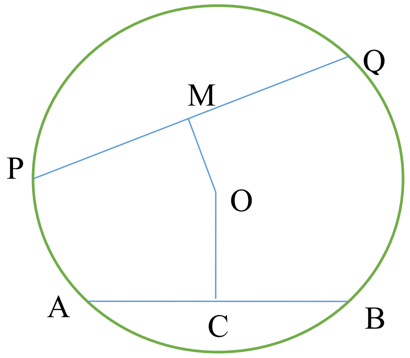
Gr. 11 MATHEMATICS 2016 REVISION EXERCISES

EUCLIDEAN GEOMETRY 50 marks from PAPER 2

1. O is the centre of the circle. PQ and AB are both chords of the circle with OM perpendicular to PQ and OC perpendicular to AB.
   OM = 5 cm, OC = 7 cm and AB = 48 cm.



Calculate the length of:

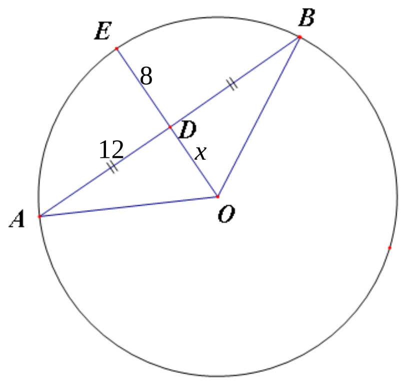
1.1 The radius of the circle

1.2 PQ

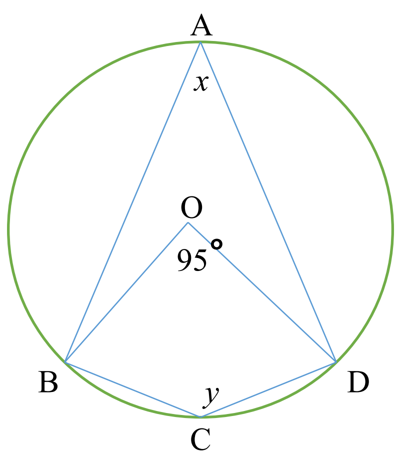
2. AB is a chord of the circle with centre O.
   OE bisects AB.
   AD = 12cm, ED = 8cm and OD = x

2.1 Determine the radius OB in terms of x.

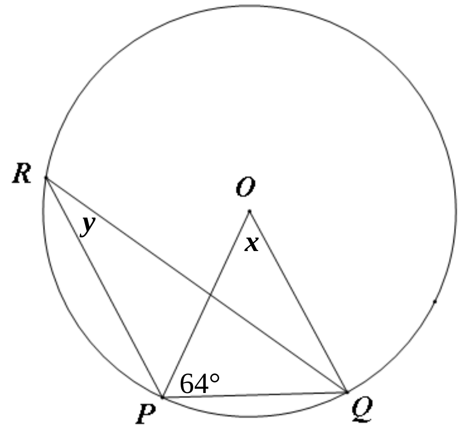
2.2 Hence, calculate the length of the radius OB.



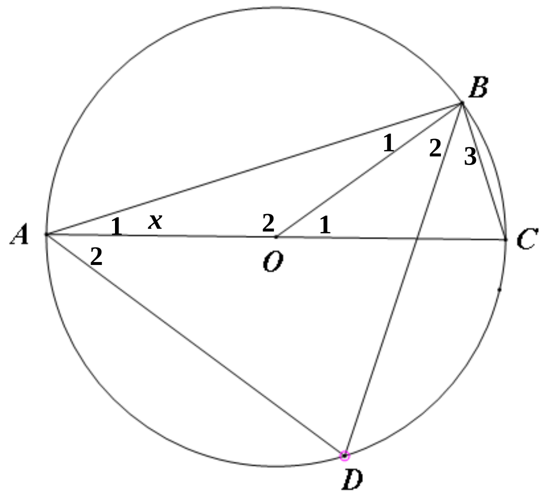
3. O is the centre of the circle. Calculate the values of x and y.



