DES EXERCICE AVEC SOLUTION EDPM

***********LMD 3*****************

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*********LADGHEM OMAR*************

$$\boxed{3e^x(\text{tg}\,y)\,dx + (2-e^x)\sec^2 y\,dy = 0}$$

$$3e^x(\text{tg}\,y)\,dx = -(2-e^x)\sec^2 y\,dy \qquad \sec y = \frac{1}{\cos y}$$

$$\frac{3e^x}{-(2-e^x)}\,dx = \frac{\sec^2 y}{\text{tg}\,y}\,dy \Rightarrow \int \frac{3e^x\,dx}{e^x-2} = \int \frac{1}{\cos^2 y}\,\frac{1}{\text{tg}\,y}\,dy$$

$$t = e^x-2 \Rightarrow dt = e^x\,dx \qquad u = \text{tg}\,y \Rightarrow du = \frac{1}{\cos^2 y}\,dy$$

$$3\int \frac{dt}{t} = \int \frac{du}{u} \Rightarrow 3\ln|t| = \ln u + C$$

$$3\ln|e^x-2| = \ln|\text{tg}\,y| + C$$

$$\ln|e^x-2|^3 = \ln|\text{tg}\,y| + C$$

$$e^{\ln|e^x-2|^3} = e^{\ln|\text{tg}\,y|+C} = e^{\ln|\text{tg}\,y|}\cdot\overbrace{e^C}^{=D}$$

$$|e^x-2|^3 = \text{tg}\,y\cdot D \Rightarrow \text{tg}\,y = \overbrace{\frac{1}{D}}^{=E}\cdot|e^x-2|^3 = E|e^x-2|^3$$

$$\boxed{y(x) = \text{arctg}\left(E\,|e^x-2|^3\right)}$$

$$\boxed{xy\,dx + (1+x^2)\,dy = 0}$$

$$xy\,dx = -(1+x^2)\,dy \Rightarrow \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \int -\frac{1}{y}\,dy$$

$$\frac{1}{2}\ln(1+x^2) = -\ln y + C \Rightarrow \ln(1+x^2)^{1/2} = \ln y^{-1} + C$$

$$e^{\ln(1+x^2)^{1/2}} = e^{\ln y^{-1} + C} \Rightarrow (1+x^2)^{1/2} = e^{\ln y^{-1}} \cdot e^{C} = D$$

$$\left(\sqrt{1+x^2}\right)^2 = (y^{-1} \cdot D)^2 \Rightarrow (1+x^2) = y^{-2} D$$

$$\frac{1+x^2}{y^{-2}} = D \Rightarrow \boxed{(1+x^2)\,y^2 = D}$$

$(1+e^x)\, y y' = e^x$ Hallar la solución que pasa por $(0,1)$

$$(1+e^x)\, y \frac{dy}{dx} = e^x$$

$$y\,dy = \frac{e^x}{(1+e^x)}\,dx \Rightarrow \int y\,dy = \int \frac{e^x}{1+e^x}\,dx$$

Sea $t = 1+e^x \Rightarrow dt = e^x\,dx$

$$\frac{y^2}{2} = \int \frac{dt}{t} \Rightarrow \frac{y^2}{2} = \ln|t| + C$$

$$\frac{y^2}{2} = \ln(1+e^x) + C \Rightarrow y = \pm\sqrt{2\ln(1+e^x) + D}$$

$y(0)=1 \Rightarrow$ $1 = \sqrt{2\ln(1+e^0) + D}$

$1 = \sqrt{2\ln 2 + D} \Rightarrow 2\ln 2 + D = 1 \Rightarrow$

$D = 1 - 2\ln 2$

$$y = \sqrt{2\ln(1+e^x) + 1 - 2\ln 2} = \sqrt{2(\ln(1+e^x) - \ln 2) + 1}$$

$$y = \sqrt{2\ln\left(\frac{1+e^x}{2}\right) + 1}$$

$$\boxed{x^3dx + (y+1)^2dy = 0}$$

$$\int x^3dx = \int -(y+1)^2dy \quad \Rightarrow \frac{x^4}{4} = -\frac{(y+1)^3}{3} + C$$

