

Using identities, find the value of

(i) 101^2

(ii) 98^2

(iii) $(0.98)^2$

(iv) 11×9

(v) $190 \times 190 - 10 \times 10$

Solution:

$$\begin{aligned} \text{(i) } 101^2 &= (100 + 1)^2 \\ &= 100^2 + 2 \times 100 \times 1 + 1^2 \\ &= 10000 + 200 + 1 = 10201 \end{aligned}$$

$$\begin{aligned} \text{(ii) } 98^2 &= (100 - 2)^2 \\ &= 100^2 - 2 \times 100 \times 2 + 2^2 \\ &= 10000 - 400 + 4 = 9604 \end{aligned}$$

$$\begin{aligned} \text{(iii) } 0.98^2 &= (1 - 0.02)^2 \\ &= 1^2 - 2 \times 1 \times 0.02 + (0.02)^2 \\ &= 1 - 0.04 + 0.0004 \\ &= 0.9604 \end{aligned}$$

$$\begin{aligned} \text{(iv) } 11 \times 9 &= (10+1)(10-1) \\ &= 10^2 - 1^2 \\ &= 99 \end{aligned}$$

$$\begin{aligned} \text{(v) } 190 \times 190 - 10 \times 10 &= 190^2 - 10^2 \\ &= (190 + 10)(190 - 10) \\ &= 200 \times 180 \\ &= 36000 \end{aligned}$$

Factorize

(i) $9x^2 - y^2$

(ii) $(3 - x)^2 - 36x^2$

(iii) $(2x - 3y)^2 - (3y + 4y)^2$

(iv) $16x^4 - y^4$

Solution:

$$\begin{aligned}(i) \quad 9x^2 - y^2 &= (3x)^2 - (y)^2 \\ &= (3x + y)(3x - y)\end{aligned}$$

$$\begin{aligned}(ii) \quad (3 - x)^2 - 36x^2 &= (3 - x)^2 - (6x)^2 \\ &= (3 - x + 6x)(3 - x - 6x) \\ &= (3 + 5x)(3 - 7x)\end{aligned}$$

$$\begin{aligned}(iii) \quad (2x - 3y)^2 - (3x + 4y)^2 &= (2x - 3y + 3x + 4y)(2x - 3y - 3x - 4y) \\ &= (5x + y)(-x - 7y) \\ &= -(5x + y)(x + 7y)\end{aligned}$$

$$\begin{aligned}(iv) \quad 16x^4 - y^4 &= (4x^2)^2 - (y^2)^2 \\ &= (4x^2 + y^2)(4x^2 - y^2) \\ &= (4x^2 + y^2)(2x + y)(2x - y)\end{aligned}$$

Factorise

(i) $2x^2(x + y) - 3(x + y)$

(ii) $5xy(5x + y) - 5y(5x + y)$

(iii) $x(x^2 + y^2 - z^2) + y(x^2 + y^2 - z^2) + z(x^2 + y^2 - z^2)$

(iv) $ab(a^2 + b^2 - c^2) + bc(a^2 + b^2 - c^2) + ca(a^2 + b^2 - c^2)$

Solution:

(i) $2x^2(x + y) - 3(x + y)$

$$= (x + y) (2x^2 - 3)$$

(ii) $5xy(5x + y) - 5y(5x + y)$

$$= 5y(5x + y) (x - 1)$$

(iii) $x(x^2 + y^2 - z^2) + y(x^2 + y^2 - z^2) + z(x^2 + y^2 - z^2)$

$$= (x^2 + y^2 - z^2) (x + y + z)$$

(iv) $ab(a^2 + b^2 - c^2) + bc(a^2 + b^2 - c^2) + ca(a^2 + b^2 - c^2)$

$$= (a^2 + b^2 - c^2) (ab + bc + ca)$$

Find the remainder when $y^3 + y^2 - 2y + 5$ is divided by $y - 5$.

Solution:

$$\begin{array}{r} y^2 + 6y + 28 \\ y - 5 \overline{) y^3 + y^2 - 2y + 5} \\ \underline{-(y^3 - 5y^2)} \\ 6y^2 - 2y + 5 \\ \underline{-(6y^2 - 30y)} \\ 28y + 5 \\ \underline{-(28y - 140)} \\ 145 \end{array}$$

Remainder = 145

Again, we should evaluate $p(5)$

$$\text{Let } p(y) = y^3 + y^2 - 2y + 5$$

$$\begin{aligned} \therefore p(5) &= 5^3 + 5^2 - 2 \times 5 + 5 \\ &= 125 + 25 - 10 + 5 \\ &= 145 \end{aligned}$$

Thus, we find that $p(5)$ is the remainder when

$p(y)$ is divided by $y - 5$.