4. O is the centre of the circle. Calculate the values of the unknown variables.



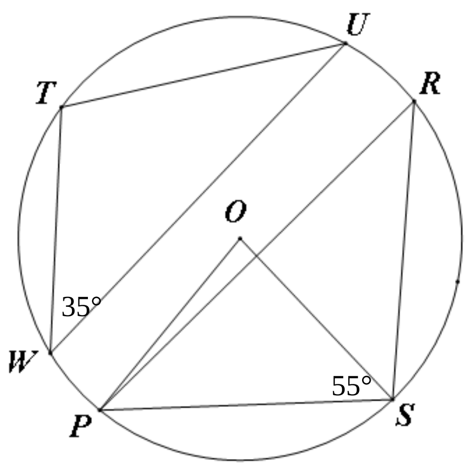
5. AOC is a diameter of the circle with centre O.
   $\hat{A}_1 = x$.



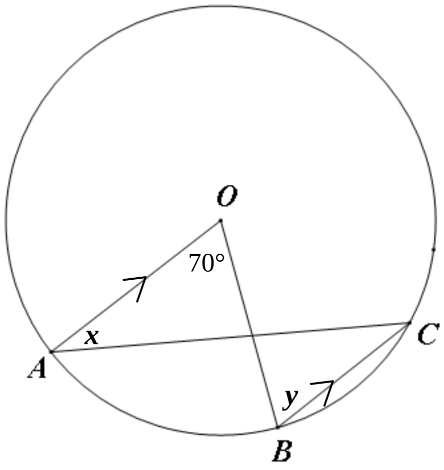
Write the following angles in terms of x:

5.1	$\hat{B}_1$;	5.2	$\hat{O}_1$
5.3	$\hat{C}$;	5.4	$\hat{D}$

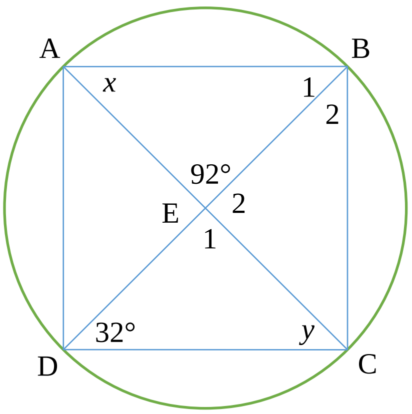
6. In the circle below O is the centre, $\hat{W} = 35^\circ$ and $O\hat{S}P = 55^\circ$.
   Prove that TU = PS.



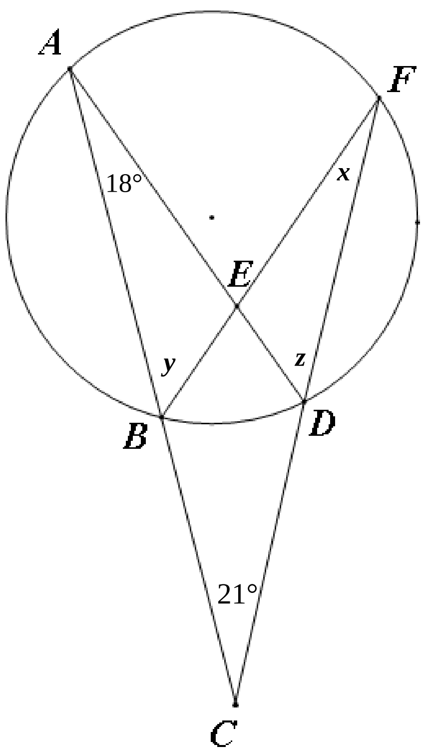
7. O is the centre of the circle. AO || BC.
and y.



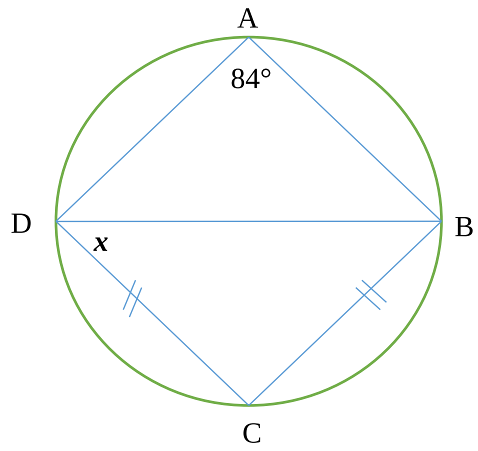
8. O is the centre of the circle. Calculate the values of x and y.



