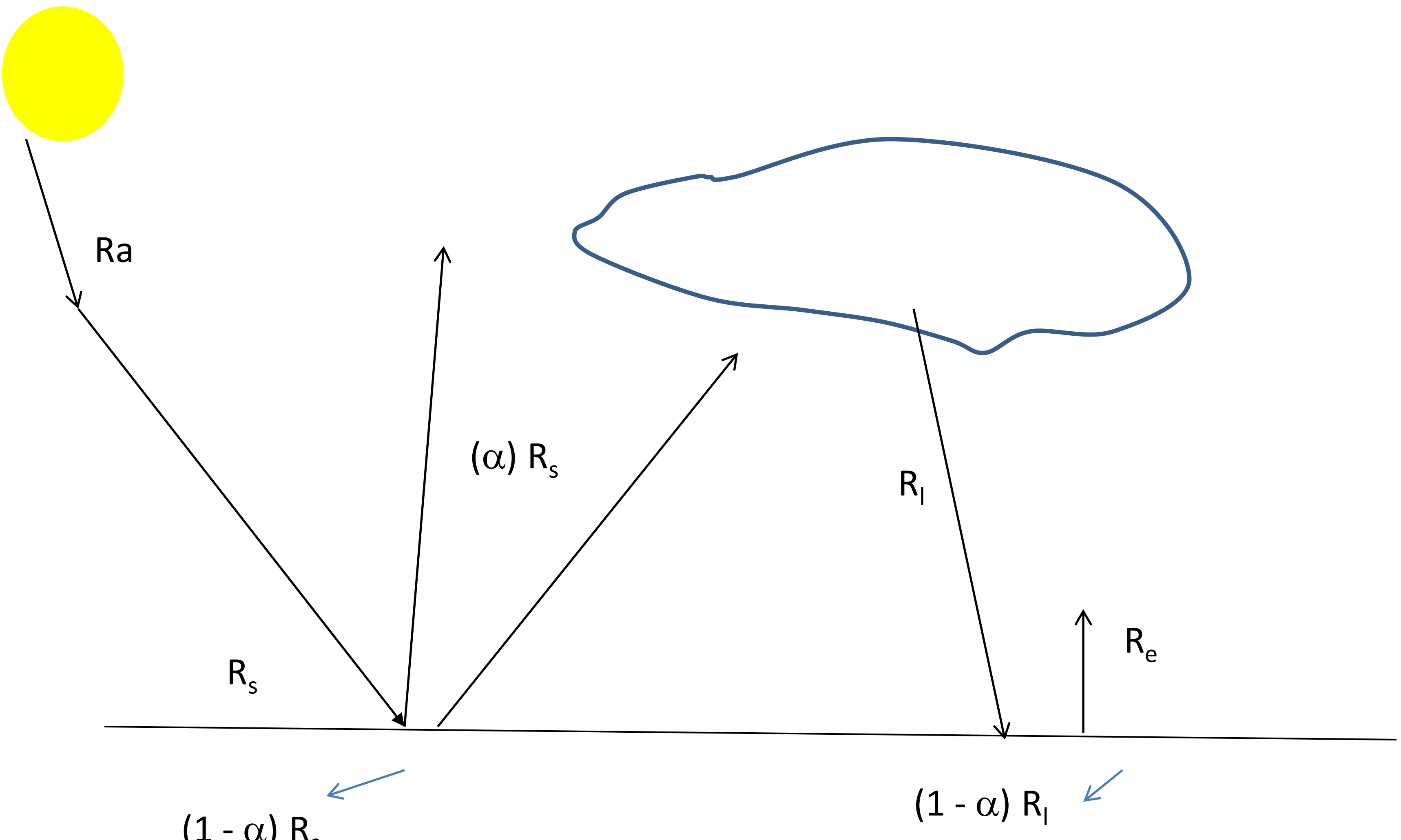
Evapotranspiration

Hydrology 604212

Energy balance



$$R_n = (1 - \alpha)(R_s + R_l) - R_e$$

Albedo

- Albedo (α): The surface reflectance of the solar radiation.
- Water absorb most of the incoming radiation, $\alpha \cong 0.06$.
- Snow reflects most of the incoming radiation, $\alpha \cong 0.90$
- As soil water content increases the α decreases.

