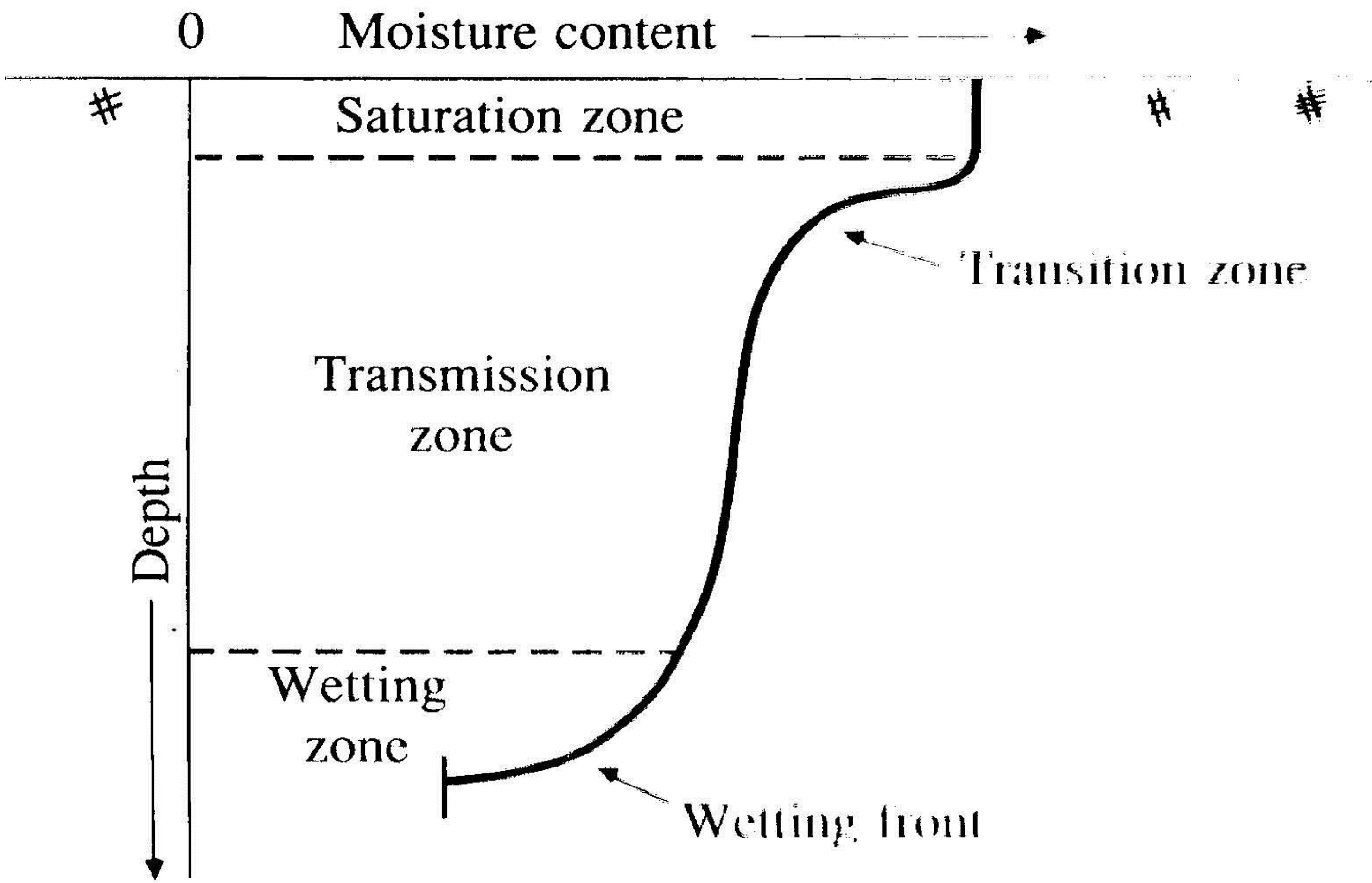
Hydrology

Surface water

Infiltration



Infiltration equations

- Horton's Equation:

$$f = f_c + (f_0 - f_c)e^{-kt}$$

Total infiltration

$$F = f_c t + \frac{(f_0 - f_c)}{k}(1 - e^{-kt})$$

Infiltration equations

- Kostiakov

$$f = akt^{a-1} + f_0$$

Total infiltration

$$F = kt^a + f_o t$$

Infiltration equations

- Philip's Equation

$$f = \frac{1}{2} S t^{-\frac{1}{2}} + K$$

Total infiltration

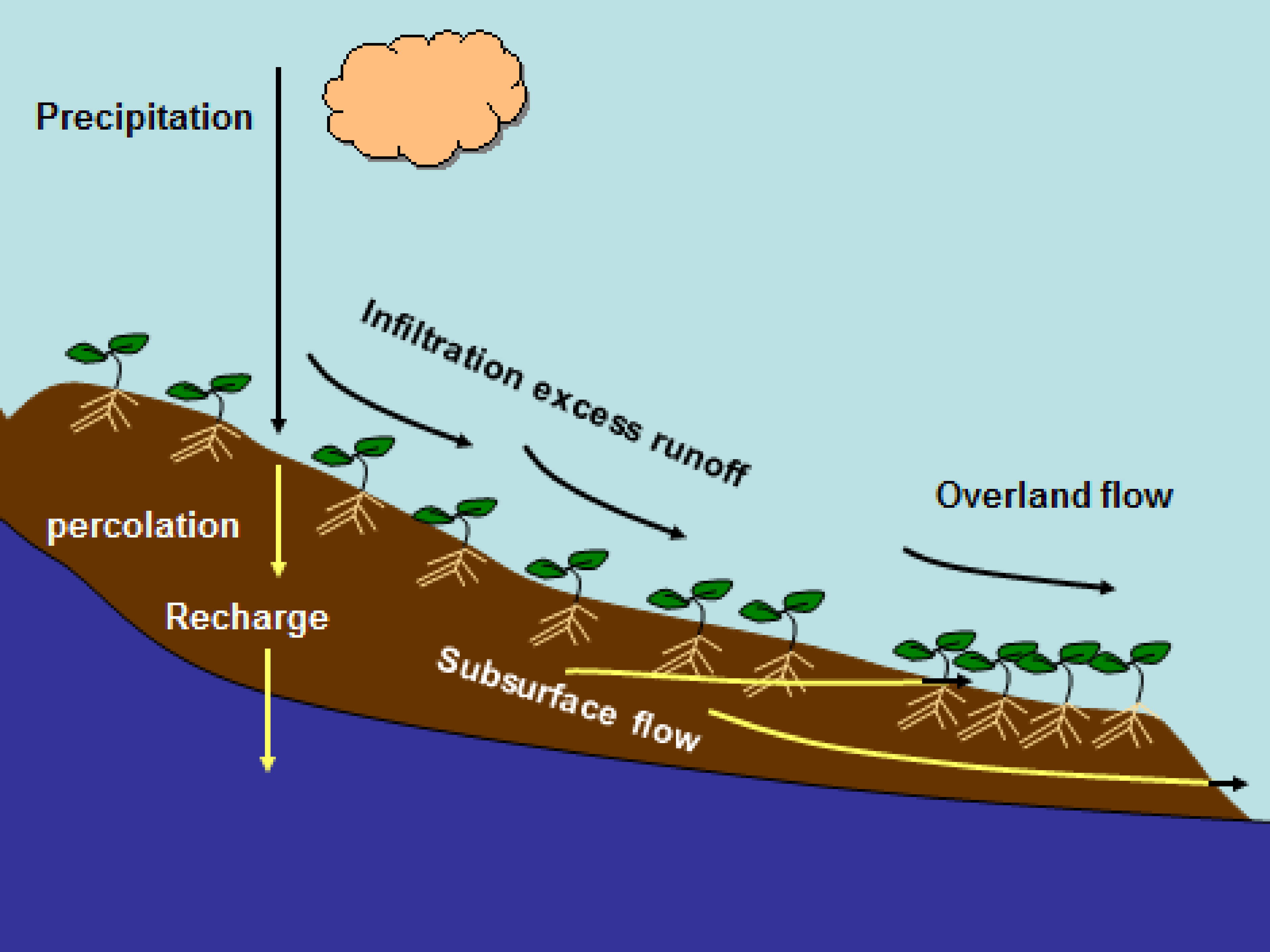
$$F = S t^{\frac{1}{2}} + K t$$

$f \cong K$ as $t \rightarrow \infty$

Sources of streamflow

1. Watershed contribution (surface water):
    a. Hortonian overland flow
    b. Subsurface flow
    c. Saturation overland flow