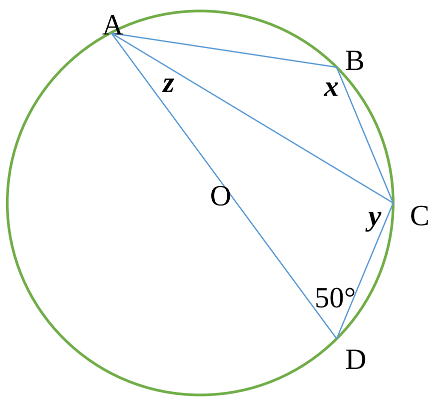
9. O is the centre of the circle. Calculate the values of x, y and z.



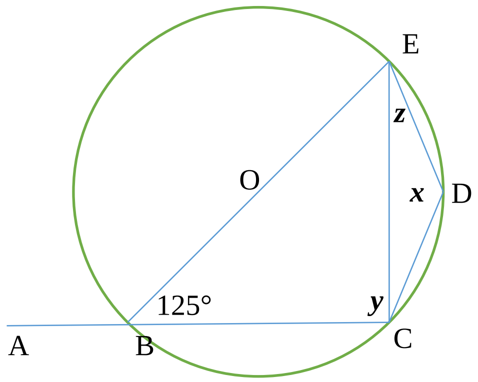
10. Calculate the value of x.



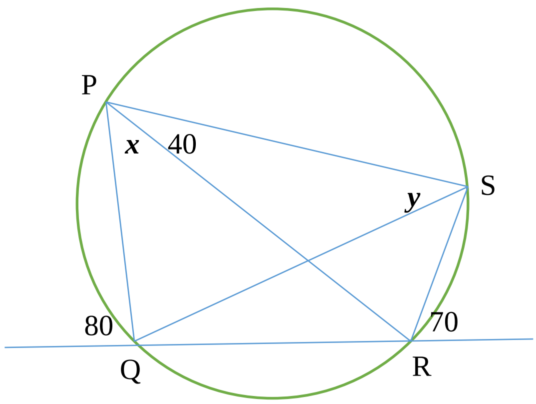
11. O is the centre of the circle. Calculate the values of x, y and z.



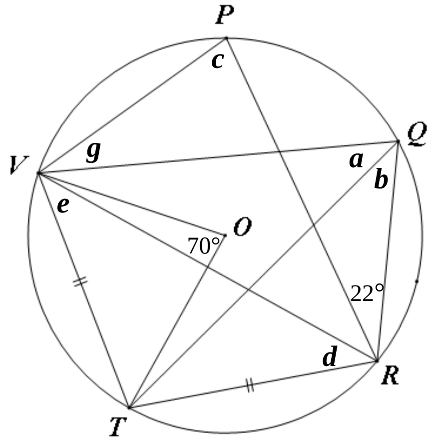
12. O is the centre of the circle. Calculate the values of x, y and z.



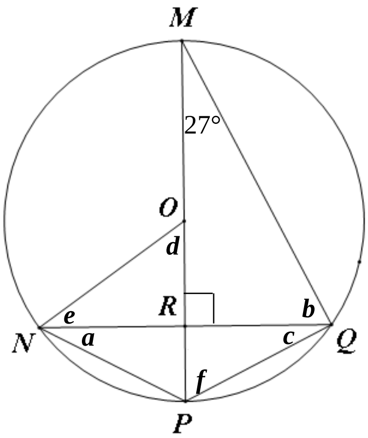
13. Calculate the values of x and y.



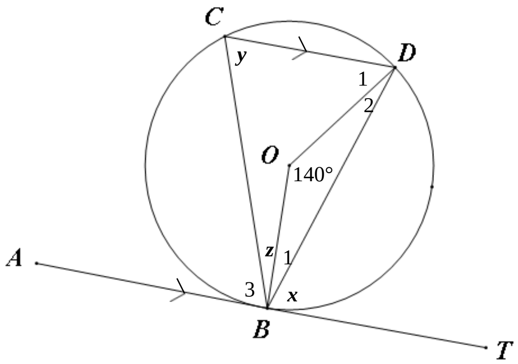
14. O is the centre of the circle. Calculate the values of the unknown variables. $V\hat{T}R = f$



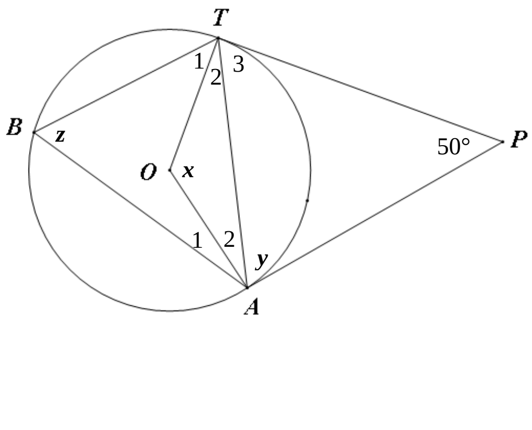
15. O is the centre of the circle. Calculate the values of the unknown variables.



