Manual for Design and Detailing of Reinforced Concrete to the Code of Practice for Structural Use of Concrete 2013

September 2013

Contents

1.0 Introduction

1.1 Promulgation of the Revised Code

The revised concrete code titled "Code of Practice for Structural Use of Concrete 2013" was formally promulgated by the Buildings Department of Hong Kong in end February 2013 which supersedes the former concrete code 2004. The revised Code, referred to as "the Code" hereafter in this Manual will become mandatory by 28 February 2014, after expiry of the grace period in which both the 2004 and 2013 versions can be used.

1.2 Overview of the Code

The Code retains most of the features of the 2004 version though there are refinements here and there, some of which are subsequent to comments obtained from the practitioners ever since the implementation of the 2004 version. The major revisions in relation to design and detailing of reinforced concrete structures are outlined as follows :

(i) Introduction of the fire limit state;
(ii) A set of Young's moduli of concrete which are "average values" is listed in the Code, as in addition to the "characteristic values" originally listed in the 2004 version. The average values can be used in determination of overall building deflection. In addition, the initial tangent in the concrete stress strain curve for design (in Figure 3.8 of the Code) has been given a separate symbol E_d which is different from the Young's modulus of concrete with the symbol E_c as the two have different formulae for determination;
(iii) The high yield bar (which is termed "ribbed steel reinforcing bar" in the Code as in CS2:2012) is upgraded to Grade 500 to CS2:2012, i.e. the yield strength is upgraded from $f_y = 460$ MPa to 500MPa;
(iv) The use of mechanical coupler Type 1 and Type 2;
(v) The determination of design force on the beam column joint has been clarified, together with revision in detailing requirements in some aspects;
(vi) The discrepancies in design provisions of cantilevers between the 2004 version and the PNAP 173 have generally been resolved in the Code;
(vii) Additional reinforcement requirements in bored piles and barrettes;
(viii) Refinement of ductility detailing in beams and columns;
(ix) Additional ductility detailing in walls.

In the aspects of design and detailing, the drafting of the Code is based on the following national and international codes, though with modifications or simplifications as appropriate:

(i) The British Standard BS8110 Parts 1 and 2 generally for most of its contents;
(ii) The Eurocode EC2 on detailing as mostly contained in Chapter 8;
(iii) The New Zealand Standard NZS 3101 in the design of beam column joint;

(iv) The New Zealand Standard NZS 3101 in most of the provisions of ductility detailing for beams and columns;
(v) The ACI Code ACI318-2011 for modifications of some of the detailing;
(vi) The China Code GB50011-2010 in some respects of detailing including that of wall.
(vii) The Eurocode BSEN 1536 for the detailing of bored pile and diaphragm wall.

However, the properties of concrete including the Young's modulus and the stress strain relationships are based on local studies by Professor Albert K.H. Kwan of the University of Hong Kong.

1.3 Outline of this Manual

This Practical Design and Detailing Manual intends to outline practice of detailed design and detailing of reinforced concrete work to the Code. Detailing of individual types of members are included in the respective sections for the types, though the Section 13 in the Manual includes certain aspects in detailing which are common to all types of members. The underlying principles in some important aspects in design and detailing have been selectively discussed. Design examples, charts are included, with derivations of approaches and formulae as necessary.

As computer methods have been extensively used nowadays in analysis and design, the contents as related to the current popular analysis and design approaches by computer methods are also discussed. The background theory of the plate bending structure involving twisting moments, shear stresses, and design approach by the Wood Armer Equations which are extensively used by computer methods are also included an Appendix (Appendix D) in this Manual for design of slabs, pile caps and footings.

To make distinctions between the equations quoted from the Code and the equations derived in this Manual, the former will be prefixed by (Ceqn) and the latter by (Eqn).

Unless otherwise stated, the general provisions and dimensioning of steel bars are based on ribbed steel reinforcing bars with $f_y = 500$ N/mm^2.

Design charts for beams, columns and walls are based on the more rigorous stress strain relationship of concrete comprising a rectangular and a parabolic portion as indicated in Figure 3.8 of the Code.

2.0 Some Highlighted Aspects in Basis of Design

2.1 Ultimate and Serviceability Limit states

The ultimate and serviceability limit states used in the Code carry the normal meaning as in other codes such as BS8110. However, the Code has incorporated an extra serviceability requirement in checking human comfort by limiting acceleration due to wind load on high-rise buildings (in Cl. 7.3.2). No method of analysis has been recommended in the Code though such accelerations can be estimated by the wind tunnel laboratory if wind tunnel tests are conducted. Nevertheless, worked examples are enclosed in Appendix A, based on empirical approach in accordance with the Australian/New Zealand code AS/NZS 1170.2:2011. The Australian/New Zealand code is the code on which the current Hong Kong Wind Code has largely relied in deriving dynamic effects of wind loads.

2.2 Design Loads

The Code has made reference generally to the "Code of Practice for Dead and Imposed Loads for Buildings 2011" for determination of characteristic gravity loads for design. However, the designer may need to check for the updated loads by fire engine for design of new buildings, as required by FSD.

The Code has placed emphasize on design loads for robustness which are similar to the requirements in BS8110 Part 2. The requirements include design of the structure against a notional horizontal load equal to 1.5% of the characteristic dead weight at each floor level and vehicular impact loads (Cl. 2.3.1.4). The small notional horizontal load can generally be covered by wind loads if wind loads are applied to the structure. Identification of key elements and designed for ultimate loads of 34 kPa, together with examination for progress collapse in accordance with Cl. 2.2.2.3 of the Code can be exempted if the buildings are provided with ties in accordance with Cl. 6.4.1 of the Code. The reinforcement provided for other purpose can also act as effective ties if continuity and adequate anchorage for rebar of ties have been provided. Fuller discussion is included in Section 14 of this Manual.

Wind loads for design should be taken from Code of Practice on Wind Effects in Hong Kong 2004.

It should also be noted that there are differences between Table 2.1 of the Code that of BS8110 Part 1 in some of the partial load factors γ_f. The beneficial partial load factor for wind, earth and water load is 0 and that for dead load is 1.0 which appear more reasonable than that in BS8110 giving 1.2 for both items. However, higher partial load factor of 1.4 is used for earth and water pressure that in BS8110 giving 1.2 and 1.0 so as to account for higher uncertainty of soil load as experienced in Hong Kong.

2.3 Materials – Concrete

Table 3.2 of the Code has tabulated Young's Moduli of concrete up to grade

C100. The listed characteristic values in the table are based on local studies which are generally smaller than that in BS8110 by more than 10%. In addition, average values (with cube strength 5N/mm^2 lower than the characteristic values) are listed which are allowed to be used to check lateral building deflections.

Table 4.2 of the Code tabulated nominal covers to reinforcements under different exposure conditions. However, reference should also be made to the "Code of Practice for Fire Safety in Buildings 2011".

The stress strain relationship of concrete has been well defined for grade up to C100. It can readily be seen that as concrete grade increases, the transition point between the parabolic and rectangular portion at $\varepsilon_0 = 1.34 f_{cu} / \gamma_m / E_d$ shifts so that the parabolic portion lengthens while the rectangular portion shortens. In addition, the ultimate strain also decreases from the value 0.0035 to $0.0035 - 0.00006\sqrt{f_{cu} - 60}$ when $f_{cu} > 60$ as illustrated in Figure 2.1 for grades C35, C60, C80 and C100. These changes are due to the higher brittleness of the concrete at higher grades which are modified from BS8110 as BS8110 does not have provisions for high grade concrete.

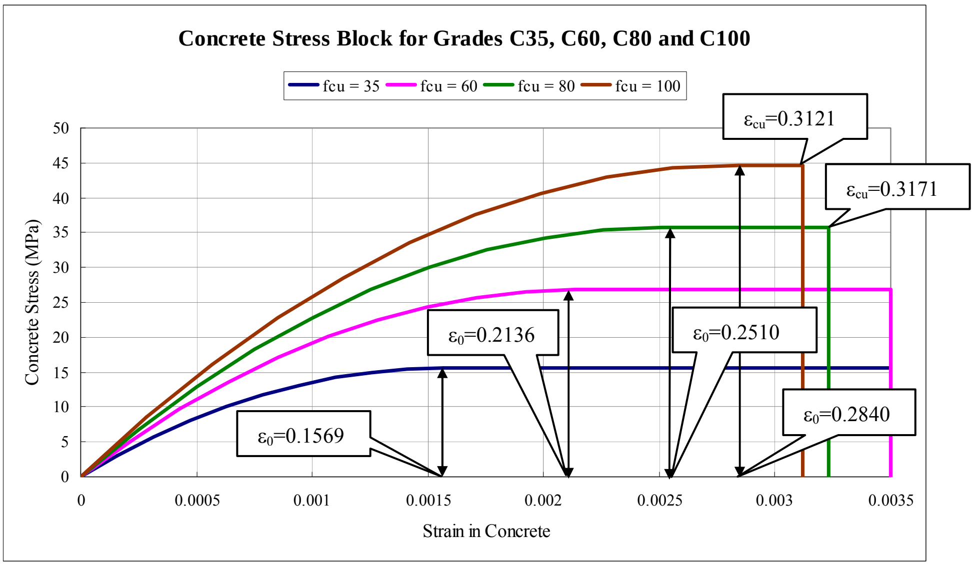


Figure 2.1 – Stress Strain Relationship of Grades C35, C60, C80 and C100 in accordance with the Code

Following the provisions in BS8110 and other codes include the Eurocode EC2, the Code has provisions for "simplified stress block" as indicated in its Figure 6.1 of the Code which is reproduced in Figure 2.2. The simplified stress block is to simulate the more rigorous stress block with the aim of simplifying design calculations. However, instead of a single stress block of 0.9 times the neutral axis as in BS8110, the Code has different factors for concrete grades higher than C45 and C70 to achieve higher accuracy. The equivalent factors for force and moments of the more rigorous stress block have been worked out as compared with that of the simplified stress block for concrete grades from C30 to C100 as shown in Figure 2.3. It can be seen that the simplified stress block tends to over-estimate both force and moments at low concrete grades but under-estimate at high concrete grades.

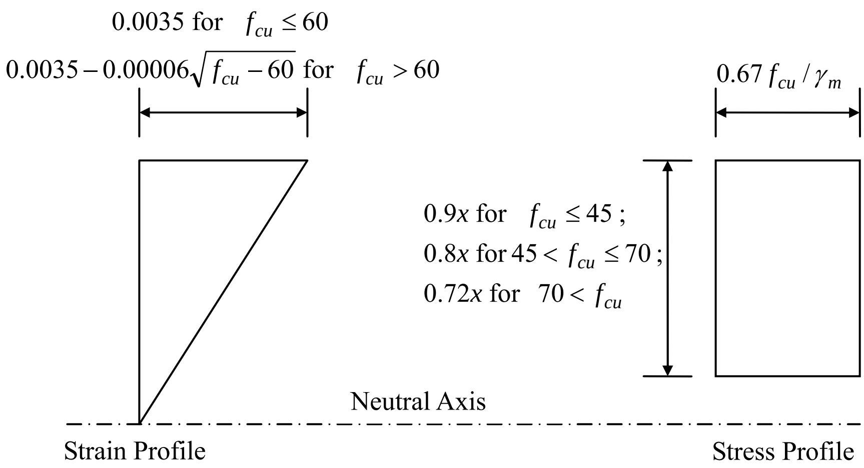


Figure 2.2 – Simplified stress block for ultimate reinforced concrete design

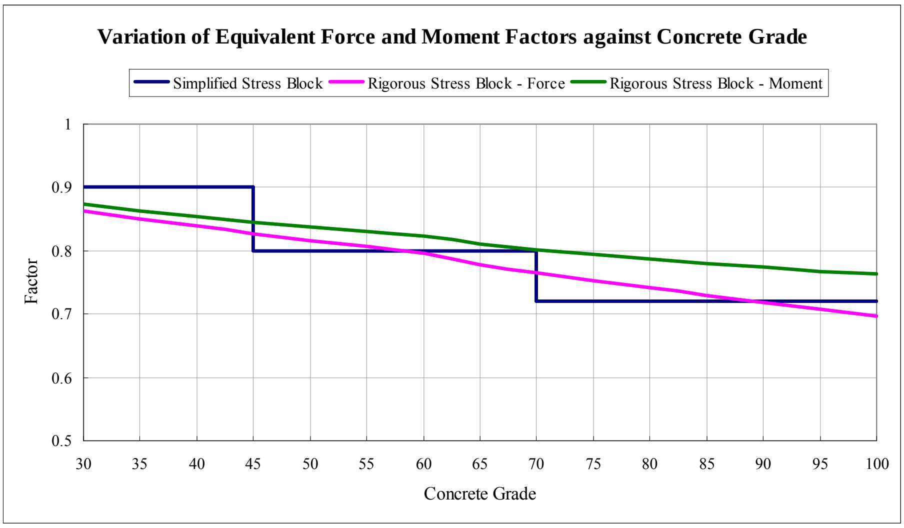


Figure 2.3 – Equivalent Factors of Rigorous Stress Blocks for Force and Moments as Compared with the Simplified Stress Block

2.4 Ductility Requirements

As discussed in para. 1.2, an important feature of the Code is the incorporation of ductility requirements which directly affects r.c. detailing. By ductility we refer to the ability of a structure to undergo "plastic deformation", which is often significantly larger than the "elastic" deformation prior to failure. Such ability is desirable in structures as it gives adequate warning to the user for repair or escape before failure. Figure 2.4 illustrates how ductility is generally quantified. In the figure, the relation of the moment of resistance of two Beams A and B are plotted against their curvatures and their factors of ductility are defined in the formula listed in the figure. It can be described that Beam B having a "flat plateau" is more ductile than Beam A having a comparatively "steep hill". Alternatively speaking, Beam B can tolerate a larger curvature, i.e. angular rotation and subsequently deflection than Beam A before failure.

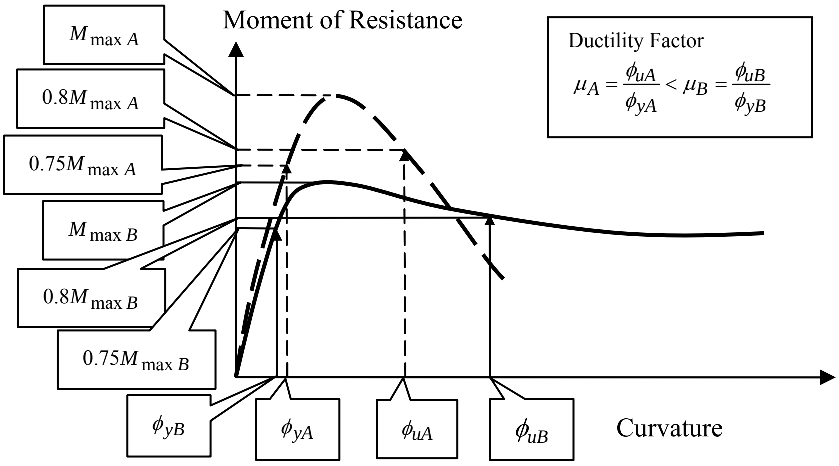


Figure 2.4 – Illustration of Plots of Ductility of Beams

The basic principles for achieving ductility by r.c. detailing as required by the Code are highlighted as follows :

(i) Use of closer and stronger transverse reinforcements to achieve better concrete confinement which increases concrete strengths and subsequently enhances both ductility and strength of concrete against compression, both in columns and beams and walls. As an illustration, a plot of the moment of resistance against curvature of the section of a 500×500 column of grade C35 with various amounts of links but with constant axial load is shown in Figure 2.5. It can be seen that both flexural strength and ductility increase with stronger links.

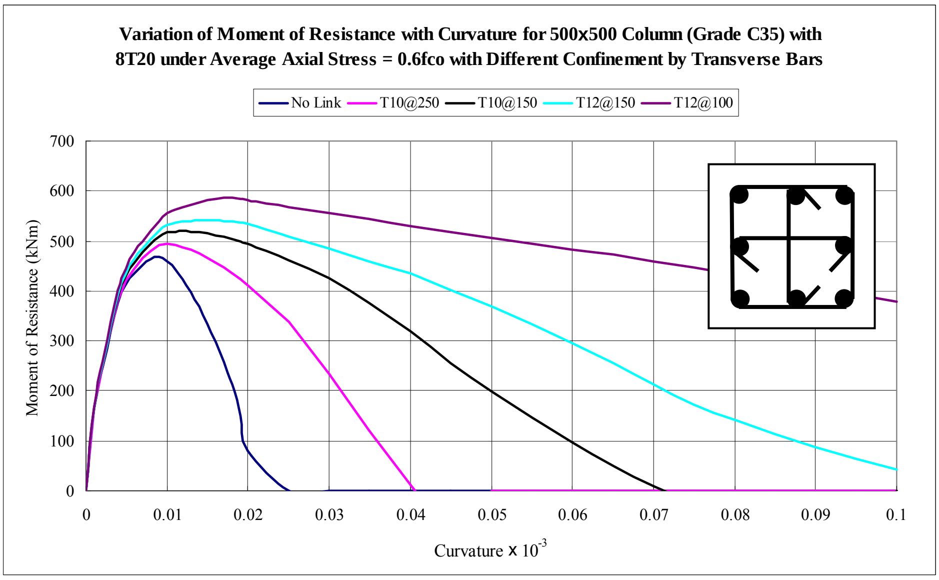


Figure 2.5 – Demonstration of Increase of Ductility by Increase of Transverse Reinforcement in Column

(ii) Stronger anchorage of transverse reinforcements in concrete by means of hooks with bent angles ≥ 135° for ensuring better performance of the transverse reinforcements. This is illustrated in Figure 2.6 by which a 135° bend performs better than a 90° bend;

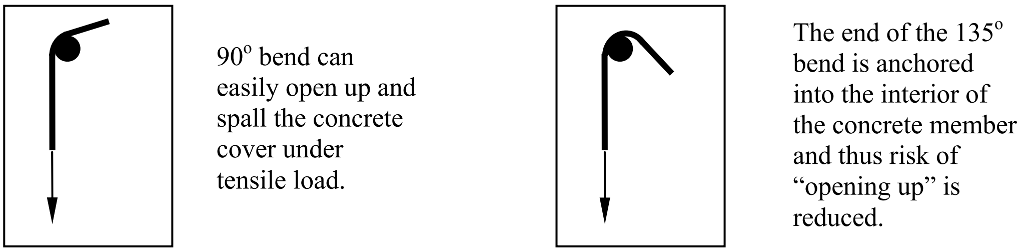


Figure 2.6 – Comparison of Anchorage between 90° and 135° bends

As discussed by Law & Mak (2013), though the 135° hook is unarguably the better option, the 90° hook is more popular as it has the relative ease of placement. The 135° hook is much more difficult to place especially when the cast-in bars are misaligned. However, if there are other physical restraints such as adjoining beams or slabs preventing the opening up of the 90° hook as illustrated in Figures 9.5 and 9.7 of the Code, the use of 90° hook should be acceptable. More examples of these options can be found in the Annex of the letter addressing to all authorized persons and registered structural and geotechnical engineers and contractors by the Buildings Department dated 29 April 2011 which are extracted in Appendix B of this Manual;

(iii) More stringent requirements in restraining and containing longitudinal reinforcing bars in compression against buckling by closer and stronger transverse reinforcements with hooks of included angles ≥ 135°. (compare Figures 5.19 and 5.21 for column of this Manual);

(iv) Longer bond and anchorage length of reinforcing bars in concrete to ensure failure by yielding prior to bond slippage as the latter failure is more brittle as illustrated in Figure 2.7;

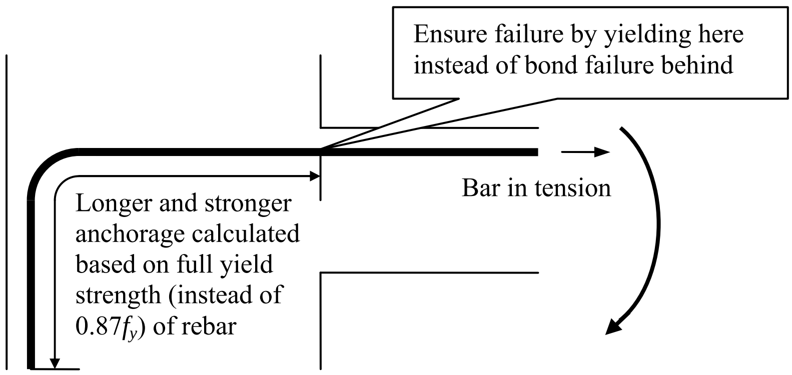


Figure 2.7 – Longer Bond and Anchorage Length of Reinforcing Bars

(v) Restraining and/or avoiding radial forces by reinforcing bars on concrete at where the bars change direction as illustrated in Figure 2.8;

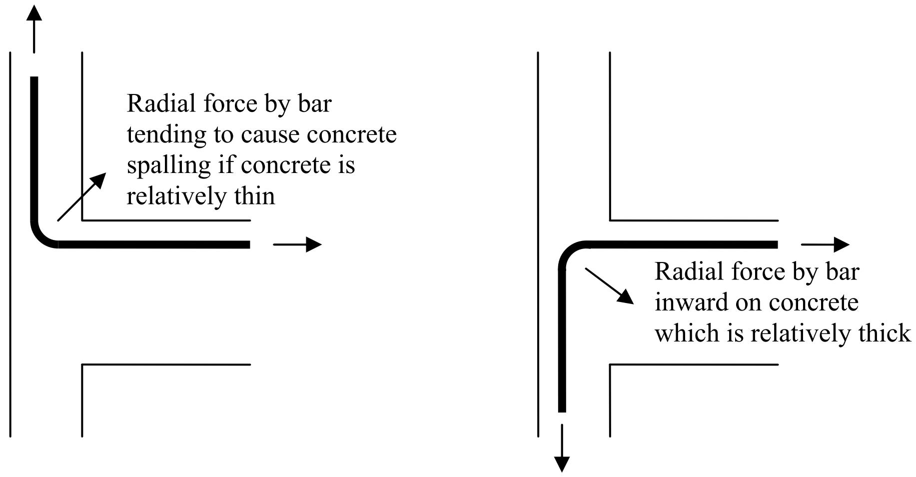


Figure 2.8 – Bars Bent Inwards to Avoid Radial Forces on Thin Concrete Cover

(vi) Limiting amounts of tension reinforcements in flexural members as over-provisions of tension reinforcements will lead to increase of neutral axis and thus greater concrete strain and easier concrete failure which is brittle as illustrated in Figure 2.9. As a result, ductility is decreased. This phenomenon has been discussed by Kwan (2006) and Law (2010) in details. However, conversely the increase of compression reinforcements will increase ductility as discussed by Law (2010). The plots of moment of resistance of a 700(D)×400(B) grade C35 beam in Figure 2.9(a) and 2.9(b) with different tension and compression steel ratios demonstrate how the ductility is affected.

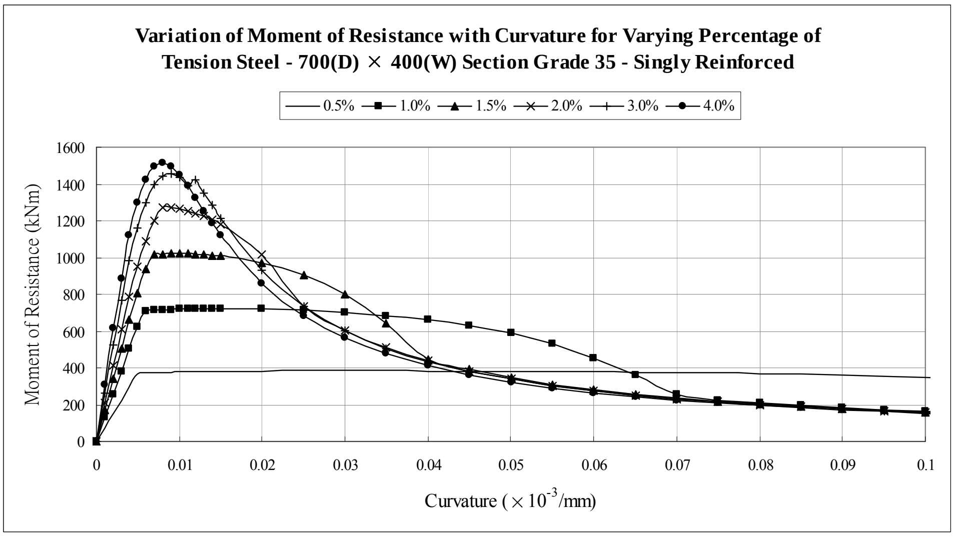


Figure 2.9(a) – Ductility of Beam Affected by Tension Bar Ratios

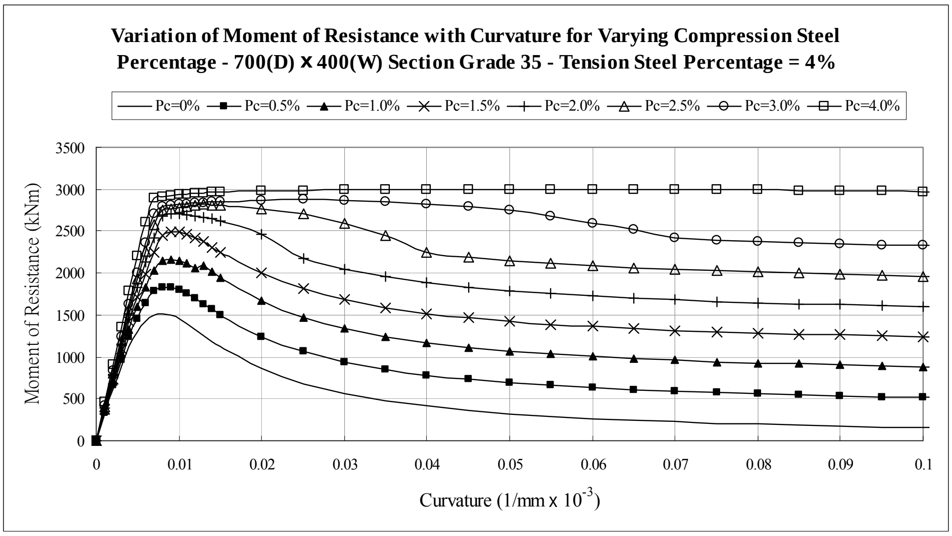


Figure 2.9(b) – Ductility of Beam Affected by Compression Bar Ratios

(vii) Limiting axial compression ratio (ratio of axial compression stress due to factored gravity load to strength of concrete limited to 0.75) in wall as per Cl. 9.9.3.3 of the Code as high axial compression ratio in wall will decrease ductility as illustrated by Figure 2.10. In the figure, the variations of flexural strengths of a 300×3000 grade C45 wall against curvature are shown. "Flat plateaus" implying high ductility for low axial stresses and "steep hills" implying low ductility for high axial stresses are demonstrated.

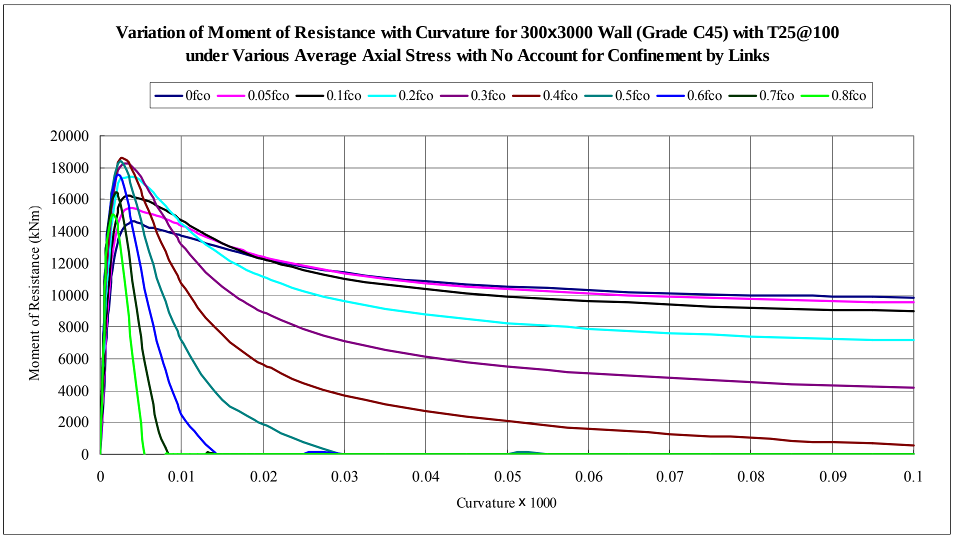


Figure 2.10 – Demonstration of Wall Ductility vs Axial Compression Stress

Though the same phenomenon should also apply to column, the Code however has not imposed similar absolute limit to column though stronger confinements by transverse reinforcements is required at high axial compressive stress as per Cl. 9.9.2.2 of the Code;

(viii) More stringent requirements on design using high strength concrete which is more brittle including (a) lowering ultimate concrete strain by $\varepsilon_{cu} = 0.0035 - \sqrt{f_{cu} - 60}$ for concrete grade exceeding C60 as per Figure 3.8 and 6.1 of the Code; (b) restricting neutral axis depth to effective depth ratios to not exceeding 0.4 for $45 < f_{cu} \le 70$ and 0.33 for $70 < f_{cu} \le 100$ as per Cl. 6.1.2.4(b); (c) no moment redistribution for concrete grade exceeding C70 as per Cl. 5.2.9.1 of the Code.

Often the ductility requirements specified in the Code are applied to zones where plastic hinges may be formed which are termed "critical zones" as in Cl. 9.9.1.1 for beam, Cl. 9.9.2.2 for column and Cl. 9.9.3.1 for wall. The sequential occurrence of plastic hinges in various zones or sections of the structure can be determined by a "push over analysis" by which a lateral load with step by step increments is added to the structure. Among the structural members met at a joint, the location (mostly within a critical zone) at which plastic hinge is first formed will be identified as the critical section of plastic hinge formation at the joint. Nevertheless, the determination can be approximated by judgment without going through such analysis. In a column beam frame with relatively strong columns and weak beams, the critical sections of plastic hinge formation should be in the beams at their interfaces with the columns. In case of a column connected into a thick pile cap, footing or transfer plate, the critical section with plastic hinge formation will be in the columns at their interfaces with the cap, footing or transfer plate as the plastic hinge will unlikely be formed in the thick cap, footing or transfer structure. The above is illustrated in Figure 2.11. Nevertheless, the Code directly or indirectly implements certain requirements such to secure some preferential sequence of failure such as "strong column weak beam", strong beam column joint over provisions of flexural strengths of adjoining beams.

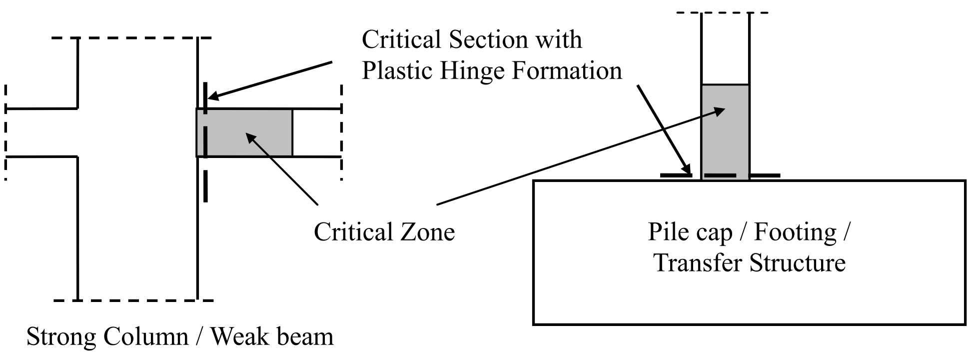


Figure 2.11 – Locations of Critical sections with Plastic Hinge Formation

Ductility requirements in detailing are specified in Cl. 9.9 of the Code which apply only to structural members contributing in the lateral load resisting system in accordance with Cl. 9.1 of the Code. In this Manual, a (D) mark will be added if the requirement is a ductility requirement.

2.5 Mechanical Couplers

The Code has introduced the use of "Type 1" and "Type 2" couplers which are originated from ACI318-11 Cl. 21.1.6.1. They are extracted in Cl. 3.2.8 of the Code with detail requirements in testing which are actually based on AC133 – Acceptance Criteria for Mechanical Connector Systems for Steel Reinforcing Bars. According to ACI318-11 Cl. 12.14.3.2, both types of couplers are required to develop tensile strength of not less than 125% yield strength of the parent bars. However, CS2:2012 only requires the steel bars tensile strength to exceed 540MPa for Grade 500B and 575MPa for Grade 500C which are respectively 108% and 115%. This is consistent with Cl. 3.2.8.3(b) of the Code that the "steel bar assembly" (coupler and the bar) shall have tensile strength exceeding 540MPa and 575MPa for Grade 500B and 500C respectively for Type 1 coupler. Nevertheless, Cl. 3.2.8.4 of the Code requires Type 2 coupler to develop tensile strength of not less than 125% yield strength of the parent bars. This requirement is consistent with ACI318-11, but exceeding the requirements for the steel bars. For the testing, Type 2 has to undergo all tests for Type 1 and extra compression test and cyclic tension / compression tests simulating that of seismic actions. Therefore Type 2 couplers are stronger and more ductile.

In accordance with ACI318-11, in a structure undergoing inelastic deformations during an earthquake, the tensile stresses in reinforcement may approach the tensile strength of the reinforcement. The requirement for Type 2 mechanical splices are intended to avoid a splice failure when the reinforcement is subjected to expected stress levels in yielding regions. Type 1 splices are not required to satisfy the more stringent requirements for Type 2 splices, and may not be capable of resisting the stress levels expected in yielding regions. Therefore the locations of Type 1 splices are restricted and not to be in regions with potential "plastic hinge formation" while Type 2 can practically be used at any locations.

2.6 Design for Robustness

The requirements for design for robustness are identical to BS8110 and fuller discussions are given in Section 14.

2.7 Definitions of Structural Elements

The Code has included definitions of beam, slab, column and wall in accordance with their dimensions in Cl. 5.2.1.1(a), (b) and (e). Additional information for shear wall and transfer structures is given in Cl. 5.4 and 5.5. They are briefly repeated as follows for ease of reference :

(a) Beam : for span ≥ 2 times the overall depth for simply supported span and ≥ 2.5 times the overall depth for continuous span, classified as shallow beam, otherwise : deep beam;
(b) Slab : the minimum panel dimension ≥ 5 times its thickness;
(c) Column : vertical member with section depth not exceeding 4 times its width and height is at least 3 times its section depth;
(d) Wall : vertical member with plan dimensions other than that of column.
(e) Shear Wall : a wall contributing to the lateral stability of the structure.
(f) Transfer Structure : a horizontal element which redistributes vertical

loads where there is a discontinuity between the vertical structural elements above and below.

This Manual is based on the above definitions in delineating structural members for discussion.

2.8 Fire Limit State

Fire limit state is the state related to the structural effects of a fire in a building or part of a building. Generally Tables E2 to E7 of the "Code of Practice for Fire Safety in Buildings 2011" in relation to the minimum structural sizes and concrete covers to reinforcements against various "Fire Resistance Rating (FRR)" (which is the period referring to the period of time that a building element is capable of resisting the action of fire when tested in accordance with ISO 834, BS 476: Parts 20 to 24 or equivalent as defined in the "Code of Practice for Fire Safety in Buildings 2011") are deemed-to-satisfy requirements. However, if a performance-based approach using fire engineering is adopted to formulate an alternative solution, the structures may have to be designed for "elevated temperatures" as determined from the performance-based approach under reduced strength factors as indicated in Tables 3.5 to 3.7 of the Code.

3.0 Beams

3.1 Analysis (Cl. 5.2.5.1 & 5.2.5.2)

Normally continuous beams are analyzed as sub-frames by assuming no settlements at supports by walls, columns (or beams) and rotational stiffness by supports provided by walls or columns as $4EI/L$ (far end of column / wall fixed) or $3EI/L$ (far end of column / wall pinned).