$$\Rightarrow \boxed{3x^4 + 4(y+1)^3 = D}$$

$$\boxed{y - xy' = 1 + x^2 y'}$$

$$y - 1 = xy' + x^2 y' \Rightarrow y - 1 = x(1+x)\,y' \Rightarrow y - 1 = x(1+x)\frac{dy}{dx}$$

$$\int \frac{1}{x(1+x)}\,dx = \int \frac{1}{y-1}\,dy \Rightarrow \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx = \ln(y-1) + C$$

$$\ln x - \ln(x+1) = \ln(y-1) + C \Rightarrow \ln\left(\frac{x}{x+1}\right) = \ln(y-1) + C$$

$$e^{\ln\left(\frac{x}{x+1}\right)} = e^{\ln(y-1) + C} = e^{\ln(y-1)} \cdot \underbrace{e^{C}}_{=D}$$

$$\frac{x}{x+1} = (y-1)D \Rightarrow \underbrace{\frac{1}{D}}_{=E}\,\frac{x}{x+1} + 1 = y \Rightarrow \boxed{y(x) = \frac{Ex}{x+1} + 1}$$

$$\boxed{(1+y)dx - (1+x)dy = 0}$$

$$(1+y)dx = (1+x)dy \Rightarrow \int \frac{1}{1+x}dx = \int \frac{1}{1+y}dy$$

$$\ln(1+x) = \ln(1+y) + C \Rightarrow e^{\ln(1+x)} = e^{\ln(1+y)+C}$$

$$1+x = e^{\ln(1+y)} \cdot \underbrace{e^{C}}_{=D} \Rightarrow 1+x = (1+y)D$$

$$\frac{1+x}{D} = 1+y \Rightarrow y = \frac{1+x}{D} - 1 = \underbrace{\frac{1}{D}}_{=E}(1+x) - 1 \Rightarrow \boxed{y = E(1+x) - 1}$$

$$\boxed{\frac{dy}{dx} = \frac{x-y}{x}} \qquad \frac{dy}{dx} = x\frac{dz}{dx} + z = \frac{x}{x} - \frac{y}{x} = 1 - z$$

$$\Rightarrow x\frac{dz}{dx} + z = 1 - z \Rightarrow x\frac{dz}{dx} = 1 - 2z$$

$$\frac{1}{-2}\int \frac{(-2)\,1}{1-2z}\,dz = \int \frac{1}{x}\,dx \Rightarrow -\frac{1}{2}\ln(1-2z) = \ln x + C$$

$$\ln(1-2z)^{-1/2} = \ln x + C \Rightarrow e^{\ln(1-2z)^{-1/2}} = e^{\ln x + C} = e^{\ln x} \cdot \underbrace{e^{C}}_{=D}$$

$$\left((1-2z)^{-1/2}\right)^2 = (D.x)^2 \Rightarrow (1-2z)^{-1} = D^2x^2 = Ex^2$$

$$\frac{1}{1-2z} = Ex^2 \Rightarrow \underbrace{\frac{1}{E}}_{=F} \cdot \frac{1}{x^2} = 1-2z \Rightarrow \frac{F}{x^2} = 1 - 2\frac{y}{x}$$

$$\Rightarrow \frac{2y}{x} = 1 - \frac{F}{x^2} \Rightarrow 2y = x - \frac{F}{x} \Rightarrow y = \frac{x}{2} - \underbrace{\frac{F}{2}}_{G}\frac{1}{x} = \boxed{\frac{x}{2} - \frac{G}{x}}$$

$$\boxed{(2x^3+y^3)dx - 3xy^2\,dy = 0}$$

$$z = \frac{y}{x}$$

$$(2x^3+y^3)\,dx = 3xy^2 dy \Rightarrow \frac{2x^3+y^3}{3xy^2} = \frac{dy}{dx}$$

$$y' = x\frac{dz}{dx} + z = \frac{2x^{\cancel{3}\,2}}{3\cancel{x}y^2} + \frac{y^{\cancel{3}\,1}}{3x\cancel{y^2}} = \overbrace{\frac{2x^2}{3y^2}}^{=\frac{1}{z^2}} + \overbrace{\frac{y}{3x}}^{=z}$$

$$x\frac{dz}{dx} + z = \frac{2}{3z^2} + \frac{z}{3} \Rightarrow x\frac{dz}{dx} = \frac{2}{3z^2} + \overbrace{\frac{z}{3} - z}^{=-\frac{2}{3}z}$$