2. Ground water contribution



Precipitation
Infiltration excess runoff
Overland flow
percolation
Recharge
Subsurface flow

Hortonian overland

- Rainfall intensity > infiltration rate
- Beginning of the storm the infiltration rate is high enough to absorb rainfall
- Runoff start to occur when infiltration rate decreases
- Only a fraction of runoff reaches the streams because of depressions and interception by plant
- Important in arid regions.

Subsurface flow

- As the soil becomes saturated, the soil water will move laterally to the stream flow.
- Flow based on Darcy’s law (matrix flow) is slow compared to the overall flow to streams.
- Tracer studies showed that subsurface is faster than what is predicted by Darcy’s equation indicating preferential flow, still contribution limited to area close to the stream.
- Important in humid area where infiltration is usually high.

$$q = -K\left(\frac{\partial h}{\partial z} - \cos(\propto)\right)$$

Saturation overland flow

- As a result of subsurface flow the soil beneath the end of the slope or near the stream will become saturated.
- Saturation will also occur as a result of the rising water table.
- Precipitation over the saturated areas will be converted to overland

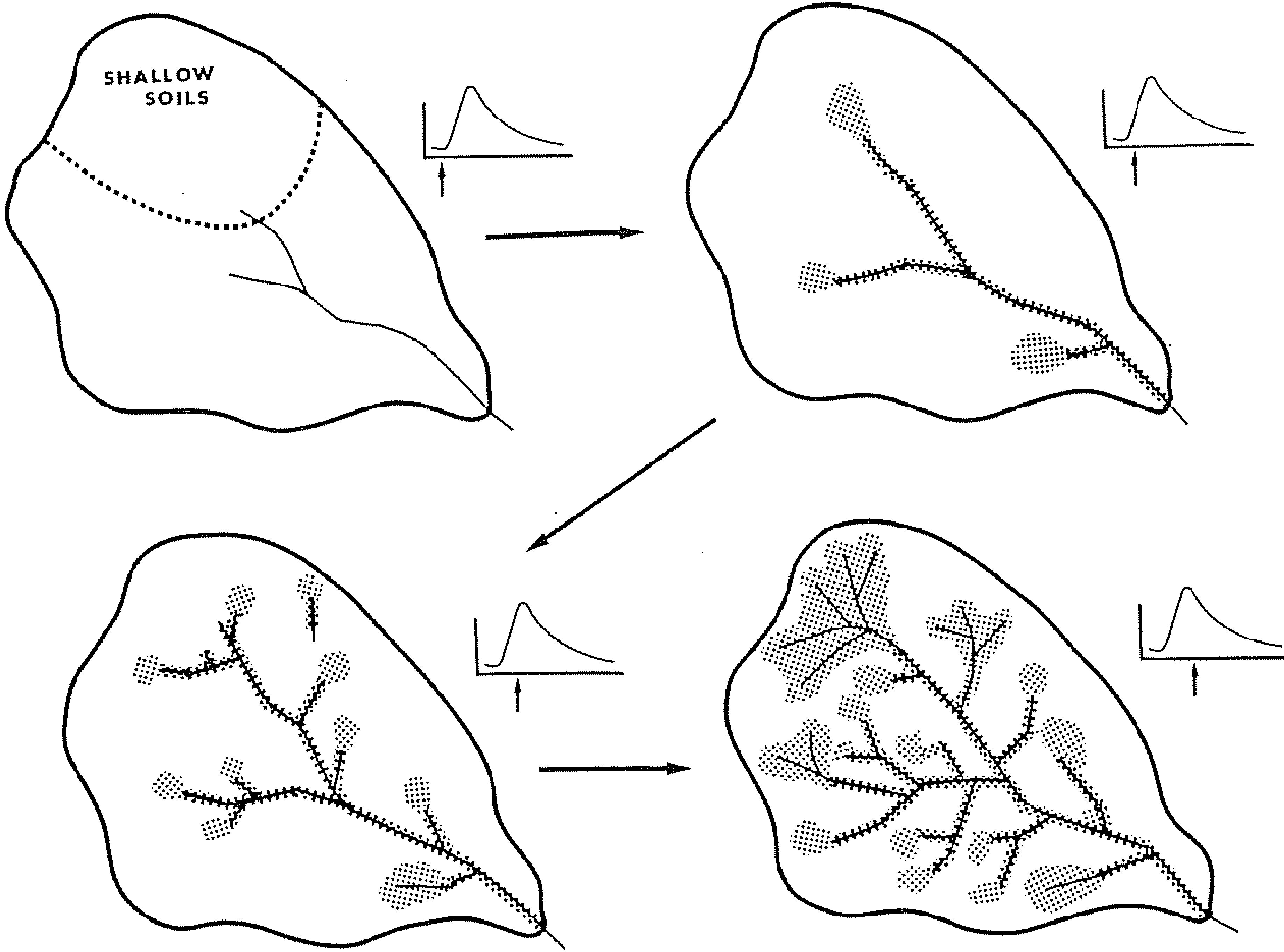
Hortonian vs Saturation overland flow

Hortonian overland flow ➔ saturation from above

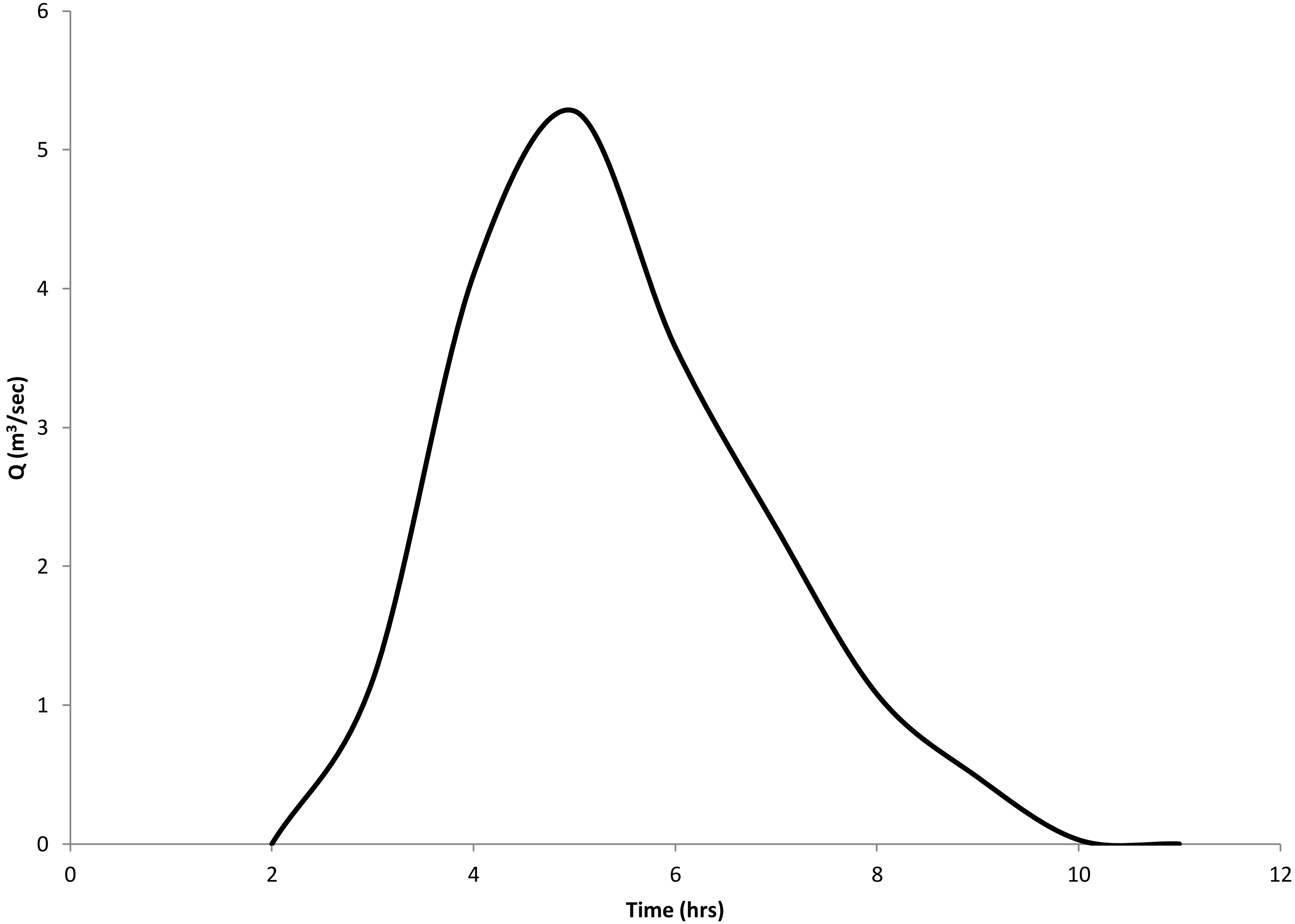
Saturation overland flow ➔ saturation from below

Contributing area

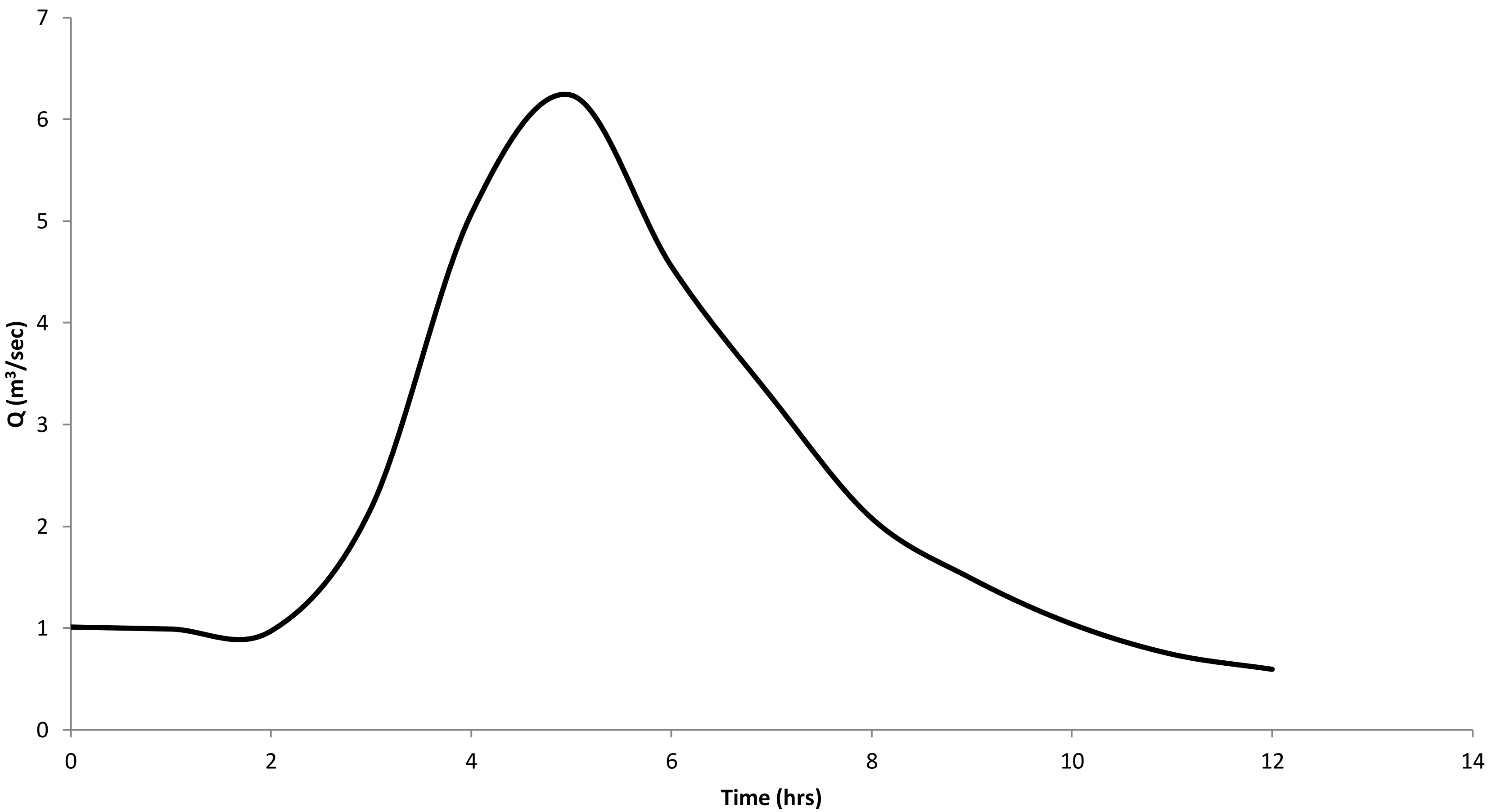
Does all the watershed contribute to streamflow during any given rainfall event ?