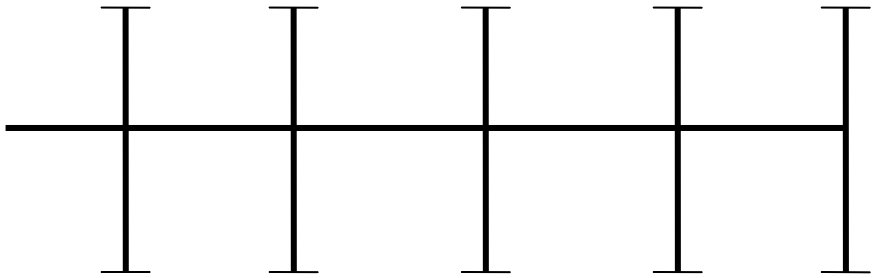


Figure 3.1 – continuous beam analyzed as sub-frame

In analysis as sub-frame, Cl. 5.2.5.2 of the Code states that the following loading arrangements will be adequate to determine the design moments which are identical to BS8110-1:1997 Cl. 3.2.1.2.2 :

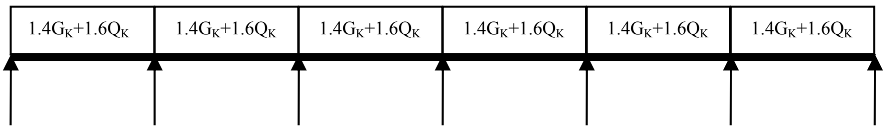


Figure 3.2(a) – To search for Maximum Support Reactions

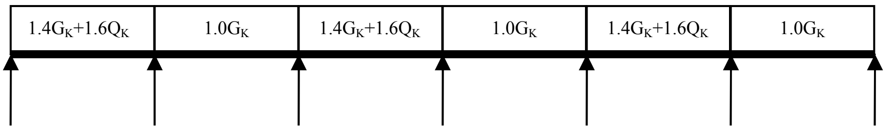


Figure 3.2(b) – To search for Maximum Sagging Moment in Spans Loaded by $1.4G_K+1.6Q_K$

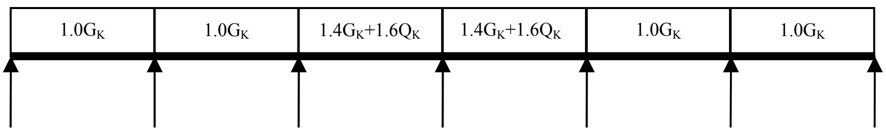


Figure 3.2(c) – To search for maximum Hogging moment at Support with Adjacent Spans Loaded with $1.4G_K+1.6Q_K$

However, nowadays most of the commercial softwares can actually analyze individual load cases, each of which is having dead load or live load on a single span and the effects on the span and others are analyzed. The enveloped design value of shears and moments at any location will be the summation of the critical values created by the individual cases. Therefore the patterned load arrangements may not be necessary.

The Code has no explicit provisions for wind loads under this clause. But with reference to Table 2.1 of the Code, it should be logical to apply generally load factors of 0 for beneficial cases and 1.2 or 1.4 for adverse cases with and without live load respectively on the span under consideration, e.g. $1.0G_K–1.4W_K$ (search for maximum sagging in span) and $1.2(G_K+Q_K+W_K)$ or $1.4(G_K +W_K)$ (search for maximum hogging at support).

3.2 Moment Redistribution (Cl. 5.2.9 of the Code)

Moment redistribution is allowed for concrete grade not exceeding C70 under conditions 1, 2 and 3 as stated in Cl. 5.2.9.1 of the Code. Nevertheless, it should be noted that there would be further limitation of the neutral axis depth ratio x/d if moment redistribution is employed as required by (Ceqn 6.4) and (Ceqn 6.5) of the Code which is identical to the provisions in BS8110. The rationale of the limitation is discussed in Kwan (2006) 6.1.2.

3.3 Highlighted Aspects in Determination of Design Parameters of Shallow Beam

(i) Effective Span (Cl. 5.2.1.2(b) and Figure 5.3 of the Code)

For simply supported beam, continuous beam and cantilever, the effective span can be taken as the clear span plus the lesser of half of the structural depth and half support width except that on bearing where the centre of bearing should be used to assess effective span;

(ii) Effective Flange Width of T- and L-beams (Cl. 5.2.1.2(a))

Effective flange widths of T- and L-beams are as illustrated in Figure 5.2. of the Code as reproduced as Figure 3.3 of this Manual:

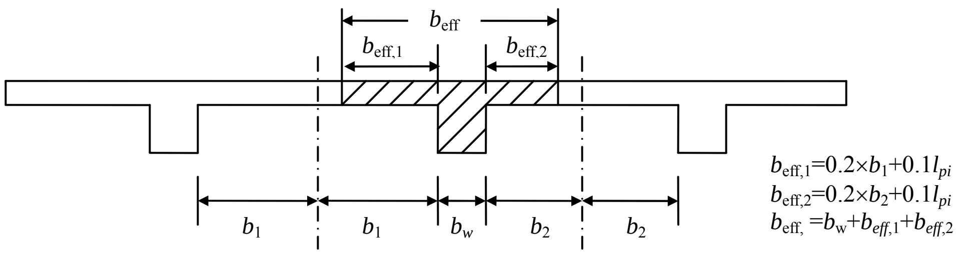


Figure 3.3 – Effective Flange Parameters

Effective width (b_{eff}) = width of beam (b_w) + $\sum$(0.2 times of half the centre to centre width to the next beam ($0.2b_i$) + 0.1 times the span of zero moment ($0.1l_{pi}$), with the sum of the latter two not exceeding 0.2 times the span of zero moment and l_{pi} taken as 0.7 times the effective span of the beam). An example for illustration as indicated in Figure 3.4 is as follows :

Worked Example 3.1

To determine effective flange width of the beams in Figure 3.4 where the effective spans are 5m and they are continuous beams.

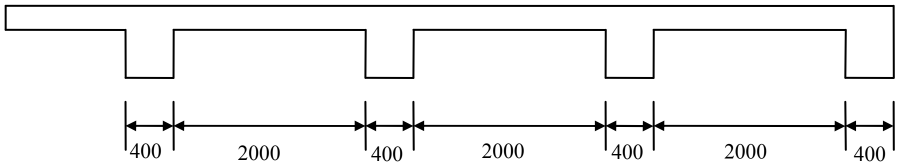


Figure 3.4 – Example Illustrating Effective Flange Width Determination

The effective width of the T-beam is, by (Ceqn 5.1) of the Code
$l_{pi} = 0.7 \times 5000 = 3500$;
$b_{eff,1} = b_{eff,2} = 0.2 \times 1000 + 0.1 \times 3500 = 550$
As $b_{eff,1} = b_{eff,2} = 550 < 0.2 \times 3500 = 700$, $b_{eff,1} = b_{eff,2} = 550$;
$b_{eff} = 400 + 550 \times 2 = 400 + 1100 = 1500$
So the effective width of the T-beam is 1500 mm.

Similarly, the effective width of the L-beam at the end is
$b_w + b_{eff,1} = 400 + 550 = 950$.

(iii) Support Moment Reduction (Cl. 5.2.1.2 of the Code)

The Code allows design moment of beam (and slab) monolithic with its support providing rotational restraint to be that at support face if the support is rectangular and 0.2Ø if the column support is circular with diameter Ø. But the design moment after reduction should not be less than 65% of the support moment. A worked example 3.2 as indicated by Figure 3.5 for illustration is given below :

<u>Worked Example 3.2</u>

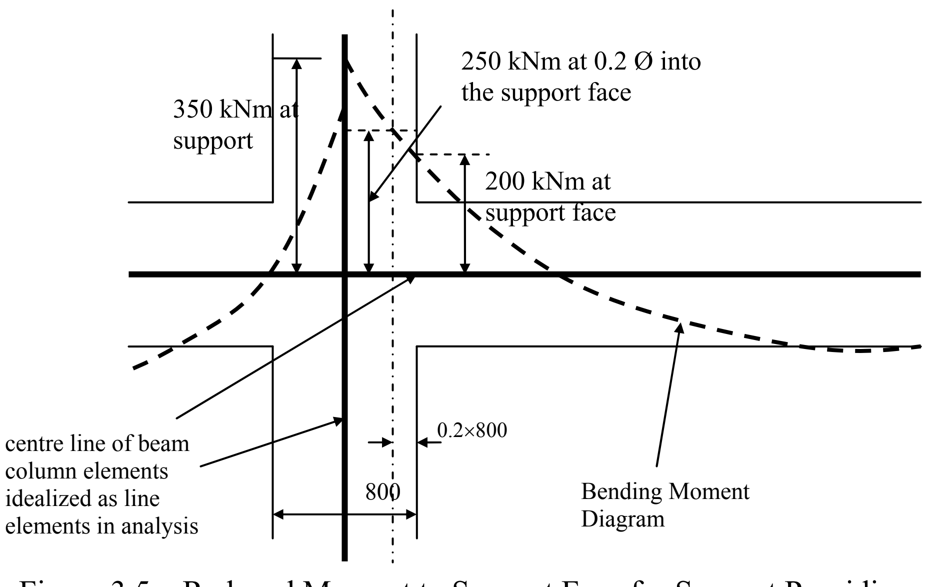


Figure 3.5 – Reduced Moment to Support Face for Support Providing Rotational Restraint

In Figure 3.5, the bending moment at support face is 200 kNm which can be the design moment of the beam if the support face is rectangular. However, as it is smaller than 0.65×350 = 227.5 kNm. 227.5 kNm should be used for design.

If the support is circular and the moment at 0.2Ø into the support and the bending moment at the section is 250 kNm, then 250 kNm will be the design moment as it is greater than 0.65×350 = 227.5 kNm.

For beam (or slab) spanning continuously over a support assumed not providing rotational restraint (e.g. wall support), the Code allows moment reduction by support shear times 1/8 of the support width to the moment obtained by analysis. Figure 3.6 indicates a numerical Worked Example 3.3.

Worked Example 3.3

By Figure 3.6, the design support moment at the support under consideration can be reduced to $250 - 200 \times 0.8/8 = 230$ kNm.

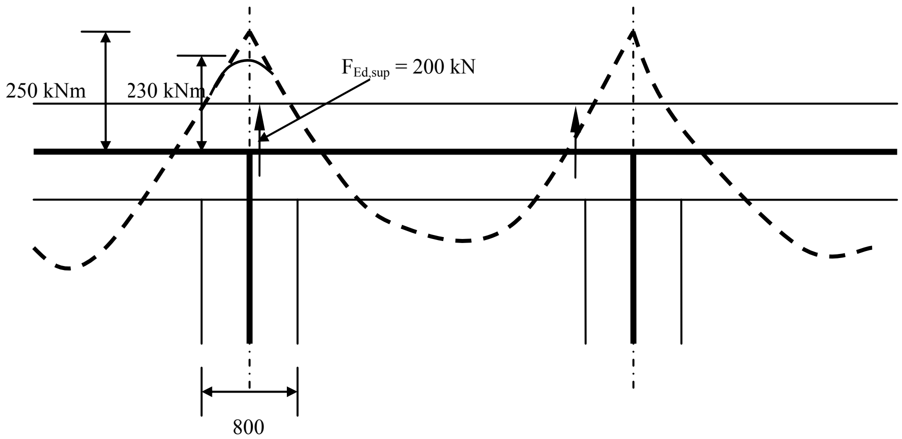


Figure 3.6 – Reduction of Support Moment by Support Shear for Support considered not Providing Rotational Restraint

(iv) Slenderness Limit (Cl. 6.1.2.1 of the Code)

The provision is identical to BS8110 as

1. Simply supported or continuous beam :
   Clear distance between restraints ≤ $60b_c$ or $250b_c^2/d$ if less; and
2. Cantilever with lateral restraint only at support :
   Clear distance from cantilever to support ≤ $25b_c$ or $100b_c^2/d$ if less

where b_c is the breadth of the compression face of the beam and d is the beam effective depth.

Usually the slenderness limits need be checked for inverted beams or bare beam (without slab).

(v) Span Effective Depth ratio (Cl. 7.3.4.2 of the Code)

Table 7.3 under Cl. 7.3.4.2 tabulates basic span depth ratios for various types of beam / slab which are "deemed-to-satisfy" requirements against deflection. The table has provisions for "slabs" and "end spans" which are not specified in BS8110 Table 3.9. Nevertheless, calculation can be carried out to justify deflection limits not to exceed span / 250 if the "deemed-to-satisfy" conditions are not satisfied. In addition, the basic span depth ratios can be modified due to provision of tensile and compressive steels as given in Tables 7.4 and 7.5 of the Code which are identical to BS8110. Modification of the factor by 10/span for span > 10 m except for cantilever as similar to BS8110 is also included.

Support condition	Rectangular Beam	Flanged Beam $b_w/b < 0.3$	One or two-way spanning solid slab
Cantilever	7	5.5	7
Simply supported	20	16	20
Continuous	26	21	26
End span	23	18.5	23(2)

Note :
1. The values given have been chosen to be generally conservative and calculation may frequently show shallower sections are possible;
2. The value of 23 is appropriate for two-way spanning slab if it is continuous over one long side;
3. For two-way spanning slabs the check should be carried out on the basis of the shorter span.

Table 3.1 – Effective Span / Depth Ratio of Beam

(vi) Maximum Spacing between Reinforcing Bars in Tension Near Surface

By Cl. 9.2.1.4 of the Code, the clear spacing of reinforcing bars in tension near surface should be $\le \frac{70000\beta_b}{f_y} \le 300$ mm where β_b is the ratio of moment redistribution. Or alternatively, the clear spacing $\le \frac{47000}{f_s} \le$ 300mm. So the simplest rule is $\frac{70000\beta_b}{f_y} = \frac{70000 \times 1}{500} = 140$ mm when using ribbed steel reinforcing bars and under no moment redistribution.

(vii) Concrete Covers to Reinforcements (Cl. 4.2.4 and Cl. 4.3 of the Code)

Cl. 4.2.4 of the Code lists the nominal covers required in accordance with Exposure conditions. However, we can, as far as our building structures are concerned, roughly adopt condition 1 (Mild) for the structures in the interior of our buildings (except for bathrooms and kitchens which should be condition 2), and to adopt condition 2 for the external structures. Nevertheless, the "Code of Practice for Fire Safety in Buildings 2011" should also be checked for different fire resistance rating (FRR) (Tables E2 to E7 of the Fire Code). So, taking into account our current practice of using concrete not inferior than grade C30 and maximum aggregate sizes not exceeding 20 mm, we may generally adopt the provision in our DSEG Manual (DSEG-104 Table 1) with updating by the Code except for compartment of 240 minutes FRR. The recommended covers are summarized in the Table 3.2 :

Description	Nominal Cover (mm)
Internal	30 (to all rebars)
External	40 (to all rebars)
Simply supported (4 hours FRR)	80 (to main rebars)
Continuous (4 hours FRR)	60 (to main rebars)

Table 3.2 – Nominal Cover of Beams

3.4 Sectional Design for Rectangular Beam against Bending

3.4.1 Design in accordance with the Rigorous Stress Strain Curve of Concrete

The stress strain block of concrete as indicated in Figure 3.8 of the Code is different from Figure 2.1 of BS8110. Reference can be made to Figure 2.1 of this Manual for grades C35, C60, C80 and C100 for the rigorous stress block. Based on this rigorous concrete stress strain block, design formulae for beam can be worked out as per the strain distribution profile of concrete and steel as indicated in Figure 3.7 below.

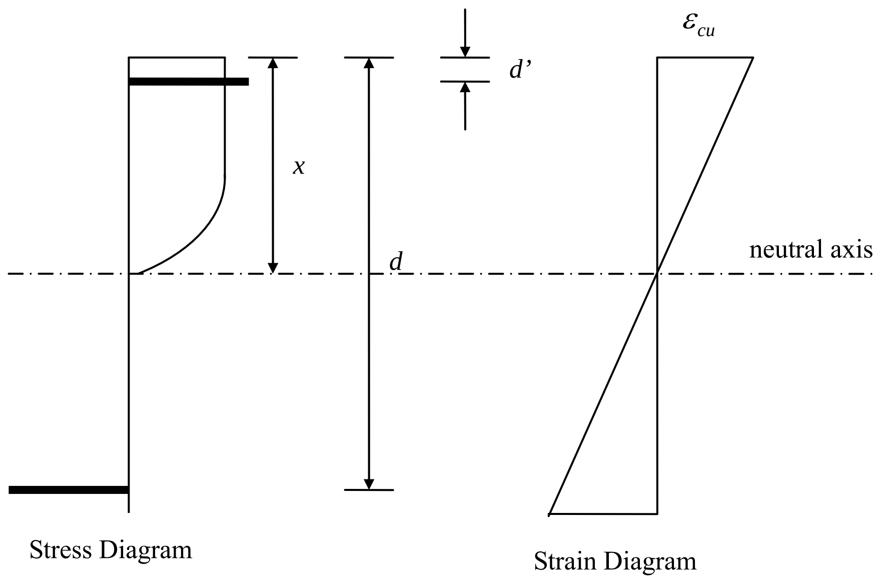


Figure 3.7 – Stress Strain Profiles of Rebars in Beam

The solution for the neutral axis depth ratio $\frac{x}{d}$ for singly reinforced beam is the root of the following quadratic equation (Re Appendix C for detailed derivation) :

$$\frac{0.67 f_{cu}}{\gamma_m}\left[-\frac{1}{2}+\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{cu}}\right)^2\right]\left(\frac{x}{d}\right)^2+\frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{d}-\frac{M}{bd^2}=0$$

(Eqn 3-1)

With neutral axis depth ratio determined, the steel ratio can be determined by

$$\frac{A_{st}}{bd}=\frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{d}$$ (Eqn 3-2)

As $\frac{x}{d}$ is limited to 0.5 for singly reinforcing sections for grades up to C45 under moment redistribution not greater than 10% (Cl. 6.1.2.4 of the Code), by (Eqn 3-1), $\frac{M}{bd^2 f_{cu}}$ will be limited to K' values as in

$K' = 0.151$ for grade C30; $K' = 0.149$ for grade C35
$K' = 0.147$ for grade C40; $K' = 0.146$ for grade C45
which are all smaller than 0.156 under the simplified stress block.

However, for concrete grades exceeding C45 and not exceeding C70 where neutral axis depth ratio is limited to 0.4 for singly reinforced sections under moment redistribution not greater than 10% (Clause 6.1.2.4 of the Code), again by (Eqn 3-1) $\frac{M}{bd^2 f_{cu}}$ will be limited to

$K' = 0.121$ for grade C50; $K' = 0.119$ for grade C60
$K' = 0.115$ for grade C70.
which are similar to 0.12 under the simplified stress block as the Code has reduced the x/d factor to 0.4. Re discussion in Appendix C.

It should be noted that the $\frac{x}{d}$ ratio will be further limited if moment redistribution exceeds 10% by (Ceqn 6.4) and (Ceqn 6.5) of the Code (with revision by Amendment No. 1) as

$\frac{x}{d} \le (\beta_b - 0.4)$ for $f_{cu} \le 45$; and

$\frac{x}{d} \le (\beta_b - 0.5)$ for $45 < f_{cu} \le 70$

where β_b us the ratio of the moment after and before moment redistribution.

When $\frac{M}{bd^2 f_{cu}}$ exceeds the limited value for single reinforcement, compression reinforcements at d' from the surface of the compression side should be added. The compression reinforcements will take up the difference between the applied moment and $K'bd^2 f_{cu}$ and the compression reinforcement ratio is

$$\frac{A_{sc}}{bd} = \frac{\left(\dfrac{M}{bd^2 f_{cu}} - K'\right) f_{cu}}{0.87 f_y \left(1 - \dfrac{d'}{d}\right)} \qquad \text{(Eqn 3-3)}$$

And the same amount of reinforcement will be added to the tensile reinforcement so the tension steel reinforcement ratio becomes :

$$\frac{A_{st}}{bd} = \frac{1}{0.87 f_y} \frac{0.67 f_{cu}}{\gamma_m} \left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right) \eta + \frac{A_{sc}}{bd} \qquad \text{(Eqn 3-4)}$$

where η is the limit of neutral axis depth ratio which is 0.5 for $f_{cu} \le 45$; 0.4 for $45 < f_{cu} \le 70$ and 0.33 for $70 < f_{cu} \le 100$ where moment redistribution does not exceed 10%.

It is interesting to note that more compressive reinforcements will be required for grade C50 than C45 due to the sudden drop in the limitation of neutral axis depth ratio, as illustrated by the following Chart 3-1 in which compression reinforcement decreases from grade C30 to C45 for the same $\dfrac{M}{bd^2}$, but increases at grade C50. The same phenomenon applies to tensile steel also. With moment redistribution exceeding 10%, the same trend of change will also take place.

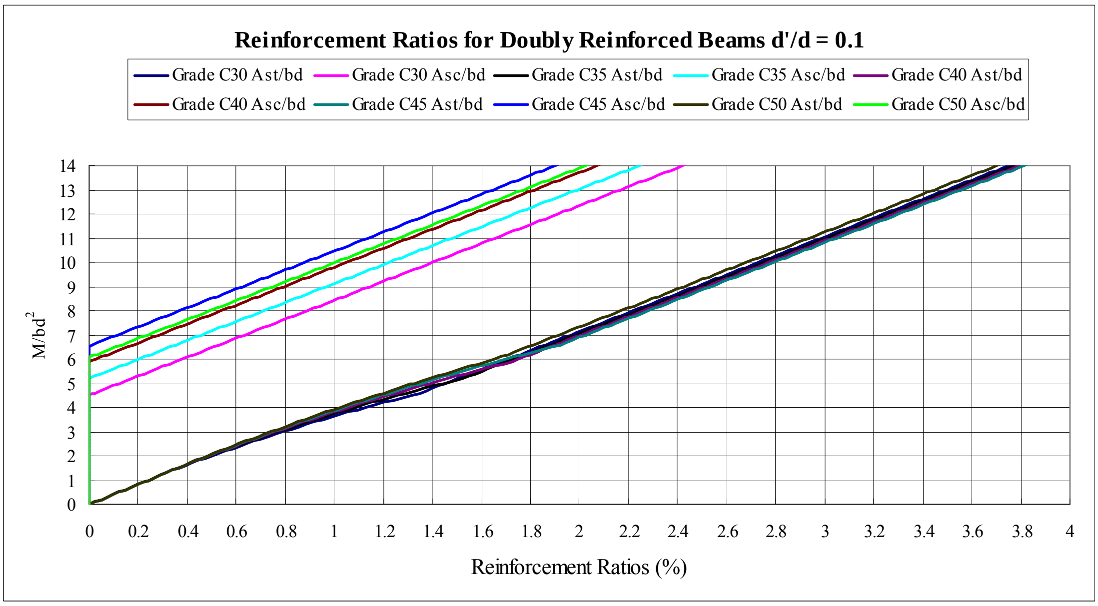


Chart 3-1 – Reinforcement Ratios of Doubly Reinforced Beams for Grade C30 to C50 with Moment Redistribution Limited to 10% or below

As similar to BS8110, there is an upper limit of "lever arm ratio" $\dfrac{z}{d}$ (which is the depth of the centroid of the compressive force of concrete) to the effective depth of the beam section of not exceeding 0.95. Thus for calculated values of $\dfrac{z}{d} \ge 0.95$ or $\dfrac{x}{d} \le 0.111$ in accordance with the simplified stress block approach, $\frac{A_{st}}{bd} = \frac{M}{0.87 f_y (0.95d) bd}$

Design Charts for grades C30 to C50 comprising tensile steel and compression steel ratios $\frac{A_{st}}{bd}$ and $\frac{A_{sc}}{bd}$ are enclosed at the end of Appendix C.

3.4.2 Design in accordance with the Simplified Stress Block

The design will be simpler and sometimes more economical if the simplified rectangular stress block as given by Figure 6.1 of the Code is adopted. The design formula becomes :

For singly reinforced sections where $K = \frac{M}{f_{cu} bd^2} \le K'$ where $K' = 0.156$ for grades C45 and below and $K' = 0.120$ for grades over C45 and not exceeding C70,

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \le 0.95;$$

$$\frac{x}{d} = \left(1 - \frac{z}{d}\right)\frac{1}{0.45} = \left(0.5 - \sqrt{0.25 - \frac{K}{0.9}}\right)\frac{1}{0.45}; \quad A_{st} = \frac{M}{0.87 f_y z} \qquad \text{(Eqn 3-5)}$$

For doubly reinforced sections $K = \frac{M}{f_{cu} bd^2} > K'$;

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K'}{0.9}} \qquad \frac{x}{d} = \left(1 - \frac{z}{d}\right)\frac{1}{0.45}$$

$$A_{sc} = \frac{(K - K') f_{cu} bd^2}{0.87 f_y (d - d')} \qquad A_{st} = \frac{K' f_{cu} bd^2}{0.87 f_y z} + A_{sc} \qquad \text{(Eqn 3-6)}$$

The above equations are based on the assumption that both the tension and compression steel are stressed to $0.87 f_y$ which requires their strains to exceed $0.87 f_y / E_s$. However, for large compression bars covers to neutral axis depth ratios, such strains may not be achievable as illustrated in Figure 3.8. Under such circumstances, the stresses of the compression bars have to be worked out from first principles as illustrated by Worked Example 3.4.

Worked Example 3.4

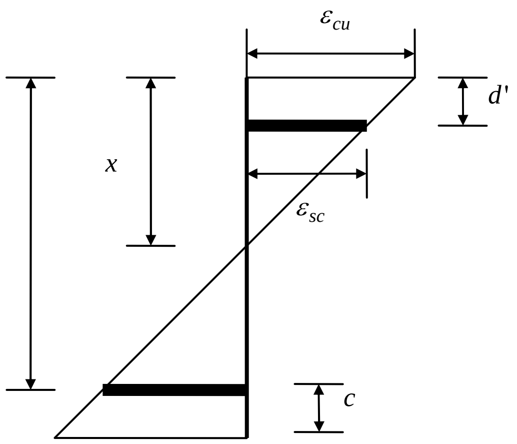
For a grade C35 beam of section 350(D)×300(B) resisting a moment 180kNm, $d = 350 - 50 - 25/2 = 287.5$ mm. $d' = 50 + 20/2 = 60$ mm.

As $K = \frac{M}{f_{cu} bd^2} = 0.207 > 0.156$, A_{sc} is required and $\frac{x}{d} = 0.5 \Rightarrow x = 143.75$.

As $\frac{d'}{x} = \frac{60}{143.75} = 0.42 > 1 - \frac{2.175 \times 10^{-3}}{0.0035} = 0.38$, the strain of the compression bar is $\varepsilon_{sc} = \left(1 - \frac{d'}{x}\right)\varepsilon_{cu} = 0.002039$ and the stress $\sigma_{sc} = E_s\varepsilon_{sc} = 407.83$ MPa.

So $A_{sc} = \frac{(K - K')f_{cu}bd^2}{\sigma_{sc}(d - d')} = \frac{(0.207 - 0.156)\times 35 \times 300 \times 287.5^2}{407.83 \times (287.5 - 60)} = 477\ \text{mm}^2$

$$A_{st} = \frac{K' f_{cu}bd^2}{0.87 f_y z} + A_{sc} = \frac{0.156 \times 35 \times 300 \times 287.5^2}{0.87 \times 500 \times 0.775 \times 287.5} + \frac{477 \times 407.83}{0.87 \times 500} = 1844\ \text{mm}^2$$



$\varepsilon_{sc} = \frac{x - d'}{x}\varepsilon_{cu} = \left(1 - \frac{d'}{x}\right)\varepsilon_{cu}$;

For $\varepsilon_{sc} < \frac{0.87 f_y}{E_s} = 2.175 \times 10^{-3}$ so that compressive stress cannot reach $0.87 f_y$,

$\varepsilon_{sc} = \left(1 - \frac{d'}{x}\right)\varepsilon_{cu} < 2.175 \times 10^{-3}$

$\Rightarrow \frac{d'}{x} > 1 - \frac{2.175 \times 10^{-3}}{\varepsilon_{cu}}$

Figure 3.8 – Illustration for Steel Bars in Beam not Reaching $0.87 f_y$

3.4.3 Ductility Requirement on Amounts of Compression Reinforcement

In accordance with Cl. 9.9.1.2(a) of the Code, within a "critical zone" of a beam contributing in the lateral load resisting system, the compression reinforcement should not be less than one-half of the tension reinforcement at the same section. A "critical zone" is defined in Cl. 9.9.1.1 to be a zone where a plastic hinge is likely to be formed and thus generally refers to sections near supports. The rationale for the requirement is based on the phenomenon that increase in tension reinforcement will decrease ductility while the increase in compression reinforcement will increase ductility as discussed by Law (2011). So, longitudinal reinforcement on the compressive side of the beam of area not less than half of that of the tension reinforcement is required even if compression reinforcement is not required for strength purpose at all, so as to ensure certain level of ductility. The adoption of the clause will likely result in providing more compression reinforcements in beams in critical zones.

3.4.4 Worked Examples for Determination of Steel Reinforcements in Rectangular Beam with Moment Redistribution < 10%

Unless otherwise demonstrated in the following worked examples, the requirement in Cl. 9.9.1.2(a) of the Code as discussed in para. 3.4.3 by requiring compression reinforcements be at least one half of the tension reinforcement is not included in the calculation of required reinforcements.

Worked Example 3.5

Section : 500 (D) × 400 (W), $f_{cu} = 35\,\text{MPa}$
cover = 40 mm (to main reinforcement)

(i) $M_1 = 286\,\text{kNm}$;

$d = 500 - 40 - 16 = 444$

$$\varepsilon_0 = \frac{1.34 f_{cu}}{\gamma_m E_d} = \frac{1.34\times 35}{1.5\times 19923} = 0.001569 \qquad \frac{\varepsilon_0}{\varepsilon_{cu}} = 0.4484$$

$$\frac{M_1}{f_{cu}bd^2} = \frac{286\times 10^6}{35\times 400\times 444^2} = 0.104 < 0.149\text{, so singly reinforced}$$

Solving the neutral axis depth ratio by (Eqn 3-1) $\dfrac{x}{d}$

$$\frac{0.67 f_{cu}}{\gamma_m}\left[-\frac{1}{2}+\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{cu}}\right)^2\right] = -5.742\,;$$

$$\frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right) = 13.297\,; \qquad -\frac{M_1}{bd^2} = \frac{286\times 10^6}{400\times 444^2} = -3.627$$

$$\frac{x}{d} = \frac{-13.297+\sqrt{13.297^2-4\times(-5.742)\times(-3.627)}}{2\times(-5.742)} = 0.316 \le 0.5$$

$$\frac{A_{st}}{bd} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{d} = \frac{1}{0.87\times 500}\times 13.297\times 0.316 = 0.00966$$

$\Rightarrow A_{st} = 1716\,\text{mm}^2$ Use 2T32 + I T25

(ii) $M_2 = 486\,\text{kNm}$;

$d = 500 - 40 - 20 = 440$

$$\varepsilon_0 = \frac{1.34 f_{cu}}{\gamma_m E_d} = \frac{1.34\times 35}{1.5\times 19923} = 0.001569 \qquad \frac{\varepsilon_0}{\varepsilon_{cu}} = 0.4484$$

$$\frac{M_2}{f_{cu}bd^2} = \frac{486\times 10^6}{35\times 400\times 440^2} = 0.179 > 0.149\text{, so doubly reinforced}$$

$d' = 40 + 10 = 50 \quad \dfrac{d'}{d} = \dfrac{50}{440} = 0.114$ (assume T20 bars)

By (Eqn 3-3) $$\frac{A_{sc}}{bd} = \frac{\left(\dfrac{M_2}{bd^2 f_{cu}} - K'\right)f_{cu}}{0.87 f_y\left(1-\dfrac{d'}{d}\right)} = \frac{(0.179-0.149)\times 35}{0.87\times 500\times(1-0.114)} = 0.272\,\%$$

$A_{sc} = 0.00272\times 400\times 440 = 479\,\text{mm}^2$ <u>Use 2T20</u>

By (Eqn 3-4) $$\frac{A_{st}}{bd} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\eta + \frac{A_{sc}}{bd}$$

$$\frac{A_{st}}{bd} = \frac{1}{0.87\times 500}13.297\times 0.5 + 0.00272 = 1.800\,\%$$

$A_{st} = 0.018\times 400\times 440 = 3168\,\text{mm}^2$ <u>Use 3T40</u>

Worked Example 3.6

(i) and (ii) of Worked Example 3.5 are re-done in accordance with Figure 6.1 of the Code (the simplified stress) block by (Eqn 3-5) and (Eqn 3-6)

(i) $\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} = 0.5 + \sqrt{0.25 - \frac{286 \times 10^6}{35 \times 400 \times 444^2 \times 0.9}} = 0.867$

$$\frac{A_{st}}{bd} = \frac{M_1}{bd^2 \times 0.87 f_y (z/d)} = \frac{286 \times 10^6}{400 \times 444^2 \times 0.87 \times 500 \times 0.867} = 0.00962$$

$\Rightarrow A_{st} = 1707\text{ mm}^2$ Use 2T32 + 1T25

(ii) $K = \frac{M_2}{f_{cu} bd^2} = \frac{486 \times 10^6}{35 \times 400 \times 440^2} = 0.179 > 0.156$, so doubly reinforcing section required, $\frac{z}{d} = 1 - 0.5 \times 0.9 \times 0.5 = 0.775$

$A_{sc} = \frac{(K - K') f_{cu} bd^2}{0.87 f_y (d - d')} = \frac{(0.179 - 0.156) \times 35 \times 400 \times 440^2}{0.87 \times 500 \times (440 - 50)} = 367\text{ mm}^2 <$ $0.2\% \to 400\text{mm}^2$ in accordance with Table 9.1 of the Code, Use 2T16

$$A_{st} = \frac{K' f_{cu} bd^2}{0.87 f_y z} + A_{sc} = \frac{0.156 \times 35 \times 400 \times 440^2}{0.87 \times 500 \times 0.775 \times 440} + 367 = 3217\text{ mm}^2$$

Use 3T40

Results of comparison of results from Worked Examples 3.5 and 3.6 (with the omission of the requirement in Cl. 9.9.1.2(a) that compressive reinforcements be at least half of that of tension reinforcements) are summarized in Table 3.3, indicating differences between the "Rigorous Stress Block" and "Simplified Stress Block" Approaches :

	Singly Reinforced	Doubly Reinforced		
	A_{st} (mm^2)	A_{sc} (mm^2)	A_{st} (mm^2)	Total (mm^2)
Based on Rigorous Stress Approach	1716	479	3168	3647
Based on Simplified stress Approach	1707	367 (min. 400)	3217	3584 (3617)

Table 3.3 – Summary of Results for Comparison of Rigorous Stress and Simplified Stress Blocks Approaches

Results by the two approaches are very close. The approach based on the simplified stress block is even slightly more economical.

3.4.5 Worked Example 3.7 for Rectangular Beam with Moment Redistribution > 10%

If the Worked Example 3.4 (ii) has undergone a moment redistribution of 20% > 10%, i.e. $\beta_b = 0.8$, by (Ceqn 6.4) of the Code, the neutral axis depth is limited to $\frac{x}{d} \le (\beta_b - 0.4) \Rightarrow \frac{x}{d} \le 0.8 - 0.4 = 0.4$,

and the lever arm ratio becomes $\frac{z}{d} = 1 - 0.4 \times 0.9 \times 0.5 = 0.82$.

So the $K = \frac{M_2}{bd^2 f_{cu}}$ value become $0.5 + \sqrt{0.25 - \frac{K'}{0.9}} = 0.82 \Rightarrow K' = 0.132$

$$A_{sc} = \frac{(K - K')f_{cu}bd^2}{0.87 f_y (d - d')} = \frac{(0.179 - 0.132) \times 35 \times 400 \times 440^2}{0.87 \times 500 \times (440 - 50)} = 751\,\text{mm}^2;$$

$$A_{st} = \frac{K' f_{cu} bd^2}{0.87 f_y z} + A_{sc} = \frac{0.132 \times 35 \times 400 \times 440^2}{0.87 \times 500 \times 0.82 \times 440} + 751 = 3031\,\text{mm}^2$$

So the total amount of reinforcement is greater.

3.5 Sectional Design of Flanged Beam against Bending

3.5.1 Slab structure adjacent to the beam, if in flexural compression, can be used to act as part of compression zone of the beam, thus effectively widen the structural width of the beam. The use of flanged beam will be particularly useful in eliminating the use of compressive reinforcements, as in addition to reducing tensile steel due to increase of lever arm. The principle of sectional design of flanged beam follows that rectangular beam with an additional flange width of $b_{eff} - b_w$ as illustrated in Figure 3.9.

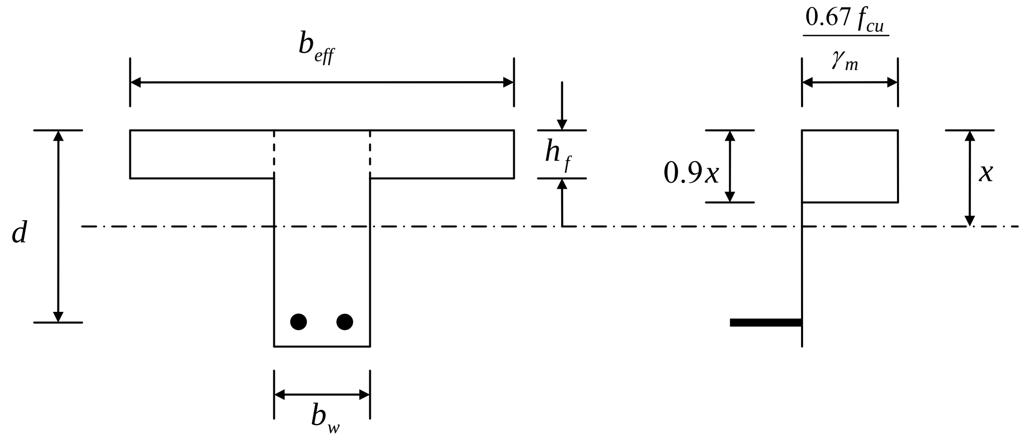


Figure 3.9 – Analysis of a T or L beam section

Design formulae based on the simplified stress block are derived in Appendix C which are summarized as follows :

(i) Singly reinforcing section where 0.9 × neutral axis depth is inside flange depth by checking

$$0.9\frac{x}{d} = 2\left(0.5 - \sqrt{0.25 - \frac{K}{0.9}}\right) \le \frac{h_f}{d} \quad \text{where} \quad K = \frac{M}{f_{cu} b_{eff} d^2} \quad \text{(Eqn 3-7)}$$

If so, carry out design as if it is a rectangular beam of width b_{eff} .

