**Prime Factorization**

**Prime Numbers**

A [Prime Number](https://www.mathsisfun.com/prime-composite-number.html) can be divided exactly **only** by 1 or itself.
And it must be a whole number greater than 1.

The first few prime numbers are: 2, 3, 5, 7, 11, 13, and 17 ..., and there is a prime number chart with more.

A [Prime Number](https://www.mathsisfun.com/prime-composite-number.html) is:

a whole number that **cannot** be made by multiplying other whole numbers

(if we **can** make it by multiplying other whole numbers it is a **Composite Number**)

Here we see it in action:



2 is Prime, 3 is Prime, 4 is Composite (=2×2), 5 is Prime, and so on...

 Here is a list of all the prime numbers up to 1,000:



**Factors**

"**Factors**" are the numbers you multiply together to get another number:



**Prime Factorization**

"Prime Factorization" is finding **which prime numbers** multiply together to make the original number.

Here are some examples:

Example 1: What are the prime factors of 12?

It is best to start working from the smallest prime number, which is 2, so let's check:

12 ÷ 2 = 6

Yes, it divided exactly by 2. We have taken the first step!

But 6 is not a prime number, so we need to go further. Let's try 2 again:

6 ÷ 2 = 3

Yes, that worked also. And 3 **is** a prime number, so we have the answer:

**12 = 2 × 2 × 3**

As you can see, **every factor** is a **prime number**, so the answer must be right.

Note: **12 = 2 × 2 × 3** can also be written using [exponents](https://www.mathsisfun.com/exponent.html) as **12 = 22 × 3**

**Your Notes:**

Example 2: What is the prime factorization of 147?

Can we divide 147 exactly by 2?

147 ÷ 2 = 73½

No it can't. The answer should be a whole number, and 73½ is not.

Let's try the next prime number, 3:

147 ÷ 3 = 49

That worked, now we try factoring 49, and find that 7 is the smallest prime number that works:

49 ÷ 7 = 7

And that is as far as we need to go, because all the factors are prime numbers.

**147 = 3 × 7 × 7**

(or **147 = 3 × 72** using exponents)

Example 3: What is the prime factorization of 17?

Hang on ... **17 is a Prime Number**.

So that is as far as we can go. **17 = 17**

**Another Method**

We showed you how to do the factorization by starting at the smallest prime and working upwards.

But sometimes it is easier to break a number down into **any factors** you can ... then work *those* factors down to primes.

Example: What are the prime factors of 90?

Break 90 into 9 × 10

* The prime factors of 9 are **3 and 3**
* The prime factors of 10 are **2 and 5**

So, the prime factors of 90 are **3, 3, 2 and 5**

**Factor Tree**

And a "Factor Tree" can help: find **any factors** of the number, then the factors of those numbers, etc, until we can't factor any more.

Example: 48

**48 = 8 × 6**, so we write down "8" and "6" below 48

Now we continue and factor 8 into **4 × 2**

Then 4 into **2 × 2**

And lastly 6 into **3 × 2**

We can't factor anymore, so we have found the prime factors.

Which reveals that **48 = 2 × 2 × 2 × 2 × 3**

(or **48 = 24 × 3** using exponents)

**Why find Prime Factors?**

A prime number can only be divided by 1 or itself, so it cannot be factored any further!

Every other whole number can be broken down into prime number factors.

|  |  |  |
| --- | --- | --- |
|  |   | It is like the Prime Numbers are the **basic building blocks** of all numbers. |

This idea can be very useful when working with big numbers, such as in **Cryptography**.

**Your Notes:**

**Cryptography**

**Cryptography** is the study of secret codes. **Prime Factorization** is very important to people who try to make (or break) secret codes based on numbers.

That is because factoring very large numbers is very hard, and can take computers a long time to do.

If you want to know more, search the subject is "encryption" or "cryptography".

**Unique**

And here is another thing:

**There is only one (unique!) set of prime factors for any number.**

Example: The prime factors of 330 are 2, 3, 5 and 11:

330 = 2 × 3 × 5 × 11

There is no other possible set of prime numbers that can be multiplied to make 330.

In fact, this idea is so important it is called the [**Fundamental Theorem of Arithmetic**](https://www.mathsisfun.com/numbers/fundamental-theorem-arithmetic.html).

## The Fundamental Theorem of Arithmetic

Let us start with the definition:

Any integer greater than 1 is either a **prime number**, or can be written as a **unique product of prime numbers** (ignoring the order).

What does it mean?

Let's build up the ideas piece by piece:

"Any [integer](https://www.mathsisfun.com/whole-numbers.html) greater than 1" means the numbers **2, 3, 4, 5, 6, ...** etc.

Integers = { ..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ... }

Negative Integers = { ..., -5, -4, -3, -2, -1 }

Positive Integers = { 1, 2, 3, 4, 5, ... }

Non-Negative Integers = { 0, 1, 2, 3, 4, 5, ... } *(includes zero, see?)*

A [Prime Number](https://www.mathsisfun.com/prime-composite-number.html) is a number that cannot be exactly divided by any other number (except 1 or itself).

"...product of prime numbers" means that we **multiply prime numbers together**.

So, by multiplying prime numbers we can create any other whole number.

### Example: 42

Can we make 42 by multiplying **only prime numbers?** Let's see:

2 × 3 × 7 = 42

Yes, **2**, **3** and **7** are prime numbers, and when multiplied together they make **42**.

Try some other examples for yourself. How about 30? Or 33?

|  |  |
| --- | --- |
|  | It is like the Prime Numbers are the **basic building blocks** of all numbers. |

"... **unique** product of prime numbers" means there is only one (unique!) set of prime numbers that will work

### Example: we just showed that 42 is made by the prime numbers 2, 3 and 7:

2 × 3 × 7 = 42

**No other prime numbers will work!**

We could try 2 × 3 × 5, or 5 × 11, but none of them will work:

Only 2, 3 and 7 make 42

**Your Notes:**