SHALLOW SOILS

Ephemeral streams



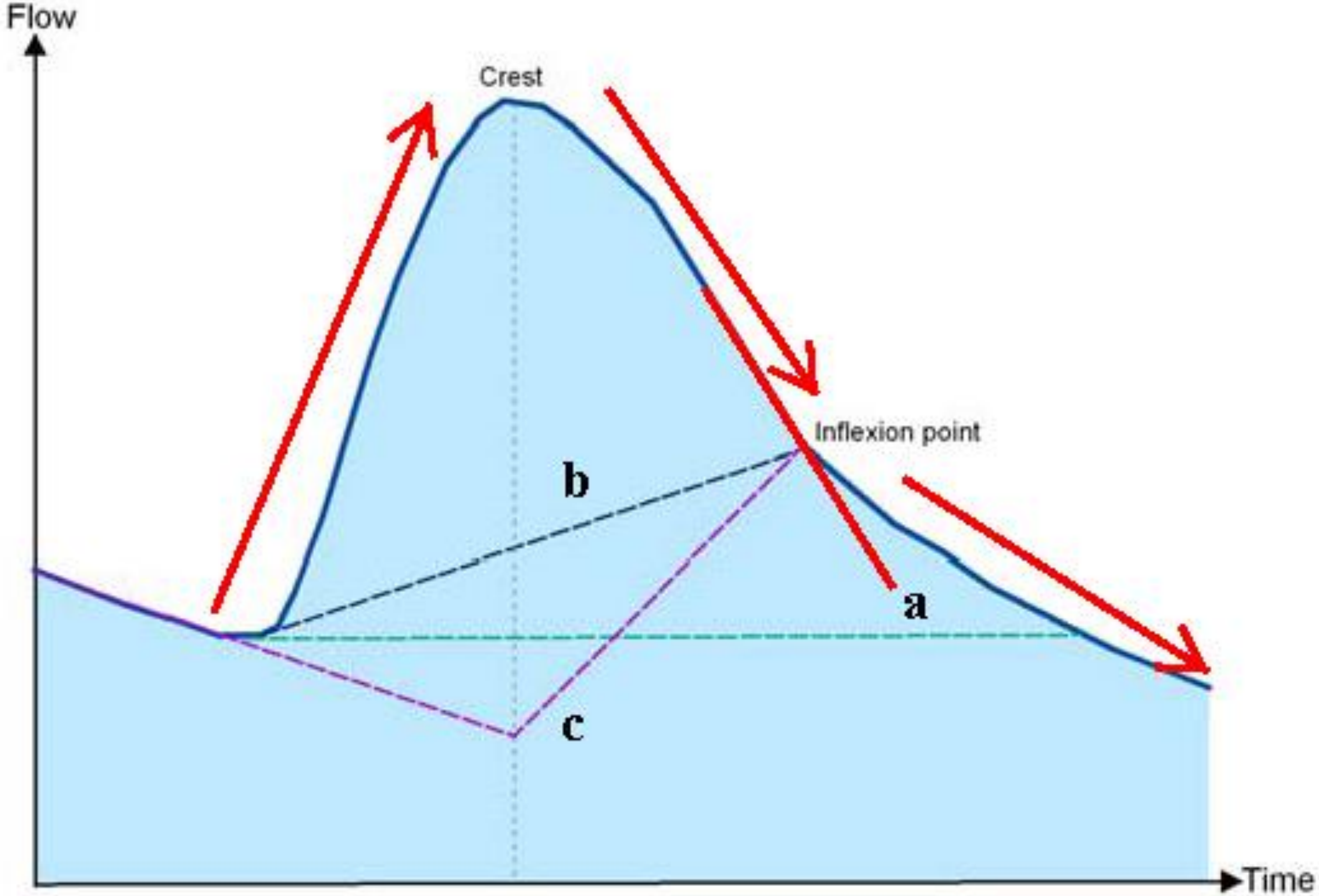
Perennial streams



Base flow separation

a. Straight line method
b. Constant slope method
c. Concave method

Base flow separation



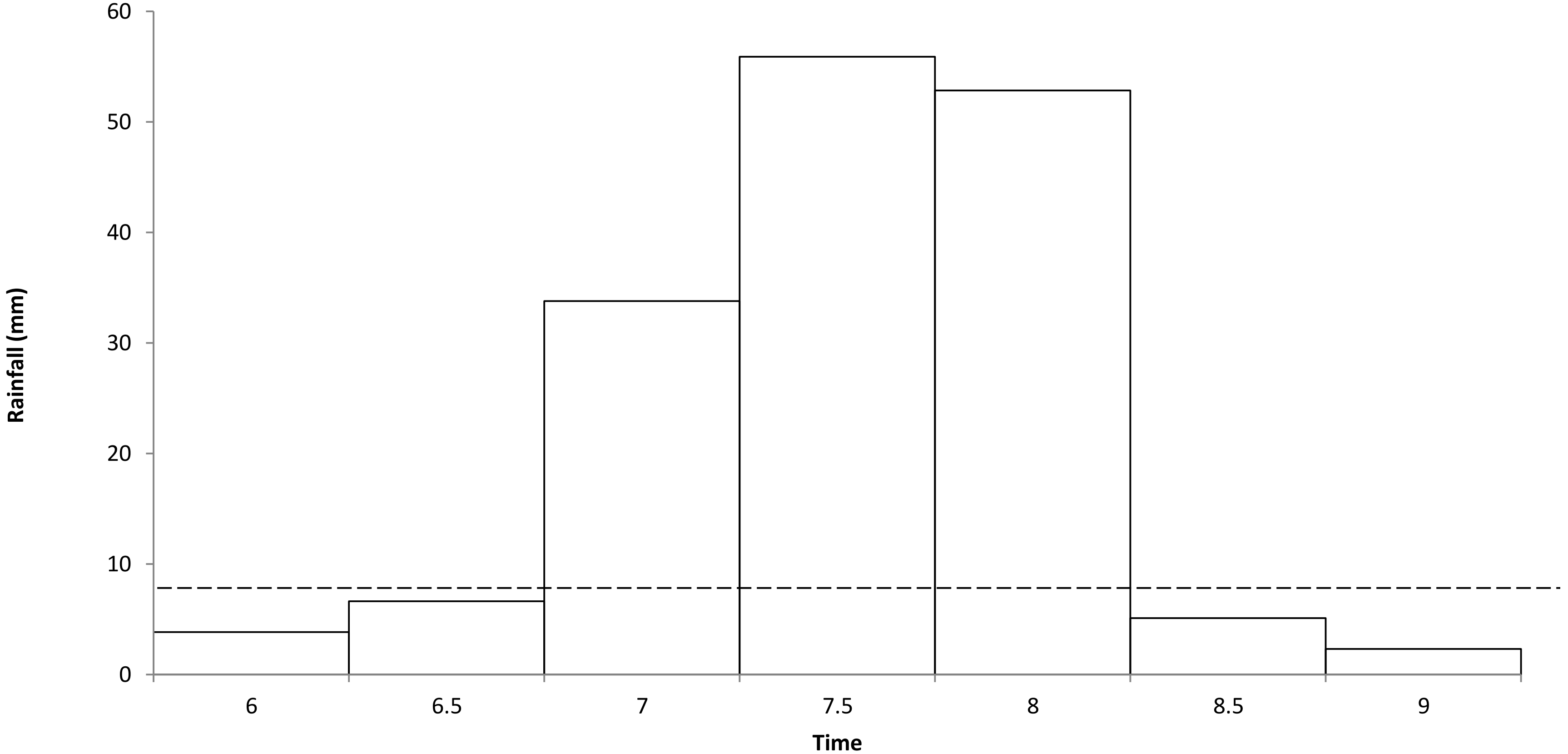
“Excess Rainfall”

1. $\emptyset$ $-$Index
2. Green-Ampt
3. SCS – curve number method

$\emptyset$ –Index

Definition: constant rate (mm/hr) of abstraction

Ø −Index



Example

Time	Rainfall (mm)	Direct runoff (m^3/sec)
1:00 a.m.	3.8	
1:30 a.m.	6.6	
2:00 a.m	33.8	12.1
2:30 a.m.	55.9	54.5
3:00 a.m.	52.8	150.0
3:30 a.m.	5.1	258.6
4:00 a.m.	2.3	300.9
4:30 a.m.		221.8
5:00 a.m.		111.0
5:30 a.m.		52.3
6:00 a.m.		39.7
6:30 a.m.		23.5
7:00 a.m.		8.9

Example

Total area of watershed is 18.2 km^2.