(ii) Singly reinforcing section where 0.9 × neutral axis depth is outside flange depth, i.e. $0.9\times\frac{x}{d}>\frac{h_f}{d}$ and

$$\frac{M}{b_w d^2 f_{cu}}\le K'=K_f'+K_w'=\frac{0.67}{\gamma_m}\frac{h_f}{d}\left(\frac{b_{eff}}{b_w}-1\right)\left(1-\frac{1}{2}\frac{h_f}{d}\right)+K_w'$$

where $K_w'=0.156$ for $f_{cu}\le 45$ and $K_w'=0.120$ for $45<f_{cu}\le 70$

$\frac{x}{d}$ be solved by the quadratic equation :

$$0.1809 f_{cu}\left(\frac{x}{d}\right)^2-0.402 f_{cu}\frac{x}{d}+\frac{M-M_f}{b_w d^2}=0 \quad \text{(Eqn 3-8)}$$

where $$\frac{M_f}{b_w d^2}=\frac{0.67 f_{cu}}{\gamma_m}\frac{h_f}{d}\left(\frac{b_{eff}}{b_w}-1\right)\left(1-\frac{1}{2}\frac{h_f}{d}\right) \quad \text{(Eqn 3-9)}$$

If $0.9\times\frac{x}{d}>\frac{h_f}{d}$, $$\frac{A_{st}}{b_w d}=\frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left[\left(\frac{b_{eff}}{b_w}-1\right)\frac{h_f}{d}+0.9\frac{x}{d}\right]$$

(Eqn 3-10)

(Eqn 3-10) is more economical than (Ceqn 6.17) which actually assumes the maximum value of the neutral axis depth ratio.

(iii) Doubly reinforcing section :

By following the procedure in (ii), if $\frac{x}{d}$ obtained by (Eqn 3-8) exceeds 0.5 for concrete grades up to and including C45, then double reinforcements will be required with required A_{sc} and A_{st} as

$$\frac{A_{sc}}{b_w d}=\frac{\left(\frac{M}{b_w d^2 f_{cu}}-K_f'\right)f_{cu}}{0.87 f_y\left(1-\frac{d'}{d}\right)} \quad \text{(Eqn 3-11)}$$

$$\frac{A_{st}}{b_w d}=\frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left[\left(\frac{b_{eff}}{b_w}-1\right)\frac{h_f}{d}+0.45\right]+\frac{A_{sc}}{b_w d}$$

(Eqn 3-12)

3.5.2 Cl. 5.2.1.2 of the Code requires that unless $b_{eff,i}$ (which is the width of the flange on either side of the web) is taken as less than $0.1l_{pi}$ (l_{pi} is the distance between points of zero moments), the shear stress between the web and flange should be checked and provided with transverse reinforcement. The requirement is explained by Figure 3.10 which serves to ensure effectiveness in mobilizing the flange in resisting bending. In the figure where the bending moment along the flanged varies along the span, the difference in compressive force (ΔF_d) in the flange has to be balanced by the shear force along the interfaces between the flange and the web. Transverse reinforcement of area (each bar) A_{sf} and spacing s_f should be used such that

$$0.87 f_y \frac{A_{sf}}{s_f} \geq v_{sf} h_f \qquad \text{(Eqn 3-13)}$$

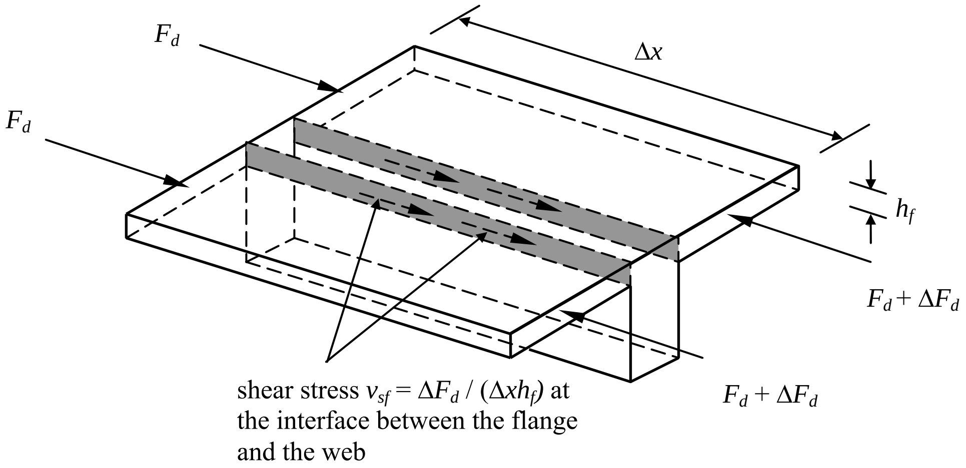


Figure 3.10 – Diagrammatic Illustration of the Shear at the Flange / Web Interface of Flanged Beam

Δx should be so chosen to arrive at the maximum value of v_{sf} in Figure 3.10 which is tedious. Nevertheless, we may adopt the requirement in EC2 Cl. 6.2.4 that the maximum value of Δx is half distance between the section where the moment is 0 and the section where the moment is maximum. Where point loads are applied, Δx should not exceed the distance between the point loads. This will be illustrated by Worked Example 3.11.

3.5.3 Worked Examples for Flanged Beam

(i) Worked Example 3.8 : Singly reinforced section where $0.9\frac{x}{d} \leq \frac{h_f}{d}$

Consider the previous example done for a rectangular beam 500(D) × 400(W), $f_{cu} = 35$ MPa, under a moment 486 kNm, with a flanged section of width = 1200 mm and depth = 150 mm :

$b_w = 400$, $d = 500 - 40 - 20 = 440$, $b_{eff} = 1200$ $h_f = 150$

First check if $0.9\frac{x}{d} \leq \frac{h_f}{d}$ based on beam width of 1200,

$$K = \frac{M_2}{f_{cu} b_{eff} d^2} = \frac{486 \times 10^6}{35 \times 1200 \times 440^2} = 0.0598$$

By (Eqn 3-5), $\frac{x}{d} = \left(0.5 - \sqrt{0.25 - \frac{K}{0.9}}\right)\frac{1}{0.45} = 0.159$;

$\therefore 0.9\frac{x}{d} = 0.143 < \frac{h_f}{d} = \frac{150}{440} = 0.341$. $\frac{z}{d} = 1 - 0.45\frac{x}{d} = 0.928$; Thus

$$\frac{A_{st}}{b_{eff} d} = \frac{M}{b_{eff} d^2 \times 0.87 f_y (z/d)} = \frac{486 \times 10^6}{1200 \times 440^2 \times 0.87 \times 500 \times 0.928} = 0.005182$$

$\therefore A_{st} = 2736\,\text{mm}^2$. Use 2T40 + 1T25

As in comparison with the previous example based on rectangular section, it can be seen that there is also saving in tensile steel (2736 mm² vs 3217 mm²). In addition, the compression reinforcements are eliminated in the flanged beam.

(ii) Worked Example 3.9 – Singly reinforced section where $0.9\frac{x}{d} > \frac{h_f}{d}$

Beam Section : 1000 (h) × 600 (w), flange width = 2000 mm,
flange depth = 150 mm $f_{cu} = 35$ MPa under a moment 4000 kNm

$b_w = 600$, $d = 1000 - 50 - 60 = 890$, $b_{eff} = 2000$ $h_f = 150$

$\frac{h_f}{d} = \frac{150}{890} = 0.169$; $\frac{b_{eff}}{b_w} = \frac{2000}{600} = 3.333$

First check if $0.9\frac{x}{d} \le \frac{h_f}{d}$ based on beam width of $b_w = b_{eff} = 2000$

$$K = \frac{M}{f_{cu} b_{eff} d^2} = \frac{4000\times10^6}{35\times2000\times890^2} = 0.0721$$

By (Eqn 3-7)

$$0.9\frac{x}{d} = 2\left(0.5 - \sqrt{0.25 - \frac{K}{0.9}}\right) = 0.176 > \frac{h_f}{d} = \frac{150}{890} = 0.169$$

So 0.9 × neutral axis depth extends below flange.

$$\frac{M_f}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m}\frac{h_f}{d}\left(\frac{b_{eff}}{b_w} - 1\right)\left(1 - \frac{1}{2}\frac{h_f}{d}\right) \Rightarrow M_f = 2675.65\text{ kNm}$$

Solve $\frac{x}{d}$ by (Eqn 3-8)

$$0.1809 f_{cu}\left(\frac{x}{d}\right)^2 - 0.402 f_{cu}\frac{x}{d} + \frac{M - M_f}{b_w d^2} = 0$$

$$\Rightarrow 0.1809\times35\left(\frac{x}{d}\right)^2 - 0.402\times35\frac{x}{d} + \frac{(4000 - 2675.65)\times10^6}{600\times890^2} = 0;$$

$$\Rightarrow \frac{x}{d} = 0.2198;$$

By (Eqn 3-10)

$$\frac{A_{st}}{b_w d} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left[\left(\frac{b_{eff}}{b_w} - 1\right)\frac{h_f}{d} + 0.9\times0.2198\right] = 0.02128$$

$A_{st} = 11364\text{ mm}^2$, Use 10-T40 in 2 layers

(iii) Worked Example 3.10 – Doubly reinforced section

Beam Section : 1000 (h) × 600 (w), flange width = 1250 mm,
flange depth = 150 mm $f_{cu} = 35$ MPa under a moment 4000 kNm

$b_w = 600$, $d = 1000 - 50 - 60 = 890$, $b_{eff} = 1250$ $h_f = 150$

$$\frac{h_f}{d} = \frac{150}{890} = 0.169 ; \qquad \frac{b_{eff}}{b_w} = \frac{1250}{600} = 2.083$$

First check if $0.9\frac{x}{d} \le \frac{h_f}{d}$ based on beam width of $b_{eff} = 1250$

$$K = \frac{M}{f_{cu} b_{eff} d^2} = \frac{4000 \times 10^6}{35 \times 1250 \times 890^2} = 0.115$$

By (Eqn 3-7),

$$0.9\frac{x}{d} = 2\left(0.5 - \sqrt{0.25 - \frac{K}{0.9}}\right) = 0.302 > \frac{h_f}{d} = \frac{150}{890} = 0.169$$

So 0.9 × neutral axis depth extends below flange.

$$\frac{M_f}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m}\frac{h_f}{d}\left(\frac{b_{eff}}{b_w} - 1\right)\left(1 - \frac{1}{2}\frac{h_f}{d}\right) \Rightarrow M_f = 1242.26\,\text{kNm}$$

Solve $\frac{x}{d}$ by (Eqn 3-8)

$$0.1809 f_{cu}\left(\frac{x}{d}\right)^2 - 0.402 f_{cu}\frac{x}{d} + \frac{M - M_f}{b_w d^2} = 0$$

$$\Rightarrow 0.1809 f_{cu}\left(\frac{x}{d}\right)^2 - 0.402 \times 35\frac{x}{d} + \frac{(4000 - 1242.26) \times 10^6}{600 \times 890^2} = 0$$

$\frac{x}{d} = 0.547 > 0.5$. Double reinforcement required. $d' = 50 + 20 = 70$

$$K' = K_f' + K_w' = \frac{0.67}{\gamma_m}\frac{h_f}{d}\left(\frac{b_{eff}}{b_w} - 1\right)\left(1 - \frac{1}{2}\frac{h_f}{d}\right) + 0.156 = 0.2308$$

By (Eqn 3-11) $$\frac{A_{sc}}{b_w d} = \frac{\left(\frac{M}{b_w d^2 f_{cu}} - K_f'\right) f_{cu}}{0.87 f_y\left(1 - \frac{d'}{d}\right)} = 0.084\% \quad A_{sc} = 451\,\text{mm}^2$$

Use 0.4% on flange as per Table 9.1 of the Code

$A_{sc} = 0.004 \times 1250 \times 150 = 750\,\text{mm}^2$ (0.125%). Use <u>6T20</u>

$$\frac{A_{st}}{b_w d} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left[\left(\frac{b_{eff}}{b_w} - 1\right)\frac{h_f}{d} + 0.9\frac{x}{d}\right] + 0.00084 = 0.0236$$

$\Rightarrow A_{st} = 12602\,\text{mm}^2$, Use <u>10T40 + 2T32 in 2 layers</u> (2.66%)

(iv) <u>Worked Example 3.11</u> – Transverse Reinforcements in Flange for Transmission of Interface Shear between Flange and Web of Flanged Beam

Consider a simply supported flanged beam of span 8m of section

500(D) × 400(W), $f_{cu} = 35$ MPa with a symmetrical flange width = 2160mm and flange depth = 150mm carrying a total factored uniformly distributed load of 62.5kN/m. So the maximum moment is $\frac{1}{8}\times 62.5\times 8^2 = 500$ kNm at mid-span and zero at support.

By Cl. 5.2.1.2 of the Code, as $b_{eff,1} = b_{eff,2} = (2160-400)/2 = 880$ mm > $0.1l_{pi} = 800$ mm, shear stress between the flange and the web should be checked.

EC2 Cl. 6.2.4 requires Δx to be half distance between the section where the moment is 0 and the section where the moment is maximum, $\Delta x = 0.5\times 4 = 2$ m.

The next step is to calculate ΔF_d. The greatest value should be that between the support where $F_d = 0$ (zero moment) and the section at 2m from support having 0.75 of the maximum moment which is $0.75\times 500 = 375$ kNm.

Assuming the compression zone does not fall below the flange at moment = 375kNm, $K = \frac{375\times 10^6}{2160\times 440^2\times 35} = 0.0256$;

$z = \left(0.5+\sqrt{0.25-\frac{K}{0.9}}\right)d = 427.1$ mm > $0.95d = 418$ mm;

$0.9x = (440-418)\times 2 = 44$ mm < 150mm, compression zone in flange.

$\therefore \Delta F_d = (M/z)\times 880/2160 = (375\times 10^6/418)\times(880/2160) = 365497$ N

The shear stress is $v_{sf} = \frac{\Delta F_d}{\Delta x h_f} = \frac{365497}{2000\times 150} = 1.218$ N/mm^2

By (Eqn 3-13) $0.87 f_y \frac{A_{sf}}{s_f} \geq v_{sf} h_f \Rightarrow \frac{A_{sf}}{s_f} \geq 0.42$

Use T12@250c/c, or 0.3% > 0.15% (required by Table 9.1 of the Code).

3.6 Detailing of Longitudinal Reinforcement for Bending in Beam

The followings should be observed in placing of longitudinal steel bars for bending. Re Cl. 9.2.1 and 9.9.1 of the Code. The requirements arising from "ductility" requirements are marked with "D":

(i) Minimum tensile steel percentage : 0.13% for rectangular beam generally (Table 9.1 of the Code) and 0.3% in accordance with Cl. 9.9.1 of the Code for ductility requirements (D); except for beams subject to pure tension which requires 0.45% as in Table 9.1;

(ii) Maximum tension steel percentage : 2.5% within critical section (Cl.

9.9.1.2(a)) (D). Maximum tension steel percentage : 4% at location outside critical section (Cl. 9.2.1.3 of the Code);

(iii) Minimum compressive steel percentage : When compressive steel is required for ultimate design, Table 9.1 of the Code should be followed by providing 0.2% for rectangular beam and different percentages for others. In addition, at any section of a beam within a critical zone (a zone extending from the column face to twice the beam depth for beam contributing in the lateral resisting system as described in Cl. 9.9.1.1 which is a potential plastic hinge zone as discussed in Section 2.4 and illustrated in Figure 3.11) the compression reinforcement ≥ one-half of the tension reinforcement in the same region (Cl. 9.9.1.2(a) of the Code) (D). The reason of limiting tension steel in (ii) and at the same time requiring compression steel not less than half of that of tension steel in "critical regions" is because tensile steel decreases ductility while compression steel increases ductility as discussed by Law (2011). The requirements thus ensure certain level of ductility in the beam;

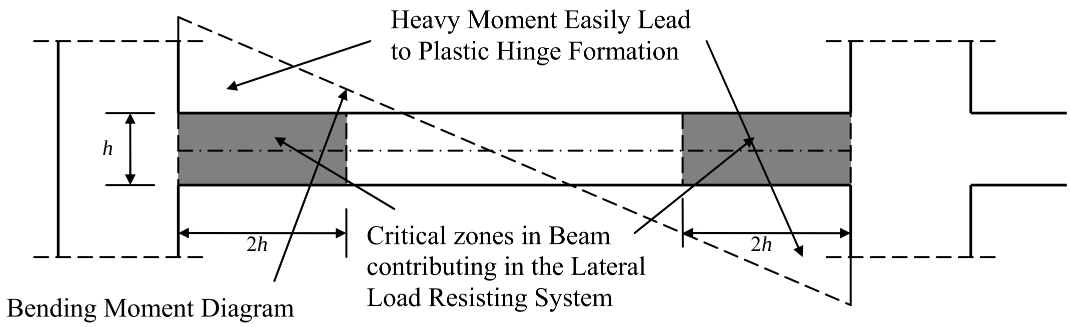


Figure 3.11 – Location of "Critical Zone" in Beam

(iv) For flanged beam, Figure 3.12 is used to illustrate the minimum percentages of tension and compression steel required in various parts of flanged beams (Table 9.1 of the Code);

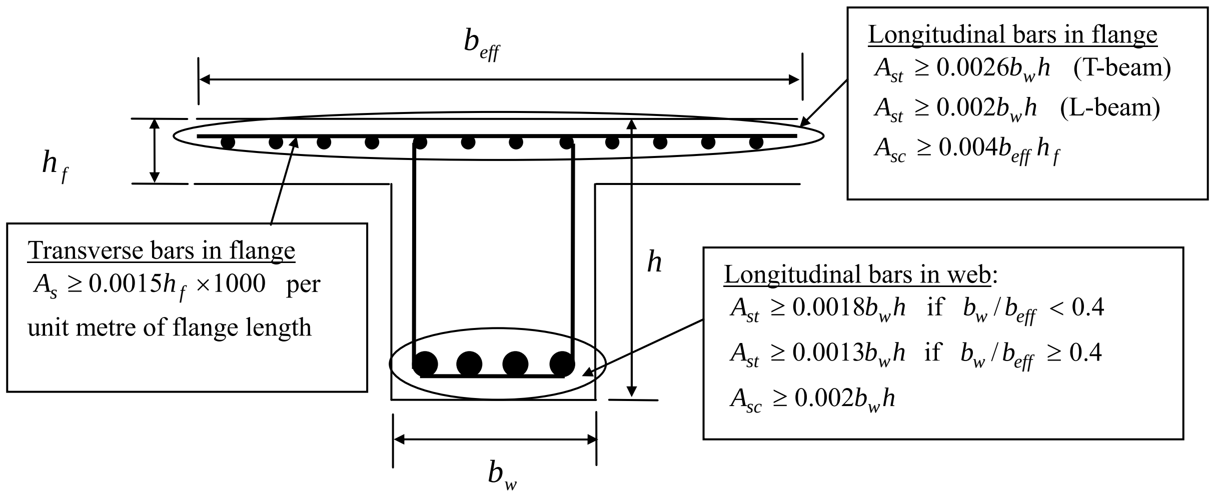


Figure 3.12 – Minimum Steel Percentages in Various Parts of Flanged Beams

(v) For calculation of anchorage lengths of longitudinal bars anchored into exterior columns, bars must be assumed to be stressed to f_y as a ductility requirement according to Cl 9.9.1.2(c) of the Code. That is, stresses in the steel should be f_y instead of $0.87f_y$ in the assessment of anchorage length. As such, the anchorage length as indicated in Table 8.4 of the Code should be increased by 15% as per (Ceqn 8.4) of the Code in which $l_b \geq \frac{f_y \phi}{4 f_{bu}}$ (which is modified from increasing stress in steel from $0.87f_y$ to f_y) where $f_{bu} = \beta\sqrt{f_{cu}}$ and β is 0.5 for tension anchorage and 0.63 for compression anchorage for ribbed steel reinforcing bars in accordance with Table 8.3 of the Code (D). An illustration is shown in Figure 3.20, as in addition to the restriction on the start of the anchorage;

(vi) For laps and type 1 mechanical couplers (Re 2.5 of this Manual) no portion of the splice shall be located within the beam/column joint region or within one effective depth of the beam from the column/wall face as per Cl. 9.9.1.2(d) (D) and illustrated in Figure 3.13;

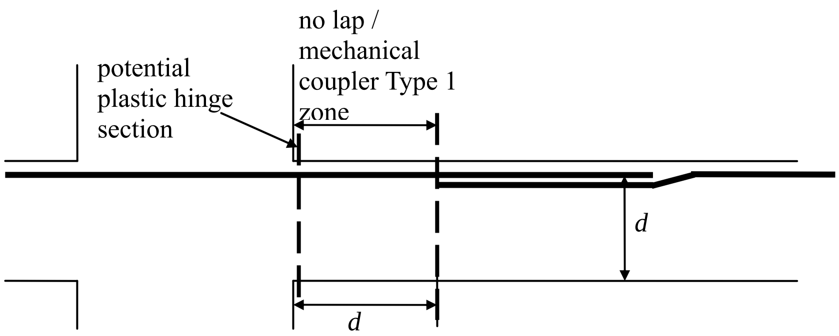


Figure 3.13 – Location of No Lap / Mechanical Coupler Type 1 Zone in Beam

(vii) Type 2 couplers, can however, be used anywhere in the beam as per Cl. 9.9.1.2(e) of the Code (D). The reason is that the Type 2 couplers are stronger and are tested to be able to resist cyclic tension and compression loads which are simulating seismic actions;

(viii) By Cl. 9.9.1.2(f) of the Code, a general requirement is stated that distribution and curtailment of longitudinal bar shall be such that the flexural overstrength will be within the critical section. (D)

(ix) At laps in general, the sum of reinforcement sizes in a particular layer should not exceed 40% of the beam width as illustrated by a numerical example in Figure 3.14 (Cl. 8.7.2 and Cl. 9.2.1.3 of the Code);

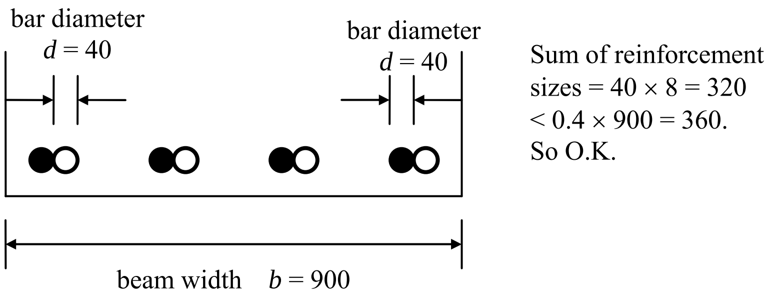


Figure 3.14 – Illustration of Sum of Reinforcement Sizes at Laps < 0.4 of Beam Width

(x) Minimum clear spacing of bars should be the greatest of bar diameter, 20 mm and aggregate size + 5 mm (Cl. 8.2 of the Code);

(xi) Maximum clear spacing between adjacent bars near tension face of a beam ≤ $70000\beta_b/f_y$ ≤ 300 mm where β_b is the ratio of moment redistribution (ratio of moment after redistribution to moment before redistribution) or alternatively ≤ $47000/f_s$ ≤ 300 mm where $f_s = \frac{2f_y A_{s,req}}{3A_{s,prov}} \times \frac{1}{\beta_b}$. Based on the former with $\beta_b = 1$ (no moment redistribution), the maximum clear spacing is 140mm (Cl. 8.7.2 and 9.2.1.4 of the Code) as illustrated in Figure 3.15;

(xii) By Cl. 9.2.1.9 of the Code, requirements for containment of compression steel bars in beam is identical to that of column as described in Cl. 9.5.2 of the Code :

(1) Every corner bar and each alternate bar (and bundle) in an outer layer should be supported by a link passing around the bar and having an included angle ≤ 135^o;
(2) Links should be adequately anchored by means of hook through a bent angle ≥ 135^o;
(3) No bar within a compression zone be more than 150 mm from a restrained bar supported / anchored by links as stated in (1) or (2) as illustrated in Figure 3.15.

In addition, in accordance with Cl. 9.5.2 of the Code, spacing of links along the beam should not exceed the least of :

(1) 12 times smallest longitudinal bar diameter;
(2) The lesser beam dimension; and
(3) 400mm;

(xiii) No tension bars should be more than 150 mm from a vertical leg (link) as illustrated in Figure 3.15 (Cl. 6.1.2.5(d) and Cl. 9.2.2 of the Code);

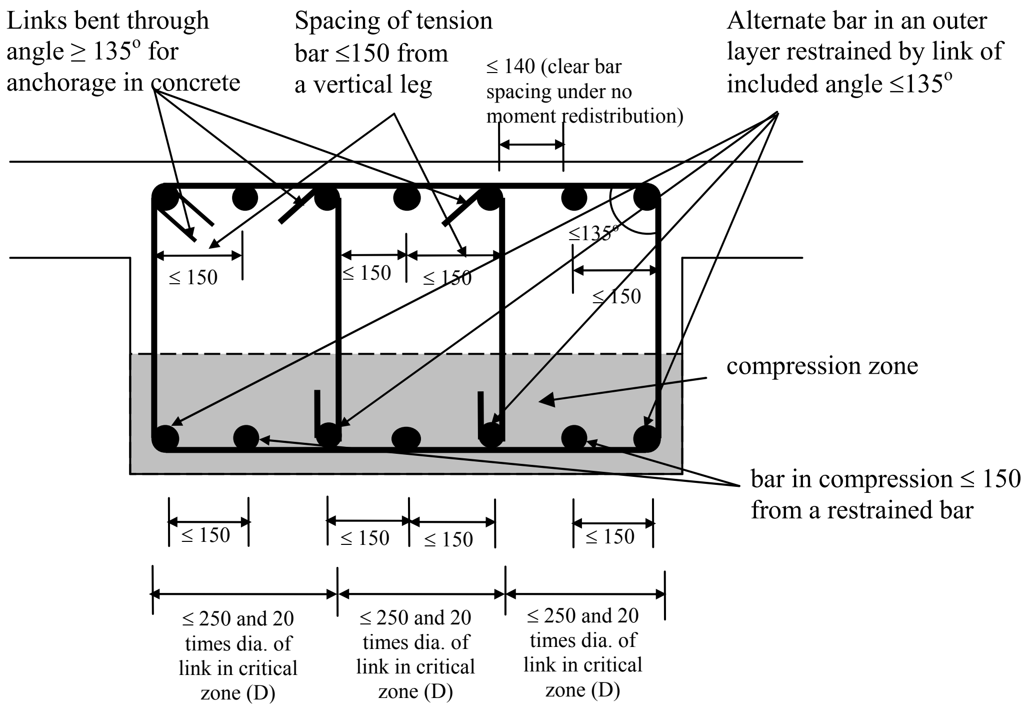


Figure 3.15 – Anchorage of Longitudinal Bar in Beam Section

(xiv) At an intermediate support of a continuous member, at least 30% of the calculated mid-span bottom reinforcement should be continuous over the support as illustrated in Figure 3.16 (Cl. 9.2.1.8 of the Code);

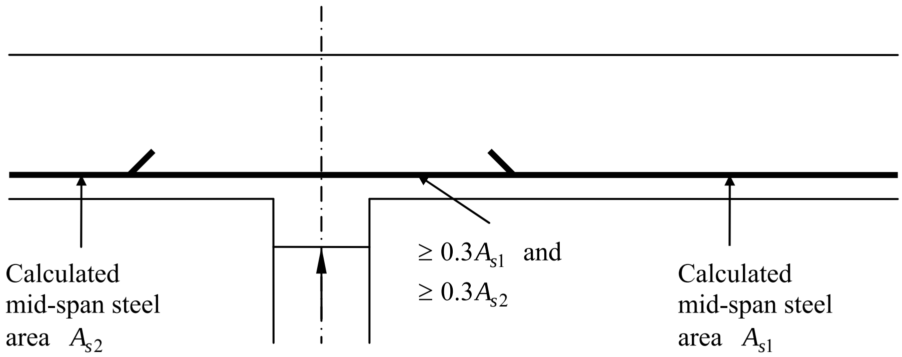


Figure 3.16 – At least 30% of the Calculated Mid-span Bottom Bars be Continuous over Intermediate Support

(xv) In monolithic construction, simple supports top reinforcements should be designed for 15% of the maximum moment in span as illustrated in Figure 3.17 (Cl. 9.2.1.5 of the Code) to allow for partial fixity;

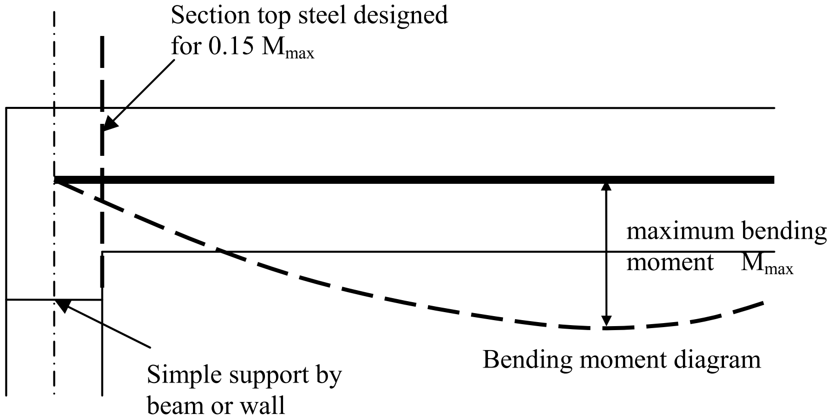


Figure 3.17 – Simple Support be Designed for 15% of the Maximum Span Moment

(xvi) For flanged beam over intermediate supports, the total tension reinforcements may be spread over the effective width of the flange with at least 85% inside the web as shown in Figure 3.18 reproduced from Figure 9.1 of the Code;

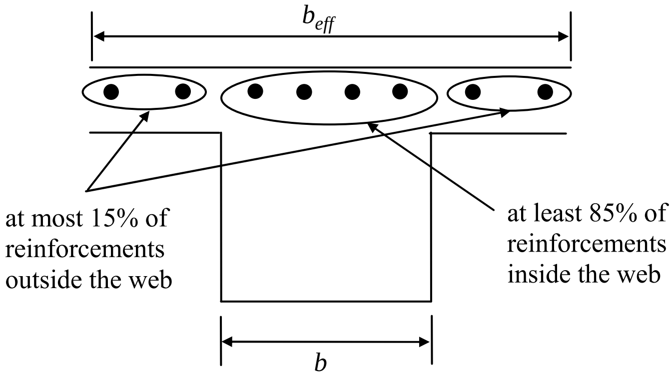
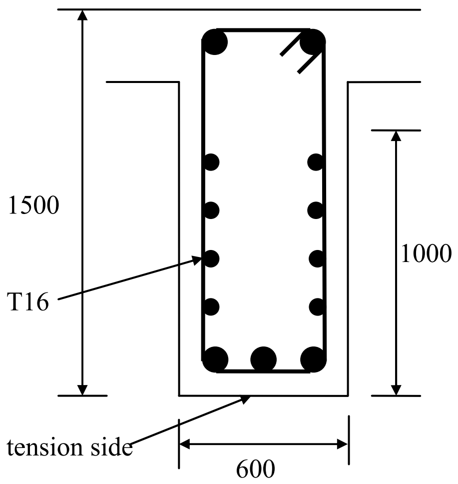


Figure 3.18 – Distribution of Tension Reinforcement Bars of Flanged Beam over Support

(xvii) For beam with depths > 750 mm, provision of sides bars of size (in mm) ≥ $\sqrt{s_b b / f_y}$ where s_b is the side bar spacing (in mm) and b is the lesser of the beam (in mm) breadth under consideration and 500 mm. f_y is in N/mm^2. In addition, it is required that $s_b \leq 250$ mm and side bars be distributed over two-thirds of the beam's overall depth measured from its tension face. Figure 3.19 illustrate a numerical example (Cl. 9.2.1.2 of the Code);



b is the lesser of 600 and 500, so
$b = 500$
s_b chosen to be 200 mm ≤ 250mm,
So size of side bar is
$\sqrt{s_b b / f_y} = \sqrt{200 \times 500 / 460}$
$= 14.74$
Use T16.
The side bars be distributed over
$\frac{2}{3} \times 1500 = 1000$ from bottom
which is the tension side

Figure 3.19 – Example of Determination of Side Bars in Beam

(xviii) When longitudinal beam bars are anchored in cores of exterior columns or beam studs, the anchorage for tension shall be deemed to commence at the lesser of 1/2 of the relevant depth of the column or 8 times the bar diameter as indicated in Figure 3.20. In addition, notwithstanding the adequacy of the anchorage of a beam bar in a column core or a beam stud, no bar shall be terminated without a vertical 90° standard hook or equivalent anchorage device as near as practically possible to the far side of the column core, or the end of the beam stud where appropriate, and not closer than 3/4 of the relevant depth of the column to the face of entry. Top beam bars shall be bent down and bottom bars must be bent up also indicated in Figure 3.20. (Cl. 9.9.1.2(c) of the Code) (D);

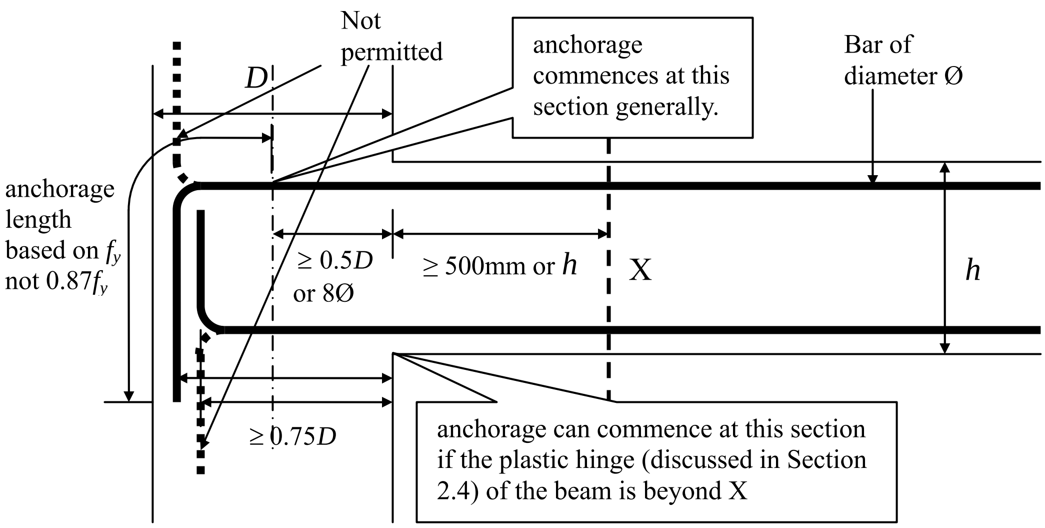


Figure 3.20 – Anchorage of Reinforcing Bars at Support

(xix) Beam should have a minimum support width by a supporting beam, wall, column as shown in Figure 3.21 as per Cl. 8.4.8 of the Code. The requirement stems from the practice that bend of bar not to begin before the centre of support as specified in Cl. 9.2.1.7(a). As such, the requirement should be that in Figure 3.20 which does not quite agree with Cl. 8.4.8 of the Code which has omitted for the case Ø ≤ 12.

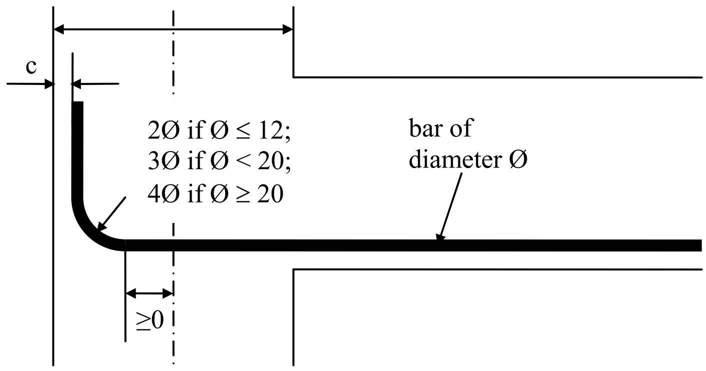


Figure 3.21 – Support Width Requirement

(xx) Curtailment of flexural reinforcements except at end supports should be in accordance with Figure 3.22 (Cl. 9.2.1.6(a) to (c) of the Code).