$$x\frac{dz}{dx} = \frac{2}{3z^2} - \frac{2}{3}z \Rightarrow x\frac{dz}{dx} = \frac{2-2z^3}{3z^2}$$

$$-\frac{1}{2}\int \frac{2 \cdot 3z^2}{2-2z^3}\,dz = \int \frac{1}{x}\,dx \qquad \text{Sea } u = 2-2z^3$$
$$du = -6z^2 dz$$

$$-\frac{1}{2}\int \frac{1}{u}\,du = \ln x + C \Rightarrow -\frac{1}{2}\ln u = \ln x + C$$

$$-\frac{1}{2}\ln(2-2z^3) = \ln x + C \Rightarrow \ln(2-2z^3)^{-1/2} = \ln x + C$$

$$e^{\ln(2-2z^3)^{-1/2}} = e^{\ln x + C} \Rightarrow (2-2z^3)^{-1/2} = e^{\ln x}\cdot \overbrace{e^{c}}^{=D} = x \cdot D$$

$$(2-2z^3)^{-1} = x^2 \overbrace{D^2}^{=E} \Rightarrow \frac{1}{2-2z^3} = Ex^2 \Rightarrow 1 = Ex^2(2-2z^3)$$

$$1 = Ex^2\left(2 - 2\frac{y^3}{x^3}\right) \Rightarrow 1 = E\cancel{x^2}\,\frac{2x^3-2y^3}{x^{\cancel{3}\,1}} \Rightarrow 1 = \frac{2E(x^3-y^3)}{x}$$

$$\underbrace{\frac{1x}{2E}}_{=F} = x^3 - y^3 \Rightarrow \boxed{Fx = x^3 - y^3}$$

$$\boxed{(1+2y)\,dx-(4-x)\,dy=0}$$

$$(1+2y)\,dx=(4-x)\,dy \Rightarrow \int\frac{1}{4-x}\,dx=\int\frac{1}{1+2y}\,dy$$

$$-\int\frac{-1}{4-x}\,dx=\frac{1}{2}\int\frac{2}{1+2y}\,dy \Rightarrow -\ln(4-x)=\frac{1}{2}\ln(1+2y)+C$$

$$\ln(4-x)^{-1}=\ln(1+2y)^{1/2}+C$$

$$e^{\ln(4-x)^{-1}}=e^{\ln(1+2y)^{1/2}}\cdot \underbrace{e^{C}}_{=D}$$

$$(4-x)^{-1}=(1+2y)^{1/2}\cdot D \Rightarrow \left(\frac{1}{4-x}\right)^2=\left(\cancel{\sqrt{1+2y}}\right)^{\cancel{2}}\cdot \underbrace{D^2}_{=E}$$

$$\frac{1}{(4-x)^2}=(1+2y)E \Rightarrow \underbrace{\frac{1}{E}}_{=F}=(1+2y)(4-x)^2 \Rightarrow \boxed{F=(1+2y)(4-x)^2}$$

$$\boxed{xy^2\,dy = (x^3+y^3)\,dx}$$

$$\frac{dy}{dx} = \frac{x^3+y^3}{xy^2} = \frac{x^{\cancel{3}\,2}}{\cancel{x}y^2} + \frac{y^{\cancel{3}}}{xy^{\cancel{2}}} = \frac{x^2}{y^2} + \frac{y}{x} = \frac{1}{\frac{y^2}{x^2}} + \frac{y}{x} = \frac{1}{z^2} + z$$

$$\parallel$$

$$x\frac{dz}{dx} + \cancel{z} = \frac{1}{z^2} + \cancel{z} \Rightarrow x\frac{dz}{dx} = \frac{1}{z^2} \Rightarrow \int z^2\,dz = \int \frac{1}{x}\,dx$$

$$\frac{z^3}{3} = \ln x + C \Rightarrow z^3 = 3\ln x + \underbrace{3C}_{D} \Rightarrow \left(\frac{y}{x}\right)^3 = 3\ln x + D$$

$$\frac{y^3}{x^3} = 3\ln x + D \Rightarrow y^3 = 3x^3\ln x + Dx^3$$

$$y = \sqrt[3]{3x^3\ln x + Dx^3} = \sqrt[3]{x^3(3\ln x + D)}$$

$$y = \sqrt[\cancel{3}]{x^{\cancel{3}}} \cdot \sqrt[3]{3\ln x + D} \Rightarrow \boxed{y = x\sqrt[3]{3\ln x + D}}$$