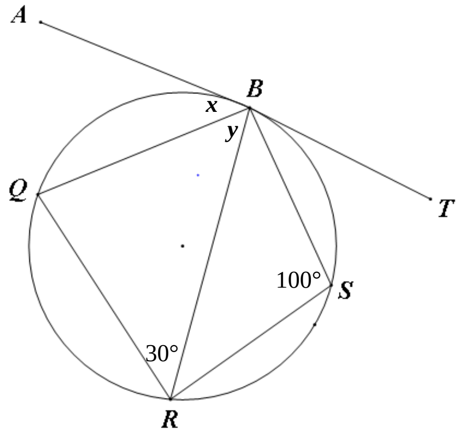
16. O is the centre of the circle and ABT a tangent. Calculate the values of the unknown variables.



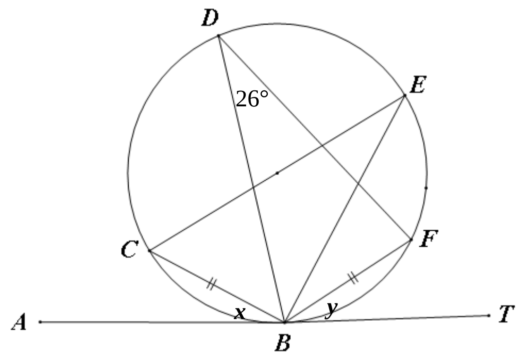
17. O is the centre of the circle and AP and PT are tangents. Calculate the values of the unknown variables.



18. ABT is a tangent. Calculate the values of the unknown variables.

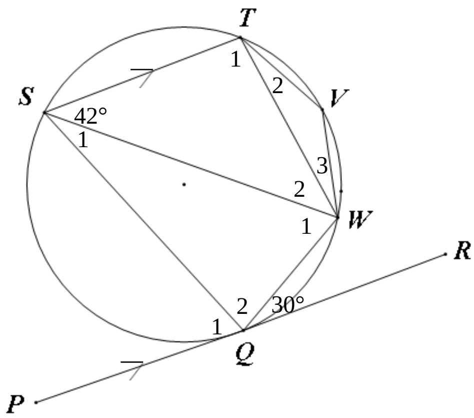


19. ABT is a tangent. Calculate the values of the unknown variables.

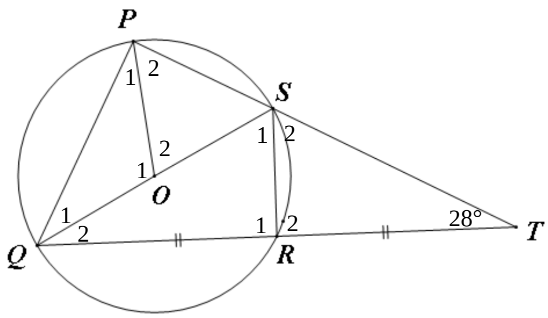


20. PQR is a tangent at Q. ST||QW. $W\hat{Q}R = 30^\circ$ and $T\hat{S}W = 42^\circ$. Calculate the sizes of the following angles:

20.1 $\hat{V}$; 20.2 $\hat{Q}_1$; 20.3 $\hat{T}_1$; 20.4 $\hat{W}_2$



21. QOS is a diameter of circle centre O. QR = RT and $\hat{T} = 28^\circ$. Calculate 21.1 $\hat{R}_2$; 21.2 $\hat{S}_2$; 21.3 $\hat{Q}_2$; 21.4 $\hat{Q}_1$; 21.5 $\hat{P}_1$