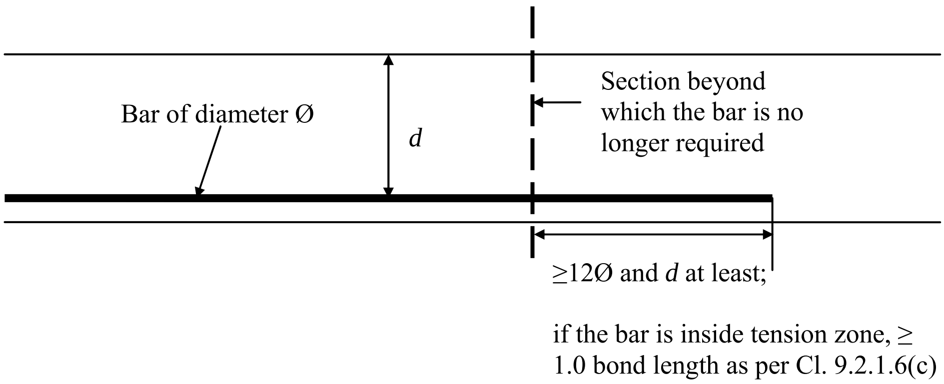


Figure 3.22 – Curtailment of Reinforcement Bars

<u>Worked Example 3.12</u>

Worked example 3.12 is used to illustrate the arrangement of longitudinal bars and the anchorages on thin support for the corridor slab beam of a typical housing block which functions as coupling beam between the shear walls on both sides. Plan, section and dimensions are shown in Figure 3.23. Concrete grade is C35. The design ultimate moment at support due to combined gravity and wind load is 352kNm.

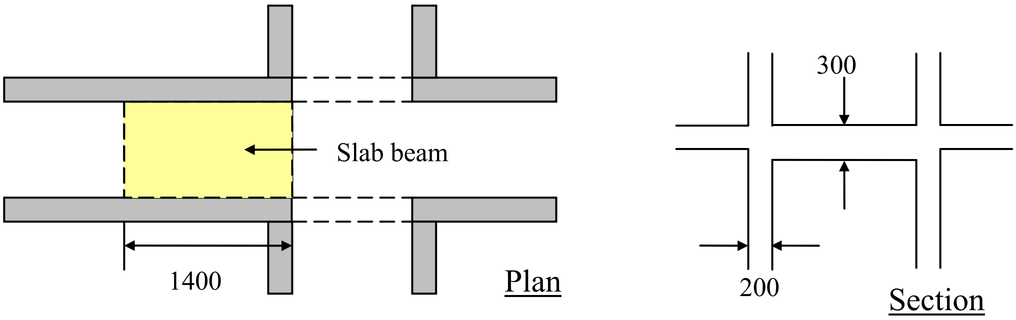


Figure 3.23 – Layout of the slab beam in Worked Example 3.12

The designed moment is mainly due to wind load which has resulted in required longitudinal steel area of 3763 mm^2 (each at top and bottom). The 200 mm thick wall can accommodate at most T16 bars as $2(4\times16+25)=178<200$ as per 3.6(xix). So use 19T16 (A_{st} provided is 3819 mm^2. Centre to centre bar spacing is $(1400-25\times2-16)/18=74$ mm.

For anchorage on support, anchorage length should be $38\times16=608$ mm. The factor 38 is taken from Table 8.4 of the Code which is used in assessing anchorage length. Anchorage details of the longitudinal bars at support are shown in Figure 3.24;

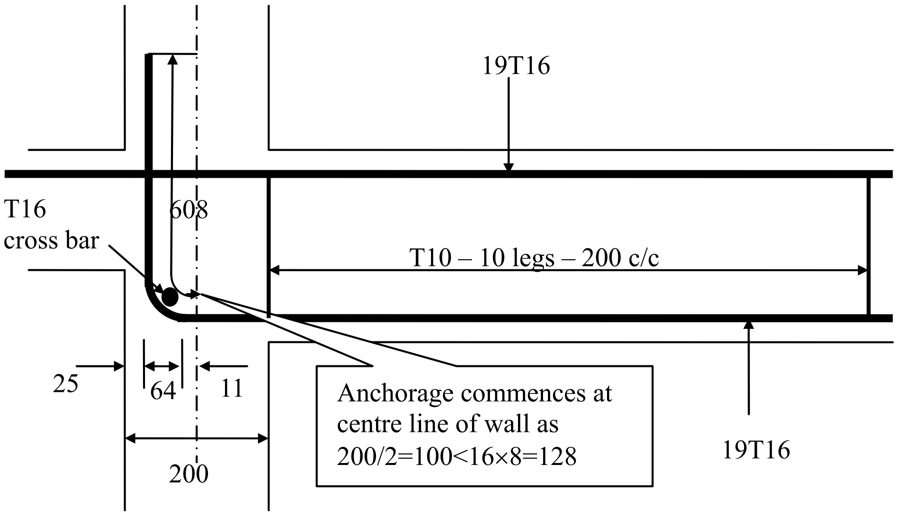


Figure 3.24 – Anchorage Details at Support for Worked Example 3.12

3.7 Design against Shear

3.7.1 Checking of Shear Stress and Provision of Shear Reinforcements

Checking of shear in beam is based on the averaged shear stress calculated from (Ceqn 6.19)

$$v=\frac{V}{b_v d}$$

where v is the average shear stress, V is the ultimate shear, d is the effective depth of the beam and b_v is beam width. b_v should be taken as the averaged width of the beam below flange in case of flanged beam)

If v is greater than the values of v_c, termed "design concrete shear stress" in Table 6.3 of the Code which is determined by the formula $v_c = 0.79\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}\left(\frac{100A_s}{b_v d}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}}\frac{1}{\gamma_m}$ listed in Table 6.3 of the Code with the following limitations :

(i) $\gamma_m = 1.25$;

(ii) $\frac{100A_s}{b_v d}$ should not be taken as greater than 3;

(iii) $\left(\frac{400}{d}\right)^{\frac{1}{4}}$ should not be taken as less than 0.67 for member without shear reinforcements and should not be taken as less than 1 for members with links;

Then shear links of $\frac{A_{sv}}{s_v} = \frac{b_v(v - v_c)}{0.87 f_{yv}} \geq \frac{b_v v_r}{0.87 f_{yv}}$ should be provided (Table 6.2 of the Code) where $v_r = 0.4$ for $f_{cu} \leq 40$ MPa and $0.4(f_{cu}/40)^{2/3}$ for $80 \geq f_{cu} > 40$. Alternatively, less than half of the shear resistance can be taken up by bent up bars by $0.5V \geq V_b = A_{sb}(0.87 f_{yv})(\cos\alpha + \sin\alpha\cot\beta)\frac{d - d'}{s_b}$ as per Ceqn 6.20 and Cl. 6.1.2.5(e) of the Code and the rest by vertical links. Maximum shear stress should not exceed $v_{tu} = 0.8\sqrt{f_{cu}}$ or 7 MPa, whichever is the lesser by Cl. 6.1.2.5(a).

3.7.2 Minimum Shear Reinforcements (Table 6.2 of the Code)

If $v < 0.5v_c$ throughout the beam, no shear reinforcement is required in beams of minor structural importance while minimum shear links be in other beams;

If $0.5v_c < v \leq (v_c + v_r)$, minimum shear links of $\frac{A_{sv}}{s_v} = \frac{b_v v_r}{0.87 f_{yv}}$ along the whole length of the beam be provided where $v_r = 0.4$ for $f_{cu} \leq 40$ and $0.4(f_{cu}/40)^{2/3}$ for $f_{cu} > 40$, but not greater than 80;

3.7.3 Enhanced Shear Strength close to Support (Cl. 6.1.2.5(g))

At sections of a beam at distance $a_v \leq 2d$ from a support, the shear strength can be increased by a factor $\frac{2d}{a_v}$, bounded by the absolute maximum of v_{tu}

which is the lesser of $0.8\sqrt{f_{cu}}$ and 7 MPa as illustrated by Figure 3.25.

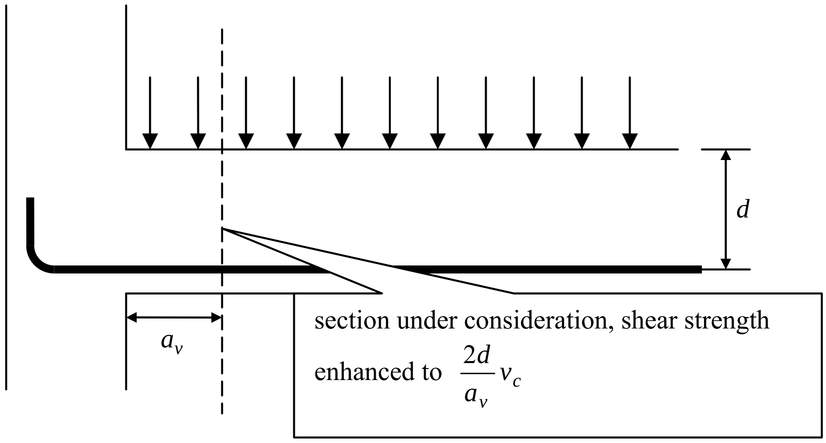


Figure 3.25 – Shear enhancement near support

3.7.4 Where load is applied to the bottom of a section, sufficient vertical reinforcement to carry the load should be provided in addition to any reinforcements required to carry shear as per Cl. 6.1.2.5(j) and shown in Figure 3.26;

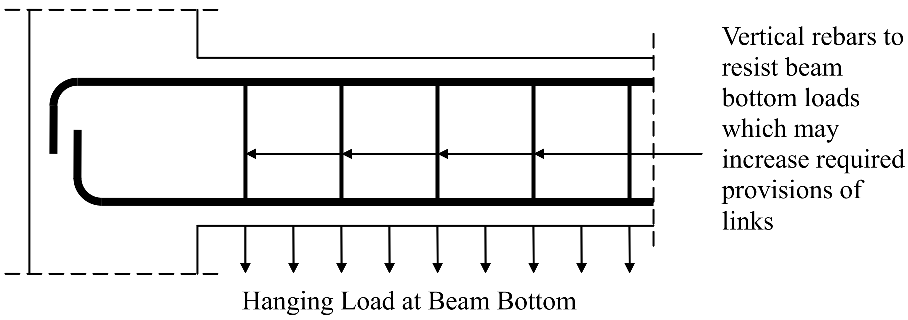


Figure 3.26 – Vertical Rebars to Resist Hanging Load at Beam Bottom (e.g. Inverted Beam)

3.7.5 Worked Examples for Shears

(i) Worked Example 3.13 – Shear design without shear enhancement in concrete

Beam Section : $b = 400$ mm;

$d = 700 - 40 - 16 = 644$ mm; $\frac{100A_{st}}{bd} = 1.5$; $f_{cu} = 35$ MPa;

$V = 700$ kN;

$$v_c = 0.79\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}\left(\frac{100A_s}{b_v d}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}}\frac{1}{\gamma_m} = 0.81\,\text{MPa};$$

where $\left(\frac{400}{d}\right)^{\frac{1}{4}}$ taken as 1.

$$v = \frac{700\times10^3}{400\times644} = 2.72\text{ MPa},$$

$$\frac{A_{sv}}{s_v} = \frac{b(v-v_c)}{0.87f_{yv}} = \frac{400(2.72-0.81)}{0.87\times500} = 1.76\text{ ; Use } \underline{\text{T12 – 250 c/c d.s.}}$$

(ii) Worked Example 3.14 – shear design with shear enhancement in concrete.

Re Figure 3.27 in which a section at 750mm from a support where a heavy point load is acting so that from the support face to the point load along the beam, the shear is more or less constant at 700 kN.

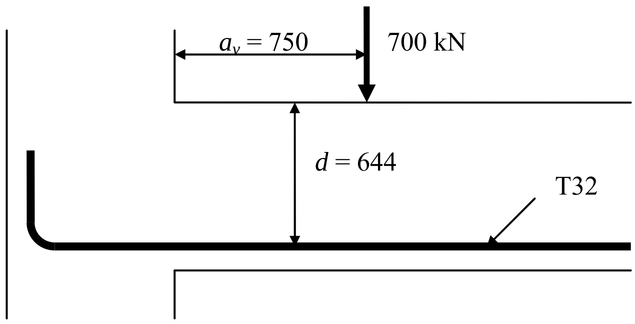


Figure 3.27 – Worked Example 3.14

Section : $b = 400$ mm; cover to main reinforcement = 40 mm;

$d = 700 - 40 - 16 = 644$ mm; $\frac{100A_{st}}{bd} = 1.5$; $f_{cu} = 35$ MPa; $V = 700$ kN;

$v_c = 0.79\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}\left(\frac{100A_s}{b_v d}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}}\frac{1}{\gamma_m} = 0.81\text{ MPa}$ where $\left(\frac{400}{d}\right)^{\frac{1}{4}}$ taken as 1.

Concrete shear strength enhanced to $\frac{2d}{a_v}v_c = \frac{2\times644}{750}\times0.81 = 1.39\text{ MPa}$ < 7 MPa and $0.8\sqrt{f_{cu}} = 0.8\sqrt{35} = 4.7\text{ MPa}$

$$v = \frac{700\times10^3}{400\times644} = 2.72\text{ MPa} < 4.7\text{MPa}$$

$$\frac{A_{sv}}{s_v} = \frac{b(v-v_c)}{0.87f_{yv}} = \frac{400(2.72-1.39)}{0.87\times500} = 1.22\text{ ; Use T12 – 175 c/c s.s}$$

(iii) Worked Example 3.15 – inclusion of bent-up bars (Cl. 6.1.25(e) of the Code)

If half of the shear in Worked Example 3.14 is to be taken up by bent-up bars, i.e. $0.5\times(2.72-0.81)\times400\times644\times10^{-3} = 246$ kN to be taken up by bent-up bars as shown in Figure 3.28.

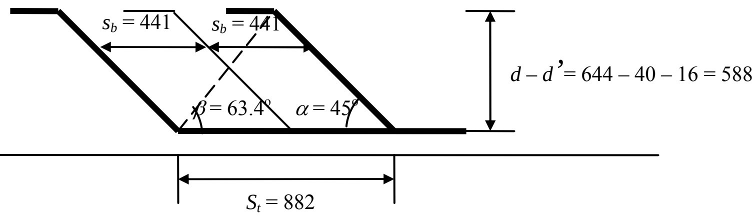


Figure 3.28 – Worked Example 3.15

By (Ceqn 6.20) of the Code,

$$V_b = A_{sb}\left(0.87 f_{yv}\right)\left(\cos\alpha + \sin\alpha\cot\beta\right)\frac{d-d'}{s_b}$$

$$\Rightarrow A_{sb} = \frac{246000 \times 441}{0.87 \times 500\left(\cos 45^0 + \sin 45^0 \cot 63.4^0\right) \times 588} = 400\,\text{mm}^2$$

Use 6 nos. of T10 at spacing of s_b = 441 mm as shown.

3.8 Placing of the Transverse Reinforcements

The followings should be observed for the placing of shear reinforcements :

(i) The minimum provision of shear reinforcements (links or bent up bars) in beams should be given by $A_{sv} \geq \dfrac{v_r b_v s_v}{0.87 f_{yv}}$ where $v_r = 0.4$ for $f_{cu} \leq 40$ and $v_r = 0.4\left(f_{cu}/40\right)^{2/3}$ for $80 \geq f_{cu} > 40$ (Re Cl. 6.1.2.5(b) of the Code);

(ii) At least 50% of the necessary shear reinforcements be in form of links (Re Cl. 6.1.2.5(e) of the Code);

(iii) In accordance with Cl. 6.1.2.5(d) of the Code, the maximum spacing of links in the direction of the span of the beam is 0.75*d*, the lateral spacing of links in the direction perpendicular to the span should not exceed the beam effective depth and be that no longitudinal tension bar be more than 150 mm from a vertical leg as illustrated in Figure 3.29 (also shown in Figure 3.15);

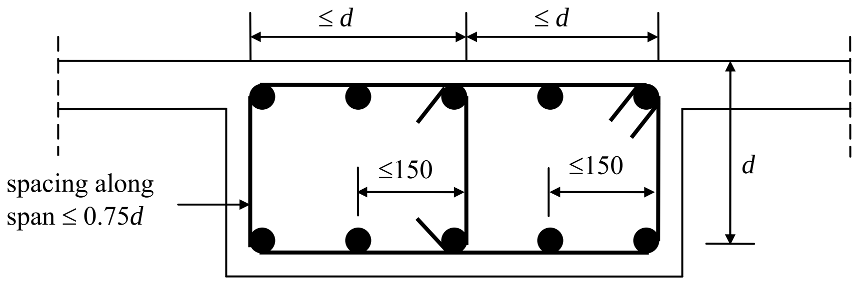


Figure 3.29 – Shear Links for Beams

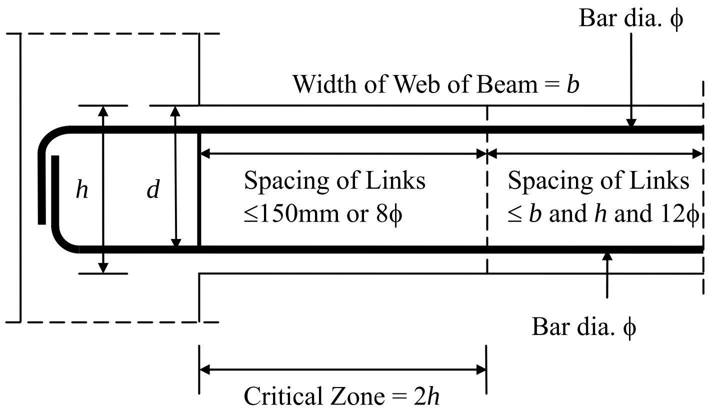
(iv) In addition to (iii), for beams contributing in the lateral load resisting system, the maximum spacing of links in the direction of the span of the beam not within critical zone) is the lesser of

(a) the least lateral dimension of the beam (D);
(b) 12 times the longitudinal bar diameter (D);

and that within critical zone is the greater of

(c) 150mm (D);
(d) 8 times longitudinal bar diameter (D);

The requirements stem from Cl. 9.9.1.3(a) and are illustrated in Figure 3.30 with a worked example.



Example : Beam of width $b = 400$, depth, $h = 700$, $d = 644$ and beam bar $\phi = 32$; link spacing $\leq 0.75d = 483$mm along the whole beam

Outside Critical Zone
(a) $b = 400$;
(b) $12 \times 32 = 384$;
Within Critical Zone;
(a) 150mm;
(b) $8 \times 32 = 256$mm
So maximum link spacing should be 384 mm outside critical zone and 256mm inside critical zone

Figure 3.30 – Maximum Spacing of Shear Links in the Span Direction of Beam Contributing in the Lateral Load Resisting System

(v) By Cl. 9.2.1.9 of the Code which requires containment of compression reinforcements in beams to follow that of column as in Cl. 9.5.2, links or ties shall be arranged so that every corner and alternate longitudinal bar that is required to function as compression reinforcement shall be restrained by a leg with an included angle ≤ 135° as illustrated Figure 3.15. In addition, the requirement that no bar within a compression zone be more than 150 mm from a restrained bar (anchored by links of bent angle ≥ 135°) as illustrated in Figure 3.15 also applies. Spacing of links along the beam should not exceed the least of (1) 12 times smallest longitudinal bar diameter; (2) the lesser beam dimension; and (3) 400mm also in accordance with Cl. 9.5.2. Also, link spacing transverse to beam direction should not exceed 20 times longitudinal bar diameter and 250mm as shown in Figure 3.15 (D) (Re Cl. 9.9.1.3(a) of the Code);

(vi) By Cl. 9.9.1.3(b) of the Code, links in beams should be adequately anchored by means of 135° or 180° hooks in accordance with Cl. 8.5 of the Code. Illustration of the better performance in anchorage between the 90° and 135° hooks has been included in Figure 2.6. Anchorage by welded cross bars is not permitted. Where there is adequate confinement to prevent the end anchorage of the link from "kick off", the 135° hood can be replaced by other standard hoods, say 90° hook as indicated in Figure 9.7 of the Code in which the adjacent slab acts as confinement to prevent link "kick-off". Links with different angles of hooks are shown in Figure 3.31 (D);

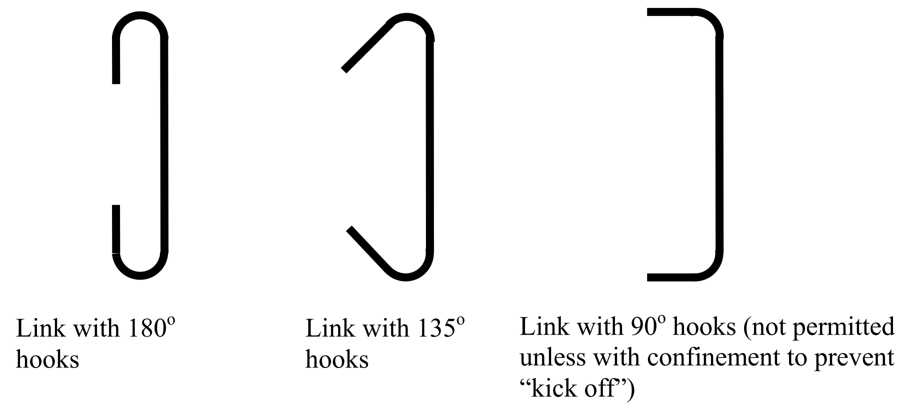


Figure 3.31 – Links with Hooks with different Bent Angles in Beam

3.9 Design against Torsion

3.9.1 By Cl. 6.3.1 of the Code, in normal slab-and-beam and framed construction, checking against torsion is usually not necessary. However, checking needs be carried out if the design relies entirely on the torsional resistance of a member such as that indicated in Figure 3.32.

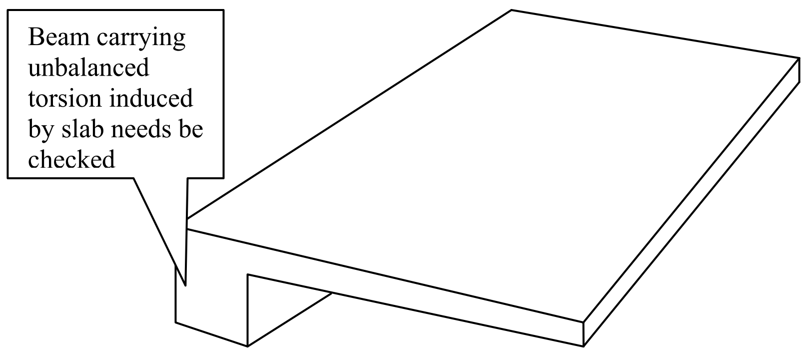


Figure 3.32 – Illustration for necessity of checking against torsion

3.9.2 Calculation of Torsional Rigidity of a Rectangular Section for Analysis (in grillage system) is by (Ceqn 6.64) of the Code

$C = \frac{1}{2}\beta h_{min}^{3} h_{max}$ where β is to be read from Table 6.16 of the Code reproduced as Table 3.4 of this Manual.

h_{max}/h_{min}	1	1.5	2	3	5	>5
β	0.14	0.20	0.23	0.26	0.29	0.33

Table 3.4 – Values of the Coefficient β

3.9.3 Calculation of Torsional Shear Stress

Upon arrival of the torsion on the rectangular section, the torsional shear stress is calculated by (Ceqn 6.65)

$$v_t = \frac{2T}{h_{\min}^2 \left(h_{\max} - \frac{h_{\min}}{3} \right)}$$

and in case of a section such as T or L sections made of rectangles, the section should be broken up into two or more rectangles such that the $\sum h_{\min}^3 h_{\max}$ is maximized and the total Torsional moment T be apportioned to each rectangle in accordance with (Ceqn 6.66) of the Code as $T \times \left(\frac{h_{\min}^3 h_{\max}}{\sum h_{\min}^3 h_{\max}} \right)$.

If the torsional shear stress exceeds $0.067\sqrt{f_{cu}}$ (but not more than 0.6MPa), torsional reinforcements will be required (Table 6.17 of the Code).

Furthermore, the torsional shear stress should be added to the shear stress induced by shear force to ensure that the absolute maximum $v_{tu} = 0.8\sqrt{f_{cu}}$ or 7MPa is not exceeded, though for small section where y_1 (the larger centre-to-centre dimension of a rectangular link) < 550mm, v_{tu} will be decreased by a factor $y_1 / 550$ (Cl. 6.3.4 of the Code). Revision of section is required if the absolute maximum is exceeded (Table 6.17 of the Code).

3.9.4 Calculation of Torsional Reinforcements

Torsional reinforcement in forms of close rectangular links and longitudinal bars are to be calculated by (Ceqn 6.67) and (Ceqn 6.68) of the Code as

$$\frac{A_{sv}}{s_v} = \frac{T}{0.8 x_1 y_1 (0.87 f_{yv})} \qquad \text{(Ceqn 6.67)}$$

(A_{sv} is the area of the 2 legs of the link)

$$A_s = \frac{A_{sv} f_{yv} (x_1 + y_1)}{s_v f_y} \qquad \text{(Ceqn 6.68)}$$

It should be noted that there is no reduction by shear strength (v_c) of concrete.

The derivation of the design formula (Ceqn 6.67) of the Code for close rectangular links is under the assumption of a shear rupture length of stirrup width + stirrup depth $x_1 + y_1$ as shown in Figure 3.33, the principle of which can be found in text book such as Kong & Evans (1987). A spiral torsional failure face is along the heavy dotted line in the figure. It is also shown in the figure that the torsional moment of resistance by the stirrups within the Regions X and Y are identical and the total resistance is therefore

$T = \frac{A_{sv} 0.87 f_{yv} x_1 y_1}{s_v} \Rightarrow \frac{A_{sv}}{s_v} = \frac{T}{0.87 f_{yv} x_1 y_1}$. An additional efficiency factor of 0.8 is then added to allow for errors in assumption made about the truss behaviour and the equation becomes $\frac{A_{sv}}{s_v} = \frac{T}{0.8(0.87 f_{yv} x_1 y_1)}$ (Ceqn 6.67). The derivation of the longitudinal bars is based on the use of same quantity of longitudinal bars as that of stirrups with even distribution along the inside of the stirrups. Nevertheless, the Code allows merging of the flexural steel with these longitudinal bars by using larger diameter of bars as will be illustrated in the Worked Example 3.16.

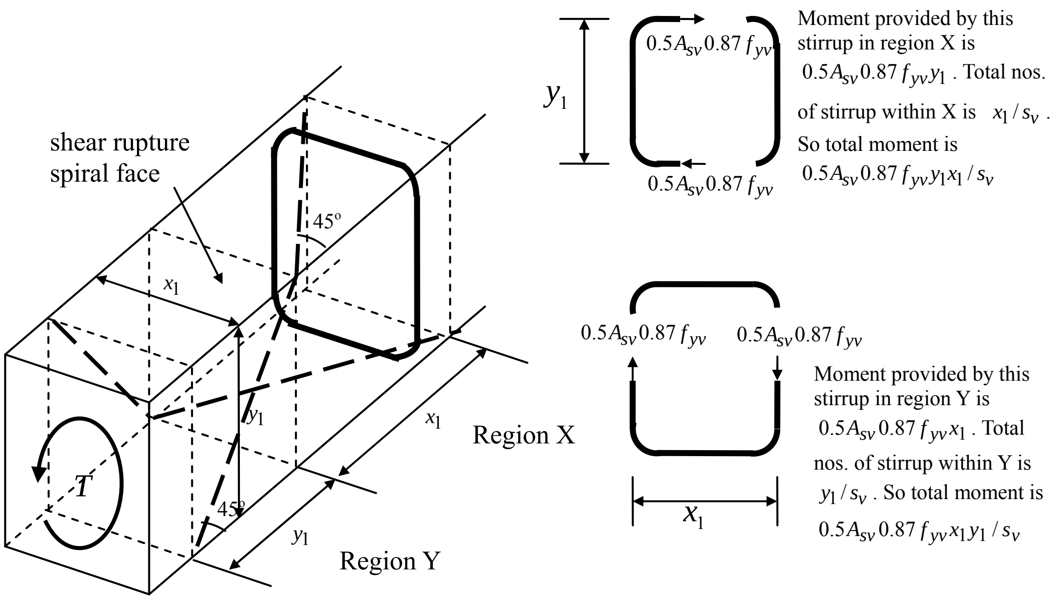


Figure 3.33 – Derivation of Formulae for Torsional Reinforcements

3.9.5 Worked Example 3.16 – Design for T-beam against torsion

A total torsion of $T = 200$ kNm on a T-section as shown in Figure 3.34 with an average vertical shear stress on the web of 0.82 N/mm^2. The section is also under bending requiring flexural steel area of 2865 mm^2 at bottom. Concrete grade is C35.

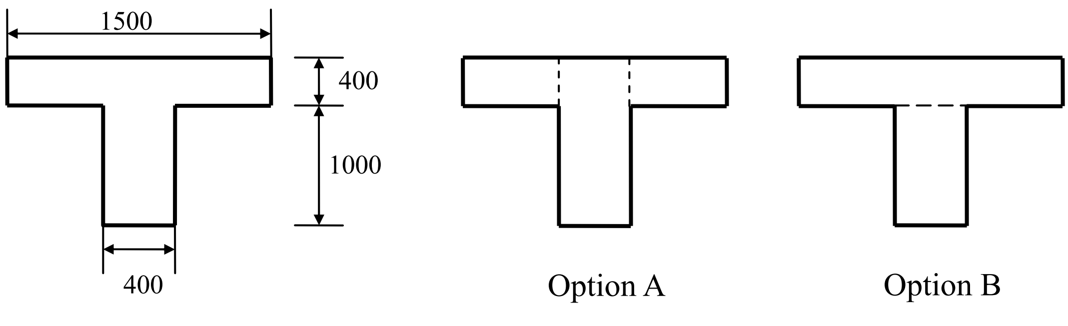


Figure 3.34 – Section of a T section resisting torsion for Worked Example 3.14

Option A is made up of two rectangles of 550×400 and one rectangle of 400×1400. From Table 6.16 of the Code, β for the 550×400 rectangle and 400×1400 are respectively 0.185 and 0.2675.
So the total stiffness of Option A is

$$2\times0.185\times550\times400^3+0.2675\times1400\times400^3=36.992\times10^9\,\text{mm}^4$$

Option B is made up of one rectangle of 1500×400 and one rectangle of 400×1000. From Table 6.16 of the Code, β for the 1500×400 rectangle and 400×1000 are respectively 0.27125 and 0.245.
So the total stiffness of Option B is

$$0.27125\times1500\times400^3+0.245\times1000\times400^3=41.72\times10^9\,\text{mm}^4$$

As Option B has a larger torsional stiffness, it is adopted for design.

The torsional moment is apportioned to the two rectangles of Option B as :
For the 1500×400 rectangle $T_1=200\times\dfrac{26.04}{41.72}=124.832\,\text{kNm}$;
Torsional shear stress is

$$v_{t1}=\frac{2T_1}{{h_{\min}}^2\left(h_{\max}-\dfrac{h_{\min}}{3}\right)}=\frac{2\times124.832\times10^6}{400^2\left(1500-\dfrac{400}{3}\right)}=1.142\,\text{N/mm}^2$$

$$>0.067\sqrt{f_{cu}}=0.396\,\text{N/mm}^2$$

So torsional reinforcement is required

$$x_1=400-40\times2-6\times2=308;\quad y_1=1500-40\times2-6\times2=1408$$

$$\frac{A_{sv}}{s_v}=\frac{T_1}{0.8x_1y_1(0.87f_{yv})}=\frac{124.832\times10^6}{0.8\times308\times1408\times0.87\times500}=0.827\,\text{mm}$$

Use <u>T12 – 200 s.s.</u> 200 ≤ 200, $x_1=308$, $y_1/2=1408/2=704$ as per Cl. 6.3.7 of the Code.

$$A_s=\frac{A_{sv}f_{yv}(x_1+y_1)}{s_vf_y}=\frac{0.827\times500\times(308+1408)}{500}=1419\,\text{mm}^2$$

Use <u>4T16 + 8T12</u>
For the 1000×400 rectangle $T_2=200\times\dfrac{15.68}{41.72}=75.168\,\text{kNm}$

$$v_{t2}=\frac{2T_2}{{h_{\min}}^2\left(h_{\max}-\dfrac{h_{\min}}{3}\right)}=\frac{2\times75.168\times10^6}{400^2\left(1000-\dfrac{400}{3}\right)}=1.084\,\text{N/mm}^2$$

The total shear stress is $1.084+0.82=1.904\,\text{N/mm}^2<v_{tu}=4.73\,\text{MPa}$
As $1.084>0.067\sqrt{f_{cu}}=0.396\,\text{N/mm}^2$, torsional reinforcement is required.

$$x_1=400-40\times2-6\times2=308\,\text{mm};\quad y_1=1000-40\times2-6\times2=908\,\text{mm}$$

$$\left(\frac{A_{sv}}{s_v}\right)_{Torsion} = \frac{T_2}{0.8x_1y_1(0.87f_{yv})} = \frac{75.168\times10^6}{0.8\times308\times908\times0.87\times500} = 0.772\,\text{mm}$$

For the vertical shear where $v_c = 0.575$ MPa (determined by Table 6.3 of the Code as per 2865mm^2 longitudinal steel as given),

$$\left(\frac{A_{sv}}{s_v}\right)_{shear} = \frac{b(v-v_c)}{0.87f_{yv}} = \frac{(0.82-0.575)\times400}{0.87\times500} = 0.225 < \frac{0.4\times400}{0.87\times500} = 0.368$$

($v_r = 0.4$), the total required $\frac{A_{sv}}{s_v} = 0.772 + 0.368 = 1.14$.

So use Use <u>T12 – 175 s.s.</u> 175 < 200 < $x_1 = 308$, 175 < $y_1/2 = 908/2 = 454$ as per Cl. 6.3.7 of the Code.

It should be noted that the torsional shear link should be closed links of shape as indicated in Figure 9.3 of the Code.

$$A_s = \frac{A_{sv}f_{yv}(x_1+y_1)}{s_vf_y} = \frac{0.772\times500\times(308+908)}{500} = 939\,\text{mm}^2.$$ Use <u>9T12</u>

Incorporating the bottom 3T12 into the required flexural steel, the bottom steel area required is $2865 + 113.1\times3 = 3205\,\text{mm}^2$. So use <u>4T32</u> at bottom and 6T12 at sides. The sectional detail is shown in Figure 3.35.

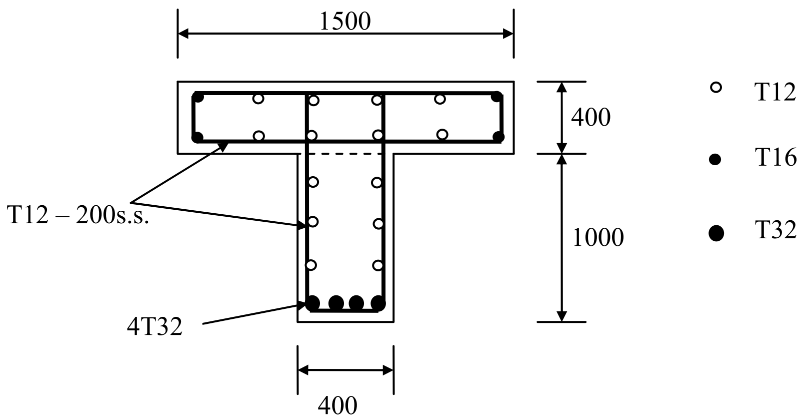


Figure 3.35 – Arrangement of torsional reinforcements

It should be borne in mind that these torsional reinforcements are in addition to others required for flexure and shear etc.

3.10 Placing of Torsional Reinforcements

The followings (in Cl. 6.3.7, Cl. 6.3.8 and Cl. 9.2.3 of the Code) should be observed for the placing of torsional reinforcements :

(i) The torsional shear link should form the closed shape as in Figure 9.1 of the Concrete Code Handbook reproduced as Figure 3.36. It should be noted that the second configuration in the Figure is not included in Figure 9.3 of the Code though it should also be acceptable;

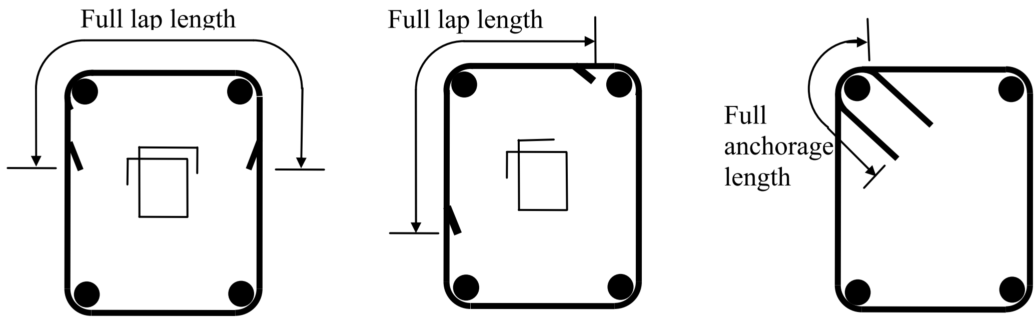


Figure 3.36 – Shape of Torsional shear links

Nevertheless, the Code adds in its Cl. 9.2.3 that where there is adequate confinement to prevent the end anchorage of the link from "kick off", alternative anchor details can be used. This arises from the phenomenon that torsion will create diagonal compression along the surface of the beam and outward thrust at the corner as discussed by Law & Mak (2013) which is extracted in Figure 3.37.