Scanned by CamScanner

$$\boxed{\operatorname{ctg}\theta \, d\rho + \rho \, d\theta = 0}$$

$$\operatorname{ctg}\theta \, d\rho = -\rho \, d\theta \Rightarrow \int \frac{1}{\rho} d\rho = \int -\frac{1}{\operatorname{ctg}\theta} d\theta$$

$$\Rightarrow \ln \rho = -\int \frac{\operatorname{sen}\theta}{\cos\theta} d\theta \qquad CV: \; u = \cos\theta \,, \; du = -\operatorname{sen}\theta \, d\theta$$

$$\ln \rho = \int \frac{1}{u} du = \ln u + C \quad \Rightarrow \ln \rho = \ln \cos\theta + C$$

$$e^{\ln \rho} = e^{\ln \cos\theta + C} \quad \Rightarrow \rho = e^{\ln \cos\theta} \cdot \underbrace{e^{C}}_{=D} \Rightarrow \boxed{\rho = D \cos\theta}$$

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$$\boxed{y' + 3y = 8 \quad , \quad y(0) = 2}$$

$$y' = 8 - 3y \Rightarrow \frac{dy}{dx} = 8 - 3y \Rightarrow \int \frac{1}{8-3y}\, dy = \int dx$$

$$\frac{1}{-3} \int \frac{1(-3)}{8-3y}\, dy = x + C \qquad \Rightarrow \qquad -\frac{1}{3} \ln|8-3y| = x + C$$

$$\ln|8-3y| = -3(x+c) \Rightarrow e^{\ln|8-3y|} = e^{-3(x+c)}$$

$$|8-3y| = e^{-3(x+c)} \qquad \Rightarrow \qquad \frac{8 - e^{-3(x+c)}}{3} = y$$

$$y(0) = 2 = \frac{8 - e^{-3c}}{3} \Rightarrow 6 = 8 - e^{-3c} \Rightarrow e^{-3c} = 2$$

$$y(x) = \frac{8 - e^{-3x} \cdot \overbrace{e^{-3c}}^{=2}}{3} \Rightarrow \boxed{y(x) = \frac{8 - 2e^{-3x}}{3}}$$

$$\boxed{x^2(y+1)dx + y^2(x-1)dy = 0}$$

$$x^2(y+1)\,dx = -y^2(x-1)\,dy = 0 \Rightarrow \int \frac{x^2}{x-1}\,dx = -\int \frac{y^2}{y+1}\,dy$$

$$\left(\begin{array}{ll} \begin{array}{r|l} x^2 & x-1 \\ \cline{2-2} -x^2+x & x+1 \\ \cline{1-1} x & \\ -x+1 & \\ \cline{1-1} 1 & \end{array} \Rightarrow \frac{x^2}{x-1} = x+1+\frac{1}{x-1} & \begin{array}{r|l} y^2 & y+1 \\ \cline{2-2} -y^2-y & y-1 \\ \cline{1-1} -y & \\ y+1 & \\ \cline{1-1} 1 & \end{array} \Rightarrow \frac{y^2}{y+1} = y-1+\frac{1}{y+1} \end{array}\right)$$

$$\Rightarrow \int\left(x+1+\frac{1}{x-1}\right)dx = -\int\left(y-1+\frac{1}{y+1}\right)dy$$

$$\frac{x^2}{2} + x + \ln(x-1) = -\left(\frac{y^2}{2} - y + \ln(y+1)\right) + C$$

