GR 11 REVISION: NATURE OF THE ROOTS

Exercise 7

1. Determine the nature of the roots of:

   1.1 $2x^2 - 5x + 3 = 0$ 1.2 $2x^2 + 3x = -7$

   1.3 $7x^2 - 6x = 0$ 1.4 $3x(x - 8) = 1$

   1.5 $x^2 - 3x(4x - 3) = (x - 5)^2$

2. For which values of k will the following equations have equal roots?

   2.1 $2x^2 + 3x - 2 = k$ 2.2 $x^2 - 3x + 4k^2 = 0$

3. Determine the value(s) of k if the roots of $x^2 - 2x = 4 - k$ are real.

4. Prove that the roots of $(a - 1)x^2 + 2ax - x + 2 = 0$ are real and rational.

5. If the equation $x^2 + k + 1 = 0$ has equal roots, determine the nature of the roots of $x^2 + kx + 1 = 0$.

6. Determine the smallest value of k for which the roots of $kx^2 - 2x - kx = 4$ will be rational if k is a positive integer.

7. Prove that the roots of the equations $(x - m)(x - n) = a^2$ are real.

8. Calculate the value(s) of p for which $px^2 + (p + 1)x = -1$ has equal roots.

9. Prove that if c and d are rational, the roots of $(c + d)x^2 - (c + 2d)x - 2c = 0$ will be rational.

Exercise 8: Complex procedures

1. Describe the nature of the roots of $ax^2 + bx = bx^2 + a$.

2. Determine the nature of the roots of $ax^2 + bx + c = 0$ if:

   2.1 $a < 0, b > 0, c = 0$ 2.2 $b^2 = 4(ac + 2)$ 2.3 $b^2 = ac, a \neq 0$

3. If k is a positive integer, find the lowest value of k so that the equation $5x^2 - kx = 2$, has real, rational roots.

4. Show that the roots of $x^2 - kx + k + x = 3$ are real for all values of k.

5. Given that $3x^2 - 5x = \frac{-k}{4}$, calculate k if:

   5.1 the roots are real

   5.2 the roots are rational and k is a positive integer.

6. Prove that the equation $\frac{3x^2 + x - 2}{x^2 - 3x + 1} = k$ has real roots for all real values of k.

7. Given: $k = \frac{x^2 + x + 1}{x^2 - x + 1}$. For what values of k will the equation have equal roots?

8. **8.1** Show that the equation $3x^2 - p(2x - p) = 2(2x - 1)$ has non-real roots for all real values of p, except if $p = 1$.

    8.2 What is the nature of the roots if $p = 1$?

9. Prove (with reasons) that the roots of $x^2 - 2x = 2 = kx - \frac{k^2}{2}$ can only be real for one real value of k.

10. For which values of k will $\frac{x-3}{(x-1)^2} = k$ have real roots?

11. Prove that the roots of $(x - 3)(x + 1) = x(p - 3) - p$ are real for all values of p.

12. If $k \in \mathbb{R}$, $x \neq 6$, show that $k = \frac{x^2 + 4x - 24}{x - 6}$ can be any real value except values between 4 and 28.

13. The roots of the equation $(x + 2)(x + k) = 2 + 3x$ are non-real. Determine the possible values of k.