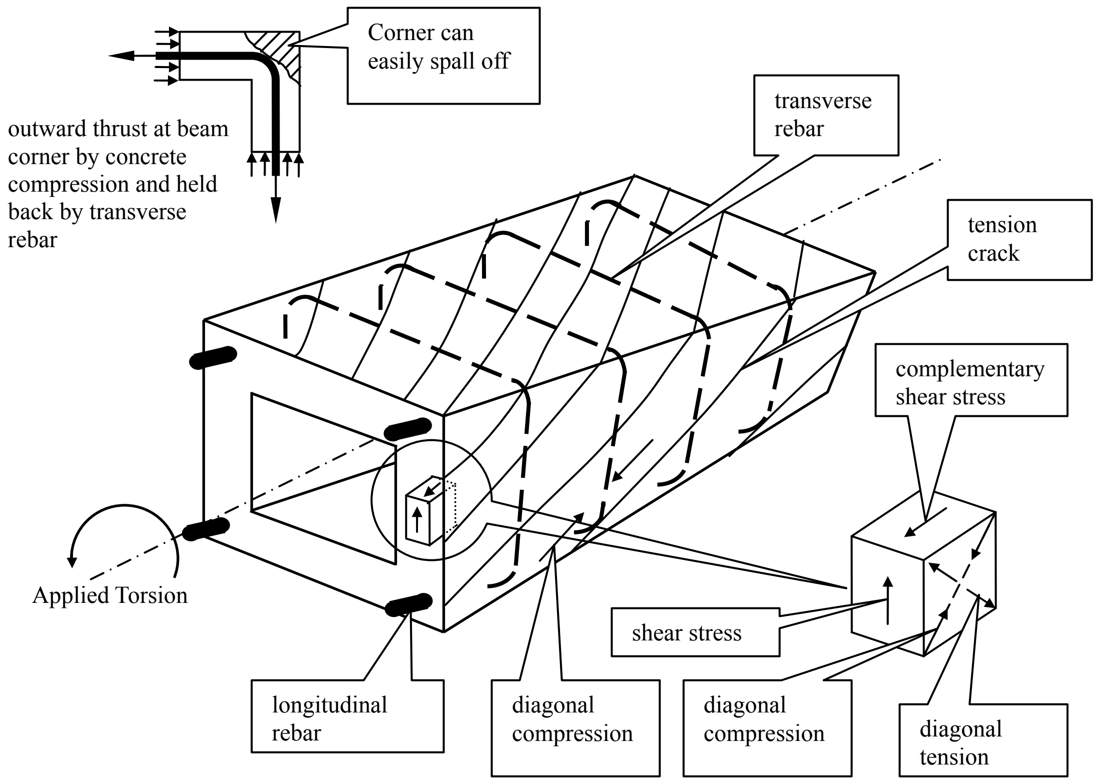


Figure 3.37 – The Diagonal Compression Stress Field Model for Torsion

The stirrups are therefore required to be well anchored so as to "hold" the inner "core" of the concrete. Demonstration of failures of "kick off" by 90^{o} anchorage of links and U-bars are shown in Figures 3.38(a) and 3.38(b).

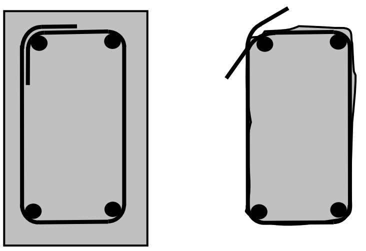


Figure 3.38(a) – Loss of Anchorage of Stirrups formed by 90° Bend End

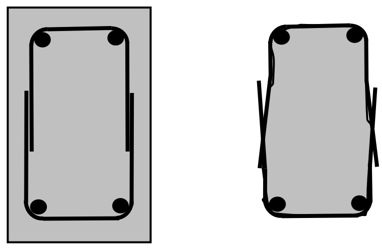


Figure 3.38(b) – Loss of Anchorage of Stirrups formed by Lapped U-bars

So stirrups which are well anchored so that "kick-off" at their ends will not take place are acceptable. Alternative examples are shown in Figure 3.39.

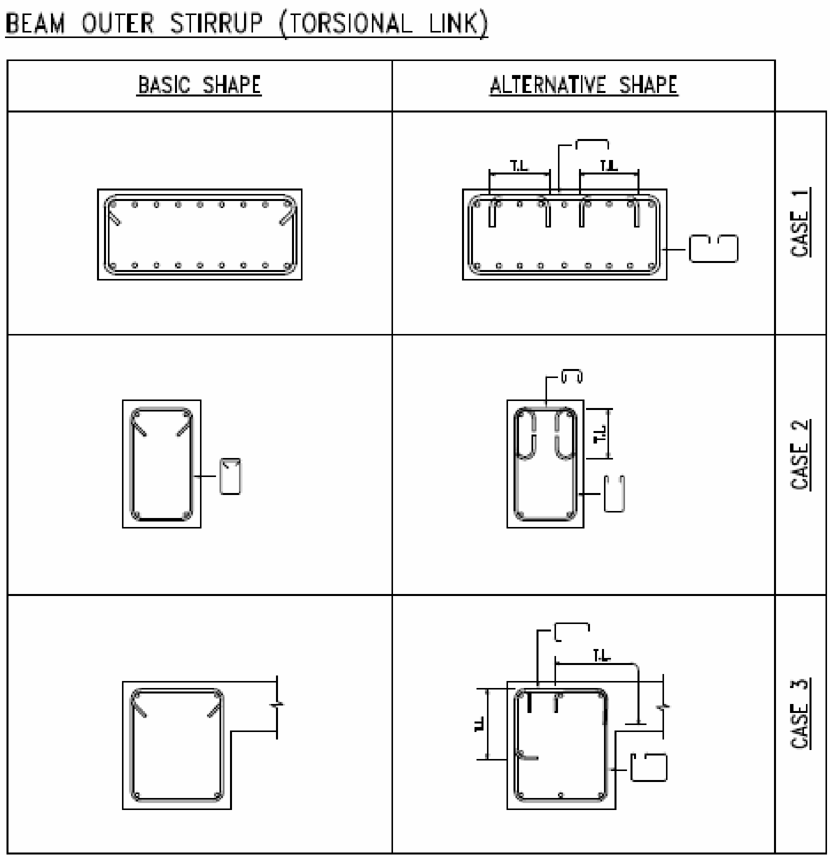


Figure 3.39 – Alternative Form of Torsional Links

(ii) The value s_v for the closed link should not exceed the least of x_1, $y_1/2$ or 200 mm as per Cl. 6.3.7 of the Code;

(iii) In accordance with Cl. 6.3.8 of the Code, provision of the longitudinal torsion reinforcement should comply with the followings :

(a) The bars distributed should be evenly round the inside perimeter of the links as illustrated in Figure 3.35;
(b) Clear distance of the bars not to exceed 300 mm;
(c) Additional longitudinal bars required at the level of the tension or compression reinforcements may be provided by using larger bars than those required for bending alone, as illustrated in Worked Example 3.16 and Figure 3.35;
(d) The longitudinal bars should extend a distance at least equal to the largest dimension of the section beyond where it theoretically ceases to be required.

4.0 Slabs

4.1 Types of Slabs

Slabs can be classified as "one way slab", "two way slab", "flat slab", "ribbed slab" with definition in Cl. 5.2.1.1 of the Code.

4.1.1 One way slab is defined by the Code as one subjected predominantly to u.d.l. either

(i) it possesses two free and parallel edge; or

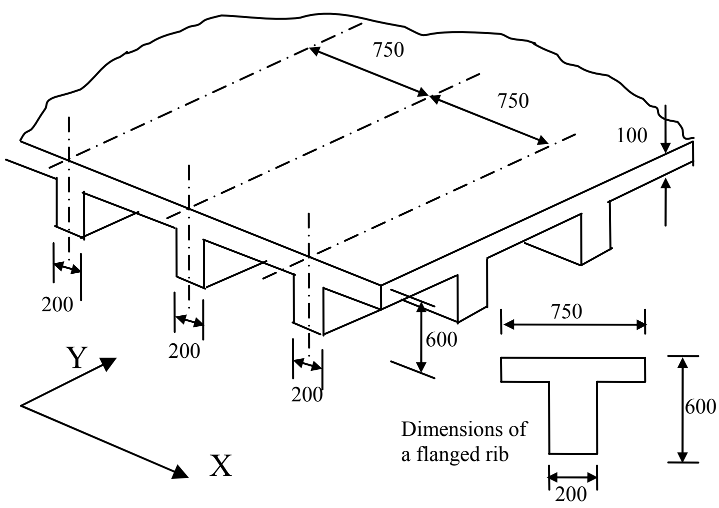
(ii) it is the central part of a rectangular slab supported on four edges with a ratio of the longer to shorter span greater than 2.

4.1.2 Two way slab is a rectangular one supported on four sides with length to breadth ratio smaller than 2.

4.1.3 Flat slab is a slab supported on columns without beams.

4.1.4 Ribbed or Waffled Slab is a slab with topping or flange supported by closely spaced ribs. The Code allows idealization of the ribbed slab or waffled slab as a single slab without treatment as discretized ribs and flanges in analysis in Cl. 5.2.1.1(d) of the Code. If the stiffness of the ribbed or waffled slab is required for input, the bending stiffness in the X and Y directions can be easily determined by summing the total bending stiffness zf the composite ribs and flange structure per unit width as illustrated in Figure 4.1. The twisting stiffness is more difficult to assess. However, it should be acceptable to set the twisting stiffness to zero which will end up with pure bending in the X and Y directions as the slab, with its ribs running in the X and Y directions which are clearly predominantly strong in bending in the X and Y directions.

Figure 4.1 illustrates the computation of "I" value of a waffle slab about the X-direction which is total stiffnesses of the nos. of "flanged ribs" within one metre. "I" value in the Y-directions can be worked out similarly.



Centroid of a flanged rib is at

$$\frac{0.55\times 0.1^2/2+0.2\times 0.6^2/2}{0.55\times 0.1+0.2\times 0.6}$$

= 0.2214m from top

I of a rib is

$$\frac{1}{12}\left(0.55\times 0.1^3+0.2\times 0.6^3\right)$$
$$+0.55\times 0.1\times(0.2214-0.05)^2$$
$$+0.2\times 0.6\times(0.3-0.2214)^2$$
$$=0.006\text{m}^4$$

Within one metre, there will be 1000/750 = 1.333 nos. of ribs.

So the I per metre width is $0.006\times 1.333 = 0.008\ \text{m}^4/\text{m}$

Figure 4.1 – Illustration of Calculation of I value about X-direction of a Waffle Slab

4.2 Analysis of Slabs without the Use of Computer Method

4.2.1 One way slab can be analyzed as if it is a beam, either continuous or single span. As we aim at simple analysis for the slab, we tend to treat it as a single element without the necessity to consider the many loading cases for continuous spans, Cl. 6.1.3.2(c) of the Code allows the design against moment and shear arising from the single-load case of maximum design load on all spans provided that :

(i) the area of each bay (defined in Figure 6.5 of the Code and reproduced in Figure 4.2) > 30 m^2;

(ii) the ratio of the characteristic imposed load to characteristic dead load ≤ 1.25; and

(iii) the characteristic imposed load ≤ 5 kN/m^2 excluding partitions.

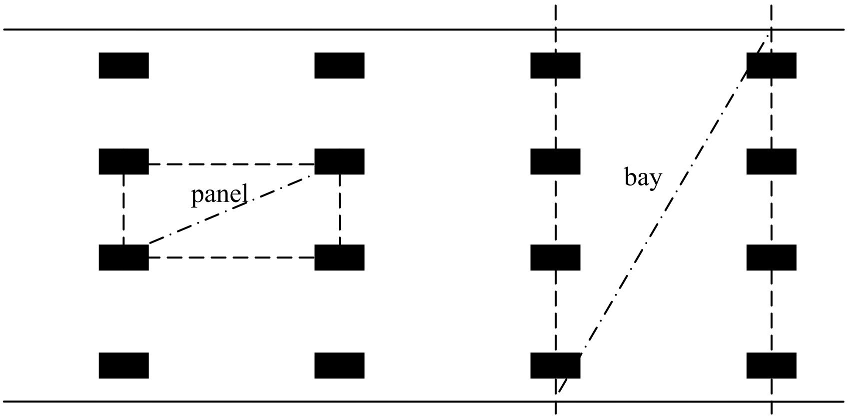


Figure 4.2 – Definition of panels and bays

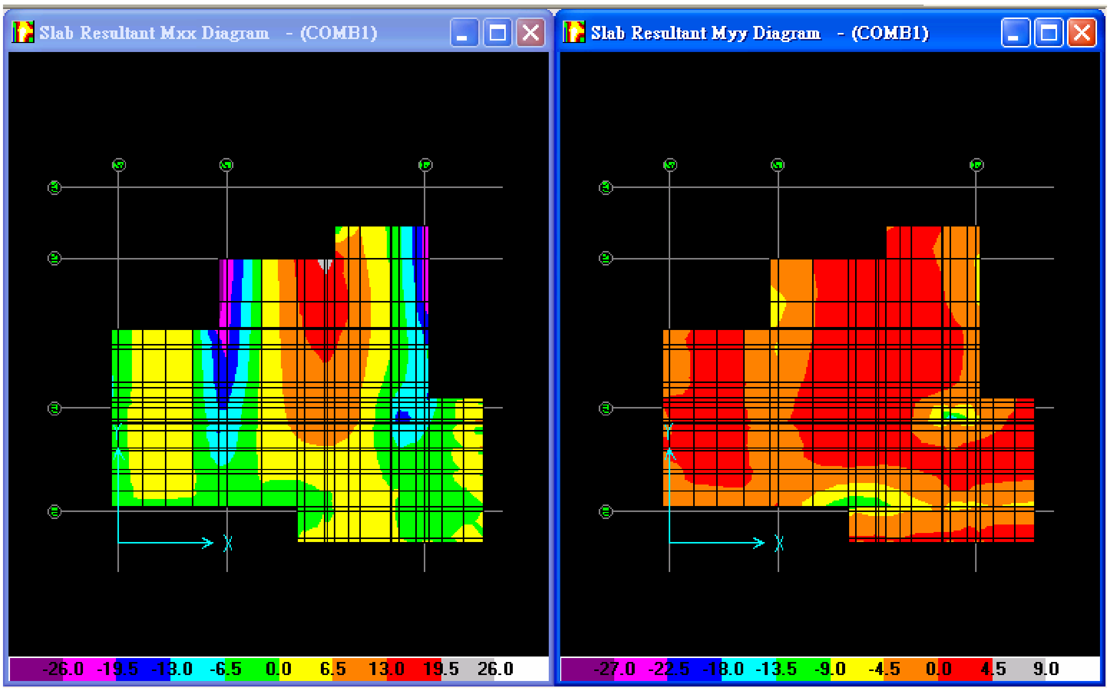
4.2.2 Two way rectangular slab is usually analyzed by treating it as if it is a single slab in the absence of computer method. Bending moment coefficients for calculation of bending moments are presented in Table 6.6 of the Code for different support restraint conditions. Nevertheless, simplified formulae for the bending coefficients in case of rectangular simply supported two way slab are available in the Code (Ceqn 6.26 and 6.27) and reproduced as follows : $m_x = \alpha_{sx} n l_x^2$ and $m_y = \alpha_{sy} n l_x^2$ where n is the u.d.l. l_x and l_y are respectively $\alpha_{sx} = \frac{(l_y / l_x)^4}{8[1+(l_y / l_x)^4]}$ and $\alpha_{sy} = \frac{(l_y / l_x)^2}{8[1+(l_y / l_x)^4]}$ for the shorter and longer spans;

4.2.3 Flat slabs, if of regular arrangement, can be analyzed as frames in the transverse and longitudinal directions by such methods as moment distribution method as if they are separate frames. Analyzed moments and shears should be apportioned to the "column strip" and "Middle strip" as per Table 6.10 of the Code. In addition, the bending moment and shear force coefficients for the one way slab can also be used as a simplified approach.

4.2.4 More bending moment and shear force coefficients of rectangular slabs with various different support and loading conditions can be found from other published handbooks, including Bareš' "Tables for the Analysis of Plates, Slabs and Diaphragms based on the Elastic Theory".

4.3 Analysis of Slabs with the Use of the Computer Method

Analysis of slabs with the use of the computer method is mainly by the finite element method in which the slab is idealized as an assembly of discrete "plate bending elements" joined at nodes. The support stiffnesses by the supporting walls and columns are derived as similar to that for beams as "sub-frames". A complete set of results including bending moments, twisting moment, shear force per unit width (known as "stress" in the terminology of the finite element method) can be obtained after analysis for design purpose. The design against flexure is most commonly done by the Wood Armer Equations which calculate "design moments" in two directions (conveniently in two perpendicular directions) and they are adequate to cater for the complete set of bending and twisting moments. The design based on node forces / moments should be avoided due to its inadequacy to cater for twisting effects which will result in under-design. A discussion of the plate bending theory including the origin of the "twisting moment" and the design approach by the Wood Armer Equations is enclosed in Appendix D, together with the "stress approach" for checking and designing against shear in the same appendix. A fuller discussion can also be found in Lam & Law (2009). An example of the mathematical modeling of a floor slab by the software SAFE and results of subsequent analysis is illustrated in Figure 4.3.



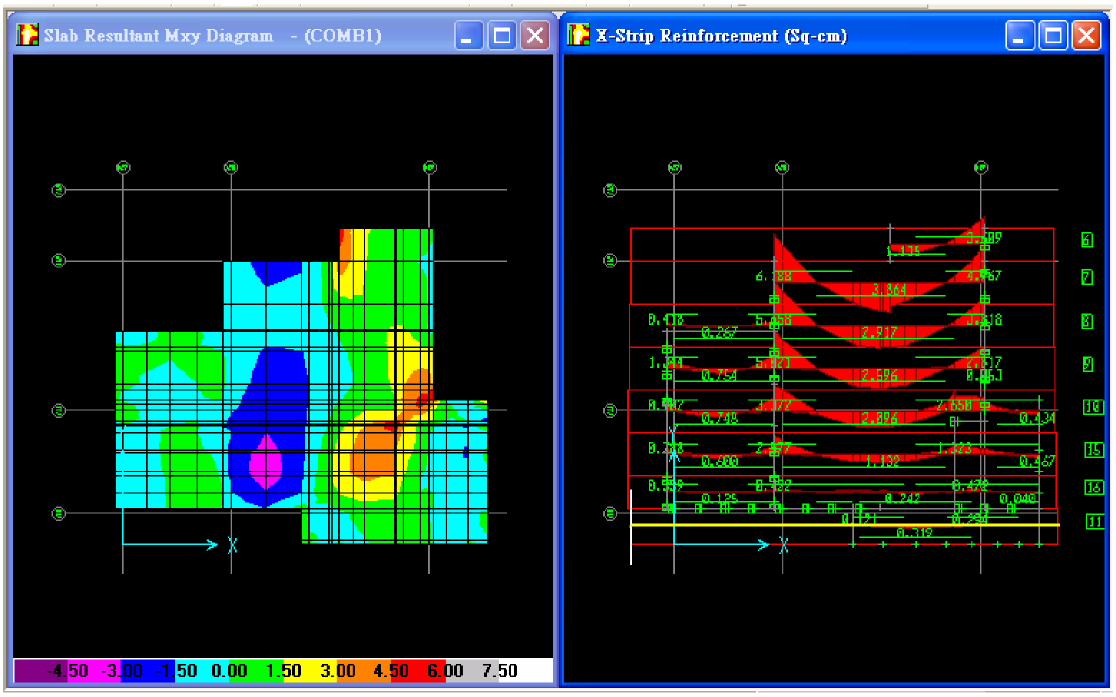


Figure 4.3 – Mathematical Modeling and Analytical Results of Slabs by SAFE

The finite element mesh of the mathematical model is often very fine. So it is a practice of "lumping" the design reinforcements of a number of nodes over certain widths and evenly distributing the total reinforcements over the widths, as is done by the popular software "SAFE". However, care must be taken in not taking widths too wide for "lumping" as local effects may not be well captured.

4.4 Detailing for Solid Slabs

Generally considerations in relation to determination of "effective span", "effective span depth ratio", "moment redistribution", "reduced design moment to support", "maximum and minimum steel percentages", "concrete covers" as discussed in Section 3.3 for design of beam are applicable to design of slab. Nevertheless, the detailing considerations for slabs are listed as follows with h as the structural depth of the slab (Re 9.3.1.1 of the Code) :

(i) Minimum steel percentage (Cl. 9.3.1.1(a) of the Code):
Main Reinforcing bars:
0.24% for $f_y = 250$ MPa and 0.13% for $f_y = 500$ MPa;
Distribution bars in one way slab ≥ 20% of the main reinforcements

(ii) Maximum reinforcements spacing (Cl. 9.3.1.1(b) of the Code):
(a) In general areas without concentrated loads :
the principal reinforcement, max. spacing $\leq 3h \leq 400$ mm; and the secondary reinforcement, max. spacing $\leq 3.5h \leq 450$ mm.
(b) In areas with concentrated loads or areas of maximum moment:
the principal reinforcement, max. spacing $\leq 2h \leq 250$ mm; and for the secondary reinforcement, max. spacing $\leq 3h \leq 400$ mm.

(iii) In addition to (ii), if either :
   (a) $h \le 250$ mm (grade 250 steel);
   (b) $h \le 200$ mm (grade 500 steel); or
   (c) the percentage of required tension reinforcement is less than 0.3%.

   no direct crack widths check by calculation is required. If none of conditions in (a), (b) & (c) is satisfied, bar spacing to comply with Cl. 9.2.1.4 of the Code as discussed in 3.3(vi) of this Manual if steel percentage > 1%. Otherwise, increase the spacing by 1/percentage;

(iv) Requirements pertaining to curtailment and anchoring of tension reinforcements should be similar to that of beams;

(v) Reinforcements at end supports (Cl. 9.3.1.3 of the Code) of simply supported slab and continuous slab as illustrated in Figure 4.4 :
   (a) At least 50% of the span reinforcements should be provided with full effective tensile anchorage into supports of simply supported slabs or end of continuous slabs;
   (b) To provide partial fixity, end supports shall be provided with total steel area capable of resisting not less than 50% of the maximum span moments and greater than the minimum reinforcement of 0.13%;
   (c) The support anchorage bar shall extend not less than 0.15 times the slab span or 45 times of the bar diameter;
   (d) If support shear stress $v < 0.5v_c$, the arrangement in Figure 4.4 can be considered as effective anchorage.

(vi) Minimum bottom reinforcements at internal supports : 40% of the calculated mid-span bottom reinforcements as illustrated in Figure 4.4. (Cl. 9.3.1.4 of the Code)

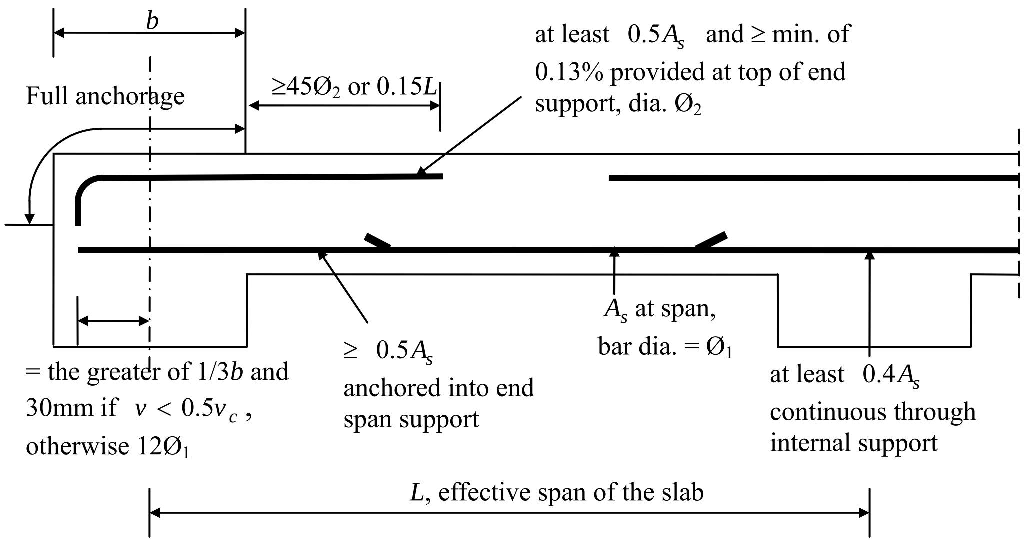


Figure 4.4 – Requirements of Slab Longitudinal Reinforcements

(vii) Reinforcements at free edge should be as shown in Figure 4.5 (Cl. 9.3.1.6 of the Code)

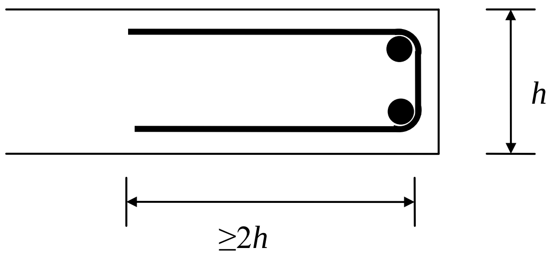


Figure 4.5 – Free edge reinforcements for Slabs

(viii) Shear reinforcements not to be used in slabs < 200 mm thick. (Cl. 9.3.2 of the Code)

4.5 <u>Structural Design of Slabs</u>

The structural of slab against flexure is similar to that of beam. The determination of reinforcements should be in accordance with Section 3.4 of this Manual listing the options of either following the rigorous or simplified "stress strain" relationship of concrete. Design against shear for slabs under line supports (e.g. one-way or two-way) is also similar to that of beam. However for a flat slab, the checking should be based on punching shear in accordance with empirical method of the Code or based on shear stresses revealed by the finite element method. They are demonstrated in the Worked Examples in the following sub-Section 4.6 :

4.6 <u>Worked Examples</u>

<u>Worked Example 4.1 – One Way Slab</u>

A one-way continuous slab with the following design data :

(i) Live Load = 4.0 kN/m^2;
(ii) Finishes Load = 1 kN/m^2;
(iii) Concrete grade : C35 with cover to rebars 25 mm;
(iv) Slab thickness : 200 mm;
(v) Fire rating : 1 hour, mild exposure;
(vi) Span : 4 m

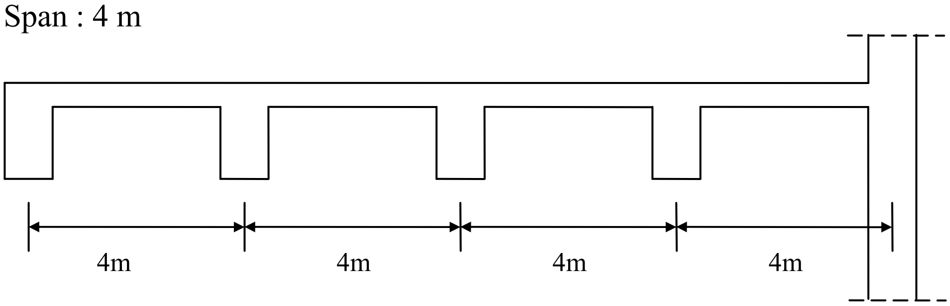


Figure 4.6 – Slab in Worked Example 4.1

Sizing : Limiting Span depth ratio = $23 \times 1.1 = 25.3$ (by Table 7.3 and Table 7.4 of the Code, assuming modification by tensile reinforcement to be 1.1 as the slab should be lightly reinforced). Assuming 10mm dia. bars under 25 mm concrete cover, effective depth is $d = 200 - 25 - 5 = 170$. Span effective depth ratio is $4000/170 = 23.5 < 25.3$. So OK.

Loading :	D.L.	O.W.	$0.2 \times 24.5 =$	4.9kN/m^2
		Fin.		1.0 kN/m^2
		Total		5.9 kN/m^2
	L.L.			4.0 kN/m^2

The factored load on a span is $F = (1.4 \times 5.9 + 1.6 \times 4.0) \times 4 = 58.64$ kN/m.

Based on coefficients of shear and bending in accordance with Table 6.4 of the Code listed as follows :

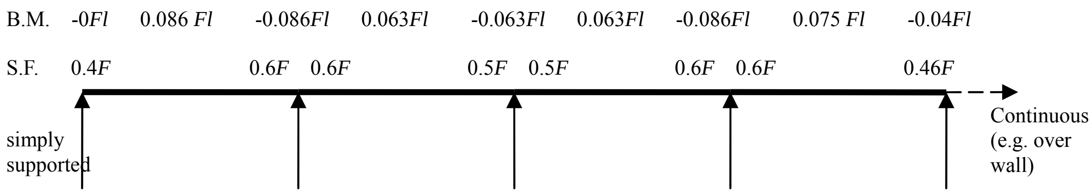


Figure 4.7 – Bending Moment and Shear Force coefficients for Continuous Slab

(a) End span support moment (continuous) $= 0.04 \times 58.64 \times 4 = 9.38$ kNm/m

$$K = \frac{M}{f_{cu}bd^2} = \frac{9.38 \times 10^6}{35 \times 1000 \times 170^2} = 0.00927$$

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} = 0.99 > 0.95$$

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{9.38 \times 10^6}{1000 \times 170^2 \times 0.87 \times 500 \times 0.95} = 0.0785\%$$

$A_{st} = 0.000785 \times 1000 \times 170 = 133\,\text{mm}^2$ < Minimum required by Table 9.1 of the Code $A_{st} = 0.13 \div 100 \times 1000 \times 200 = 260\,\text{mm}^2$ Use T10 – 300

(A_{st} provided = 262mm^2)

(b) End span moment = $0.086 \times 58.64 \times 4 = 20.17$ kNm/m

$$K = \frac{M}{f_{cu}bd^2} = \frac{20.17 \times 10^6}{35 \times 1000 \times 170^2} = 0.0199$$

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} = 0.977 > 0.95$$

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{20.17 \times 10^6}{1000 \times 170^2 \times 0.87 \times 500 \times 0.95} = 0.169\%$$

$A_{st} = 0.167 \div 100 \times 1000 \times 170 = 284\,\text{mm}^2$ > 0.13% → 260mm^2. Use T10 – 250 (A_{st} provided = 314mm^2)

(c) First interior support moment = $0.086 \times 58.64 \times 4 = 20.17$ kNm/m, same reinforcement as that of end span reinforcement.

(d) Interior span or support moment = $0.063 \times 58.64 \times 4 = 14.78$ kNm/m;

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{14.78 \times 10^6}{1000 \times 170^2 \times 0.87 \times 500 \times 0.95} = 0.124\%$$

$A_{st} = 0.00124 \times 1000 \times 170 = 211\,\text{mm}^2 < 0.13\% \rightarrow 260\text{mm}^2$.

Use T10 – 300 (A_{st} provided = 261mm^2)

(e) End span span moment to continuous support
= $0.075 \times 58.64 \times 4 = 17.59$ kNm/m

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{17.59 \times 10^6}{1000 \times 170^2 \times 0.87 \times 500 \times 0.95} = 0.147\%$$

$A_{st} = 0.00147 \times 1000 \times 170 = 250\,\text{mm}^2 < 0.13\% \rightarrow 260\text{mm}^2$.

Use T10 – 250 (A_{st} provided = 314mm^2)

(f) Check Shear
Maximum shear = $0.6 \times 58.64 = 35.18$ kN/m.

By Table 6.3 of the Code $v_c = 0.79\left(\frac{100A_s}{bd}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}}\frac{1}{\gamma_m}\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}$

$v_c = 0.79(0.13)^{\frac{1}{3}}\left(\frac{400}{170}\right)^{\frac{1}{4}}\frac{1}{1.25}\left(\frac{35}{25}\right)^{\frac{1}{3}} = 0.44$ N/mm^2, based on minimum steel 0.13%; $v = \frac{35180}{1000 \times 170} = 0.207$ N/mm^2 < $v_c = 0.44$ N/mm^2.

No shear reinforcement required.

<u>Worked Example 4.2 – Two Ways Slab (4 sides simply supported)</u>

A two-way continuous slab with the following design data :

(i) Live Load = 4.0 kN/m^2;
(ii) Finishes Load = 1 kN/m^2;
(iii) Concrete grade : C35;
(iv) Slab thickness : 200 mm
(v) Fire rating : 1 hour, mild exposure, cover = 25mm;
(vi) Span : Long way : 4 m, Short way, 3 m

Sizing : Limiting Span depth ratio = 20 (by Table 7.3). So effective depth taken as $d = 200 - 25 - 5 = 170$ as 3000/170 = 17.65 < 20.

Loading :	D.L.	O.W.	$0.2 \times 24.5 =$	4.9kN/m^2
		Fin.		1.0 kN/m^2
		Total		5.9 kN/m^2
	L.L.			4.0 kN/m^2

The factored load is $\omega = (1.4 \times 5.9 + 1.6 \times 4.0) = 14.66\,\text{kN/m}^2$

(Ceqn 6.26) and (Ceqn 6.27) of the Code are used to calculate the bending moment coefficients along the short and long spans :

$$\alpha_{sx} = \frac{(\ell_y/\ell_x)^4}{8\left[1+(\ell_y/\ell_x)^4\right]} = 0.095; \qquad \alpha_{sy} = \frac{(\ell_y/\ell_x)^2}{8\left[1+(\ell_y/\ell_x)^4\right]} = 0.053$$

So the bending moment along the short span is

$M_x = 0.095 \times 14.66 \times 3^2 = 12.53\,\text{kNm/m}$

$$K = \frac{M}{f_{cu}bd^2} = \frac{12.53 \times 10^6}{35 \times 1000 \times 170^2} = 0.0124$$

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} = 0.986 > 0.95$$

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{12.53 \times 10^6}{1000 \times 170^2 \times 0.87 \times 500 \times 0.95} = 0.105\%$$

$A_{st} = 0.13 \div 100 \times 1000 \times 200 = 260\,\text{mm}^2$ Use T10 – 250 (Re Cl. 9.3.1.1(b)(iii) of the Code) (A_{st} provided = 314mm^2)

$M_y = 0.053 \times 14.66 \times 3^2 = 6.99\,\text{kNm/m}$

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{6.99 \times 10^6}{1000 \times 158^2 \times 0.87 \times 500 \times 0.95} = 0.068\%$$

Use T10 – 300 (0.13% control) (A_{st} provided = 262mm^2)

<u>Worked Example 4.3 – Two Ways Slab (3 sides supported)</u>

A two-way slab with the following design data :

(i) Live Load = 4.0 kN/m^2;
(ii) Finishes Load = 1 kN/m^2;
(iii) Concrete grade : C35;
(iv) Slab thickness : 200 mm
(v) Span : Long way : 5 m, Short way, 4 m
(iv) Fire rating : 1 hour, mild exposure, cover = 25mm;

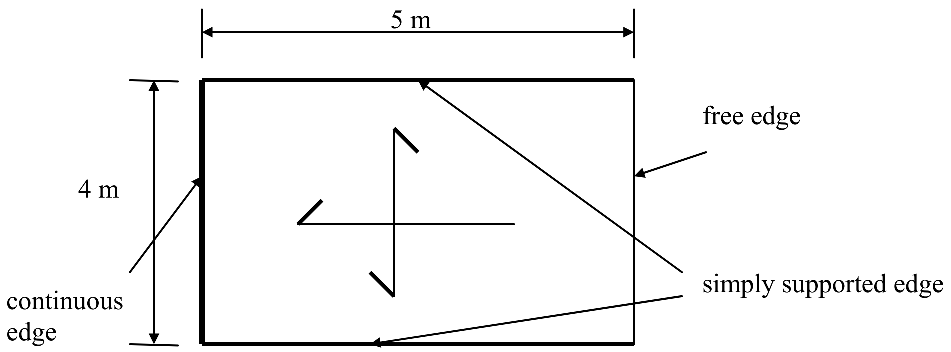


Figure 4.8 – Plan of 3-sides supported slab for Worked Example 4.3

Loading : D.L. O.W. $0.2 \times 24.5 =$ 4.9kN/m^2

Fin.	1.0 kN/m^2
Sum	5.9 kN/m^2
L.L.	4.0 kN/m^2

The factored load is $\omega = (1.4 \times 5.9 + 1.6 \times 4.0) = 14.66$ kN/m^2

From Table 1.38 of "Tables for the Analysis of Plates, Slabs and Diaphragms based on Elastic Theory" where $\gamma = 4/5 = 0.8$, the sagging bending moment coefficient for short way span is maximum at mid-span of the free edge which 0.1104 (linear interpolation between $\gamma = 0.75$ and $\gamma = 1.0$)

$$M_x = 0.1104 \times 14.66 \times 4^2 = 25.90 \text{ kNm/m}$$

$$K = \frac{M}{f_{cu}bd^2} = \frac{25.90 \times 10^6}{35 \times 1000 \times 170^2} = 0.0256$$

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} = 0.971 > 0.95$$

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{25.90 \times 10^6}{1000 \times 170^2 \times 0.87 \times 500 \times 0.95} = 0.217\%$$

$A_{st} = 0.217 \div 100 \times 1000 \times 170 = 369$ mm^2 > 0.13%→260mm^2; Use T10 – 200

(A_{st} provided = 393mm^2)

At 2 m and 4 m from the free edge, the sagging moment reduces to $0.0844 \times 14.66 \times 4^2 = 19.797$ kNm/m and $0.0415 \times 14.66 \times 4^2 = 9.734$ kNm/m and A_{st} required are reduced to 282 mm^2 and 139 mm^2.
Use T10 – 250 and T10 – 300 respectively.