Depth of direct runoff:

$\frac{12.1+54.5}{2} \times 1800 + \frac{54.5+150}{2} \times 1800 + \cdots + \frac{23.5+8.9}{2} \times 1800$ = 2200852 m^3

$$Runoff\ depth = \frac{2200852}{18.2\ \times 10^6} = 0.121\,m$$
$$= 121\,mm$$

Example

$R_d = \sum_{m=1}^{M}(R_m - \emptyset\Delta t)$

Trial 1: (include all rainfall increments)

121 = 160.3-Ø x 7 x 0.5

Ø = 11.22 mm/hr ➔ Ø x 0.5 =6.55 mm

Total effective rainfall = 126.7 mm (Ø need to be adjusted)

Note: negative values are meaningless ➔ need to adjust until there is no –ve when subtract the involved incremental rainfall depth from Ø

Example

Trial 2 (only rainfall > 6.6 m)

121 = 142.5 – 3 x Ø x 0.5

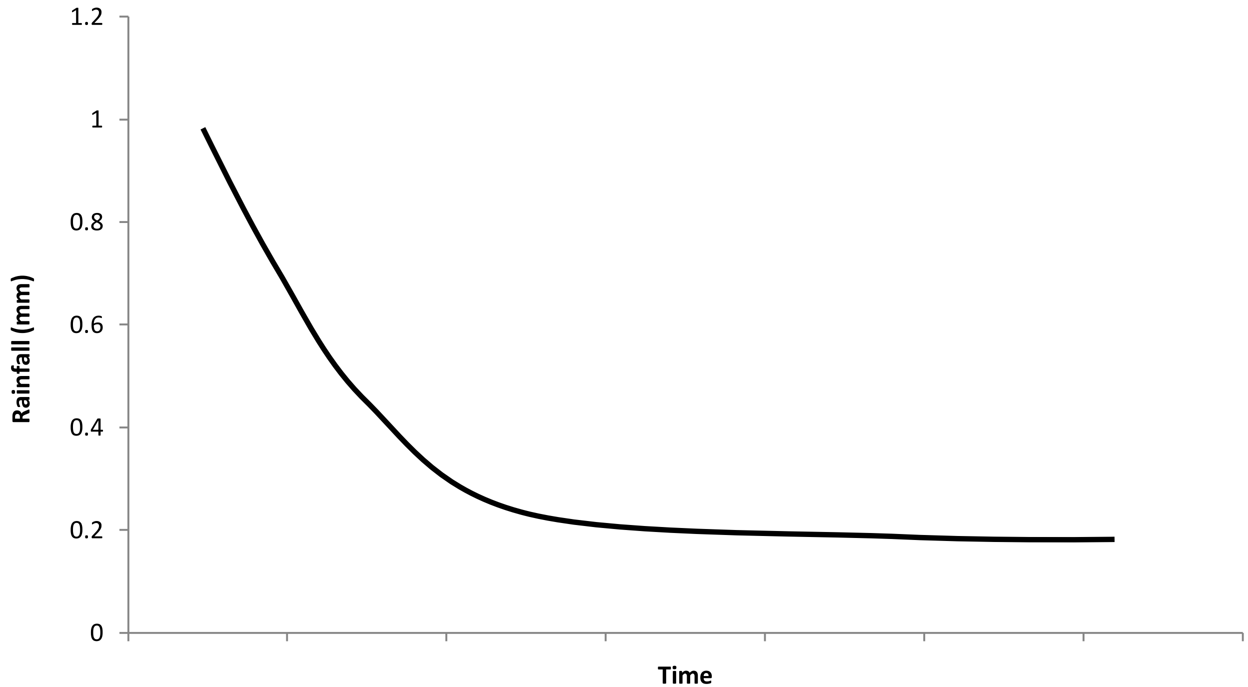
Ø = 14.33 mm/hr ➔ 14.33 x 0.5 = 7.165 mm

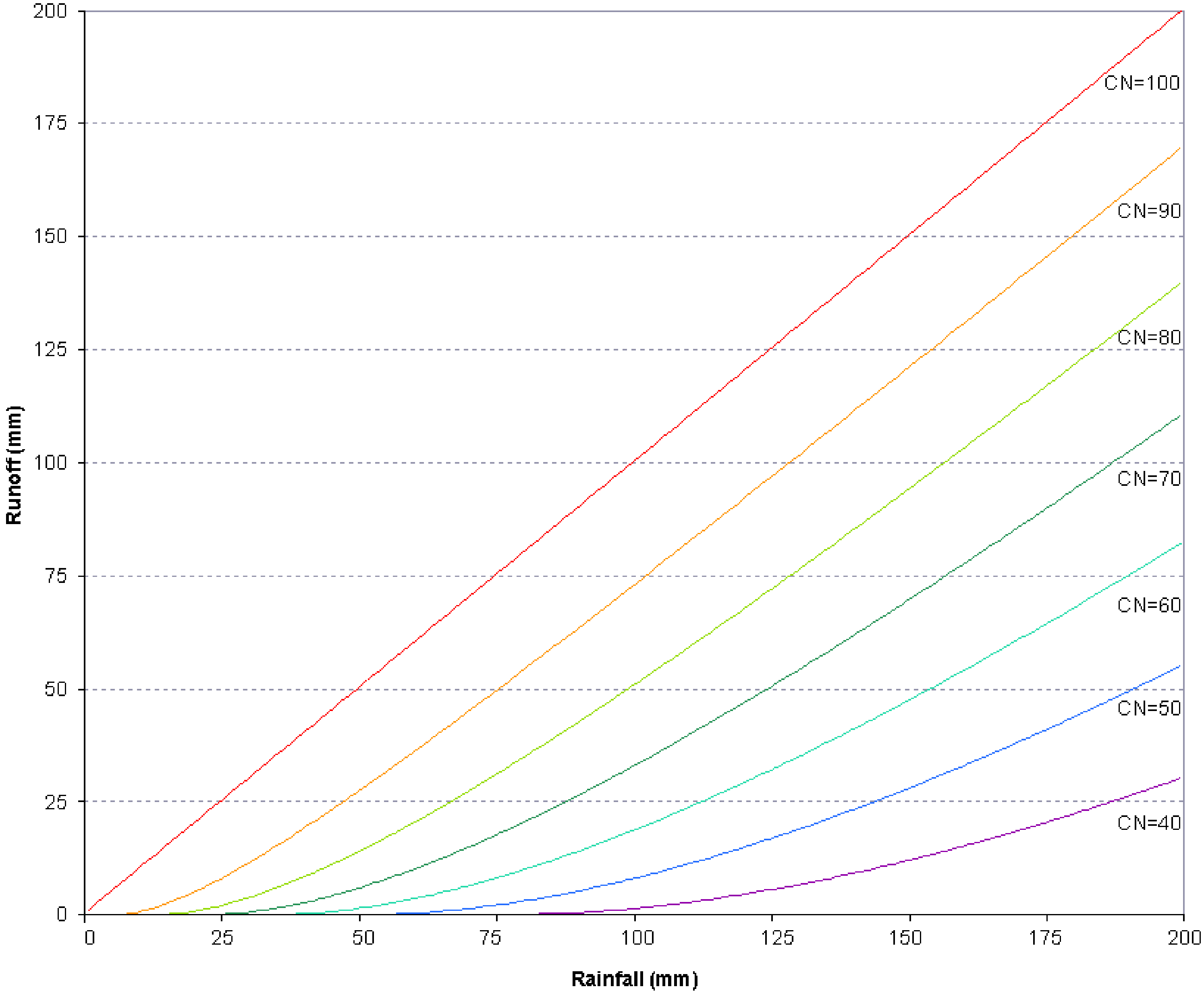
Total effective rainfall = 121 mm

Example

Time	Effective Rainfall (mm)	Direct runoff (m^3/sec)
1:00 a.m.		
1:30 a.m.		
2:00 a.m	26.64	12.1
2:30 a.m.	48.74	54.5
3:00 a.m.	45.64	150.0
3:30 a.m.		258.6
4:00 a.m.		300.9
4:30 a.m.		221.8
5:00 a.m.		111.0
5:30 a.m.		52.3
6:00 a.m.		39.7
6:30 a.m.		23.5
7:00 a.m.		8.9

SCS Curve Number (CN)





Runoff (mm)
200
175
150
125
100
75
50
25
0
CN=100
CN=90
CN=80
CN=70
CN=60
CN=50
CN=40
0
25
50
75
100
125
150
175
200
Rainfall (mm)