Units

- Solar Radiation unit is Watt/m^2
- Watt = N. m/s ➔ J/s.
- Langley = 41840 J/m^2
- Calorie = 4.184 J

Net radiation balance

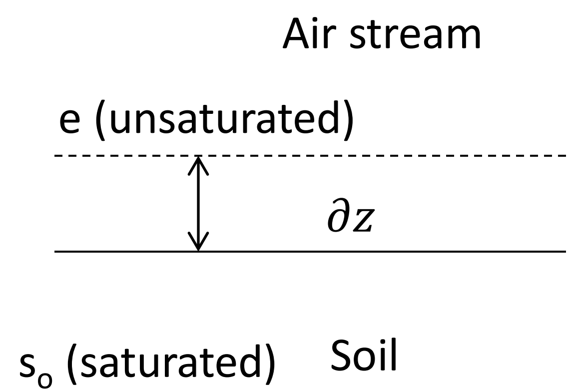
- $R_n = \lambda E + G + H$
- E : Evaporative flux (Kg /(m^2 s)
- λ : Latent heat of vaporization (J/kg)
- G: Ground heat flux (Watt / m^2)
- H: Sensible heat flux (Watt / m^2)
- $\lambda = 2.501 \times 10^6 - 2370 \times T$
- T: Temperature (^{O}c)

Evaporative flux

- For evaporation to occur :
  - Water
  - Energy (R_n)
  - Vapor pressure gradient between the air stream and inside the soil.

Evaporative flux

- $E = -\rho_a K_w \frac{\partial q_v}{\partial z}$
- ➔ $E = -\frac{0.622\rho_a}{P r_{av}} \partial e$
- Where:
- $r_{av} = \frac{\partial z}{K_w}$
- $\partial e = e_o - e$



K_w: Diffusivity coefficient (m^2/s)

r_{av} : aerodynamic resistance for vapor pressure

Sensible heat flux

- $H = -\rho_a C_p K_H \frac{\partial T}{\partial z}$
- ➔ $H = -\frac{\rho_a C_p}{r_{ah}} \partial T$
- Where:
- $r_{ah} = \frac{\partial z}{K_H}$
- $\partial T = T_o - T$
- c_p is the specific heat for the moist air at constant pressure. The average value is 1013= J/(kg . ^{0}C)
- r_{ah} : aerodynamic resistance for heat transfer.

Bowen ratio

- $\beta = \frac{H}{\lambda E}$
- $\beta = \frac{-\frac{\rho_a C_p}{r_{ah}} \partial T}{-\frac{0.622 \lambda \rho_a}{P r_{av}} \partial e}$
- If $r_{ah} \cong r_{av}$ ➔ r_a
- $\beta = \frac{P C_p \partial T}{0.622 \lambda \partial e}$

- $\beta = \gamma \frac{\partial T}{\partial e}$
- where
- $\gamma = \frac{P C_p}{0.622 \lambda}$
- γ : psychrometric constant

Back to R_n

- $R_n = \lambda E + G + H$
- Using Bowen ratio
- $R_n = \lambda E + G + \beta \lambda E$
- Rearrangement:
- $\lambda E = \frac{R_n - G}{\beta + 1}$
- ➔ $\lambda E = \frac{R_n - G}{\gamma \frac{\partial T}{\partial e} + 1}$

(cont’d)

- Recall $\frac{\partial e_s}{\partial T} = \Delta = \frac{4098 e_s}{(237.3+T)^2}$
- Note that ∂e_s represents the change in *saturated* vapour pressure
- $\partial e_s \neq \partial e$
- But $\frac{\partial e_s}{\Delta} = \partial T$
- Thus
- $\lambda E = \frac{R_n - G}{\gamma \frac{\partial e_s}{\Delta \partial e} + 1}$

(Cont'd)

- $\partial e_s = e_o - e_s$
- $\partial e = e_o - e$
- Also $\partial e_s = (e_o - e) - (e_s - e)$
- $\frac{\partial e_s}{\partial e} = \frac{(e_o - e) - (e_s - e)}{e_o - e} = 1 - \frac{(e_s - e)}{e_o - e}$
- $E = -\frac{0.622\rho_a}{Pr_a}\partial e \Rightarrow \lambda E = \frac{C_p \rho_a}{\gamma r_a}(e_o - e)$
- Also the fictitious value E_a