The maximum hogging moment (bending along long-way of the slab) is at mid-way along the supported edge of the short-way span

$$M_y = 0.0729 \times 14.66 \times 5^2 = 26.72 \text{ kNm/m}$$

$$K = \frac{M}{f_{cu}bd^2} = \frac{26.72 \times 10^6}{35 \times 1000 \times 170^2} = 0.0264$$

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} = 0.97 > 0.95$$

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{26.72 \times 10^6}{1000 \times 170^2 \times 0.87 \times 500 \times 0.95} = 0.224\%$$

$A_{st} = 0.224 \div 100 \times 1000 \times 170 = 381$ mm^2> 0.13%→260mm^2; Use T10 – 200

(A_{st} provided = 393mm^2)

The maximum sagging moment along the long-way direction is at 2 m from the free edge which is

$M_y = 0.01876 \times 14.66 \times 5^2 = 6.88$ kNm/m. The moment is small.

Use T10 – 300

Back-check compliance of effective span ratio (Re Tables 7.3 and 7.4 of the

Code) by considering only the short span which is simply supported,

$$f_s = \frac{2 f_y A_{st,req}}{3 A_{st,prov}} \times \frac{1}{\beta_b} = \frac{2 \times 500 \times 369}{3 \times 393} \times \frac{1}{1} = 313 \text{ N/mm}^2;$$

The modification factor (Table 7.4) is

$$0.55 + \frac{(477 - f_s)}{120\left(0.9 + \frac{M}{bd^2}\right)} = 0.55 + \frac{(477 - 313)}{120(0.9 + 0.0256 \times 35)} = 1.31$$

Allowable effective span depth ratio is $1.31 \times 20 = 26.2 > \frac{4000}{170} = 23.5$. O.K.

Finally the reinforcement arrangement on the slab is (Detailed curtailment, top support reinforcements at simple supports ($0.5A_s$) omitted for clarity.)

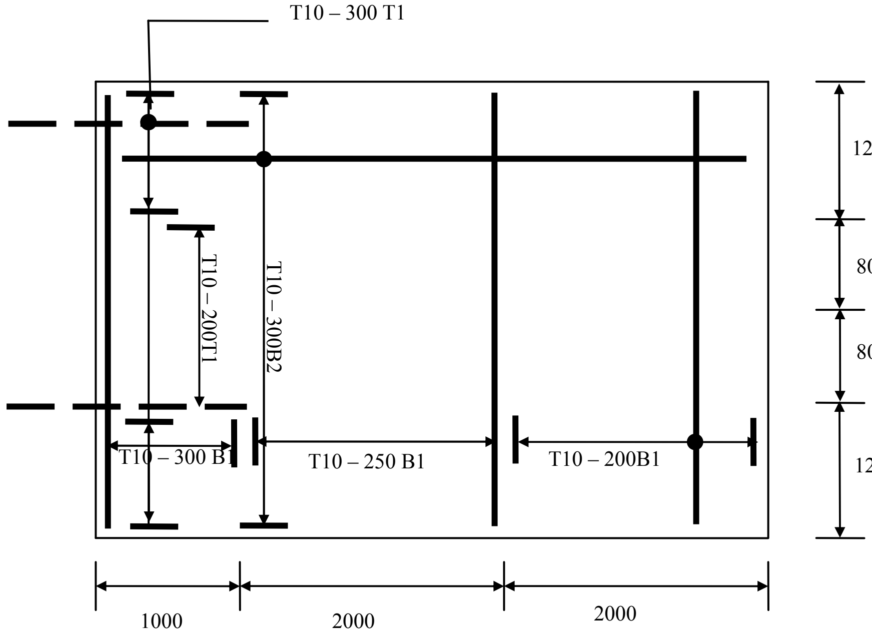


Figure 4.9 – Reinforcement Details for Worked Example 4.3

Worked Example 4.4 – Flat Slab by Simplified Method (Cl. 6.1.5.2(g))

Flat slab arrangement on rectangular column grid of 7.5 m and 6 m as shown in Figure 4.10 with the following design data :

(i) Finish Load = 1.5 kPa
(ii) Live Load = 5.0 kPa.
(iii) Column size = 550 × 550
(iv) Column Drop size = 3000 × 3000 with d_h = 200 mm
(v) Fire rating : 1 hour, mild exposure, cover = 25 mm
(vi) Concrete grade C35;

As the number of panel is more than 3 and of equal span, the simplified method for determining moments in accordance with Cl. 6.1.5.2(g) of the Code is applicable and is adopted in the following analysis.

Effective dimension of column head $l_{h\max} = l_c + 2(d_h - 40)$ (Ceqn 6.37)

$= 550 + 2(200 - 40) = 870$ mm

Effective diameter of column head (Cl. 6.1.5.1(c))

$$h_c = \sqrt{\frac{870^2 \times 4}{\pi}} = 982 \text{ mm} < \frac{1}{4} \times 6000 = 1500 \text{ mm}$$

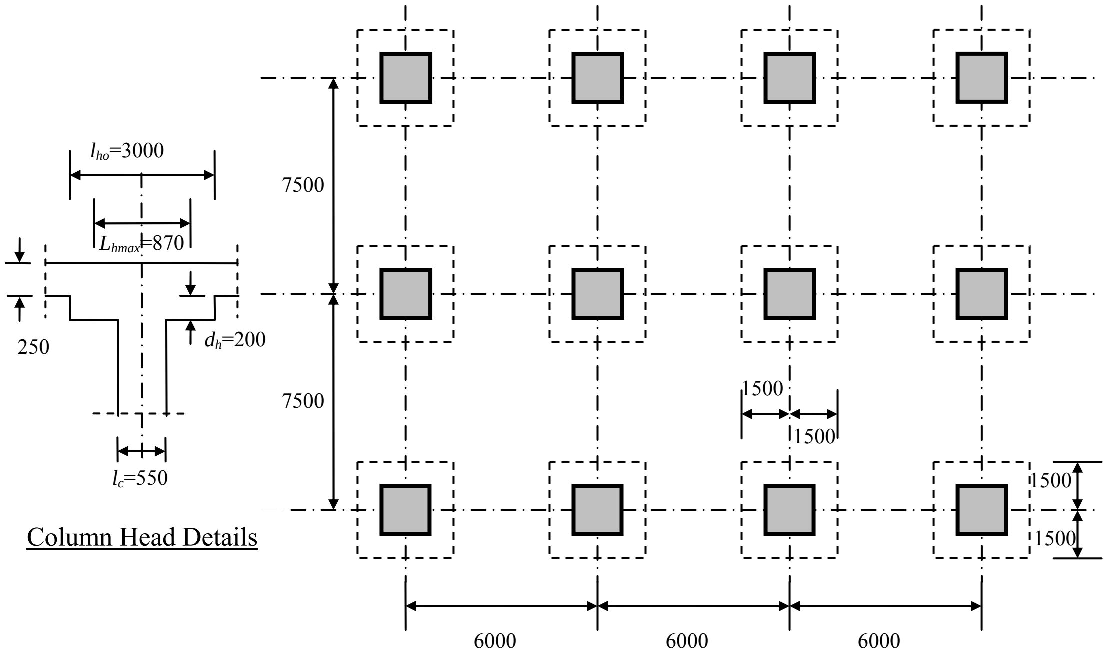


Figure 4.10 – Flat Slab Plan Layout for Worked Example 4.4

In the simplified method, the flat slab is effectively divided into (i) "column strips" containing the columns and the strips of the linking slabs and of "strip widths" equal to the widths of the column drops; and (ii) the "middle strips" between the "column strips". (Re Figure 6.9 of the Code.) They are designed as beams in flexural design with assumed apportionment of moments among the strips. However, for shear checking, punching shears along successive "critical" perimeters of column are carried out instead.

Sizing : Based on the same limiting Span depth ratio for one way and two way slab which is $26 \times 1.15 = 30$ (by Table 7.3 and Table 7.4 of the Code, assuming modification by tensile reinforcement to be 1.15), $d = \frac{6000}{30} = 200$.

As cover = 25 mm, assuming T12 bars, structural depth should at least be $200 + 25 + 12 \div 2 = 231$ mm, so use structural depth of slab of 250 mm.

Loading :	D.L.	O.W.	$0.25 \times 24.5 =$	6.13kN/m^2
		Fin.		1.5 kN/m^2
		Total		7.63 kN/m^2
	L.L.			5.0 kN/m^2

The factored load is $\omega = (1.4 \times 7.63 + 1.6 \times 5.0) = 18.68$ kN/m^2

<u>Design against Flexure (Long Way)</u>

The bending moment and shear force coefficients in Table 6.4 will be used as per Cl. 6.1.5.2(g) of the Code. Total design ultimate load on the full width of panel between adjacent bay centre lines is $F = 18.68 \times 7.5 \times 6 = 840.6$ kN. Thus the reduction to support moment for design, as allowed by Cl. 6.1.5.2(g) of the Code, is $0.15Fh_c = 0.15 \times 840.6 \times 0.982 = 123.82$ kNm for internal support and $0.15Fh_c = 0.15 \times 840.6 \times 0.982 / 2 = 61.91$ kNm for outer support.

The design moment at supports are :

Total moment at outer support is $0.04 \times 840.6 \times 7.5 = 252.18$ kNm, which can be reduced to $252.18 - 61.91 = 190.27$ kNm;

Total moment at first interior support is $0.086 \times 840.6 \times 7.5 = 542.19$ kNm, which can be reduced to $542.19 - 123.82 = 418.37$ kNm

Total moment at interior support is $0.063 \times 840.6 \times 7.5 = 397.18$ kNm, which can be reduced to $397.18 - 123.82 = 273.36$ kNm

The flat slab is divided into column and mid strips in accordance with Figure 6.9 of the Code which is reproduced as Figure 4.11 in this Manual.

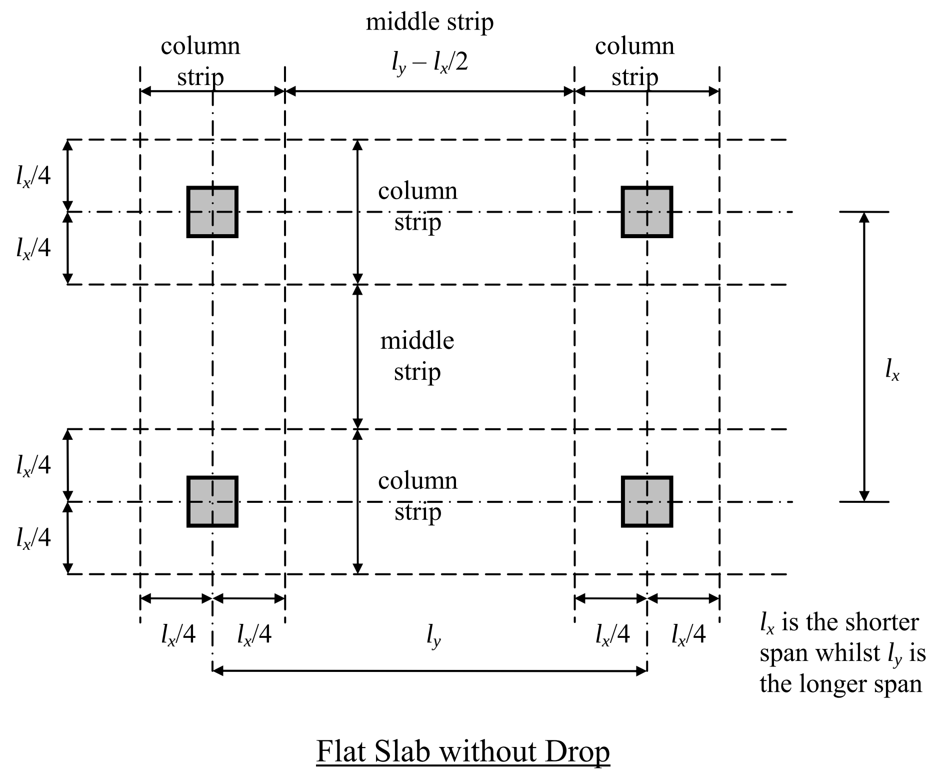
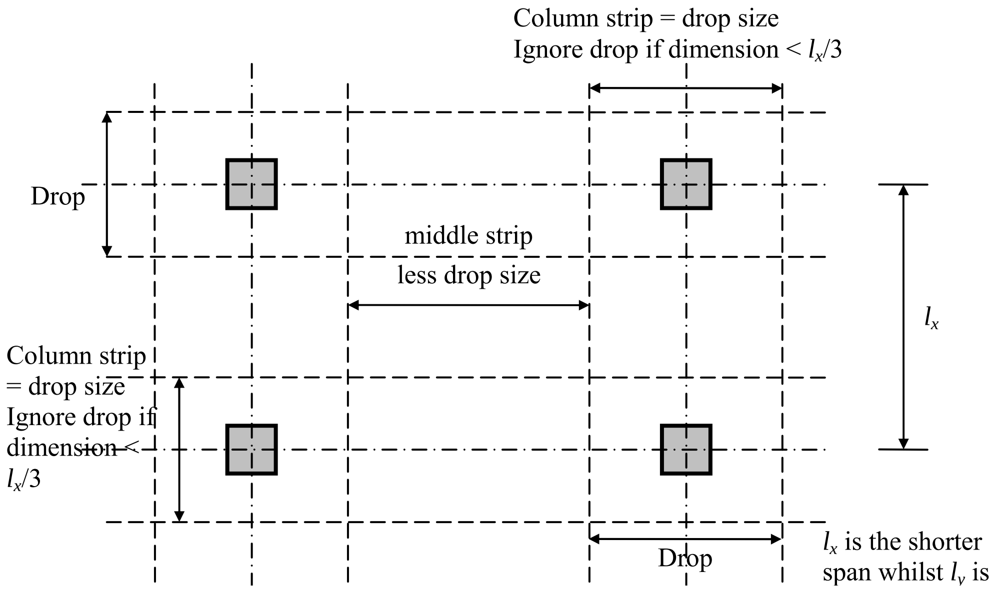


Figure 4.11(a) – Division of Panels for Flab Slab without Drop



Flat Slab with Drop

Figure 4.11(b) – Division of Panels for Flat Slab with Drop

The total support moments as arrived for the whole panel are to be apportioned to the column and middle strips with the percentages of 75% and 25% respectively as per Table 6.10 of the Code,

	Column Strip (75%)		Mid Strip (25%)	
	Total Mt	Mt/width	Total Mt	Mt/width
Outer Support	142.70	47.57	47.57	15.86
1st interior support	313.78	104.59	104.59	34.86
Middle interior support	205.02	68.34	68.34	22.78

The reinforcements – top steel are worked out as follows, (minimum of 0.13%bh in brackets) ($d = 450 - 25 - 6 = 419$ mm over column support and $d = 250 - 25 - 6 = 219$ in other locations)

	Column Strip (75%)		Mid Strip (25%)	
	Area (mm^2)/m	Steel	Area (mm^2)/m	Steel
Outer Support	274 (585)	T12 – 175	175 (585)	T12 – 175
1st interior support	604	T12 – 150	385 (585)	T12 – 175
Middle interior support	395 (585)	T12 – 175	252 (585)	T12 – 175

Sagging Moment :
Total moment near middle of end span is $0.075 \times 840.6 \times 7.5 = 472.84$ kNm
Total moment near middle of interior span $0.063 \times 840.6 \times 7.5 = 397.18$ kNm
These moments are to be apportioned in the column and mid strips in accordance with the percentages of 55% and 45% respectively as per Table 6.10, i.e.

	Column Strip (55%)		Mid Strip (45%)	
	Total Mt	Mt/width	Total Mt	Mt/width
Middle of end span	260.06	86.69	212.78	70.93
Middle of interior span	218.45	72.82	178.74	59.58

The reinforcements – bottom steel are worked out as follows :

	Column Strip (55%)		Mid Strip (45%)	
	Area (mm^2)/m	Steel	Area (mm^2)/m	Steel
Middle of end span	969	T12 – 100	784	T12 – 125
Middle of interior span	805	T12 – 125	658	T12 – 150

Design in the short way direction can be carried out similarly.

Design against Shear

Design of shear should be in accordance with Cl. 6.1.5.6 of the Code which is against punching shear by column. For the internal column support, in the absence of frame analysis, the shear for design will be $V_{eff} = 1.15V_t$ where V_t is the design shear transferred to column calculated on the assumption of all adjacent panels being fully loaded by Cl. 6.1.5.6(b) of the Code.

$V_t = 7.5 \times 6 \times 18.68 = 840.6$ kN; $\quad V_{eff} = 1.15V_t = 966.69$ kN

Check on column perimeter as per Cl. 6.1.5.6(d) of the Code :

$$\frac{V_{eff}}{ud} \le 0.8\sqrt{f_{cu}} \text{ or } 7 \qquad \frac{966.69 \times 10^3}{(4 \times 550) \times 419} = 1.05 \le 0.8\sqrt{f_{cu}} = 4.73 \text{ MPa; O.K.}$$

Check on 1st critical perimeter – $1.5d$ from column face, i.e. $1.5 \times 0.419 = 0.6285$. So side length of the perimeter is $(550 + 628.5 \times 2) = 1807$ mm

Length of perimeter is $4 \times 1807 = 7228$ mm

Shear force to be checked can be the maximum shear 966.69 kN after deduction of the loads within the critical perimeter which is

$966.69 - 18.68 \times 1.807^2 = 905.70$ kN

Shear stress = $\dfrac{905.7 \times 10^3}{7228 \times 419} = 0.299$ N/mm^2. < $v_c = 0.36$ N/mm^2 in accordance with Table 6.3 by assuming 0.13% minimum flexural reinforcement. No shear reinforcement is required. No checking on further perimeter is required.

Worked Example 4.5 – Design for shear reinforcement

(Ceqn 6.44) and (Ceqn 6.45) of the Code gives formulae for reinforcement design for different ranges of values of v.

For $v \le 1.6v_c$, $\qquad \sum A_{sv} \sin\alpha \ge \dfrac{(v - v_c)ud}{0.87 f_y}$

For $1.6v_c < v \le 2.0v_c$, $\qquad \sum A_{sv} \sin\alpha \ge \dfrac{5(0.7v - v_c)ud}{0.87 f_y}$

As a demonstration, if $v = 0.5$ N/mm^2 in the first critical perimeter which is < $1.6v_c = 0.576$ N/mm^2 but > $v_c = 0.36$ N/mm^2 in the Example 4.4. By Table 6.8 of the Code, as $v - v_c < 0.4$, $v_r = 0.4$

$\sum A_{sv} \sin\alpha \ge \dfrac{v_r ud}{0.87 f_y} = \dfrac{0.4 \times 7228 \times 419}{0.87 \times 500} = 2785$ mm^2. If vertical links is chosen as shear reinforcement, $\alpha = 90^0 \Rightarrow \sin\alpha = 1$. So the 2785mm^2 should be distributed within the critical perimeter as shown in Figure 4.12.

In distributing the shear links within the critical perimeter, there are recommendations in Cl. 6.1.5.7(f) of the Code that

(i) at least two rows of links should be used;

(ii) the first perimeter should be located at approximately 0.5*d* from the face of the loaded area (i.e. the column in this case) and should contain not less than 40% of the calculated area of reinforcements.

So the first row be determined at 200 mm from the column face with total row length $950\times4=3800$. Using T10 – 225 spacing along the row, the total steel area will be $(10^2/4)\pi\times3800/225=1326\,\text{mm}^2$ > 40% of 2785 mm^2.

The second row be at further 300 mm (≤ 0.75*d* = 314) away where row length is $1550\times4=6200$. Again using T10 – 225 spacing along the row, the total steel area will be $(10^2/4)\pi\times6200/225=2164\,\text{mm}^2$ > 60% of 2785 mm^2.

Total steel area is 1326 + 2164 = 3490 mm^2 for shear. The arrangement is illustrated in Figure 4.12.

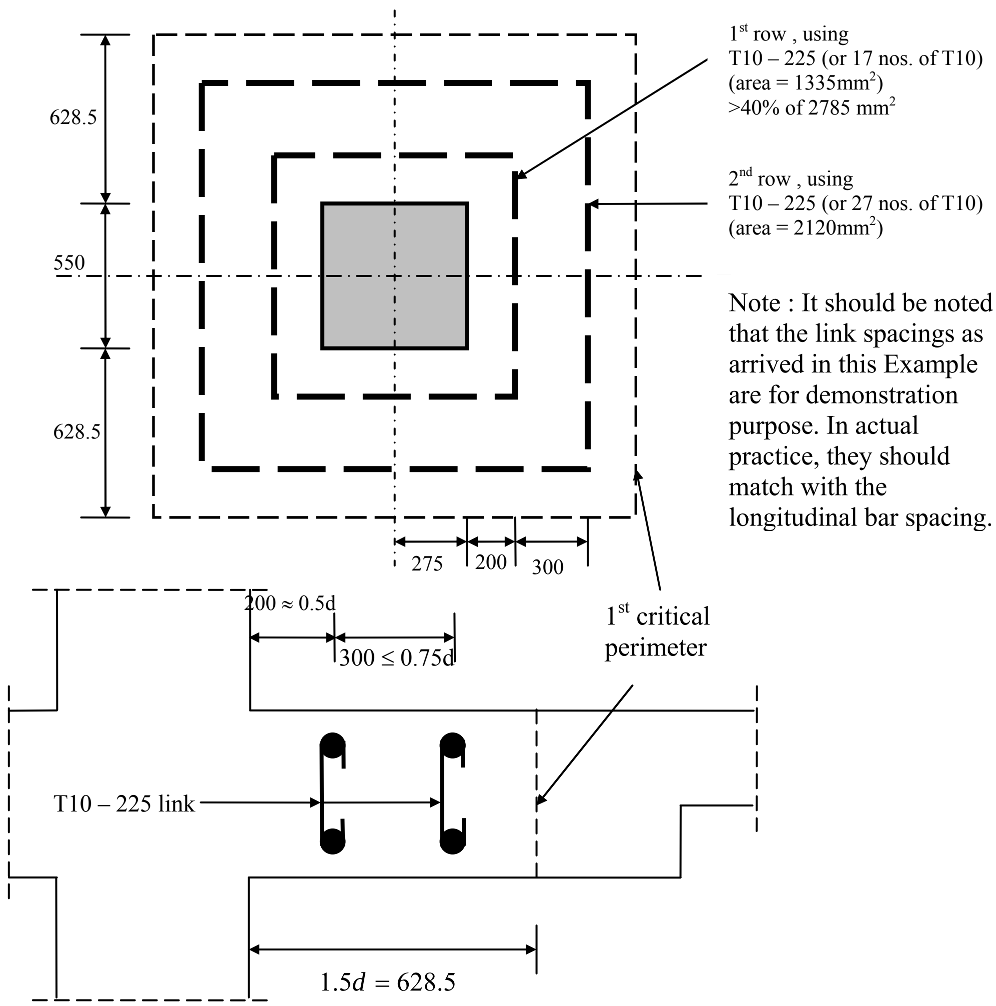


Figure 4.12 – Shear Links Arrangement in Flat Slab for Worked Example 4.5

Design for Shear when ultimate shear stress exceeds $1.6v_c$

It is stated in (Ceqn 6.45) in Cl. 6.1.5.7(e) that if $1.6_c < v \le 2.0v_c$,

$\sum A_{sv} \sin\alpha \ge \dfrac{5(0.7v - v_c)ud}{0.87 f_y}$ which effectively reduces the full inclusion of v_c for reduction to find the "residual shear to be taken up by steel" at $v = 1.6v_c$ to zero inclusion at $v = 2.0v_c$.

Worked Example 4.6 – when $1.6v_c < v \le 2.0v_c$

In the previous Example 4.5, if the shear stress $v = 0.65\,\text{N/mm}^2$ which lies between $1.6v_c = 1.6 \times 0.36 = 0.576\,\text{N/mm}^2$ and $2.0v_c = 2 \times 0.36 = 0.72\,\text{N/mm}^2$

$\sum A_{sv} \sin\alpha \ge \dfrac{5(0.7v - v_c)ud}{0.87 f_y} = \dfrac{5(0.7 \times 0.65 - 0.36) \times 7228 \times 419}{0.87 \times 500} = 3307\,\text{mm}^2$. If arranged in two rows as in Figure 4.12, use T12 – 250 for both rows : the inner row has 3800/250 = 15.2 nos. or use 15 nos. (15T12) which gives $(12^2/4)\pi \times 15 = 1695\,\text{mm}^2$ > 40% of 3307 mm^2; the outer row has 6200/250 = 24.8 or say 25 nos. (25T12) which gives $(12^2/4)\pi \times 25 = 2825\,\text{mm}^2$. The total area is 1695 + 2825 = 4520mm^2 > 3307 mm^2.

Cl. 6.1.5.7(e) of the Code says, "When $v > 2v_c$ and a reinforcing system is provided to increase the shear resistance, justification should be provided to demonstrate the validity of design." If no sound justification, the structural sizes need be revised.

5.0 Columns

5.1 Slenderness of Columns

Columns are classified as short and slender columns in accordance with their "slenderness". Short columns are those with ratios l_{ex}/h and l_{ey}/b < 15 (braced) and 10 (unbraced) in accordance with Cl. 6.2.1.1(b) of the Code where l_{ex} and l_{ey} are the "effective heights" of the column about the major and minor axes, b and h are the width and depth of the column.

As defined in Cl. 6.2.1.1(d) of the Code, a column may be considered braced in a given plane if lateral stability to the structure as a whole is provided by walls or bracing or buttressing designed to resist all lateral forces in that plane. It would otherwise be considered as unbraced.

The effective height is given by (Ceqn 6.46) of the Code as $l_e = \beta \cdot l_0$ where l_0 is the clear height of the column between restraints and the value β is given by Tables 6.11 and 6.12 of the Code which measures the restraints against rotation and lateral movements at the ends of the column.

Generally slenderness limits for column : $l_0 / b \le 60$ as per Cl. 6.2.1.1(f) of the Code. In addition, for cantilever column $l_0 = \dfrac{100b^2}{h} \le 60b$ where h and b are respectively the larger and smaller dimensions of the column.

<u>Worked Example 5.1</u> : a braced column of clear height $l_0 = 8$ m and sectional dimensions $b = 400$ mm, $h = 550$ mm with its lower end connected monolithically to a thick cap and the upper end connected monolithically to deep transfer beams in the plane perpendicular to the major direction (along the dimension h) but beam of size 300(W) by 350(D) in the other direction.

By Tables 6.11 and 6.12 of the Code
Lower end condition in both directions : 1
Upper end condition about the major axis : 1
Upper end condition about the minor axis : 2 (Cl. 6.2.1.1(e)(ii) of the Code)

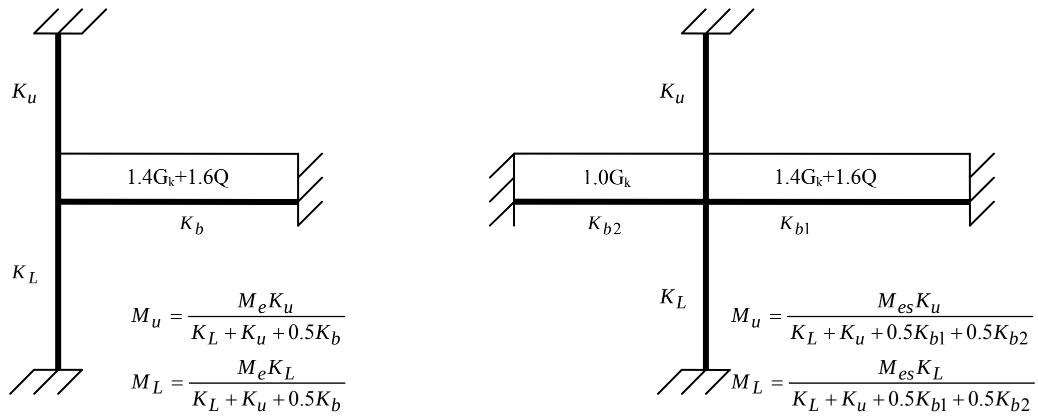
For bending about the major axis : end conditions 1 – 1, $\beta_x = 0.75$,
$l_{ex} = 0.75 \times 8 = 6$
$l_{ex} / 550 = 10.91 < 15$. $\therefore$ a short column.
For bending about the minor axis : end conditions 1 – 2, $\beta_y = 0.8$,
$l_{ey} = 0.8 \times 8 = 6.4$
$l_{ey} / 400 = 16 > 15$ $\therefore$ a slender column. $l_{ey} / 400 = 16 < 60$, O.K.

For a slender column, an additional "deflection induced moment" M_{add} will be required to be incorporated in design, as in addition to the working moment.

5.2 Design Moments and Axial Loads on Columns

5.2.1 Determination of Design Moments and Axial Loads by Sub-frame Analysis

Generally design moments, axial loads and shear forces on columns are that obtained from structural analysis. In the absence of rigorous analysis, (i) design axial load may be obtained by the simple tributary area method with beams considered to be simply supported on the column; and (ii) moment may be obtained by simplified sub-frame analysis as illustrated in Figure 5.1 :



Symbols:

M_e : Beam Fixed End Moment.

M_{es} : Total out of balance Beam Fixed End Moment.

M_u : Upper Column Design Moment

M_L : Lower Column Design Moment

K_u : Upper Column Stiffness

K_L : Lower Column Stiffness

K_{b1} : Beam 1 Stiffness

K_{b2} : Beam 2 Stiffness

Figure 5.1 – Diagrammatic illustration of determination of column design moments by Simplified Sub-frame Analysis

<u>Worked Example 5.2 (Re Column C1 in Plan shown in Figure 5.2)</u>

Design Data :

Slab thickness : 150 mm — Finish Load : 1.5 kN/m^2

Live Load : 5 kN/m^2 — Beam size : 550(D) × 400(W)

Upper Column height : 3 m — Lower Column Height : 4 m

Column size : 400(W) × 600(L)

Column Load from floors above D.L. 443 kN L.L. 129 kN

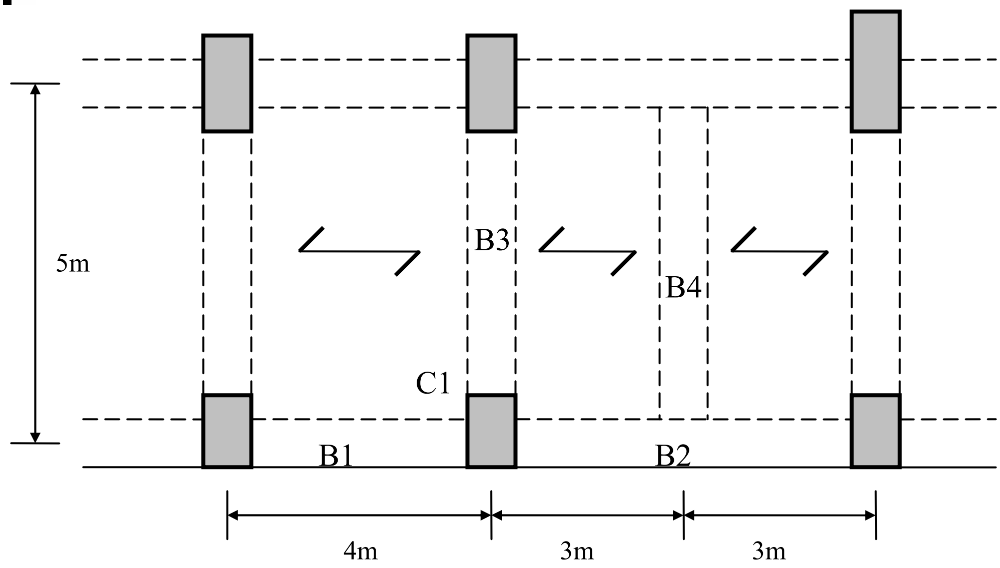


Figure 5.2 – Plan for illustration for determination of design axial load and moment on column by the Simplified Sub-frame Method

Design for Column C1 beneath the Floor (Braced)

Check for slenderness : As per Cl. 6.2.1.1(e) of the Code, the end conditions of the column about the major and minor axes are respectively 2 and 1 at the upper end and 1 at the lower end for both axes (fixed on pile cap). The clear height between restraints is $4000 - 550 = 3450$. The effective heights of the column about the major and minor axes are respectively $0.8 \times 3.45 = 2.76$ m and $0.75 \times 3.45 = 2.59$ m. So the slenderness ratios about the major and minor axes are $\frac{2760}{600} = 4.6 < 15$ and $\frac{2590}{400} = 6.475 < 15$. Thus the column is not slender in both directions.

Loads :

Slab: D.L. O.W. $0.15 \times 24.5 = 3.68$ kN/m^2
Fin. 1.5 kN/m^2
5.18 kN/m^2
L.L. 5.00 kN/m^2

Beam B1 D.L. O.W. $0.4 \times 0.55 \times 24.5 \times 4 = 21.56$ kN
End shear of B1 on C1 is D.L. $21.56 \div 2 = 10.78$ kN

Beam B3 D.L. O.W. $0.4 \times (0.55 - 0.15) \times 24.5 = 3.92$ kN/m
Slab $5.18 \times 3.5 = 18.13$ kN/m
22.05 kN/m
L.L. Slab $5.0 \times 3.5 = 17.5$ kN/m
End shear of B3 on C1 D.L. $22.05 \times 5 \div 2 = 55.13$ kN
L.L. $17.50 \times 5 \div 2 = 43.75$ kN

Beam B4 D.L. O.W. $0.4 \times (0.55 - 0.15) \times 24.5 = 3.92$ kN/m
Slab $5.18 \times 3 = 15.54$ kN/m
19.46 kN/m

L.L. Slab $5.0\times 3 = 15.0\,\text{kN/m}$

End shear of B4 on B2 D.L. $19.46\times 5\div 2 = 48.65\,\text{kN}$

L.L. $15.00\times 5\div 2 = 37.50\,\text{kN}$

Beam B2 D.L. O.W. $0.4\times 0.55\times 24.5\times 6 = 32.34\,\text{kN}$

B4 48.65 kN

80.99 kN

L.L. B4 37.50 kN

End shear of B2 on C1 , D.L. $80.99\div 2 = 40.5\,\text{kN}$

L.L. $37.5\div 2 = 18.75\,\text{kN}$

Total D.L. on C1	O.W.	$0.4\times 0.6\times 24.5\times 4 = 23.52\,\text{kN}$	
	B1 + B2 + B3	$10.78+40.5+55.13 = 106.41\,\text{kN}$	
	Floor above		443.00 kN
	Sum		572.93 kN

Total L.L. on C1	B1 + B2 +B3	$0+18.75+43.75 = 62.5\,\text{kN}$	
	Floor above		129.00 kN
	Sum		191.50 kN

So the factored axial load on the lower column is
$1.4\times 572.93+1.6\times 191.5 = 1108.50\,\text{kN}$

Factored fixed end moment bending about the major axis (by Beam B3 alone):

$$M_{ex} = \frac{1}{12}\times(1.4\times 22.05+1.6\times 17.5)\times 5^2 = 122.65\,\text{kNm}$$

Factored fixed end moment bending about the minor axis by Beam B2:

$$M_{eyb2} = 1.4\times\left(\frac{1}{12}\times 32.34+\frac{1}{8}\times 48.65\right)\times 6+1.6\times\left(\frac{1}{8}\times 37.5\right)\times 6 = 118.72\,\text{kNm}$$

Factored fixed end moment bending about the minor axis by Beam B1:

$$M_{eyb1} = 1.0\times\left(\frac{1}{12}\times 21.56\right)\times 4 = 7.19\,\text{kNm}$$

So the unbalanced fixed moment bending about the minor axis is
$M_{ey} = 118.72-7.19 = 111.53\,\text{kNm}$

The moment of inertia of the column section about the major and minor axes are $I_{cx} = \dfrac{0.4\times 0.6^3}{12} = 0.0072\,\text{m}^4$, $I_{cy} = \dfrac{0.6\times 0.4^3}{12} = 0.0032\,\text{m}^4$

The stiffnesses of the upper and lower columns about the major axis are :

$$K_{ux} = \frac{4EI_{cx}}{L_u} = \frac{4E\times 0.0072}{3} = 0.0096E$$

$$K_{Lx} = \frac{4EI_{cx}}{L_L} = \frac{4E\times 0.0072}{4} = 0.0072E$$

The stiffnesses of the upper and lower columns about the minor axis are :

$$K_{uy} = \frac{4EI_{cy}}{L_u} = \frac{4E\times 0.0032}{3} = 0.004267E$$

$$K_{Ly}=\frac{4EI_{cy}}{L_L}=\frac{4E\times0.0032}{4}=0.0032E$$

The moment of inertia of the beams B1, B2 and B3 are

$$\frac{0.4\times0.55^3}{12}=0.005546\,\text{m}^4$$

The stiffness of the beams B1, B2 and B3 are respectively

$$\frac{4E\times0.005546}{4}=0.005546E\,;\quad \frac{4E\times0.005546}{6}=0.003697E\,;\text{ and}$$

$$\frac{4E\times0.005546}{5}=0.004437E$$

Distributed moment on the lower column about the major axis is

$$M_{Lx}=\frac{M_{ex}K_{Lx}}{K_{ux}+K_{Lx}+0.5K_{b3}}=\frac{122.65\times0.0072E}{0.0096E+0.0072E+0.5\times0.004437E}$$

= 46.43 kNm

Distributed moment on the lower column about the minor axis is

$$M_{Ly}=\frac{M_{ey}K_{Ly}}{K_{uy}+K_{Ly}+0.5(K_{b1}+K_{b2})}$$

$$=\frac{111.53\times0.0032E}{0.004267E+0.0032E+0.5\times(0.005546+0.003697)E}=29.52\,\text{kNm}$$

So the lower column should be checked for the factored axial load of 1108.50kN, factored moment of 46.43 kNm about the major axis and factored moment of 29.52 kNm about the minor axis which should be the most critical case under high axial load generally.