SCS Curve Number (CN)

$$P = P_e + I_a + F_a$$

$$\frac{F_a}{S} = \frac{P_e}{P - I_a}$$

$$F_a = \frac{S(P - I_a)}{P - I_a + S}$$

$$I_a = 0.2 \times S$$

$$P_e = \frac{(P - 0.2 \times S)^2}{P + 0.8 \times S}$$

SCS Curve Number (CN)

$$S = \frac{1000}{CN} - 10 \ \ (\text{inches})$$

$$S = 25.4 \ \times \left(\frac{1000}{CN} - 10\right) \ \ (mm)$$

Time	Rainfall (mm)	Cumulative rainfall (mm)	I_a *(mm)*	F_a *(mm)*	*Cumulative Effective rainfall (mm)*	*Effective Rainfall (mm)*	Effective rainfall (mm) By *Ø-index*
1:00 a.m.	3.8	3.8	3.8	0.0	0	0	0
1:30 a.m.	6.6	10.4	7.91	2.34	0.15	0.15	0
2:00 a.m.	33.8	44.2	7.91	18.92	17.37	17.22	26.64
2:30 a.m.	55.9	100.1	7.91	27.67	64.52	47.15	48.74
3:00 a.m.	52.8	152.9	7.91	31.07	113.92	49.40	45.64
3:30 a.m.	5.1	158	7.91	31.30	118.80	4.87	0
4:00 a.m.	2.3	160.3	7.91	31.39	121	2.2	0
4:30 a.m.							
5:00 a.m.							
5:30 a.m.							
6:00 a.m.							
6:30 a.m.							
7:00 a.m.							

Example

Effective rainfall = 121 mm

Total rainfall = 160.3 mm

$$121 = \frac{(160.3 - 0.2 \times S)^2}{160.3 + 0.8 \times S}$$

S = 39.54 mm

I_a = 0.2 x 39.54 = 7.91 mm

Example

$$39.54 = 25.4 \times \left(\frac{1000}{CN} - 10\right)$$

CN = 86.53

Hydrologic groups

A: (Low runoff potential). The soils have a high infiltration rate even when thoroughly wetted. They chiefly consist of deep, well drained to excessively drained sands or gravels. They have a high rate of water transmission.

Hydrologic groups

B: The soils have a moderate infiltration rate when thoroughly wetted. They chiefly are moderately deep to deep, moderately well-drained to well-drained soils that have moderately fine to moderately coarse textures. They have a moderate rate of water transmission.

Hydrologic groups

C: The soils have a slow infiltration rate when thoroughly wetted. They chiefly have a layer that impedes downward movement of water or have moderately fine to fine texture. They have a slow rate of water transmission.

Hydrologic groups

D. (High runoff potential). The soils have a very slow infiltration rate when thoroughly wetted. They chiefly consist of clay soils that have a high swelling potential, soils that have a permanent water table, soils that have a claypan or clay layer at or near the surface, and shallow soils over nearly impervious material. They have a very slow rate of water transmission.

CN- Antecedent soil water

- Three conditions :
  I. Dry condition (PWP) (CN1)
  II. Average (CN2)
  III. Wet (FC or above) CN3

CN- Antecedent soil water

$$CN1 = CN2 - \frac{20\times(100-CN2)}{100-CN2+\exp[2.533-0.0636\times(100-CN2)]}$$

$$CN3 = CN2 \times \exp[0.00673 \times (100 - CN2)]$$

$$CN1 = \frac{4.2 \times CN2}{10 - 0.058 \times CN2}$$

$$CN3 = \frac{23 \times CN2}{10 + 0.13 \times CN2}$$

CN2 examples

What is the CN2 for:

- Well drained sandy clay loam, cultivated with barley. Contoured seeding.
- Well drained planted with grasses. Soil :clay loam. Moderately grazed.
- Residential area. Impervious: 25%. Heavy clay claypan present.

Unit Hydrograph

- Definition:

Direct runoff resulting from 1 in (or 1 cm) of effective rainfall.

Unit hydrograph

n	U_n (m^3 /s. cm)
1	4.54
2	12.15
3	26.30
4	28.14
5	16.40
6	5.03
7	4.36
8	3.04
9	1.88

Convolution

$$Q_n = \sum_{m=1}^{n\,\leq M} P_m \times U_{n-m+1}$$

n=N-M+1

$Q_1 = P_1 \times U_1$

$Q_2 = P_1 \times U_2 + P_2 \times U_1$

$Q_3 = P_1 \times U_3 + P_2 \times U_2 + P_3 \times U_1$

$Q_4 = P_1 \times U_4 + P_2 \times U_3 + P_3 \times U_2$

....

$Q_N = 0{+}0 + \cdots + P_M \times U_{N-M+1}$

Example

Determine the direct runoff for a storm of effective rainfall of 5 mm in the first half an hour, 10 mm in the second hour, and 15 mm in the third half an hour.

n	U_n (m^3 /s. cm)	U_n x 0.5	U_n x 1.0	U_n x 1.5
1	4.54	2.27	4.54	6.81
2	12.15	6.075	12.15	18.225
3	26.30	13.15	26.3	39.45
4	28.14	14.07	28.14	42.21
5	16.40	8.2	16.4	24.6
6	5.03	2.515	5.03	7.545
7	4.36	2.18	4.36	6.54
8	3.04	1.52	3.04	4.56
9	1.88	0.94	1.88	2.82

n	U_n x 0.5	U_n x 1.0	U_n x 1.5	Q_n (m^3/s)
1	2.27			2.27
2	6.075	4.54		10.615
3	13.15	12.15	6.81	32.11
4	14.07	26.3	18.225	58.595
5	8.2	28.14	39.45	75.79
6	2.515	16.4	42.21	61.125
7	2.18	5.03	24.6	31.81
8	1.52	4.36	7.545	13.425
9	0.94	3.04	6.54	10.52
10		1.88	4.56	6.44
11			2.82	2.82

$$Q_n = \sum_{m=1}^{n \leq M} P_m \times U_{n-m+1}$$

$$Q_1 = P_1 \times U_{1-1+1}$$

$$12.1 = 2.664 \times U_1 \rightarrow U_1 = \frac{12.1}{2.664} = 4.54 \frac{m^3}{cm.s}$$

$$Q_2 = P_1 \times U_{2-1+1} + P_2 \times U_{2-2+1}$$

$$54.5 = 2.664 \times U_2 + 4.874 \times 4.54$$

$$U_2 = \frac{54.5 - 4.874 \times 4.54}{2.664} = 12.15$$

Example- calculate the one hour unit hydrograph

Time (hour)	Effective Rainfall (cm(	Direct runoff (m^3/sec)
1	0.254	35.32
2	0.508	423.78
3	0	1412.60
4	0.254	1977.64
5		1765.75
6		1589.18
7		882.88
8		353.15
9		176.58
Total volume		3.06×10^7 m^3
Area of watershed		3.02×10^3 km^2

$$Q_n = \sum_{m=1}^{n \leq M} P_m \times U_{n-m+1}$$

- $Q_n, n = N - M + 1 = 9 - 4 + 1 = 6$
- $Q_1 = P_1 U_1$
- $Q_2 = P_1 U_2 + P_2 U_1$
- $Q_3 = P_1 U_3 + P_2 U_2 + P_3 U_1$
- $Q_4 = P_1 U_4 + P_2 U_3 + P_3 U_2 + P_4 U_1$
- $Q_5 = P_1 U_5 + P_2 U_4 + P_3 U_3 + P_4 U_2$
- $Q_6 = P_1 U_6 + P_2 U_5 + P_3 U_4 + P_4 U_3$

One hour unit hydrograph

Time (hour)	Effective Rainfall (cm(	Ordinate (m^3/(sec. cm))
1	1	139.04
2		1390.35
3		2780.71
4		2085.53
5		1390.35
6		695.18
7		
8		
9		
Volume		2.09×10^7 m^3
Pe=Volume/Area		0.963 cm