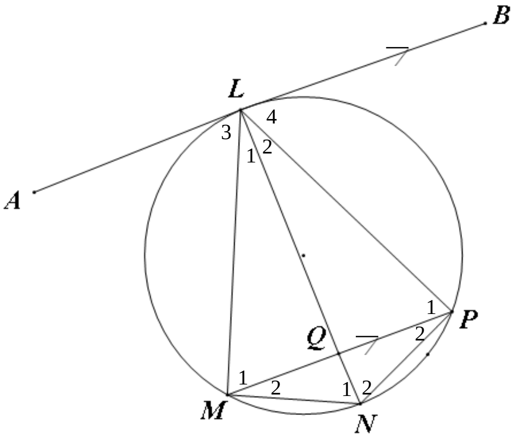


22. ALB is a tangent to circle LMNP. ALB||MP.
Prove that

22.1 LM = MP

22.2 LN bisects $M\hat{N}P$.

22.3 LM is a tangent to circle MNQ.



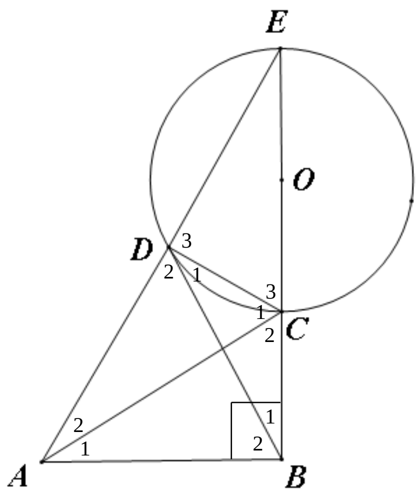
23. EC is a diameter of circle DEC. EC is produced to B. BD is a tangent at D. ED is produced to A, and AB is perpendicular to BE.
Prove that

23.1 ABCD is a cyclic quadrilateral

23.2 $\hat{A}_1 = \hat{E}$

23.3 Δ BDA is isosceles

23.4 $\hat{C}_2 = \hat{C}_3$



24. Questions from DBE CAPS Gr. 11 Papers:

24.1 Exemplar 2013 Paper 2
no. 9, 10 and 11.

24.2 Nov. 2014 Paper 2
no. 8, 9 and 10.

24.3 Nov. 2015 Paper 2
no. 9, 10 and 11.

Gr. 11 MATHEMATICS 2016 ANSWERS to REVISION EXERCISES
EUCLIDEAN GEOMETRY 50 marks from PAPER 2

1.1 AC = ½AB [line from centre ⊥ to chord]
= 24cm

$OA^2 = OC^2 + CA^2$ [Pythagoras]

$= 7^2 + 24^2$

$= 625$

$OA = 25$ cm

1.2 OP = OA [radii]
= 25cm

$PM^2 = OP^2 - OM^2$ [Pythagoras]

$= 25^2 - 5^2$

$= 600$

$PM = 24{,}49$ cm

PQ = 2 PM [line from centre ⊥ to chord]
= 48,99cm

2.1 OB = 8 + x

2.2 $OD \perp AB$ [line from centre to midpoint of chord]

$OA^2 = AD^2 + DO^2$ [Pythagoras]

$(8 + x)^2 = 12^2 + x^2$

$64 + 16x + x^2 = 144 + x^2$

$16x = 80$

$x = 5$

and OA = 13cm = OB

3. $x = \frac{1}{2}\hat{O}$ [∠ at centre = 2 x ∠ at circumf.]
= 47,5°

$y = 180^\circ - 47{,}5^\circ$ [opp. ∠ 's of cyclic quad.]
= 132,5°

4. OP = OQ [radii]

$O\hat{P}Q = O\hat{Q}P$ [∠ 's opp. = sides]

$x = 180^\circ - 2(64^\circ)$ [sum of ∠ 's of Δ]
= 52°

$y = \frac{1}{2}x$ [∠ at centre = 2 x ∠ at circumf.]
= 26°

5.1 AO = BO [radii]

$\hat{B}_1 = x$ [∠ 's opp. = sides]

5.2 $\hat{O}_1 = 2x$ [ext. ∠ of Δ]

5.3 $\hat{O}_2 = 180^\circ - 2x$ [∠ 's on a straight line]

$\hat{C} = 90^\circ - x$ [∠ at centre = 2 x ∠ at circumf.]

5.4 $\hat{D} = 90^\circ - x$ [∠ 's in same segment]

6. OS = OP [radii]

$O\hat{P}S = 55^\circ$ [∠ 's opp. = sides]

$\hat{O} = 70^\circ$ [sum of ∠ 's of Δ]

$\hat{R} = 35^\circ$ [∠ at centre = 2 x ∠ at circumf.]