5.2.2 Minimum Eccentricity

A column section should be designed for the minimum eccentricity equal to the lesser of 20 mm and 0.05 times the overall dimension of the column in the plane of bending under consideration (Re Cl. 6.2.1.2(d)). Consider Worked Example in 5.2, the minimum eccentricity about the major axis is 20 mm as $0.05\times600=30>20$ mm and that of the minor axis is $0.05\times400=20$ mm. So the minimum eccentric moments to be designed for about the major and minor axes are both $1108.5\times0.02=22.17$ kNm. As they are both less than the design moment of 46.43 kNm and 29.52 kNm, they can be ignored.

5.2.3 Check for Slenderness

In addition to the factored load and moment as discussed in 5.2.1, it is required by Cl. 6.2.1.3 of the Code to design for an additional moment M_{add} if the column is found to be slender by Cl. 6.2.1.1. M_{add} is an eccentric moment created by the ultimate axial load N multiplied by a pre-determined lateral deflection a_u as indicated by the following equations of the Code.

$$M_{add}=Na_u \qquad \text{(Ceqn 6.52)}$$

$$a_u=\beta_a Kh \qquad \text{(Ceqn 6.48)}$$

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 \qquad \text{(Ceqn 6.51)}$$

$$K = \frac{N_{uz} - N}{N_{uz} - N_{bal}} \le 1 \text{ (conservatively taken as 1)} \qquad \text{(Ceqn 6.49)}$$

or by $N_{uz} = 0.45 f_{cu} A_{nc} + 0.87 f_y A_{sc}$; $N_{bal} = 0.25 f_{cu} bd$ for symmetrically reinforced column. (Ceqn 6.50)

Final design moment M_t will therefore be the greatest of

(1) M_2, the greater initial end moment due to design ultimate load;

(2) $M_i + M_{add}$ where $M_i = 0.4M_1 + 0.6M_2 \ge 0.4M_2$ (with M_2 as positive and M_1 negative.)

(3) $M_1 + M_{add}/2$
in which M_1 is the smaller initial end moment due to ultimate load.

(4) $N \times e_{min}$ (discussed in Section 5.2.2 of this Manual)

where the relationship between M_1, M_2, M_{add} and the arrival of the critical combination of design moments due to M_{add} are illustrated in Figure 5.3 reproduced from Figure 6.16 of the Code.

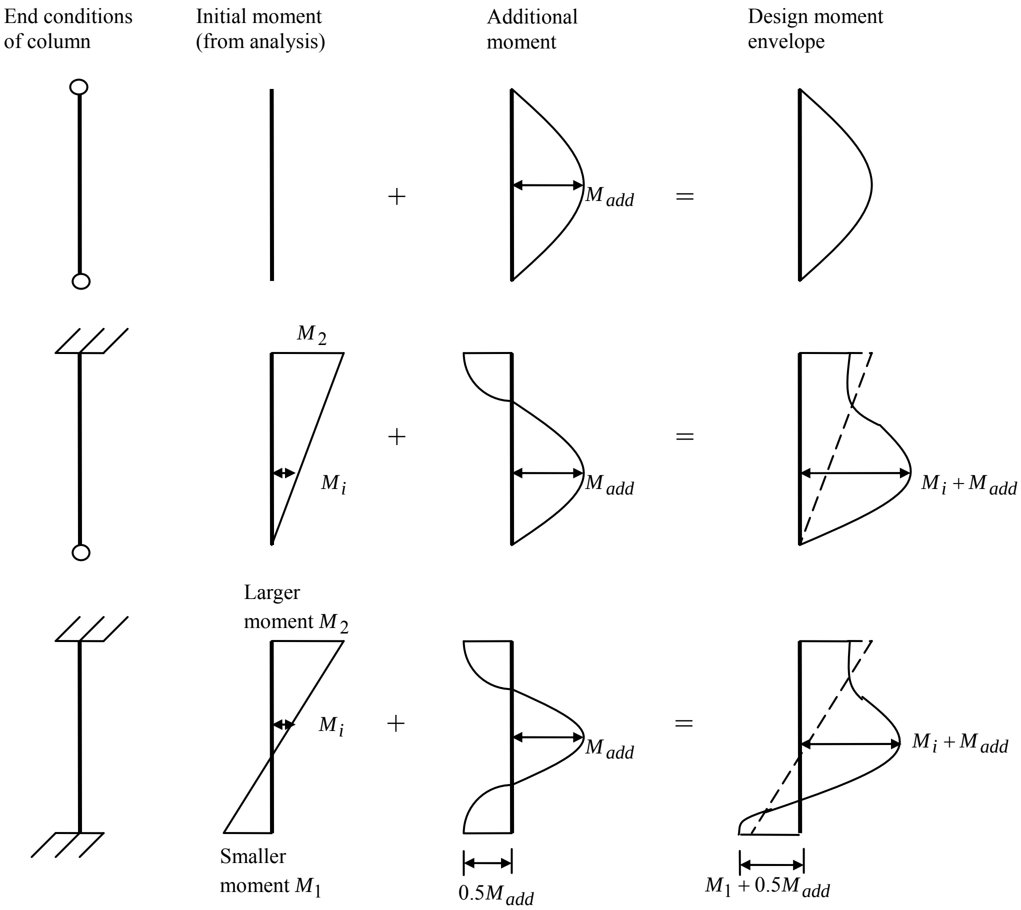


Figure 5.3 – Braced slender columns

In addition to the above, the followings should be observed in the determination of M_t as the enveloping moment of the 4 criteria described in the previous paragraph (Re Cl. 6.2.1.3 of the Code) :

(i) In case of biaxial bending (moment significant in two directions), M_t should be applied separately for the major and minor directions with b in Table 6.13 of the Code be taken as h, the dimension of the column in the plane considered for bending. Re Ceqn 6.48 and Cl. 6.2.1.3(f);

(ii) In case of uniaxial bending about the major axis where $l_e/h \le 20$ and longer side < 3 times shorter side, M_t should be applied only in the major axis (Cl. 6.2.1.3(c) of the Code);

(iii) In case of uniaxial bending about the major axis only where either $l_e/h \le 20$ or longer side < 3 times shorter side is not satisfied, the column should be designed as biaxially bent, with zero initial moment about the minor axis (Cl. 6.2.1.3(d) and (e) of the Code);

(iv) In case of uniaxial bending about the minor axis, M_{add} obviously be applied only about the minor axis only.

Worked Example 5.3 :

A slender braced column of grade C35, cross section $b = 400$, $h = 500$
$l_{ex} = l_{ey} = 8$ m, $N = 1500$ kN

(i) Moment due to ultimate load about the major axis only, the greater and smaller bending moments due to ultimate load are respectively
$M_{2x} = 153$ kNm and $M_{1x} = 96$ kNm

As $l_{ex}/h = 16 \le 20$; $h = 500 < 3b = 1200$
So needs to check for additional bending about the major axis but with β_a based on the smaller dimension of the column.
Take $K = 1$

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 = \frac{1}{2000}\left(\frac{8000}{400}\right)^2 = 0.2$$

$a_u = \beta_a K h = 0.2 \times 1 \times 0.5 = 0.1$
$M_{addx} = N a_u = 1500 \times 0.1 = 150$
$M_{ix} = 0.4M_1 + 0.6M_2 = 0.4(-96) + 0.6 \times 153 = 53.4 < 0.4M_2 = 0.4 \times 153 = 61.2$
So $M_{ix} = 61.2$

The design moment about the major axis will be the greatest of :
(1) $M_{2x} = 153$
(2) $M_{ix} + M_{addx} = 61.2 + 150 = 211.2$

(3) $M_{1x} + M_{addx} / 2 = 96 + 150 / 2 = 171$

(4) $N \times e_{min} = 1500 \times 0.02 = 30$ as $e_{min} = 20 < 0.05 \times 500 = 25$

So the greatest design moment is case (2) $M_{ix} + M_{addx} = 211.2$

Thus the section needs only be checked for uniaxial bending with $N = 1500$ kN and $M_{tx} = 211.2$ kNm bending about the major axis.

(ii) Moments due to ultimate loads about the minor axis only, the greater and smaller moments due to ultimately are as follows :

$M_{2y} = 143$ kNm and $M_{1y} = 79$ kNm

Repeating the same procedure as in (i) :

As $l_{ex} / h = 16 \le 20$; $h = 500 < 3b = 1200$

So needs to checked for additional bending about the minor axis.

Take $K = 1$

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 = \frac{1}{2000}\left(\frac{8000}{400}\right)^2 = 0.2$$

$a_u = \beta_a K b = 0.2 \times 1 \times 0.4 = 0.08$

$M_{addy} = N a_u = 1500 \times 0.08 = 120$

$M_{iy} = 0.4 M_{1y} + 0.6 M_{2y} = 0.4 \times (-79) + 0.6 \times 143 = 54.2 < 0.4 M_{2y} = 0.4 \times 143 = 57.2$

So $M_{iy} = 57.2$

The design moment will be the greatest of :

(1) $M_{2y} = 143$

(2) $M_{iy} + M_{addy} = 57.2 + 120 = 177.2$

(3) $M_{1y} + M_{addy} / 2 = 79 + 120 / 2 = 139$

(4) $N \times e_{min} = 1500 \times 0.02 = 30$ as $e_{min} = 20 \le 0.05 \times 400 = 20$

So the greatest design moment is case (2) $M_{iy} + M_{addy} = 177.2$

Thus the section need only be checked for uniaxial bending with $N = 1500$ kN and $M_{ty} = 177.2$ kNm bending about the minor axis.

(iii) Biaxial Bending, there are also moments of $M_{2x} = 153$ kNm and $M_{1x} = 96$ kNm; $M_{2y} = 143$ kNm and $M_{1y} = 79$ kNm. By Cl. 6.1.2.3(f), M_{addx} about the major axis will be revised as follows :

Bending about the major axis :

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{h}\right)^2 = \frac{1}{2000}\left(\frac{8000}{500}\right)^2 = 0.128$$

$a_u = \beta_a K h = 0.128 \times 1 \times 0.5 = 0.064$

$M_{addx} = N a_u = 1500 \times 0.064 = 96$ kNm.

Thus items (2) and (3) in (i) are revised as

(2) $M_{ix} + M_{addx} = 61.2 + 96 = 157.2$

(3) $M_{1x} + M_{addx} / 2 = 96 + 96 / 2 = 144$

So the moment about major axis for design is 157.2 kNm

Bending about the minor axis :

$M_{2y} = 143$ kNm and $M_{1y} = 79$ kNm; same as (ii);

Thus the ultimate design moment about the major axis is 157.2 kNm and that about the minor axis is 177.2 kNm.

Worked Example 5.4 :

A slender braced column of grade C35, cross section
$b = 400$, $h = 1200$
$l_{ex} = l_{ey} = 8$ m, $N = 1500$ kN, $M_{2x} = 153$ kNm and $M_{1x} = 96$ kNm

As $3b = h$, Cl. 6.2.1.3(e) should be used. The column is to be designed as biaxially bending. Take $K = 1$

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 = \frac{1}{2000}\left(\frac{8000}{400}\right)^2 = 0.2$$

$a_u = \beta_a Kb = 0.2 \times 1 \times 0.4 = 0.08\,\text{m} > 20$ mm

$M_{addy} = Na_u = 1500 \times 0.08 = 120$ kNm

So the minor axis moment is 120 kNm

As $\frac{l_e}{h} = \frac{8000}{1200} = 6.67$, the column is not slender about the major axis.

So the major axis moment is simply 153 kNm.

5.3 Sectional Design

Generally the sectional design of column utilizes both the strengths of concrete and steel in the column section in accordance with stress strain relationship of concrete and steel in Figures 3.8 and 3.9 of the Code respectively. Alternatively, the simplified stress block for concrete in Figure 6.1 of the Code can also be used.

5.3.1 Design for Axial Load only

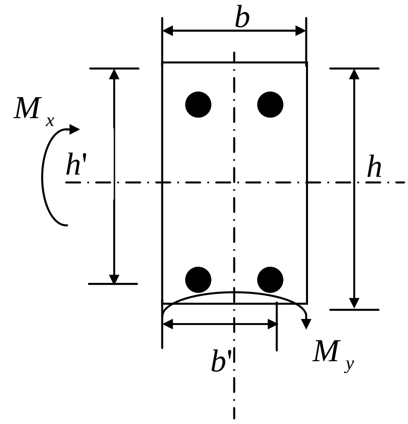
(Ceqn 6.55) of the Code can be used which is $N = 0.4 f_{cu} A_c + 0.75 A_{sc} f_y$. The equation is particularly useful for a column which cannot be subject to significant moments in such case as the column supporting a rigid structure or very deep beams. There is a reduction of approximately 10% in the axial load carrying capacity as compared with the normal value of $0.45 f_{cu} A_c + 0.87 A_{sc} f_y$ accounting for the eccentricity of $0.05h$.

Furthermore, (Ceqn 6.56) reading $N = 0.35 f_{cu} A_c + 0.67 A_{sc} f_y$ which is applicable to columns supporting an approximately symmetrical arrangement of beams where (i) beams are designed for u.d.l.; and (ii) the beam spans do not differ by more than 15% of the longer. The further reduction is to account for extra moment arising from asymmetrical loading.

5.3.2 Design for Axial Load and Biaxial Bending :

The general section design of a column is account for the axial loads and biaxial bending moments acting on the section. Nevertheless, the Code has reduced biaxial bending into uniaxial bending in design. The procedure for determination of the design moment, either M_x' or M_y' bending about the major or minor axes is as follows :

Determine b' and h' as defined by the diagram. In case there are more than one row of bars, b' and h' can be measured to the centre of the group of bars.



(i) Compare $\frac{M_x}{h'}$ and $\frac{M_y}{b'}$.

If $\frac{M_x}{h'} \geq \frac{M_y}{b'}$, use $M_x' = M_x + \beta\frac{h'}{b'}M_y$

If $\frac{M_x}{h'} < \frac{M_y}{b'}$, use $M_y' = M_y + \beta\frac{b'}{h'}M_x$

where β is to be determined from Table 5.1 which is reproduced from Table 6.14 of the Code under the pre-determined $\frac{N}{bhf_{cu}}$;

$N/(bhf_{cu})$	0	0.1	0.2	0.3	0.4	0.5	≥ 0.6
β	1.00	0.88	0.77	0.65	0.53	0.42	0.30

Table 5.1 – Values of the coefficients β

(ii) The M_x' or M_y' will be used for design by treating the section as either (a) resisting axial load N and moment M_x' bending about major axis; or (b) resisting axial load N and moment M_y' bending about minor axis as appropriate.

5.3.3 Concrete Stress Strain Curve and Design Charts

The stress strain curve for column section design is in accordance with Figure 3.8 of the Code where ε_0 to $\frac{1.34(f_{cu}/\gamma_m)}{E_d}$. The detailed design formulae and design charts have been formulated and enclosed in Appendix E. The design charts are the typical PM interaction diagrams relating *N/bh* and *M/bh*^{2}. The PM diagrams in Appendix E for rectangular column section are the 4-bar configurations which can be used to approximate the many bar configurations as will be demonstrated later. By the 4-bar configuration, it simply means two row of bars at the far ends of a column which can be quantified by the steel quantities and its capacity against axial and flexure can be completely defined by the column size, cover to reinforcement and quantities of reinforcement. However, the PM diagrams for the circular column section are for the 36-bar configuration. The intention is to simulate the reinforcements to almost a "continuous ring". Though they are more accurate for large column or bored pile, they are also good for column with only a few bars. Apart from the derivation for the normal 4-bar rectangular column, the derivation in Appendix E can also be used to determine column load capacity for reinforcement bars in any locations.

Worked Example 5.5 :

Consider a column of sectional size $b = 400$ mm, $h = 600$ mm, concrete grade C35 and under an axial load and moments of

$N = 4000\,\text{kN}$, $M_x = 250\,\text{kNm}$, $M_y = 150\,\text{kNm}$,

cover to longitudinal reinforcements = 40 mm

Assume a 4-bar column and T40 bars, $h' = 600 - 40 - 20 = 540\,\text{mm}$;

$b' = 400 - 40 - 20 = 340$ mm;

$$\frac{N}{f_{cu}bh} = \frac{4000000}{35 \times 400 \times 600} = 0.476\,;$$

$\beta = 0.446$ from Table 5.1 or Table 6.14 of the Code;

$$\frac{M_x}{h'} = 0.463 > \frac{M_y}{b'} = 0.441\,;$$

$$\therefore M_x' = M_x + \beta\frac{h'}{b'}M_y = 250 + 0.446 \times \frac{540}{340} \times 150 = 356\,\text{kNm}$$

$$\frac{N}{bh} = 16.67\,; \qquad \frac{M}{bh^2} = \frac{356 \times 10^6}{400 \times 600^2} = 2.47\,; \qquad \frac{d}{h} = \frac{540}{600} = 0.9$$

Using the relevant chart in Appendix E as extracted in Figure 5.6(a), 1.75% steel is approximated which amounts to $400 \times 600 \times 0.0175 = 4200\,\text{mm}^2$, or 6-T32 (Steel provided is 4826mm^2) The section design is also shown in Figure 5.6(b), with two additional T20 bars to avoid wide bar spacing.

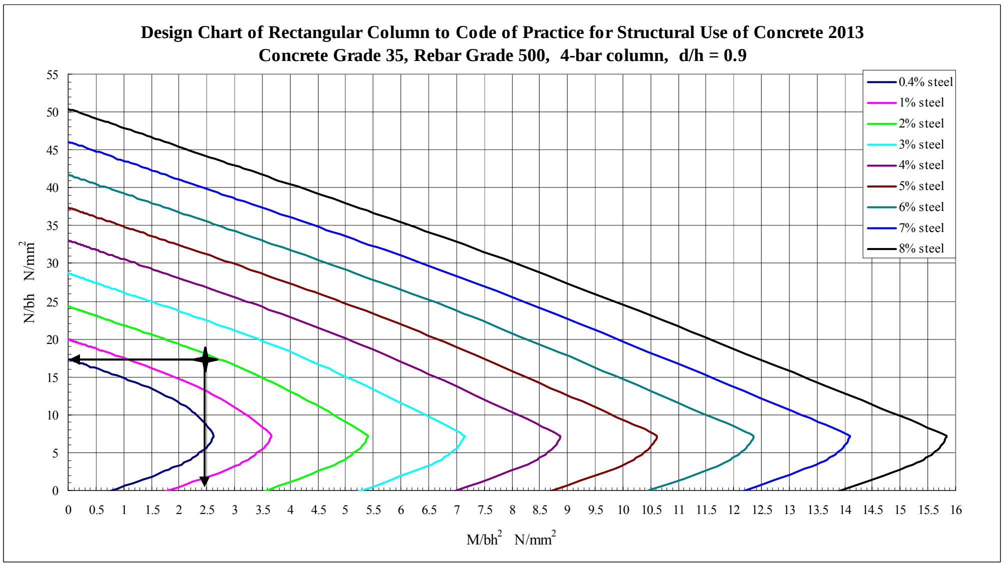


Figure 5.6(a) – Design Chart and Worked Re-bar Details for Worked Example 5.5

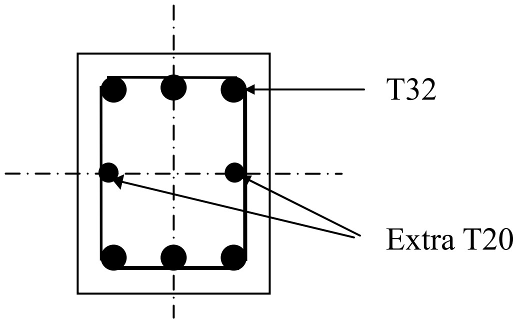


Figure 5.6(b) – Design Chart and Worked Re-bar details for Worked Example 5.5

Worked Example 5.6 :

Consider a column of sectional size $b = 800$ mm, $h = 1000$ mm, concrete grade C40 and under an axial load and moments
$N = 14400$ kN, $M_x = 2000$ kNm, $M_y = 1500$ kNm,
concrete cover to longitudinal reinforcement = 50 mm;

The design is first carried out by approximating the column as a 4 bar (or 2 layer column). First assume $d/h = 0.8$.

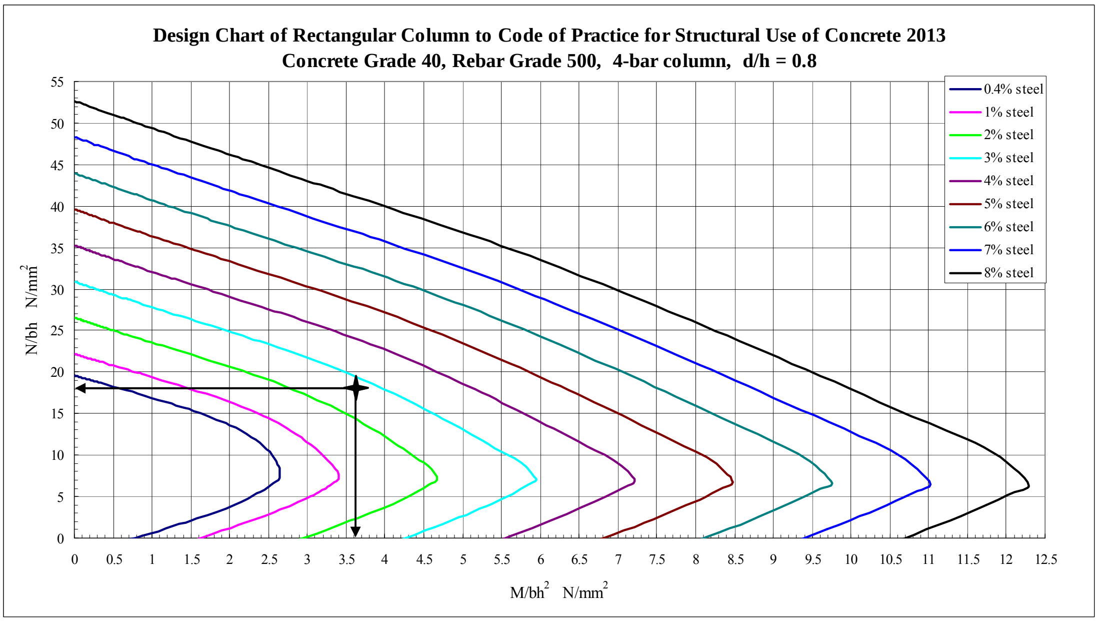
$b' = 0.8 \times 800 = 640$ mm; $h' = 0.8 \times 1000 = 800$ mm;
$\frac{N}{f_{cu}bh} = \frac{14400000}{40 \times 800 \times 1000} = 0.45$; $\beta = 0.475$ from Table 5.1 or Table 6.14 of the Code;

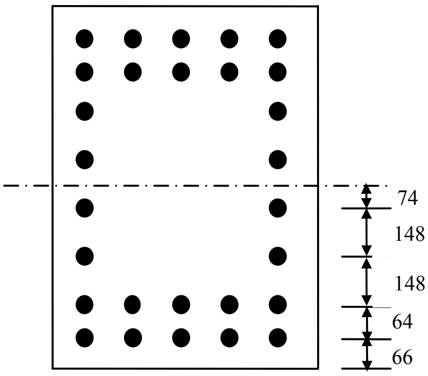
$$\frac{M_x}{h'} = \frac{2000}{800} = 2.5 > \frac{M_y}{b'} = \frac{1500}{640} = 2.34;$$

$$\therefore M_x' = M_x + \beta \frac{h'}{b'} M_y = 2000 + 0.475 \times \frac{800}{640} \times 1500 = 2890.6 \text{ kNm}$$

$$\frac{N}{bh} = 18; \qquad \frac{M}{bh^2} = \frac{2890.6 \times 10^6}{800 \times 1000^2} = 3.61;$$

Using the relevant chart in Appendix E as extracted in Figure 5.7, 2.7% steel is approximated which amounts to $0.027 \times 800 \times 1000 = 21{,}600$ mm^2, or 28-T32 (Steel provided is 22,519mm^2) The arrangement of steel bars is also shown in Figure 5.7.





Centre of mass of the reinforcements in one half of the section below the centre line is

$$\frac{5\times 434+5\times 370+2\times 222+2\times 74}{14}=329.4$$

So $h'=329.4+500=816$

$$\frac{d}{h}=0.8294>0.8$$

So the original use of $\frac{d}{h}=0.8$ is OK

Figure 5.7 – Chart and Column Section for Worked Example 5.6

The back-calculation in Figure 5.7 has shown that the $\frac{d}{h}$ ratio of the steel bar arrangement is 0.829 which is greater than the original assumed value of 0.8. So the use of the chart is conservative.

With the column bar layout in Figure 5.7, a more exact analysis is carried out by using the exact bar layout. The PM diagrams of both the exact bar layout and the 4-bar (or 2 layer) layout are constructed and presented in Figure 5.8. It can be shown that the two PM curves are quite close except that the 4-bar (or 2 layer) curve shows a higher peak in the moment of resistance. So it can be concluded that the 4-bar layout is an acceptable approximation. This approach which involves locating the centre of the rebars as the centre of mass of the group of rebars is a usual practice when computer methods for design are not employed or not available.

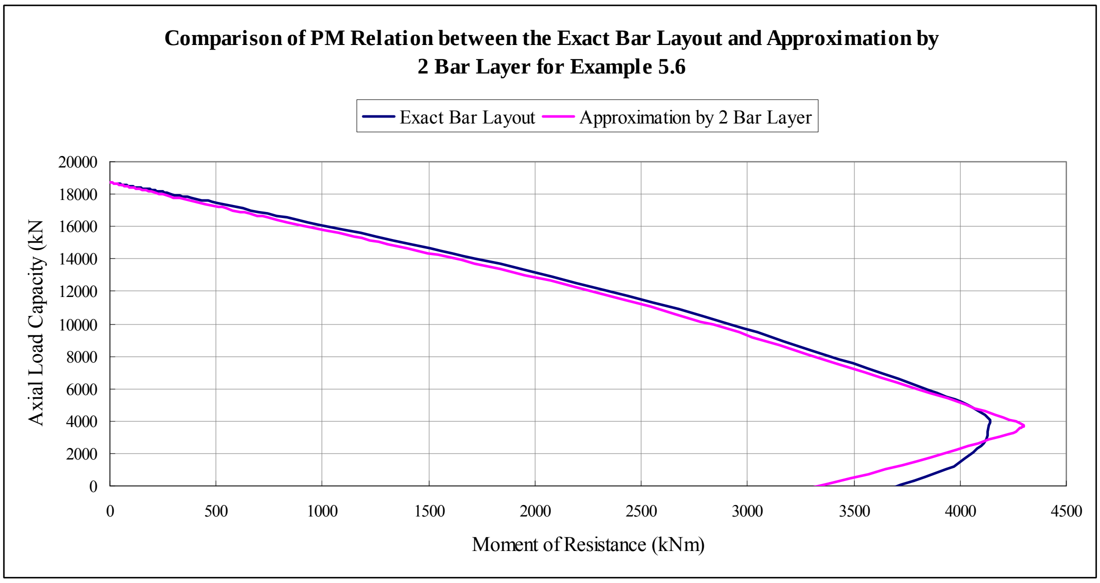
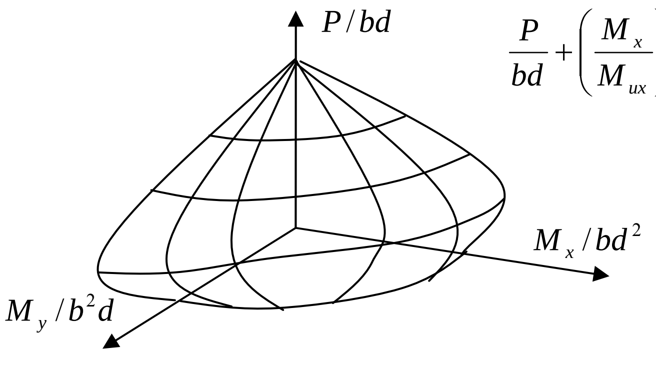


Figure 5.8 – Comparison of PM Curves between Exact Bar and 4-Bar Layout for Worked Example 5.6

5.3.4 Based on the equations derived in Appendix E, spread sheets (involving the use of visual basic in Excel) have been prepared for design of reinforced concrete column. 2 samples are enclosed at the end of the Appendix.

5.3.5 The approach by the previous British Code CP110 is based on interaction formula by which the moments of resistance in both directions under the axial loads are determined with the pre-determined reinforcements and the "interaction formula is checked". The approach is illustrated in Figure 5.9.



$$\frac{P}{bd}+\left(\frac{M_x}{M_{ux}}\right)^{\alpha}+\left(\frac{M_y}{M_{uy}}\right)^{\alpha}\le 1$$

Figure 5.9 – Interaction Formula for Design of Biaxial Bending

5.3.6 Direct sectional analysis to Biaxial Bending without the necessity of converting the biaxial bending problem into a uniaxial bending problem :

Though the Code has provisions for converting the biaxial bending problem into a uniaxial bending problem by

(i) searching for the controlling bending axis; and

(ii) aggravate the moments about the controlling bending axis as appropriate to account for the effects of bending in the non-controlling axis;

A designer can actually solve the biaxial bending problem by locating the orientation and the neutral axis depth (which generally does not align with the resultant moment except for circular section) of the column section by balancing axial load and the bending in two directions. Theoretically, by balancing axial load and the 2 bending moments, 3 equations can be obtained for solution of the neutral axis orientation, neutral axis depth and the required reinforcement. However the solution process, which is often based on trial and error approach, will be very tedious and not possible for irregular section without computer methods. Reinforcements generally need be pre-determined. Figure 5.10 illustrates the method of solution.

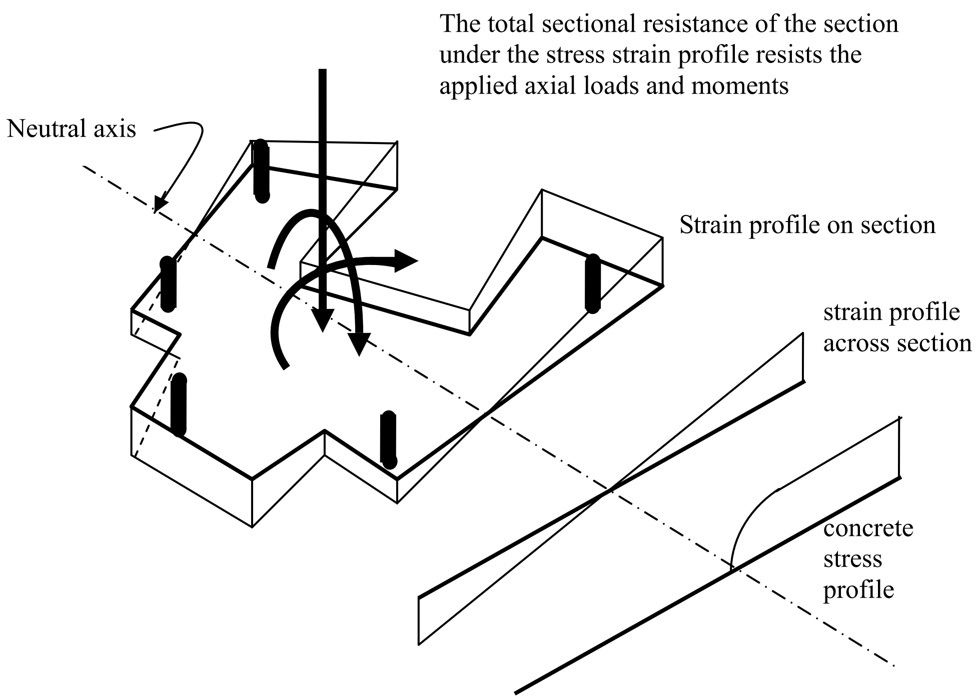


Figure 5.10 – General Biaxial Bending on Irregular Section

5.4 Detailing requirements for longitudinal bars in columns (generally by Cl. 9.5 and Cl. 9.9.2.1(a) of the Code, the ductility requirements are marked with "D")

(i) Minimum steel percentage based on gross cross-sectional area of a column is 0.8% (Cl. 9.5.1 and Cl. 9.9.2.1(a) of the Code);

(ii) Maximum steel based on gross cross-sectional area of a column is 4% for strength design except at lap which can be increased to 5.2% (D) (Cl. 9.9.2.1(a) of the Code);

(iii) Bar diameter ≥ 12 mm (Cl. 9.5.1 of the Code) ;

(iv) The minimum number of bars should be 4 in rectangular columns and 6 in circular columns. In columns having a polygonal cross-section, at least one bar be placed at each corner (Cl. 9.5.1 of the Code);

(v) In any row of bars, the smallest bar diameter used shall not be less than 2/3 of the largest bar diameter used (Cl. 9.9.2.1(a) of the Code). For example, T40 should not be used with T25 and below (D);

(vi) At laps, the sum of reinforcement sizes in a particular layer should not exceed 40% of the breadth at that section (Cl. 9.5.1 of the Code). The requirement is identical to that of beam as illustrated by Figure 3.14;

(vii) Where column bars terminate in a joint between columns, beams and foundation members with possible formation of a plastic hinge in the column, the anchorage of the column bars into the joint region should commence at 1/2 of the depth of the beam/foundation member or 8 times the bar diameter from the face at which the bars enter the beam or the foundation member. Where a column plastic hinge adjacent to the beam face cannot be formed, anchorage can commence at beam face (Cl. 9.9.2.1(c) of the Code) as illustrated in Figure 5.11 (D);

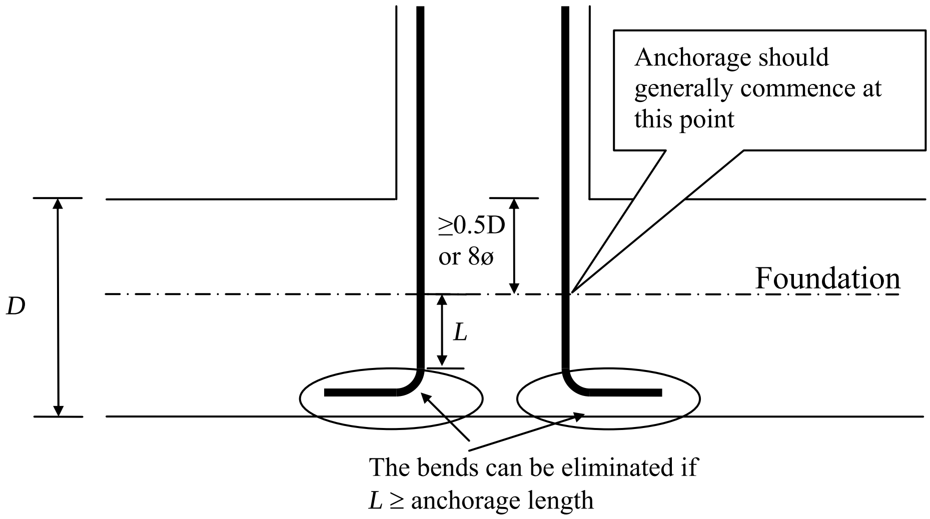


Figure 5.11 – Column Bar Anchorage in Foundation

(viii) For column bars anchoring into beam (transfer beam or roof beam), in addition to the requirement in (vii), the bars should not be terminated in a joint area without a horizontal 90^{o} standard hook or an equivalent device as near as practically possible to the far side of the beam and not closer than 3/4 of the depth of the beam to the face of entry. Unless the column is designed to resist only axial load, the direction of bend must always be towards the far face of the column (Cl. 9.9.2.1(c) of the Code) as illustrated in Figures 5.12 and 5.13 (D);

