Factorise the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$ (**Hint**: Expand $(l + m)^2$ first)

(viii) $a^4 + 2a^2b^2 + b^4$

Answer:

(i) $a^2 + 8a + 16 = (a)^2 + 2 \times a \times 4 + (4)^2$

$= (a + 4)^2$ $[(x + y)^2 = x^2 + 2xy + y^2]$

(ii) $p^2 - 10p + 25 = (p)^2 - 2 \times p \times 5 + (5)^2$

$= (p - 5)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(iii) $25m^2 + 30m + 9 = (5m)^2 + 2 \times 5m \times 3 + (3)^2$

$= (5m + 3)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

(iv) $49y^2 + 84yz + 36z^2 = (7y)^2 + 2 \times (7y) \times (6z) + (6z)^2$

$= (7y + 6z)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

(v) $4x^2 - 8x + 4 = (2x)^2 - 2\,(2x)\,(2) + (2)^2$

$= (2x - 2)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= [(2)\,(x - 1)]^2 = 4(x - 1)^2$

(vi) $121b^2 - 88bc + 16c^2 = (11b)^2 - 2\,(11b)\,(4c) + (4c)^2$

$= (11b - 4c)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(vii) $(l + m)^2 - 4lm = l^2 + 2lm + m^2 - 4lm$

$= l^2 - 2lm + m^2$

$= (l - m)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(viii) $a^4 + 2a^2b^2 + b^4 = (a^2)^2 + 2\,(a^2)\,(b^2) + (b^2)^2$

$= (a^2 + b^2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.

Answer:

$\frac{-2}{5}$ and $\frac{1}{2}$ can be represented as $-\frac{8}{20}$ and $\frac{10}{20}$ respectively.

Therefore, ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$ are

$$-\frac{7}{20}, -\frac{6}{20}, -\frac{5}{20}, -\frac{4}{20}, -\frac{3}{20}, -\frac{2}{20}, -\frac{1}{20}, 0, \frac{1}{20}, \frac{2}{20}$$

Question 5:

Find five rational numbers between

(i) $\frac{2}{3}$ and $\frac{4}{5}$

(ii) $\frac{-3}{2}$ and $\frac{5}{3}$

(iii) $\frac{1}{4}$ and $\frac{1}{2}$

Answer:

(i) $\frac{2}{3}$ and $\frac{4}{5}$ can be represented as $\frac{30}{45}$ and $\frac{36}{45}$ respectively.

Therefore, five rational numbers between $\frac{2}{3}$ and $\frac{4}{5}$ are

$$\frac{31}{45}, \frac{32}{45}, \frac{33}{45}, \frac{34}{45}, \frac{35}{45}$$

The tens and units digits of a number are the same. When the number is added to its reverse, the sum is 110. What is the number?

Solution:

Let the units digit be x.

Then the tens digit is also x.

Therefore the number is $10x + x = 11x$.

On reversing the order of the digit the number is $10x + x = 11x$.

Hence by the given condition we have,

$11x + 11x = 110$

$22x = 110$

$x = 5$

Therefore the required number is 55.

The sum of the digits of a two digit number is 12. If the new number formed by reversing the digits is greater than the original number by 18, find the original number. Check your solution.

Solution:

Let the digit in the ones place be x.

Then the digit in the tens place will be $12 - x$.

Therefore, the original number $= 10(12 - x) + x = 120 - 10x + x = 120 - 9x$.

And, the new number $= 10x + (12 - x) = 10x + 12 - x = 9x + 12$.

By the given condition,

New number = original number + 18

$9x + 12 = 120 - 9x + 18$

$9x + 12 = 138 - 9x$

$9x + 9x = 138 - 12$ (Transposing 9x and 12)

$18x = 126$

$\frac{18x}{18} = \frac{126}{18}$ (Dividing both sides by 18)

$x = 7$

Thus, ones digit is 7 and tens digit is $12 - 7 = 5$. Hence, the required number is 57.