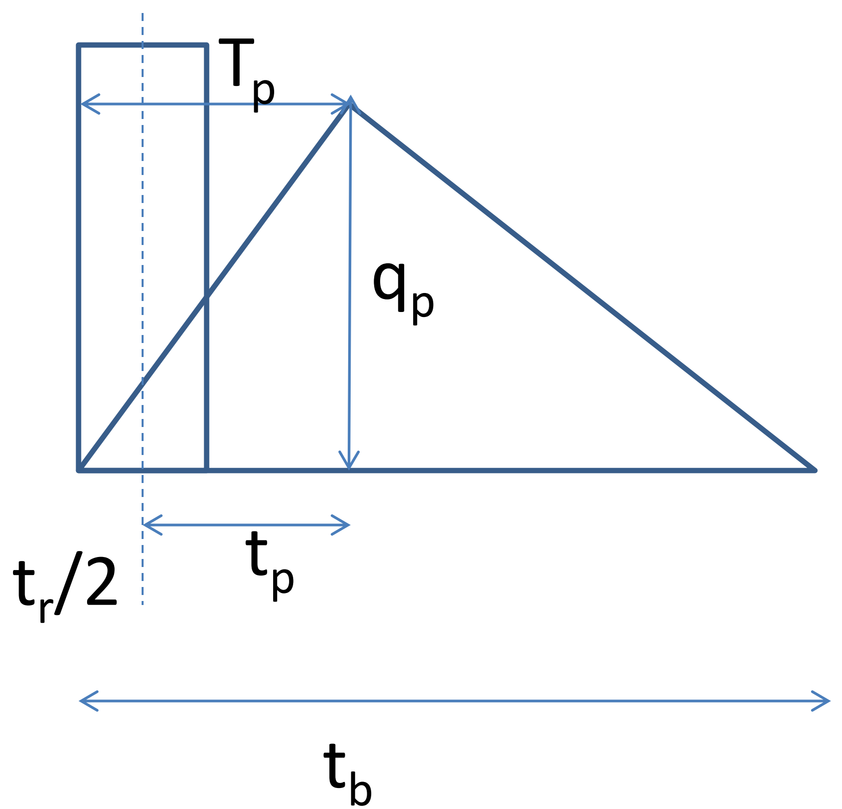
Synthetic Unit Hydrograph -SCS

$$q_p = \frac{C \times A}{T_p} = \frac{2.08 \times A}{T_p}$$

$$T_p = \frac{t_r}{2} + t_p$$

$$t_p = 0.6 \times t_c$$

$$t_b = 2.67 \times T_P$$



Example-Synthetic hydrograph

- Given
  - Watershed slope = 1.5%
  - Longest path = 2000 m
  - CN = 65
- Required
  - 0.5 hour unit hydrograph

- $t_{lag} = \frac{L^{0.8}(\frac{1000}{CN}-9)^{0.7}}{1900\ S^{0.5}}$ x1.67=3 hour
- tp=0.6 x 3 = 1.8 hours
- Tp=0.25+1.8 = 2.05
- t_b=2.67 x 2.05 =5.47
- q_p=(2.08 x 3)/2.05=3.0439 m^3/s

Check

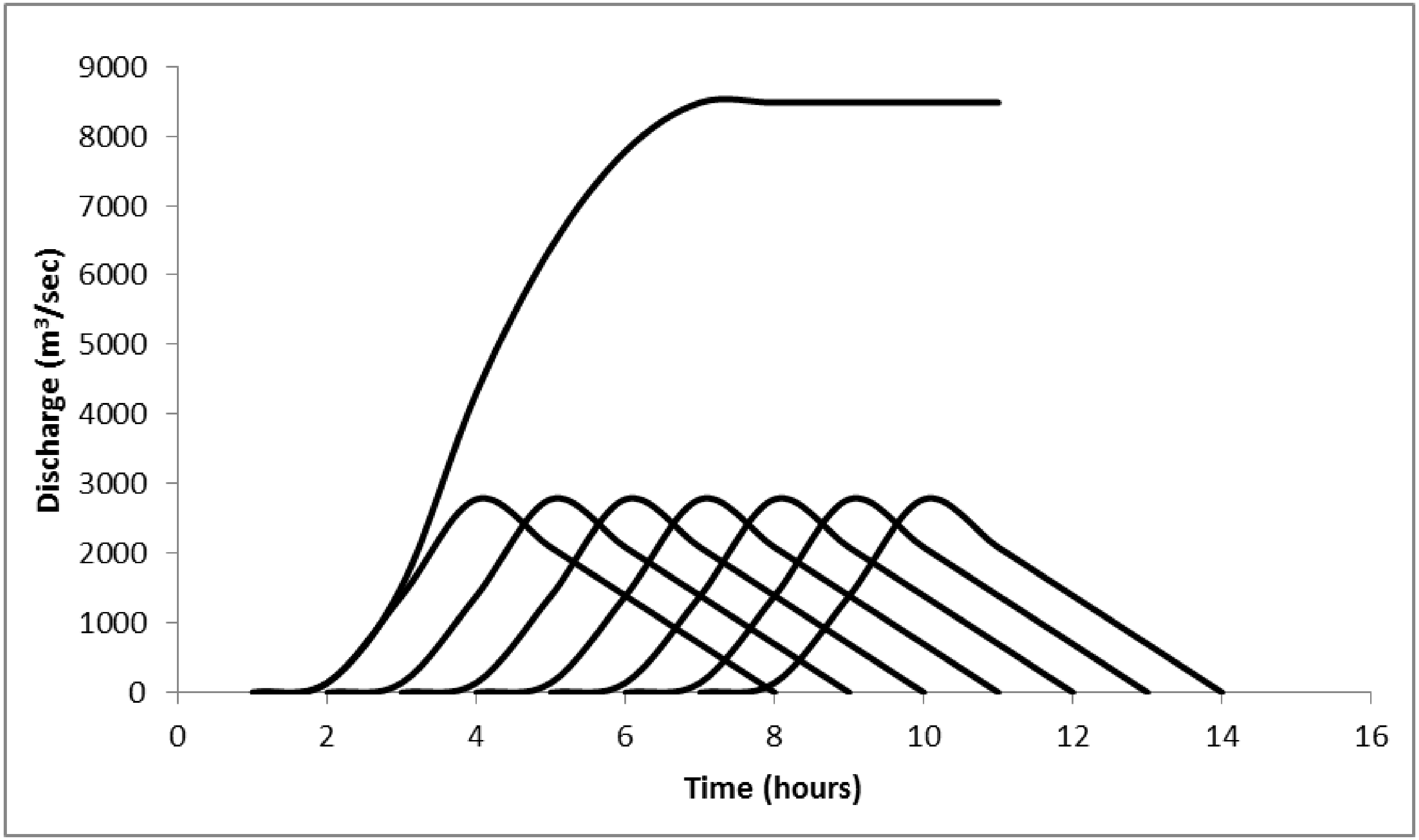
- Area =0.5 * 3.049 * 5.47 * 3600 = 30020 m^2
- = 30020/(3x 10^6 m^2) = 0.01 m = 1 cm

S - Hydrograph

- Storms come in different durations. Therefore it is necessary to change the duration of the unit hydrograph
- Mathematical definition is:

