Atmospheric Water

Hydrology 0604212

- Precipitation depend on two factors

1. Amount of water vapor
2. Mechanisms in converting water vapor into precipitation.

Quantification of water vapor

- ρ_v = density of water vapor (M/V)
- ρ_a = density of moist water (M/V)
- ρ_d = density of dry air (M/V)

Specific humidity (q_v)*:* ratio of water vapor density to density of moist air.

- $q_v = \frac{\rho_v}{\rho_a}$

Quantification of water vapor

- But the volume of air depends on the temperature so in order to calculate the mass of water as water vapor we need to use the ideal gas law:

$$PV=nRT$$

Where

P: Pressure of gas (pa)

V: volume of gas (m^3)

n: number of moles (mol)

R: ideal gas constant (= 8.314 Joule/(K.mol)

T: Temperature (K)

Some units conversion

- Pressure units

$$Pressure = \frac{Force}{Area}$$

Force = Mass x gravitational acceleration

= Kg x $\frac{m}{s^2}$ = N (Newton)

$P = \frac{N}{m^2}$ ➔ pa

Some unit conversion

- Other pressure units
- 1000 pa = 1 Kpa
- 1 bar = 100000 pa → 100 kpa
- 1 atm = 101325 pa → 101.325 kpa
- 1 bar = 1020 cm → 10.2 m of water

Energy units:

1 Joule = N x m = Pa x m^3

Back to the ideal gas law

- The law can be written as:
- $PV = \frac{M}{M_w} RT$

➔ $P = \frac{M}{V} \frac{R}{M_w} T$

➔ $P = \rho \frac{R}{M_w} T$

Back to the ideal gas law

- For water vapor

$$e = P_v = \rho_v R_v T$$

Where

$$R_v = \frac{R}{M_v}$$

Also for dry air and moist air

$$R_d = \frac{R}{M_d}; R_a = \frac{R}{M_a}$$

Back to the ideal gas law

- For dry air

$$P_d = \rho_d R_d T$$

- For moist air

$$P_a = \rho_a R_a T$$

Note that according to Dalton's partial pressure law
:

$P_a = P_d + e$ or

$P_a = \rho_d R_d T + \rho_v R_v T$

Let’s take look at the molecular weights

- Dry air consists of
- $\cong$ 78% N_2
- $\cong$ 21% O_2
- < 1% other gasses

Molecular weights

N = 14 g

O = 16 g

H =1 g

Let's take look at the molecular weights

- M_w dry air=28 x 0.78 + 0.21 x 32 ≅ 29 g
- M_w of water vapor = 18 g

$$\frac{M_v}{M_d} = \frac{18}{29} \approx 0.622$$

$$R_d = \frac{R}{M_d} = \frac{8.314}{0.029} \approx 287 \text{J/Kg.K}$$

$$R_v = \frac{R}{M_v} = \frac{8.314}{0.018} \approx 462 \text{ J/Kg.K}$$

$$\frac{R_d}{R_v} = \frac{M_v}{M_d} = 0.622 \rightarrow R_d = 0.622 \times R_v$$

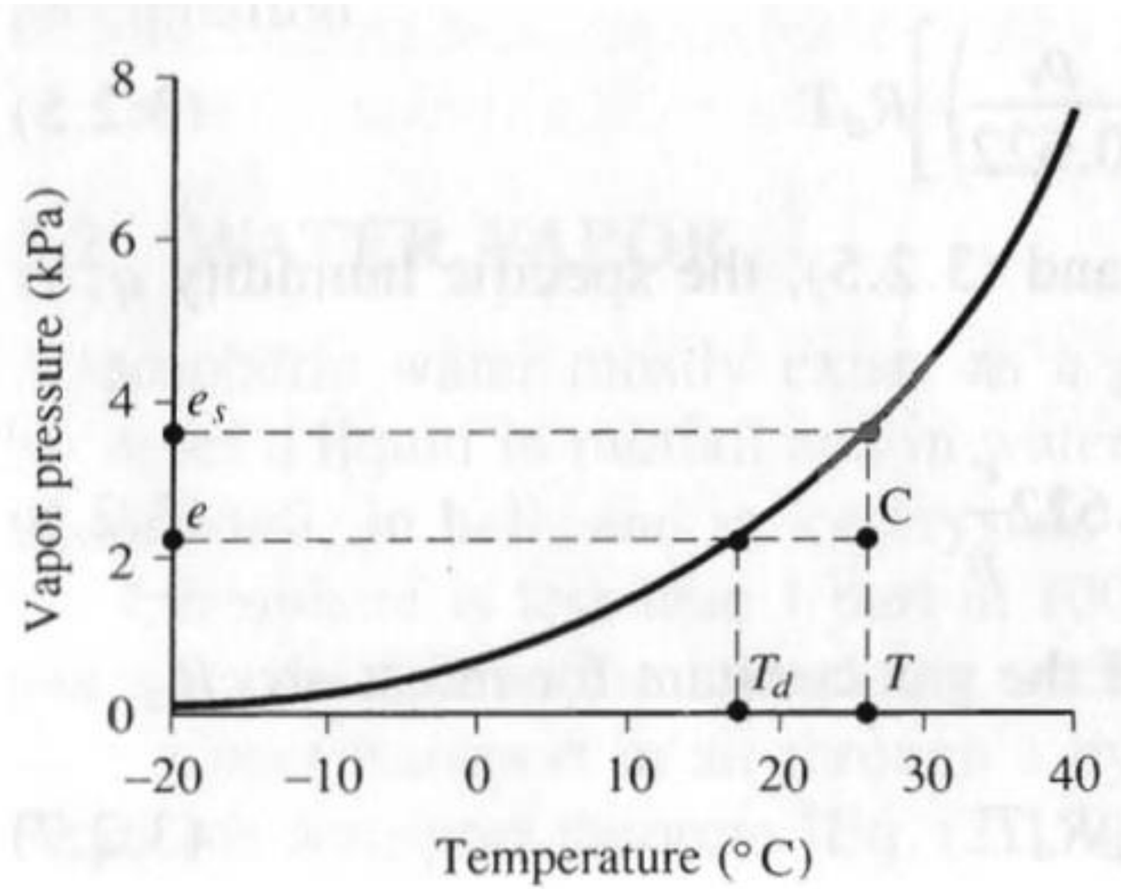
Specific humidity again

- $P_a = \rho_d R_d T + \rho_v R_v T$
- $P_a = \rho_d R_d T + \rho_v \frac{R_d}{0.622} T$
- $P_a = R_d T(\rho_d + \frac{\rho_v}{0.622})$
- $\frac{e}{p} = \frac{\rho_v R_v T}{R_d T(\rho_d + \frac{\rho_v}{0.622})} = \frac{\rho_v}{0.622\ x\ (\rho_d + \frac{\rho_v}{0.622})}$
- $q_v \cong 0.622 \frac{e}{p}$

Ra = f(Rd)

- $\rho_a R_a T = \rho_d R_d T + \rho_v R_v T$
- $R_d = 0.622 \times R_v$
- $\rho_a = \rho_d + \rho_v$
- Put these information together ➔
- $R_a = \frac{\rho_d}{\rho_a} R_d + \frac{\rho_v}{\rho_a} \frac{R_d}{0.622}$
- $R_a = \frac{\rho_d}{\rho_a} R_d + q_v \frac{R_d}{0.622}$ note that $\rho_d = \rho_a - \rho_v$
- ➔ $R_a = R_d(1 + 0.608 q_v)$

Maximum water content in air



Maximum water content in air

- Saturation vapor pressure: Maximum water content the air can hold at a given temperature.

$$e_s = 611\,\exp\left(\frac{17.27\,T}{237.3 + T}\right)$$

T: Temperature (^{o}C)

Maximum water content in air

- $\frac{\partial e_s}{\partial T} = \Delta = \frac{4098 e_s}{(237.3+T)^2}$
- Relative humidity:
- $R_h = \frac{e}{e_s}$
- Actual partial pressure can be calculated from dew-point temperature which is the temperature at which air becomes saturated and measured by wet bulb thermometer.

Examples

- Calculate *e, R_h, q_v* and ρ_a if we know *T* , and T_d and P_a.

If P_a = 1 atm, T=22 ^oC and T_d= 17 ^OC then

$$e_s = 611\exp\left(\frac{17.27\ x\ 22}{237.3+22}\right) = 2644.8\text{ pa}$$

$$e = 611\exp\left(\frac{17.27\ x\ 17}{237.3+17}\right) = 1938.4\text{ pa}$$

Examples

- $R_h = \frac{1938.4}{2644.8}$=0.73 ➔ 73%
- $\rho_a = \frac{P_a}{R_a x T}$
- $R_a = R_d(1 + 0.608 q_v)$
- $q_v = 0.622\ x\ \frac{1938.4\ pa}{101325\ pa}$ =0.0119
- $R_a = 287(1 + 0.608\ x\ 0.0119)$=
- 289.1 J/(Kg . K)

Examples

- $\rho_a = \frac{101325}{289.1\ x(22+273)} = 1.188\,\frac{kg}{m^3}$

Which is more dense a dry air or moist air ?