$$x^2 + 2x + 2\ln(x-1) = -y^2 + 2y - 2\ln(y+1) + C'$$

$$x^2 + y^2 + 2x - 2y + 2\ln(x-1) + 2\ln(y+1) = C'$$

$$\boxed{x^2 + y^2 + 2x - 2y + 2\ln(x-1)(y+1) = C'}$$

$$\boxed{4x\,dy - y\,dx = x^2 dy}$$

$$4x\,dy - x^2 dy = y\,dx$$

$$x(4-x)\,dy = y\,dx \Rightarrow \int \frac{dy}{y} = \int \frac{1}{x(4-x)}\,dx$$

$$\ln y = \frac{1}{4}\int \left(\frac{1}{x} + \frac{1}{4-x}\right) dx$$

$$\left\{ \begin{array}{l} \dfrac{1}{x(4-x)} = \dfrac{A}{x} + \dfrac{B}{4-x} \Rightarrow 1 = A(4-x) + Bx \\ x = 0 \Rightarrow 1 = 4A \Rightarrow A = 1/4 \\ x = 4 \Rightarrow 1 = 4B \Rightarrow B = 1/4 \end{array} \right. \Rightarrow \frac{1}{x(4-x)} = \frac{1}{4x} + \frac{1}{4(4-x)}$$

$$\Rightarrow \ln y = \frac{1}{4}\left(\ln x - \ln(4-x)\right) + C$$

$$\ln y = \frac{1}{4}\ln\left(\frac{x}{4-x}\right) + C \Rightarrow \ln y = \ln\left(\frac{x}{4-x}\right)^{1/4} + C$$

$$e^{\ln y} = e^{\ln\left(\frac{x}{4-x}\right)^{1/4} + C} \Rightarrow \boxed{y = \left(\frac{x}{4-x}\right)^{1/4} \cdot D}$$

$$\boxed{xy\,dy = (y+1)(1-x)\,dx}$$

$$\int \frac{y}{y+1}\,dy = \int \frac{1-x}{x}\,dx$$

$$\begin{array}{r|l} y & y+1 \\ \hline \underline{-y-1} & 1 \\ -1 & \end{array}$$

$$\frac{y}{y+1} = 1 - \frac{1}{y+1}$$

$$\int \left(1 - \frac{1}{y+1}\right) dy = \int \left(\frac{1}{x} - \overset{1}{\overset{=}{\frac{x}{x}}}\right) dx$$

$$y - \ln(y+1) = \ln x - \int 1\,dx$$

$$y - \ln(y+1) = \ln x - x + c$$

$$y + x = \ln x + \ln(y+1) + c$$

$$\boxed{y + x = \ln x(y+1) + c}$$

$$\boxed{(1+2y)\,dx + (4-x^2)\,dy = 0}$$

$$(1+2y)\,dx = -(4-x^2)\,dy = (x^2-4)\,dy$$

$$\int \frac{1}{x^2-4}\,dx = \int \frac{1}{1+2y}\,dy$$

$$\frac{1}{x^2-4} = \frac{1}{x^2-2^2} = \frac{1}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2} = \frac{A(x-2)+B(x+2)}{(x+2)(x-2)}$$

$$1 = A(x-2) + B(x+2)$$

$$x=2 \Rightarrow 1 = 4B \Rightarrow B = 1/4$$

$$x=-2 \Rightarrow 1 = -4A \Rightarrow A = -1/4$$

$$\Rightarrow \frac{1}{x^2-4} = \frac{-1}{4(x+2)} + \frac{1}{4(x-2)}$$

$$\Rightarrow \int \left[\frac{-1}{4(x+2)} + \frac{1}{4(x-2)}\right] dx = \frac{1}{2}\int \frac{2}{1+2y}\,dy$$

$$\frac{-1}{\cancel{4}_2}\ln(x+2) + \frac{1}{\cancel{4}_2}\ln(x-2) = \frac{\cancel{1}}{\cancel{2}}\ln(1+2y) + C$$

$$\ln(x+2)^{-1/2} + \ln(x-2)^{1/2} = \ln(1+2y) + C$$

$$\ln(x+2)^{-1/2}\cdot(x-2)^{1/2} = \ln(1+2y) + C$$

$$e^{\ln(x+2)^{-1/2}\cdot(x-2)^{1/2}} = e^{\ln(1+2y)+C} = e^{\ln(1+2y)}\cdot \underbrace{e^{C}}_{=D}$$

$$(x+2)^{-1/2}(x-2)^{1/2} = (1+2y)D \Rightarrow (x+2)^{-1}(x-2) = (1+2y)^2\underbrace{D^2}_{=E}$$

$$\boxed{\frac{x-2}{x+2} = (1+2y)^2 E}$$