= k^2 + 2k + 1 - 4(k^2 - 2k + 1)$

$= k^2 + 2k + 1 - 4k^2 + 8k - 4$

$= -3k^2 + 10k - 3$

$= -(3k^2 - 10k + 3)$

$= -(3k - 1)(k - 3)$

		$\frac{1}{3}$		3	
$(3k - 1)$	$-$		$+$		$+$
$(k - 3)$	$-$		$-$		$+$
$-(3k - 1)(k - 3)$	$-$		$+$		$-$

5.1 $\Delta = b^2 - 4ac$

$= 25 - 4(3)\left(\frac{k}{4}\right)$

$= 25 - 3k$

Real roots $\therefore \Delta \geq 0$

$\therefore 25 - 3k \geq 0$

$k \leq 8\frac{1}{3}$

5.2 $\Delta = 25 - 3k$

$k = 8: \Delta = 1$ $\quad (k \leq 8\frac{1}{3})$

$k = 7: \Delta = 4$

$k = 3: \Delta = 16$

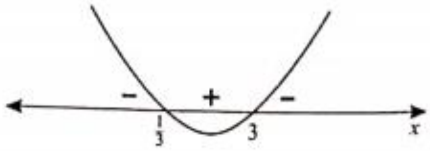
$\therefore$ Roots are rational for $k = 3, 7$ and 8

For real roots $\Delta \geq 0$

$\therefore -(3k-1)(k-3) \geq 0$

$(3k-1)(k-3) \leq 0$

$\therefore \frac{1}{3} \leq k \leq 3$



8.1

$3x^2 - p(2x - p) = 2(2x - 1)$

$3x^2 - 2px + p^2 = 4x - 2$

$3x^2 + (-2p - 4)x + p^2 + 2 = 0$

$\Delta = (-2p - 4)^2 - 12(p^2 + 2)$

$= 4p^2 + 16p + 16 - 12p^2 - 24$

$= -8p^2 + 16p - 8$

$= -8(p - 1)^2$

If $p \neq 1$ then $\Delta < 0$

$\therefore$ Roots are non-real

8.2 If $p = 1$ then $\Delta = 0$

$\therefore$ Roots are real, rational and equal.

9.

$x^2 - 2x + 2 = kx - \frac{k^2}{2}$

$x^2 - (2 + k)x + 2 + \frac{k^2}{2} = 0$

$\Delta = b^2 - 4ac$

$= (2 + k)^2 - 4\left(2 + \frac{k^2}{2}\right)$

$= 4 + 4k + k^2 - 8 - 2k^2$

$= -k^2 + 4k - 4$

$= -(k^2 - 4k + 4)$

$= -(k - 2)^2$

$\therefore \Delta < 0$ for all values of k, $k \neq 2$

$(p - 3)^2 \geq 0$

$\therefore (p - 3)^2 + 4 \geq 0$

$\therefore$ Roots are real

12.

$k = \frac{x^2 + 4x - 24}{x - 6}$

$kx - 6k = x^2 + 4x - 24$

$x^2 + 4x - kx - 24 + 6k = 0$

$x^2 + x(4 - k) - 24 + 6k = 0$

$\Delta = b^2 - 4ac$

$= (4 - k)^2 - 4(-24 + 6k)$

$= 16 - 8k + k^2 + 96 - 24k$

$= k^2 - 32k + 112$

$= (k - 28)(k - 4)$

Given: $k \in \mathbb{R}$ $\therefore$ equation has real roots

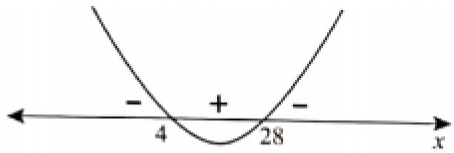
$\therefore \Delta \geq 0$

$\therefore (k - 28)(k - 4) \geq 0$

$\therefore k \leq 4$ or $k \geq 28$

If $k = 2$, then $\Delta = 0$

$\therefore$ Roots are real



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10.

$$\frac{x-3}{(x-1)^2} = k$$

$$kx^2 - 2kx + k = x - 3$$

$$kx^2 - x(2k+1) + k + 3 = 0$$

$$kx^2 + x(-2k-1) + k + 3 = 0$$

$$\therefore \Delta = (-2k-1)^2 - 4(k)(k+3)$$

$$= 4k^2 + 4k + 1 - 4k^2 - 12k$$

$$= -8k + 1$$

Roots real $\therefore \Delta \geq 0$

$$\therefore -8k + 1 \geq 0$$

$$-8k \geq -1$$

$$\therefore k \leq \frac{1}{8}$$

11.

$$(x-3)(x+1) = x(p-3) - p$$

$$x^2 - 2x - 3 = px - 3x - p$$

$$x^2 - x - px - 3 + p = 0$$

$$x^2 + x(1-p) - 3 + p = 0$$

$$\Delta = b^2 - 4ac$$

$$= (1-p)^2 - 4(-3+p)$$

$$= 1 - 2p + p^2 + 12 - 4p$$

$$= 13 - 6p + p^2$$

$$= p^2 - 6p + 9 - 9 + 13$$ (completing the square)

$$= (p-3)^2 + 4$$

13.

$$(x+2)(x+k) = 2 + 3x$$

$$x^2 + 2x + kx + 2k = 2 + 3x$$

$$x^2 - (1-k)x + 2k - 2 = 0$$

$$x^2 + (-1+k)x + 2k - 2 = 0$$

$$\Delta = b^2 - 4ac$$

$$= (-1+k)^2 - 4(2k-2)$$

$$= 1 - 2k + k^2 - 8k + 8$$

$$= k^2 - 10k + 9$$

Roots non-real $\therefore \Delta < 0$

$$\therefore k^2 - 10k + 9 < 0$$

$$(k-9)(k-1) < 0$$

$$\therefore 1 < k < 9$$