Let’s see

If R_h= *35%* at the same temperature *T=22 then*

e = 0.35 x 2644.8 pa = 925.7 pa

$$q_v = 0.622\ x\ \frac{925.7\ pa}{101325\ pa} = 0.005683$$

Examples

- $R_a = 287(1 + 0.608\ x0.005683)$=288 J/ (Kg K)
- $\rho_a = \frac{101325}{288\ x\ (22+273)} = 1.193\ \frac{kg}{m^3}$
- Decrease in water vapor increases air density
- With ρ_a is known, and for a known volume we can determine the weight of air and from q_v the weight of water vapor.

Examples

- For example for every 1 m^3 of water the weight of water vapor at $R_h = 0.73$ is :
- 1 x 1.188 x 0.0119 = 0.014137 kg =14.1 g
- at $R_h = 0.35$ is :
- 1 x 1.192 x0.005683 = 0.006777 kg = 6.8 g

Can we use the same concept to calculate the mass of water in *static* atmospheric air column ?

Precipitable water

- We know the temperature decreases with altitude ➔value *e, R_h*, q_v and ρ_a vary with altitude.

Generally

$$m = \int_{z1}^{z2} q_v \rho_a A dz$$

But we don't have a closed formulas to describe the relation of q_v and ρ_a also notice the interdependence between *P* and q_v and ρ_a

Precipitable water

- Therefore we need to approximate the solution i.e.
  - divide the atmospheric column into section
  - Calculate the atmospheric variables at the beginning and then end of each section
  - Take the average values for the beginning and the end of the section.
  - The averages approximately represents the atmospheric variables for each section

Precipitable water

- $m = A\sum_{n=1}^{N} \overline{q_{v1}}\,\overline{\rho_a}\Delta H$
- The temperature decrease linearly as the altitude increases:
- $\frac{\partial T}{\partial H} = -\propto$

Thus:

$T_2 = T_1 - \propto (H_2 - H_1)$

Precipitable water

- Pressue

$P=\rho_a gH$

Pressure also changes with elevation

$$\frac{\partial P}{\partial H} = -\rho_a g$$

And according to ideal gas law

$$P = \rho_a R_a T$$

Precipitable water

- $\frac{\partial P}{\partial H} = -\frac{Pg}{R_a T}$
- Note that R_a was treated as constant
- $\frac{\partial T}{\alpha} = -\partial H$
- ➔ $\frac{\partial P}{p} = \frac{g}{R_a \alpha} \frac{\partial T}{T}$
- Taking advantage of *ln* characteristics
- $ln\left(\frac{P_2}{P_1}\right) = \frac{g}{R_a \alpha} \ln\left(\frac{T_2}{T_1}\right)$ ➔ $\left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{g}{R_a \alpha}}$

Precipitable water

To simplify the problem further we assume the static water column is fully saturated.

Let's the divide the water column into 2 km sections, let the temperature be 20 ^oC and pressure at H=0 is 1 atm, *assume* R_a *= 287 J/ (kg K),* Temperature lapse rate (α) $=$*0.0065* $^oC/m$

Precipitable water

- Solution
- Section 1 (H_1 =0 , H_2 = 2 Km)

  At H_1

  P_1 = 101325 pa

  $e_{s1} = 2339$ *pa*

  $q_{v1} = 0.622 \times (2339/101325) = 0.0144$

  $\rho_{a1} = 101325/(287 \times 293) = 1.20$

Precipitable water

- At H_2

  $T_2 = 20 - 0.0065 \times 2000 = 7\ ^oC$

  $e_{s2} = 1002$ pa

  $$P_2 = P_1 \left(\frac{280}{293}\right)^{\frac{9.8}{287\ x\ 0.0065}}$$

$P_2 = 79832$ pa ➔ 79.83 kpa

$q_{v2} = 0.622\ x\ (1002/79832) = 0.008$

$\rho_{a2} = 79832/(287\ x\ 280) = 0.99\ kg/m^3$

Precipitable water

- ➔ $\overline{q_v}$ =0.5 x (0.008+0.0144) =0.0112
- $\overline{\rho_a}$ =0.5 x (1.2+0.99) =1.095 kg/m^3
- *M = 1 x 2000 x 1.095 x 0.0112 = 24.53 kg*

Section 2 (H_2 =2000, H_3=4000)