(Cont'd)

- Also let's define the fictitious evaporative flux E_a
- $\lambda E_a = \frac{C_p \rho_a}{\gamma r_a}(e_s - e)$
- $\frac{\partial e_s}{\partial e} = 1 - \frac{\lambda_{E_a}}{\lambda_E}$
- $\lambda E = \frac{R_n - G}{\frac{\gamma}{\Delta}\left(1 - \frac{\lambda_{E_a}}{\lambda_E}\right) + 1}$

(cont'd)

- Rearrangement
- $\lambda E = \frac{\Delta(R_n - G) + \gamma E_a}{(\gamma + \Delta)}$ ➔ combination equation
- $\lambda E = \frac{\Delta(R_n - G) + \frac{c_p \rho_a (e_s - e)}{r_a}}{(\gamma + \Delta)}$ ➔ Penman Monteith
- But this equation is for bare soil only.

Cont'd

- To consider we need to include a term for the resistance of the stomata
- $\lambda E = \frac{C_p \rho_a}{\gamma(r_a + rs)} (e_o - e)$
- $\beta = \frac{-\frac{\rho_a C_p}{r_{ah}} \partial T}{-\frac{0.622 \lambda \rho_a}{P(r_a + rs)} \partial e}$
- $\beta = \frac{\gamma \partial T}{\partial e} (\frac{(r_a + rs}{ra})$

Cont’d

- $\lambda E = \frac{R_n - G}{\gamma \frac{\partial e_s}{\Delta \partial e}(\frac{ra + rs}{ra}) + 1}$

- $\lambda E = \frac{R_n - G}{\frac{\gamma}{\Delta}\left(1 - \frac{\lambda_{Ea}}{\lambda_{E}}(\frac{ra}{ra + r_s})\right)(\frac{ra + rs}{ra}) + 1}$

- Rearrangement

- $\lambda E = \frac{\Delta(R_n - G) + \frac{C_p \rho_a (e_s - e)}{ra}}{\gamma(\frac{ra + rs}{ra}) + \Delta}$

Cont'd

- Priestley and Taylor
- $\lambda E = \alpha \frac{\Delta(R_n - G)}{\gamma\left(\frac{ra + rs}{ra}\right) + \Delta}$
- $\alpha \cong 1.3$

Cont'd

- $r_a = \frac{ln\left(\frac{z_m - d}{z_{om}}\right) ln\left(\frac{z_h - d}{z_{oh}}\right)}{k^2 u_z}$

- z_m: height of wind measurements (m)
- z_h: height of humidity measurements (m)
- d: zero plane displacement height (m)
- z_{om}: roughness length for momentum transfer (m)
- z_{oh}: roughness governing transfer of heat and vapor (m)
- *K*: von Karman's constant (0.41)
- u_z: wind speed at height z (m/s)

Cont'd

- d =2/3 . h (h is the crop height)
- z_{om} =0.123.h
- Z_{oh}=0.1.z_{om}

Thus of for a plant height of 12 cm (grass as a reference crop)

$$r_a = \frac{208}{u_2}$$

Cont'd

- $r_s = \frac{r_1}{0.5LAI}$
- r_s: bulk stomatal resistance (s/m)
- r_1:bulk stomatal resistance of will illuminated leaf (s/m)

For the reference grass r_1=100 and LAI =24.h

Then

r_s =70 s/m

Cont'd

For grass as reference crop

$$\frac{r_a + r_s}{r_a} = 1 + 0.34u_2$$

Example

- R_n=200 W/m^2

  T = 25 ^oC

  T_d =15 ^oC

  C_p= 1005 J.Kg/^oC

  P = 1atm

  z_2= 3 m/s

ET_0 grass reference evapotranspiration, using Penman Monteith and Priestly and Taylor equations ?