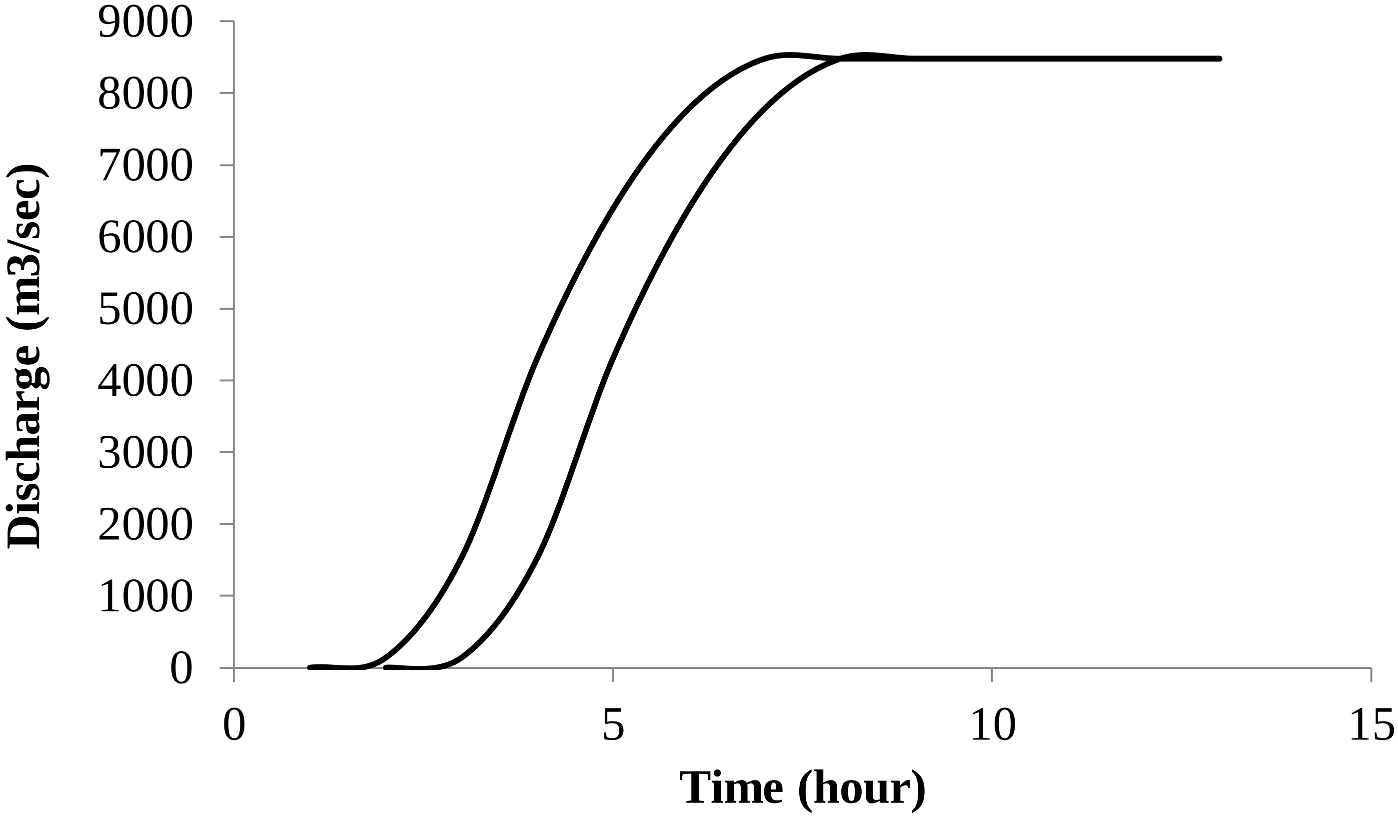
$$
\begin{aligned}
\mathrm{g}(t) \times \Delta t \\
&= \Delta t (h(t) + h(t - \Delta t) + h(t - 2\Delta t) + \cdots
\end{aligned}
$$

S-Hydrograph



UH	Lag 1 hr	2 hr	3hr	4 hr	5 hr	6 hr	7 hr	8 hr	9 hr	10 hr	S
0											0
139.04	0										139.04
1390.35	139.04	0									1529.39
2780.71	1390.35	139.04	0								4310.1
2085.53	2780.71	1390.35	139.04	0							6395.63
1390.35	2085.53	2780.71	1390.35	139.04	0						7785.98
695.18	1390.35	2085.53	2780.71	1390.35	139.04	0					8481.16
0	695.18	1390.35	2085.53	2780.71	1390.35	139.04	0				8481.16
	0	695.18	1390.35	2085.53	2780.71	1390.35	139.04	0			8481.16
		0	695.18	1390.35	2085.53	2780.71	1390.35	139.04	0		8481.16
			0	695.18	1390.35	2085.53	2780.71	1390.35	139.04	0	8481.16
				0	695.18	1390.35	2085.53	2780.71	1390.35	139.04	8481.16

Lagging S-Hydrograph



S Hydrograph

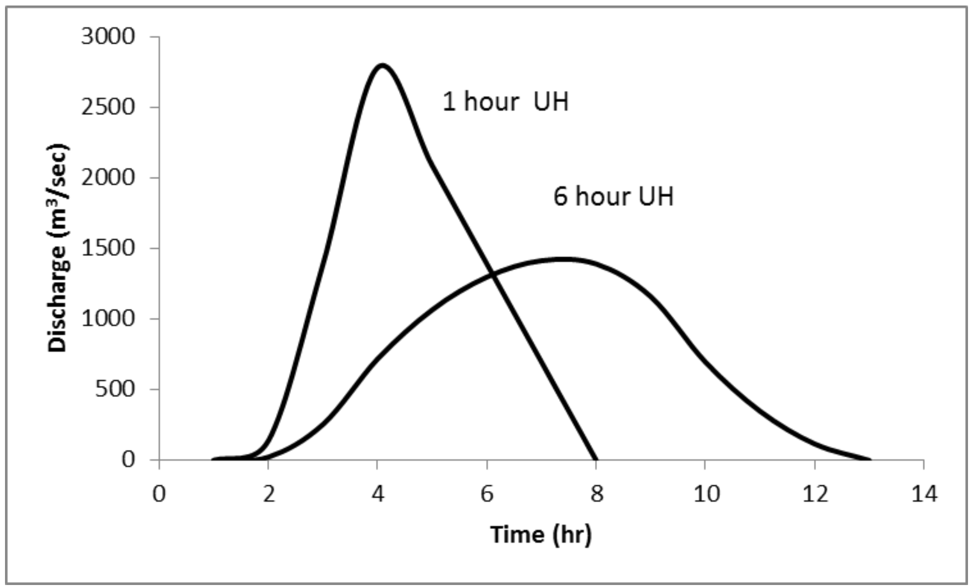
$$g(t) \times \Delta t = \Delta t(h(t) + h(t - \Delta t) + h(t - 2\Delta t) + \cdots$$

$$g(t - \Delta t) \times \Delta t = \Delta t(h(t - \Delta t) + h(t - 2\Delta t) + \cdots$$

$$h(t) = g(t)\text{-}g(t - \Delta t)$$

$$h\acute{(}t)\acute{\Delta}t = \Delta t(g(t)\text{-}g(t - \acute{\Delta}t))$$

(1) Time (hr)	(2) 1-hr S hydrograph	(3) 6 – hr lag of (2)	(4) (2)-(3)	(5) (6 hr UH) (4) * (1/6)
1	0		0	0
2	139.04		139.04	23.17333
3	1529.39		1529.39	254.8983
4	4310.1		4310.1	718.35
5	6395.63		6395.63	1065.938
6	7785.98		7785.98	1297.663
7	8481.16	0	8481.16	1413.527
8	8481.16	139.04	8342.12	1390.353
9	8481.16	1529.39	6951.77	1158.628
10	8481.16	4310.1	4171.06	695.1767
11	8481.16	6395.63	2085.53	347.5883
12	8481.16	7785.98	695.18	115.8633
13	8481.16	8481.16	0	0


3000
2500
2000
1500
1000
500
0
Discharge (m³/sec)
0
2
4
6
8
10
12
14
Time (hr)
1 hour UH
6 hour UH