At H_3

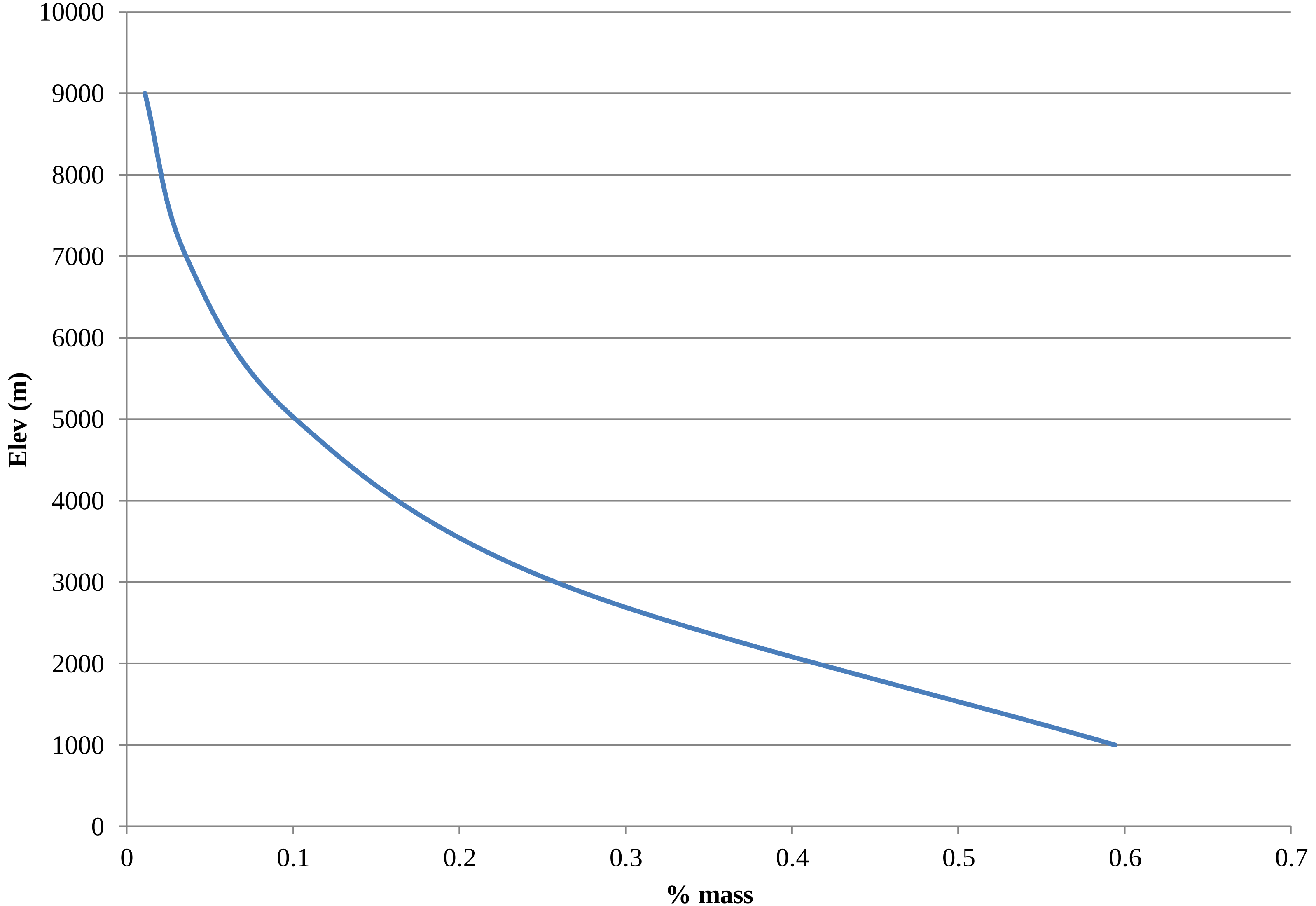
T_3 = 7 – 0.0065 x 2000 = -6

e_s = 390 pa

Precipitable water

- $P_3 = 79832 \times \left(\frac{267}{280}\right)^{\frac{9.8}{287\, x\, 0.0065}}$
- P_3 = 62189 pa
- ρ_{a3} = 62189/(287 x 267) = 0.81 kg/m^3
- q_{v3}= 0.622 x (390/62189)=0.0039
- $\overline{q_v} =$0.5 x (0.008+0.0039) =0.00595
- $\overline{\rho_a} =$0.5 x (0.99+0.81) =0.9 kg/m^3
- *M = 1 x 2000 x 0.9 x 0.00595 = 10.71 kg*

Elev (m)	T (^{o}C)	P (Kpa)	e_s (Kpa)	qv	Density (Kg/m$^{3+)}$	average qv	average density (Kg/m^3)	average mass (kg)	mass %
0	20	101.325	2.34	0.01436	1.20				
2000	7	79.83	1.00	0.00781	0.99	0.01108	1.10	24.37	0.59
4000	-6	62.19	0.39	0.00390	0.81	0.00586	0.90	10.57	0.26
6000	-19	47.84	0.14	0.00177	0.66	0.00284	0.73	4.16	0.10
8000	-32	36.31	0.04	0.00071	0.52	0.00124	0.59	1.46	0.04
10000	-45	27.13	0.01	0.00025	0.41	0.00048	0.47	0.45	0.01
								41.01	1.00



10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
Elev (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
% mass