$\therefore TU = PS$ [converse: = chords; = angles]

7. $y = 70^\circ$ [alt. ∠ 's; AO || BC]

$\hat{C} = 35^\circ$ [∠ at centre = 2 x ∠ at circumf.]

$x = 35^\circ$ [alt. ∠ 's; AO || BC]

8. $x = 32^\circ$ [∠ 's in same segment]

$\hat{B}_1 = 180^\circ - (32^\circ + 92^\circ)$ [sum of ∠ 's of Δ]

$= 56^\circ$

$y = 56^\circ$ [∠ 's in same segment]

9. $x = 21^\circ$ [∠ 's in same segment]

$y = 21^\circ + 18^\circ$ [ext. ∠ of Δ]

$= 39^\circ$

$z = 39^\circ$ [∠ 's in same segment]

10. $\hat{C} = 180^\circ - 84^\circ$ [opp. ∠ 's of cyclic quad.]

$= 96^\circ$

$x = \hat{B}_2$ [∠ 's opp. = sides]

$x = \dfrac{180^\circ - \hat{C}}{2}$ [sum of ∠ 's of Δ]

11. $x = 130^\circ$ [opp. ∠ 's of cyclic quad.]

$y = 90^\circ$ [∠ in semicircle]

$z = 180^\circ - (90^\circ + 50^\circ)$ [sum of ∠ 's of Δ]

$= 40^\circ$

12. $x = 125^\circ$ [ext. ∠ of cyclic quad.]

$y = 90^\circ$ [∠ in semicircle]

$z = 125^\circ - 90^\circ$ [ext. ∠ of Δ]

13. $Q\hat{P}S = 70^\circ$ [ext. ∠ of cyclic quad.]

$x = 30^\circ$

$P\hat{S}R = 80^\circ$ [ext. ∠ of cyclic quad.]

$Q\hat{S}R = x = 30^\circ$ [∠ 's in same segment]

$y = 80^\circ - 30^\circ = 50^\circ$

14. $a = 35^\circ$ [∠ at centre = 2 x ∠ at circumf.]

$b = 35^\circ$ [equal chords; equal ∠ 's]

$c = V\hat{Q}R$ [∠ 's in same segment]

$= 70^\circ$

$f = 110^\circ$ [opp. ∠ 's of cyclic quad.]

$d = a = 35^\circ$ [∠ 's in same segment]

$e = d = 35^\circ$ [∠ 's opp. = sides]

$g = 22^\circ$ [∠ 's in same segment]

15. $a = 27^\circ$ [$\angle$'s in same segment]

$M\hat{Q}P = 90^\circ$ [$\angle$ in semicircle]

$f = 180^\circ - (90^\circ + 27^\circ)$ [sum of $\angle$'s of Δ]

$= 63^\circ$

In ΔPNR and ΔPQR :

1) NR = RQ [line from centre $\perp$ to chord]
2) PR = PR [common]
3) $N\hat{R}P = Q\hat{R}P$ [both = 90°; given]

$\therefore \Delta PNR \equiv \Delta PQR$ [s; $\angle$;s]

$\therefore c = a = 27^\circ$ [$\equiv \Delta$'s]

$d = 2 \times c = 54^\circ$ [$\angle$ at centre=2x $\angle$ at circumf.]

$e = 180^\circ - (90^\circ + 54^\circ)$ [sum of $\angle$'s of Δ]

$= 36^\circ$

16. $y = 70^\circ$ [$\angle$ at centre=2x $\angle$ at circumf.]

$x = 70^\circ$ [tan-chord-theorem]

$A\hat{B}O = 90^\circ$ [tan $\perp$ radius]

$\hat{B}_3 = 70^\circ$ [alt. $\angle$'s; CD || AT]

$\therefore z = 90^\circ - 70^\circ = 20^\circ$

17. PT = PA [tangents from same point]

$y = \hat{T}_3$ [$\angle$'s opp. = sides]

$y = \frac{180^\circ - 50^\circ}{2}$ [sum of $\angle$'s of Δ]

$= 65^\circ$

$z = 65^\circ$ [tan-chord-theorem]

$x = 130^\circ$ [$\angle$ at centre=2x $\angle$ at circumf.]

18. $x = 30^\circ$ [tan-chord-theorem]

$\hat{Q} = 80^\circ$ [opp. $\angle$'s of cyclic quad.]

