**Greatest Common Factor**

*The highest number that divides exactly into two or more numbers.
It is the "greatest" thing for simplifying fractions!*

Let's start with an Example ...



Greatest Common Factor of 12 and 16

1. Find all the **Factors** of each number,
2. Circle the **Common** factors,
3. Choose the **Greatest** of those

**So ... what is a "Factor" ?**

Factors are numbers we can multiply together to get another number:



A number can have many factors:

Factors of 12 are **1, 2, 3, 4, 6** and **12**...

... because **2** × **6** = 12, or **4** × **3** = 12, or **1** × **12** = 12.

**What is a "Common Factor" ?**

Say we have worked out the factors of two numbers:

Example: Factors of 12 and 30

|  |
| --- |
| Factors of 12 are **1, 2, 3, 4, 6**and**12** |
| Factors of 30 are **1, 2, 3, 5, 6, 10, 15**and**30** |

Then the **common factors** are those that are found in both lists:

* Notice that **1, 2, 3** and **6** appear in both lists?
* So, the **common factors** of 12 and 30 are: **1, 2, 3**and**6**

It is a *common* factor when it is a factor of two (or more) numbers.

Here is another example with three numbers:

Example: The common factors of 15, 30 and 105

|  |
| --- |
| Factors of 15 are **1, 3, 5,**and**15** |
| Factors of 30 are **1, 2, 3, 5, 6, 10, 15**and**30** |
| Factors of 105 are **1, 3, 5, 7, 15, 21, 35**and**105** |

The factors that are common to all three numbers are **1, 3, 5** and **15**

In other words, the **common factors** of 15, 30 and 105 are **1, 3, 5** and **15**

**What is the "Greatest Common Factor" ?**

It is simply the **largest** of the common factors.

In our previous example, the largest of the common factors is 15, so the **Greatest Common Factor** of 15, 30 and 105 is **15**

The "Greatest Common Factor" is the largest of the common factors (of two or more numbers)

**Why is this Useful?**

One of the most useful things is when we want to simplify a fraction:

Example: How can we simplify $\frac{12}{30}$?

Earlier we found that the Common Factors of 12 and 30 are 1, 2, 3 and 6, and so the **Greatest Common Factor is 6**.

So, the **largest** number we can divide both 12 and 30 exactly by is **6**, like this:

|  |  |  |
| --- | --- | --- |
|   | ÷ 6 |   |
| right over arrow  |
| *12***30** |  =  | *2***5** |
| right under arrow  |
|   | ÷ 6 |   |

The Greatest Common Factor of 12 and 30 is **6**.

And so, $\frac{12}{30}$can be simplified to $\frac{2}{5}$

**Finding the Greatest Common Factor**

Here are three ways:

**1.** We can:

* find all **factors** of both numbers
* then find the ones that are **common** to both, and
* then choose the **greatest**.

Example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Two Numbers** | **Factors** | **Common Factors** | **GreatestCommon Factor** | ***Example SimplifiedFraction*** |
| 9 and 12 |  **9**: 1,3,9**12**: 1,2,3,4,6,12 | 1,3 | **3** | $\frac{9}{12}$ = $\frac{3}{4}$ |

And another example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Two Numbers** | **Factors** | **Common Factors** | **GreatestCommon Factor** | ***Example SimplifiedFraction*** |
| 6 and 18 |  **6**: 1,2,3,6**18**: 1,2,3,6,9,18 | 1,2,3,6 | **6** | $\frac{6}{18}$= $\frac{1}{3}$ |

 **2**. Or we can find the prime factors and combine the common ones together:

|  |  |  |  |
| --- | --- | --- | --- |
| **Two Numbers** | **Thinking ...** | **GreatestCommon Factor** | ***Example SimplifiedFraction*** |
| 24 and 108 | **2** × **2** × 2 × **3** = 24, and**2** × **2** × **3** × 3 × 3 = 108 | 2 × 2 × 3 = **12** | $\frac{24}{108}$= $\frac{2}{9}$ |

 **3.** Or sometimes we can just **play around** with the factors until we discover it:

|  |  |  |  |
| --- | --- | --- | --- |
| **Two Numbers** | **Thinking ...** | **GreatestCommon Factor** | ***Example SimplifiedFraction*** |
| 9 and 12 | **3** × 3 = 9 and **3** × 4 = 12 | **3** | $\frac{9}{12}$= $\frac{3}{4}$ |

But in that case, we must check that we have found the **greatest** common factor.