Example

- $\lambda E = \dfrac{\Delta(R_n - G) + \dfrac{c_p \rho_a (e_s - e)}{r_a}}{\gamma\left(\dfrac{r_a + rs}{ra}\right) + \Delta}$
- $r_a = \dfrac{208}{u_2} = \dfrac{208}{3}$ =69.3 s/m
- $\dfrac{r_a + r_s}{r_a} = 1 + 0.34u_2 = 1 + 0.34 \times 3 = 2.02$
- $\gamma = \dfrac{PC_p}{0.622\lambda}$

Example

- $\gamma = \frac{PC_p}{0.622\lambda} = \frac{101325 \times 1005}{0.622 \times \lambda}$
- λ = 2.501 x 10^6 – 2370 x 25=2441750 J/Kg
- γ =67.05 Pa/ °C
- $e_s = 611 \exp\left(\frac{17.27\, T}{237.3+T}\right) = 611 exp \frac{17.27 \times 25}{237.3+25}$
- $e_s = 3169$ Pa
- $\Delta = \frac{4098 \times 3169}{(237.3+25)^2} = 188.8$ Pa/°C

Example

- $e = 611exp\frac{17.27\times15}{237.3+15} = 1705.9\ pa$
- $q_v = 0.622 \times \frac{e}{p} = 0.622 \times \frac{1705.9}{101325} = 0.01$
- $R_a = R_d(1 + 0.608 \times 0.01) = 287 \times 1.00608 = 288.7$J/Kg.K
- $\rho_a = \frac{P}{R_aT} = \frac{101325}{288.7 \times(25+273)} = 1.18\ kg/m^3$

Example

- $\lambda E = \dfrac{188.8\ (200-0)+1005\times 1.18\dfrac{(3169-1705.9)}{69.3}}{67.05\times 2.02+188.8} = 193.68$ W/m^2
- $=6.9$ mm/day

Solar radiation

- $R_s = \left(a_s + b_s \frac{n}{N}\right) R_a$

If no calibration is available for a_s and b_s then a_s=0.25 and b_s=0.50

Solar radiation

- $R_{nl} = \sigma \left[\frac{T_{max,k}{}^4 + T_{min,k}{}^4}{2}\right] \left(0.34 - 0.14\sqrt{e_a}\right) \left(1.35\frac{R_s}{R_{so}} - 0.35\right)$
- where
- Rnl: net outgoing longwave radiation [MJ m^{-2} day^{-1}]
- σ : Stefan-Boltzmann constant [4.903 10^{-9} MJ K^{-4} m^{-2} day^{-1}]
- Tmax,K maximum absolute temperature during the 24-hour period (K)
- Tmin,K minimum absolute temperature during the 24-hour period (K)
- ea actual vapour pressure [kPa],
- Rs/Rso relative shortwave radiation radiation [MJ m^{-2} day^{-1}].

Solar radiation

- $R_n = (1 - \propto) \times R_s - R_{nl}$

Example

R_a=289 W/s, N=11 hours, e_a=2100 P_a, $\propto$*=0.25, n=7 hours, T_{max}=25 ^oC, T_{min}=19 ^oC,* determine *R_n*

$$R_s = \left(a_s + b_s \frac{n}{N}\right) R_a = \left(0.25 + 0.50 \frac{7}{11}\right) 289$$
$$= 164\ W/m^2$$

Example

- $R_{nl} = \sigma \left[\frac{T_{max,k}{}^4 + T_{min,k}{}^4}{2}\right]\left(0.34 - 0.14\sqrt{e_a}\right)\left(1.35\frac{R_s}{R_{so}} - 0.35\right)$

- = 4.903 10^{-9} $\left[\frac{(25+273)^4+(19+273)^4}{2}\right](0.34 - 0.14 \times \sqrt{2.1})(1.35\frac{164}{289\times0.75} - 0.35)$=3.42 [MJ m^{-2} day^{-1}] =39.52 W/m^2
- $R_n = (1 - 0.23) \times 164 - 39.52 = 86.76\, W/m^2$

- $\lambda E=\alpha \frac{\Delta(R_n - G)}{\gamma(\frac{ra+rs}{ra})+\Delta}$
- $\lambda E=1.3 \frac{188.8 \times(200-0)}{67.05 \times 2.02+188.8} = 151.4 \frac{W}{m^2} = 5.4\ mm/day$