$y = 180^\circ - (30^\circ + 80^\circ)$ [sum of $\angle$'s of Δ]

$= 70^\circ$

(could also use fact that $x+y = 100^\circ$ from tan-chord-theorem to find y.)

19. $y = 26^\circ$ [tan-chord-theorem]

$\hat{E} = \hat{D} = 26^\circ$ [equal chords; equal $\angle$'s]

$x = \hat{E} = 26^\circ$ [tan-chord-theorem]

20.1 $\hat{V} = 138^\circ$ [opp. $\angle$'s of cyclic quad.]

$\hat{S}_1 = 30^\circ$ [tan-chord-theorem]

$\hat{Q}_1 = T\hat{S}Q$ [alt. $\angle$'s; ST || PQR]

$= 30^\circ + 42^\circ = 72^\circ$

20.3 $\hat{Q}_2 = 180^\circ - (72^\circ + 30^\circ)$ [$\angle$'s on a straight line]

$= 78^\circ$

$\hat{T}_1 = 102^\circ$ [opp. $\angle$'s of cyclic quad.]

20.4 $\hat{W}_2 = 180^\circ - (42^\circ + 102^\circ)$ [sum of $\angle$'s of Δ]

$= 36^\circ$

21.1 $\hat{R}_2 = \hat{R}_1 = 90^\circ$ [$\angle$ in semicircle]

21.2 $\hat{S}_2 = 180^\circ - (90^\circ + 28^\circ)$ [sum of $\angle$'s of Δ]

$= 62^\circ$

21.3 In ΔQRS and ΔTRS :

1) QR = RT [line from centre $\perp$ to chord]
2) SR = SR [common]
3) $\hat{R}_1 = \hat{R}_2$ [both = 90°]

$\therefore \Delta QRS \equiv \Delta TRS$ [s; $\angle$;s]

$\therefore \hat{Q}_2 = \hat{T} = 28^\circ$ [$\equiv \Delta$'s]

21.4 $P\hat{Q}R = 62^\circ$ [ext. $\angle$ of cyclic quad.]

$\therefore \hat{Q}_1 = 62^\circ - 28^\circ = 34^\circ$

OP = OQ [radii]

$\hat{P}_1 = \hat{Q}_1$ [$\angle$'s opp. = sides]

$= 34^\circ$

22.1 $\hat{L}_3 = \hat{P}_1$ [tan-chord-theorem]

$= \hat{L}_4$ [alt. $\angle$'s; ALB || MP]

$= \hat{M}_1$ [tan-chord-theorem]

$\therefore \hat{M}_1 = \hat{P}_1$

$\therefore$ LM = LP [sides opp. = $\angle$'s]

22.2 $\hat{N}_1 = \hat{P}_1$ [$\angle$'s in same segment]

$\hat{N}_2 = \hat{M}_1$ [$\angle$'s in same segment]

$\hat{N}_1 = \hat{N}_2$ [$\hat{M}_1 = \hat{P}_1$]

22.3 $\hat{M}_1 = \hat{N}_1$ [from above]

LM is a tangent to circle MNQ [converse: tan-chord-theorem]

23.1 $\hat{D}_3 = 90^\circ$ [$\angle$ in semicircle]

$\therefore D_3 = A\hat{B}E$ [both = 90°]

$\therefore$ ABCD is a cyclic quadrilateral

[converse: ext. $\angle$ of cyclic quad.]

23.2 $\hat{D}_1 = \hat{E}$ [tan-chord-theorem]

$\hat{D}_1 = \hat{A}_1$ [$\angle$'s in same segment]

$\therefore \hat{A}_1 = \hat{E}$

23.3 Let $\hat{A}_1 = x$. $\therefore$ also $\hat{D}_1$ and $\hat{E} = x$.

$\hat{D}_2 = 90^\circ - x$ [$\angle$'s on a straight line]

$\hat{C}_3 = 90^\circ - x$ [sum of $\angle$'s of Δ]

$= D\hat{A}B$ [ext. $\angle$ of cyclic quad.]

$\therefore D\hat{A}B = \hat{D}_2$ [both $= (90^\circ - x)$]

$\therefore$ BD = BA [sides opp. = $\angle$'s]

23.4 $\hat{C}_2 = 180^\circ - (90^\circ + x)$ [sum of $\angle$'s of Δ]

$= 90^\circ - x$

$= \hat{C}_3$ [proved above]