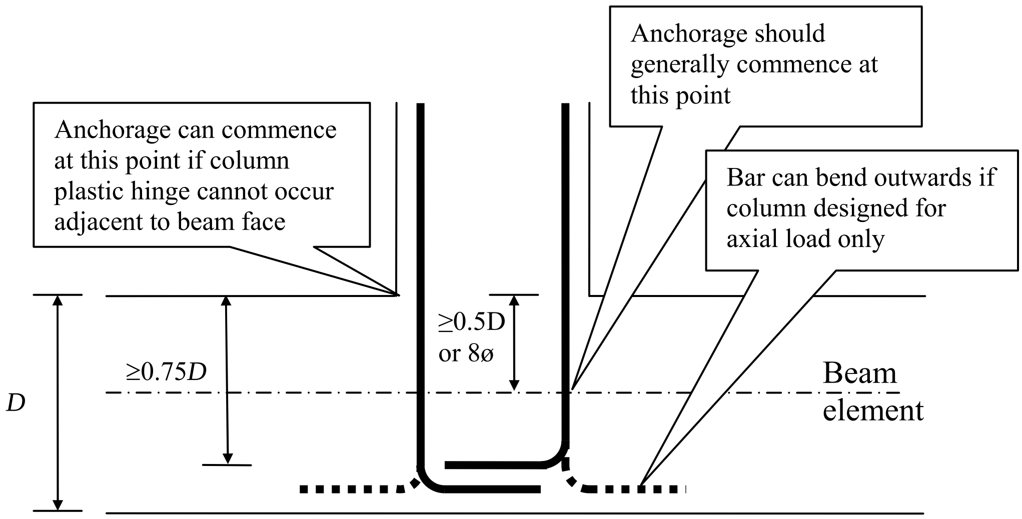


Figure 5.12 – Column Bar Anchorage in Beam

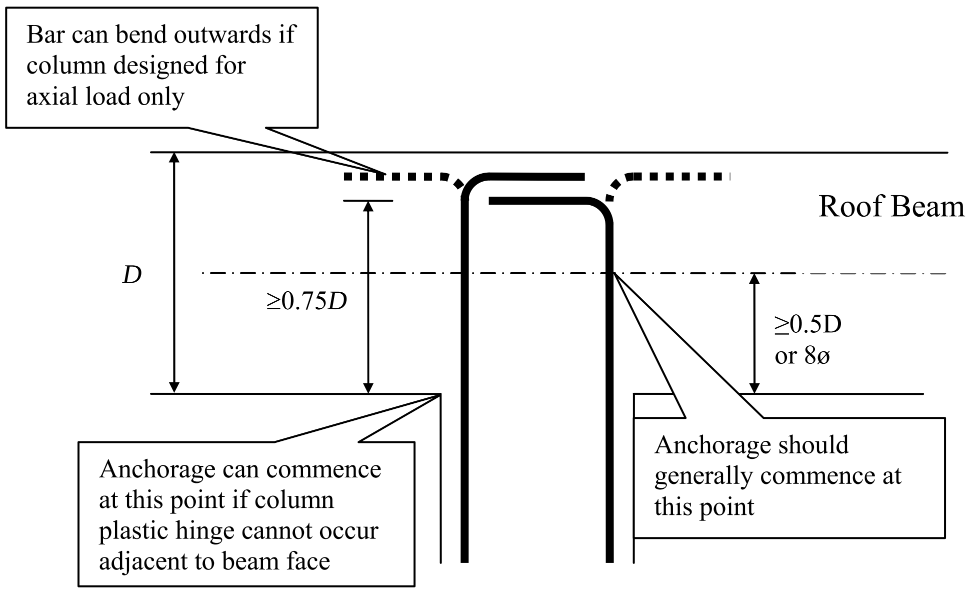


Figure 5.13 – Column Bar anchorage in Beam (Roof Beam)

(ix) For laps and Type 1 mechanical couplers in a column the centre of the splice must be within the middle half of the storey height of the column unless it can be shown that plastic hinges cannot develop in the column adjacent to the beam faces. The implied condition is $\sum M_c \geq 1.2\sum M_b$ as stated in (Ceqn 9.6) which is widely adopted in other codes such as ACI318-11. In the equation $\sum M_b$ is the sum of moments of resistance of the beams on both sides of the beam-column joint which are either both acting clockwise on the joint or anti-clockwise on the joint, i.e. one sagging and one hogging at the same time which will be created at lateral load acting on the frame as explained in Figure 5.14. Though Figure 5.14 illustrates only the case for lateral load acting from the left, the moments will all reverse in sign when the lateral load acts from the right

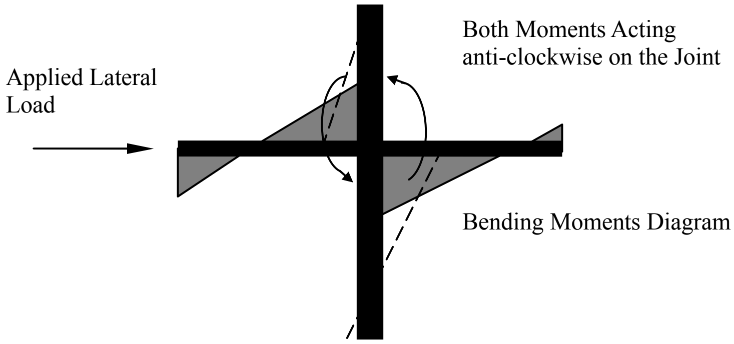


Figure 5.14 – Moments by Beam and Column on Beam-Column Joint under Lateral Load

As it is undesirable that the column will fail by bending (formation of plastic hinge in the column) prior to that of beam, the column should be made to be stronger than that of beam. The inequality in (Ceqn 9.6) serves this purpose. Therefore, upon checking failure to satisfy (Ceqn 9.6), some provisions such as forbidding lapping or mechanical splicing of the bars in the critical zones of the column should be exercised so as to minimize the chance of column failure. The following worked example 5.7 as indicated in Figure 5.15 illustrates how the condition can be fulfilled.

Worked Example 5.7

A column supporting two beams on both sides with details and loads as shown in Figure 5.15 is to be checked for the requirement of complying the "middle half rule". The column and beams form parts of a wind resisting frame and the lateral load is high enough to cause reverse of moments at the beam column junctions.

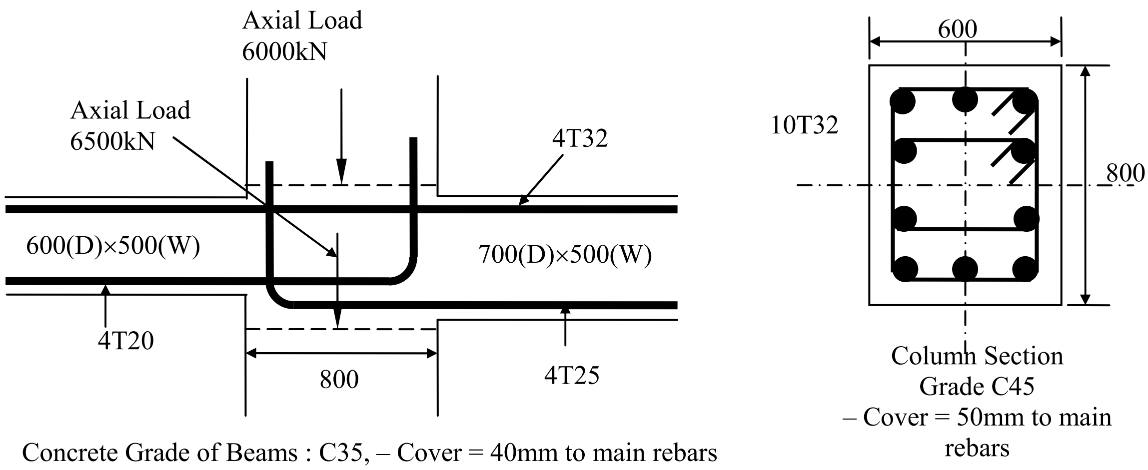


Figure 5.15 – Layout for Worked Example 5.7

The moments capacity of the beams $\sum M_b$ are first to be worked out. The clockwise (sagging) moment capacity of the left beam (acting on the joint) is first worked out as follows :

The effective depth $d = 550$ mm

The steel area provided (4T20) is $A_s = 1257\,\text{mm}^2$

Tensile strength of the steel is $T = 0.87 f_y A_s = 546795\,\text{N}$

Using the simplified stress block, the depth of the stress block is $0.9x$, then $0.45 f_{cu} b(0.9x) = T \Rightarrow 0.9x = \dfrac{546795}{0.45 \times 35 \times 500} = 69.43\,\text{mm}$

The lever arm is $z = d - 0.5(0.9x) = 515.28\,\text{mm}$

The moment capacity is $0.87 f_y z = 281754090\,\text{Nmm} = 281.75\text{kNm}$

The other clockwise and anti-clockwise moment capacity of the beams are worked out similarly and summarized in Table 5.2.

	Left Beam		Right Beam	
	Clockwise (sagging)	Anti-clockwise (hogging)	Clockwise (hogging)	Anti-clockwise (sagging)
Effective Depth (mm)	550	540	640	647.5
Steel Area (mm^2)	1257	3217	3217	1963
Steel Strength (N)	546795	1399395	1399395	853905
0.9Neutral axis depth (mm) < 0.9×0.5=0.45 effective depth	69.43	177.70	177.70	108.43
Lever arm (mm)	515.28	451.15	551.15	593.28
Moment of Resistance (kNm)	281.75	631.34	771.28	506.61

Table 5.2 – Summary of Moment Capacity of Beams in Worked Example 5.7

The moment capacity of the column sections above and below the joint can be calculated from the PM diagram constructed as in Figure 5.16 or estimated from the PM diagrams in Appendix E.

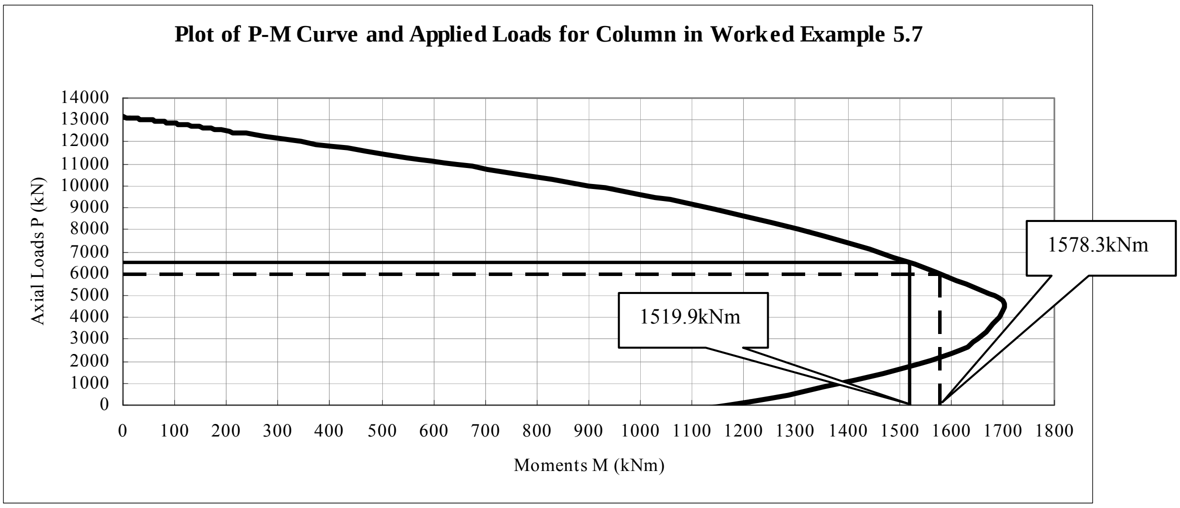


Figure 5.16 – PM Diagram for the Column in Worked Example 5.7

So $\sum M_b$ is the greater of $281.75 + 771.28 = 1053.03$ and $631.34 + 506.61 = 1137.95$. So $\sum M_b = 1137.95\,\text{kNm}$

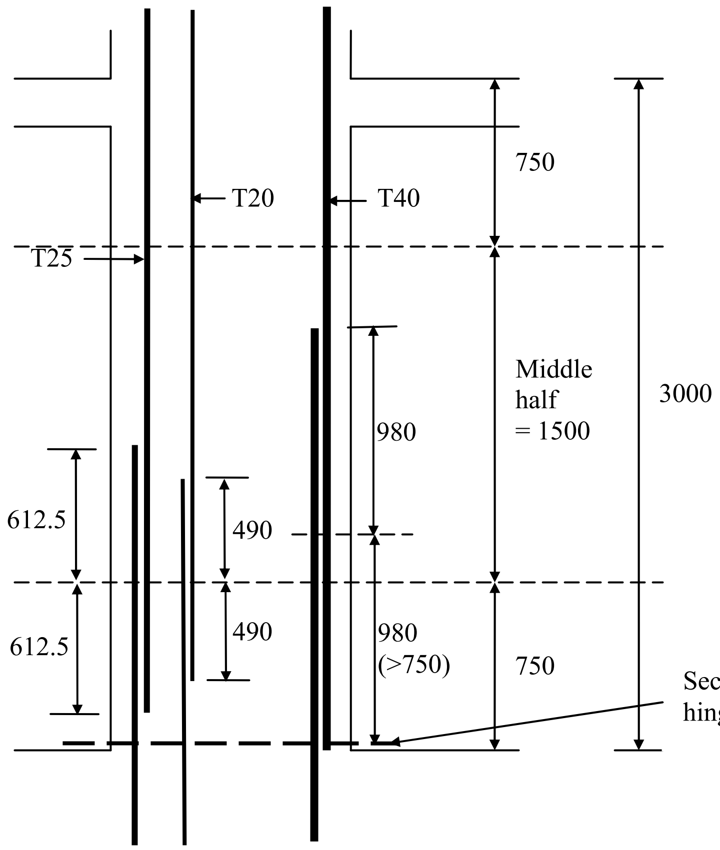
On the other hand $\sum M_c = 1519.9 + 1578.3 = 3098.2$ kNm under the greatest vertical loads creating the mode of bending (or simplest the greatest possible vertical load as a conservative design) which should lead to the smallest moment capacity generally.

So $\sum M_c = 3098.25 \geq 1.2 \sum M_b = 1365.54$.

Thus the middle half rule needs not be complied with in this example.

In addition, the Code has specified in Cl. 9.9.2.1(d) that if gravity load dominates where reversal of bending will not occur, i.e. the beam will not suffer sagging moment at the application of lateral load, the moment capacity with the bottom steel needs not be considered. In the above example, if the lateral load will not cause reverse in beam moments, $\sum M_b$ will only contain the hogging moment, i.e. the greater of 631.34kNm and 771.28kNm. Ceqn 9.6 can of course be satisfied.

In case that Ceqn 9.6 cannot be satisfied, lapping or splicing of bars by Type 1 mechanical coupler should be such that the centre of lap or splices be within the middle half of the storey height of the column. Examples of compliance with the middle half rule for lapping of bars of different bar sizes in column of storey height 3000mm are illustrated in Figure 5.17.



Consider a column of storey height 3m, grade 40 and with T20, T25 and T40 bars. Half lap lengths (tension) are respectively
1.4×35×20÷2=490;
1.4×35×25÷2=612.5;
1.4×35×40÷2=980;

The middle half of the column is at a level of 750 from the lower floor as shown. If the centre of lap is within the middle half, the T20 & T25 bars need be lapped at level higher than the floor level.

Figure 5.17 – Centre of Lapping be within Middle Half of Floor Height in Column

It can be seen that the upper T40 bar can be installed at floor level while the upper T25 and T20 bars may have to be hung above floor level. Nevertheless, it is acceptable that the upper bars be extended so that they can rest on the floor level and the lap lengths are effectively increased. If Type 1 mechanical coupler is to be used, they have to be staggered in at least 2 layers at not less than 900mm apart as illustrated in Figure 9.9 of the Code. (D)

(x) Type 2 mechanical coupler should be used in at least 2 layers not less than 300mm apart. But the lowest layer should be at least 300mm above structural floor, pile cap or transfer floor level (Cl. 9.9.2.1(e) of the Code) (D) as illustrated in Figure 9.10 of the Code;

(xi) Minimum clear spacing of bars should be the greatest of bar diameter, 20 mm and aggregate size + 5 mm (Cl. 8.2 of the Code).

5.5 Detailing requirements for transverse reinforcements in columns are given by Cl. 9.5.2 and Cl. 9.9.2.2 of the Code. Generally a column can be divided into "critical zones" and "non-critical zone" with the former having high bending moment which are often at the floor levels. More stringent requirements in the transverse reinforcements are imposed in the "critical zone" so as to achieve higher flexural capacity and ductility. The requirements are described as follows:

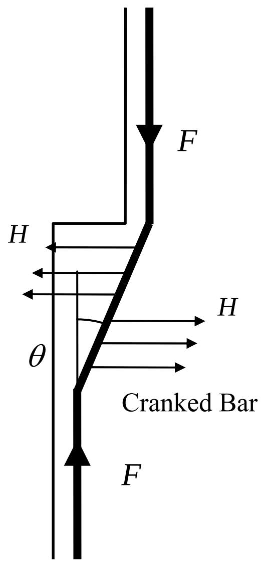
General and Non-critical Zone

(i) Diameter of transverse reinforcements ≥ the greater of 6 mm and 1/4 of largest longitudinal bar diameter (Cl. 9.5.2.1 of the Code);

(ii) The vertical centre to centre spacing of transverse reinforcement shall not exceed the least of :

(a) 12 times the diameter of the smallest longitudinal bar;
(b) the lesser dimension of the column; and
(c) 400mm as stated in Cl. 9.5.2.1 of the Code.

(iii) In accordance with Cl. 9.5.2.1 of the Code (last paragraph), where direction of the column longitudinal bar changes (e.g. at change in column size), the spacing of transverse reinforcement should be calculated to cater for the lateral force involved unless the change in direction is less than 1 in 12. The Code does not specify methods to calculate the spacing of transverse reinforcements. A conservative approach is illustrated in Figure 5.18;



As a conservative approach by which the restraint of concrete on the cranked bar is ignored, the maximum lateral force that will be induced and be taken up by links will be $H = F\tan\theta$.
So for a bar of cross sectional area A_s, cranked at a gradient $s = \tan^{-1}\theta$ and stressed to $0.87f_y$,
$H = F\tan\theta = A_s 0.87 f_y s$

If the permissible stress of the link is also $0.87f_y$, the total area of the links will be $A_{sh} = 2A_s s$. The lower half of A_{sh} serves to counteract the horizontal *H* induced by the main bar in compression while the upper half of A_{sh} is redundant (push provided by concrete instead). The action and redundancy of the transverse rebars will however reverse if the *F* reverses in direction.

Take a numerical example of a T40 bar cranked at gradient 1:5 over a length of 1000mm, the total area of links required will be 503mm^2 or 5T12 or T12@200c/c.

Figure 5.18 – Column Longitudinal Bar Changing Direction

(iv) For rectangular or polygonal columns, every corner bar and each alternate bar (or bundle) shall be laterally supported by a link passing around the bar and having an included angle ≤ 135^o. No bar within a compression zone should be further than 150 mm from a restrained bar. Links should be adequately anchored by hooks with bent angles ≥ 135^o. However, alternate cross ties as shown in Figure 5.15 d) which is reproduced from Figure 9.5 of the Code can be used. In addition, where there is adequate confinement to prevent the end anchorage from "kick off", the 135^o hook may be replaced by other standard hoods (Cl. 9.5.2.2 of the Code). See Figure 5.19e);

(v) For circular columns, loops or spiral reinforcement satisfying (i) to (ii) should be provided. Loops (circular links) should be anchored with a mechanical connection or a welded lap or by terminating each end with at least a 90^o hook bent around a longitudinal bar after overlapping the other end of the loop. Spiral should be anchored either by welding to the previous turn or by terminating each end with at least a 90^o hook bent around a longitudinal bar and at not more than 25 mm from the previous turn. Loops and spirals should not be anchored by straight lapping, which causes spalling of the concrete cover (Cl. 9.5.2.3 of the Code). The details are also illustrated in Figure 5.16 reproduced from Figure 9.5 of the Code.

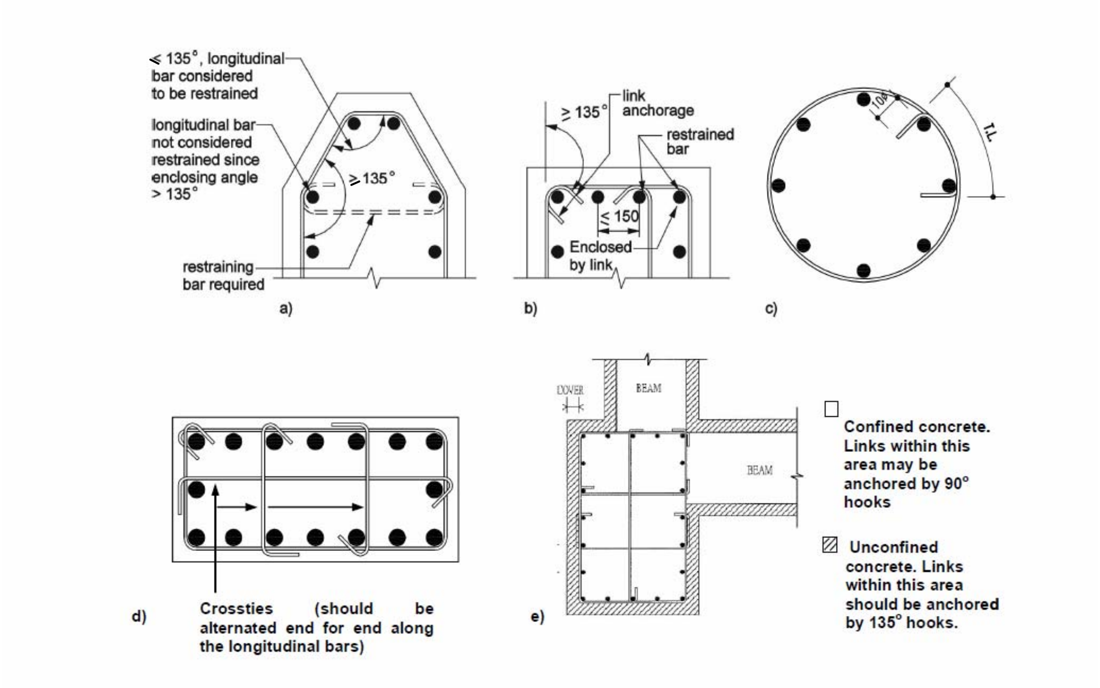


Figure 5.19 – Column Transverse Reinforcements outside "Critical Zones"

Critical Zone

(vi) "Critical zones" within a column is defined in Figure 5.20 (Re Cl. 9.9.2.2 of the Code). They are the locations having greatest moment under lateral load so that plastic hinges can be formed more readily. Denser transverse reinforcements are thus required for confinement so as to increase both the strength and ductility of the column at these critical zones. Extent of the zones are given in the clause and illustrated diagrammatically in Figure 5.20. Requirements of the denser transverse reinforcements are given in Cl. 9.9.2.2 of the Code. The denser transverse reinforcement serves the purposes of (1) confining the inner core of concrete which increases the concrete strength and its contribution to ductility; (2) prevent the reinforcement bars from buckling. To perform the function effectively, it is generally required that the links be adequately anchored into the core of the concrete by either good confinements or bent angles ≥ 135^{o} so that "kicking off" can be prevented.

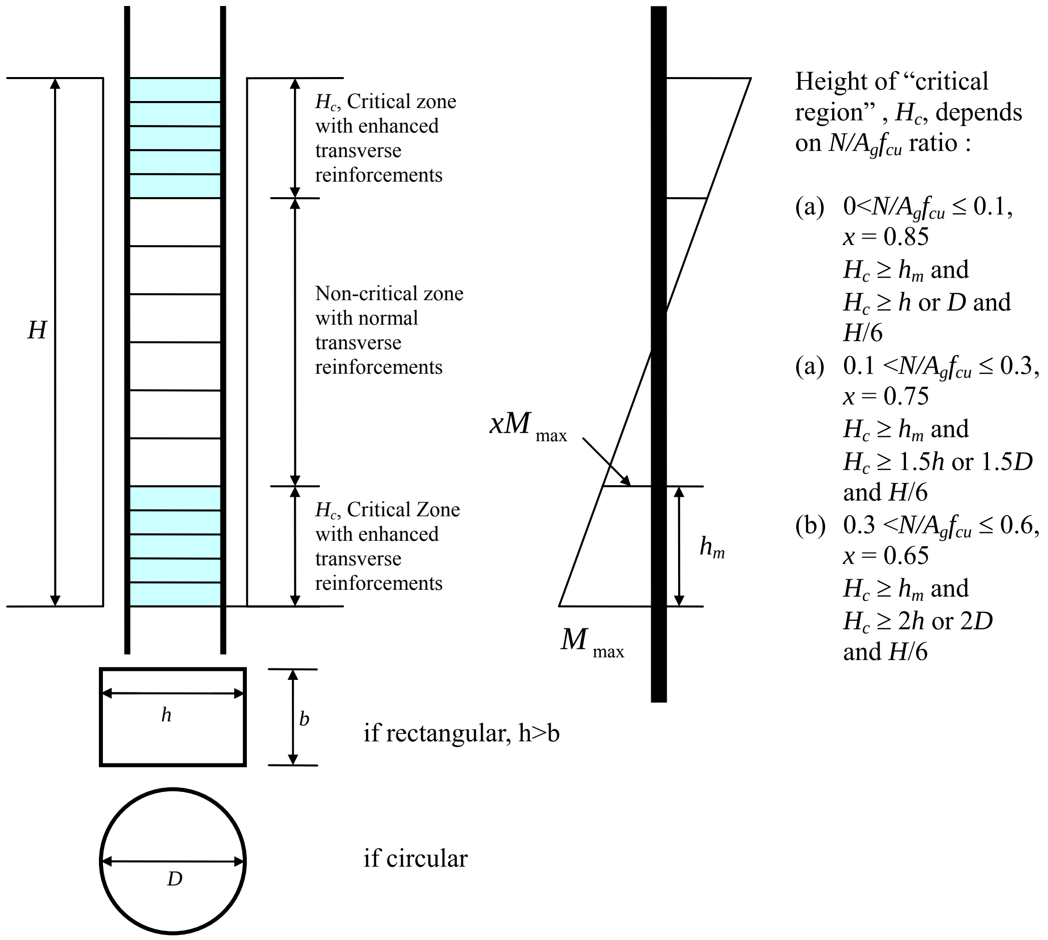


Figure 5.20 – "Critical Zones (Potential Plastic Hinge Regions)" in Columns

(vii) Within the critical zones, the diameter of the link should not be less than the greater of 10mm or 1/4 of the largest longitudinal bar diameter (D);

(viii) The vertical centre to centre spacing of transverse reinforcement should not exceed the lesser of 8 times the diameter of the longitudinal bar being restrained or 150mm (D);

(ix) The arrangement of transverse reinforcements for rectangular and polygonal columns should be so arranged that (1) every longitudinal bar (or bundle of bars) are laterally supported by a link passing around the bar, or alternatively, (2) each corner bar and alternative bar (or bundle) is laterally supported by a link passing around the bar and no bar within the compression zone shall be further than the smaller of 10 times the link bar diameter or 125mm from a restrained bar as illustrated in Figure 5.21 (D).

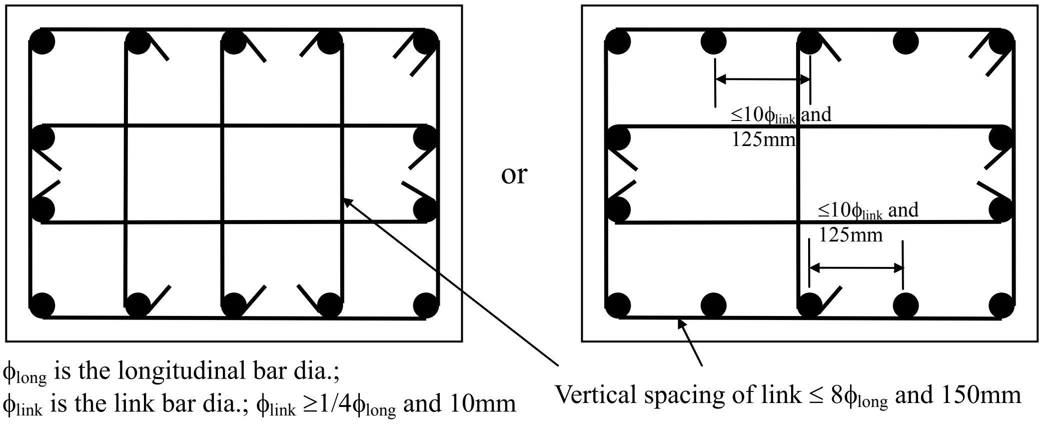


Figure 5.21 – Diagrammatical Illustration of Requirements of Links in "Critical Zones" of Column for Ductility

(x) As similar to (iv) and (v) for transverse reinforcements in the "non-critical zone", the 135^o anchorage end of link can be relaxed if there is adequate confinement that the anchorage ends will not "kick-off" as per Cl. 9.2.2.2(c) of the Code.

Worked Example 5.8 – for determination of "Critical Zone"

Consider a rectangular column of size :
Cross section 500×600 mm; clear height 3 m; grade C40; re-bars : T32

Loads and moments are as follows :
Axial Load 3000 kN
$M_x = 800$ kNm (at top), $M_x = 500$ kNm (at bottom)
$M_y = 450$ kNm (at top) $M_y = 300$ kNm (at bottom)

$$\frac{N}{A_g f_{cu}} = \frac{3000 \times 10^3}{500 \times 600 \times 40} = 0.25\text{, i.e. } 0.1 < \frac{N}{A_g f_{cu}} \le 0.3$$

So the governing criterion for determination of critical zone based on drop of moment is that the critical zone should end at where the maximum moment has dropped to 75% of its maximum value. The height h_m is worked out on Figure 5.22.

The other two criteria of the height of the critical zone are (1) 1.5 times the greater column dimension 1.5×600 = 900mm; and (2) 1/6 of the column clear height = 3000/6 = 500mm.

So the critical zones are 900mm at the top and bottom as illustrated in Figure 5.23 with the required transverse reinforcements.

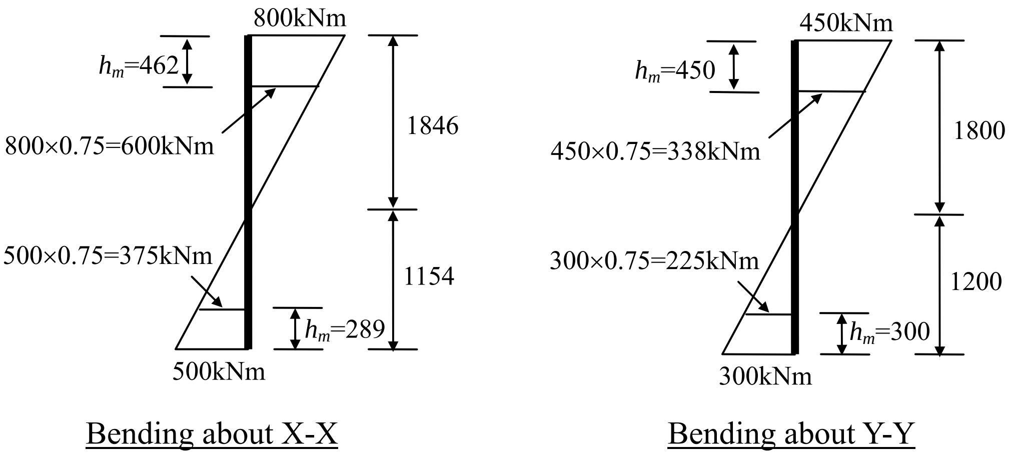
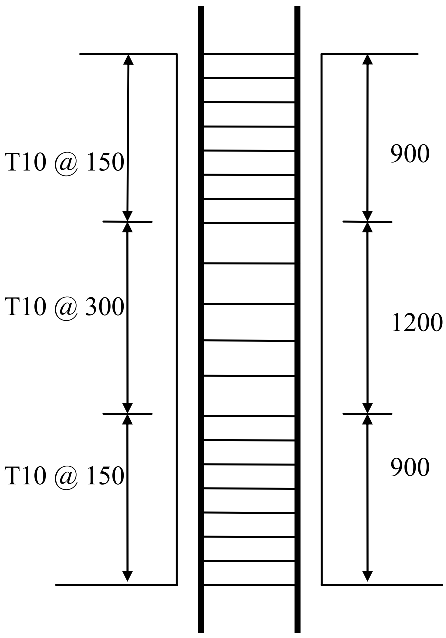


Figure 5.22 – Determination of critical heights in Worked Example 5.7



<u>Transverse re-bars</u>

(i) <u>Critical zone</u> :
Bar size 0.25×32 = 8 mm < 10 mm
Use T10 for the links
Spacing : the lesser of
8×32 = 256 mm
150mm
So spacing is 150 mm

(ii) <u>Non-critical zone</u>
Bar size 0.25×32 = 8 mm > 6 mm
Use 10mm
Spacing, the least of
(a) 12×32 = 384 mm;
(b) Lesser column size = 500mm;
(c) 400mm
Use 300mm as the spacing.

Figure 5.23 –Transverse Reinforcement arrangement to Worked Example 5.7

6.0 Beam-Column Joint

6.1 General

The design criteria of a column-beam joint comprise (i) performance not inferior to the adjoining members at serviceability limit state; and (ii) sufficient strength to resist the worst load combination at ultimate limit state. To be specific, the aim of design comprises (a) minimization of the risk of concrete cracking and spalling near the beam-column interface; and (b) checking provisions against diagonal crushing or splitting of the joint and where necessary, providing vertical and horizontal shear links within the joint and confinement to the longitudinal reinforcements of the columns adjacent to the joint.

The design provisions in the Code are largely based on the New Zealand Code NZS 3101: Part 1 and Part 2: 2006.

6.2 The Phenomenon of "Diagonal Splitting" of Joint

Diagonal crushing or splitting of a beam-column joint is resulted from direct shear from the adjoining column and shear due to unbalancing moment acting on the joints as illustrated in Figure 6.1(a) and 6.1(b) which indicate typical loadings acting on the joint. Figure 6.1(a) shows a joint with hogging moment on both sides which is the normal behaviour of a column beam joint under dominant gravity loads. On the other hand, Figure 6.1(b) shows a joint with hogging moment on the right and sagging moment on the left, which may be due to a large externally applied horizontal shear from the right. The "push" and "pull" on both sides of the joint by the reinforcement bars and/or compression zones of the beams resulted from bending do not balance generally and thus create a net "shear" on the joint.

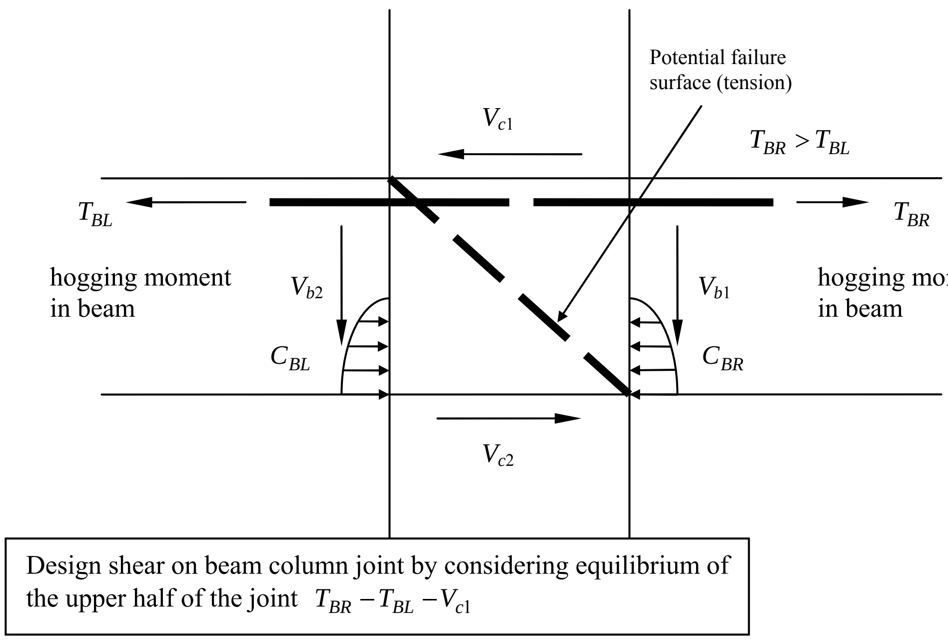


Figure 6.1(a) – Phenomenon of Diagonal Joint Splitting by Moments of Same Sign (hogging) on Opposite Sides of Beam-Column Joint

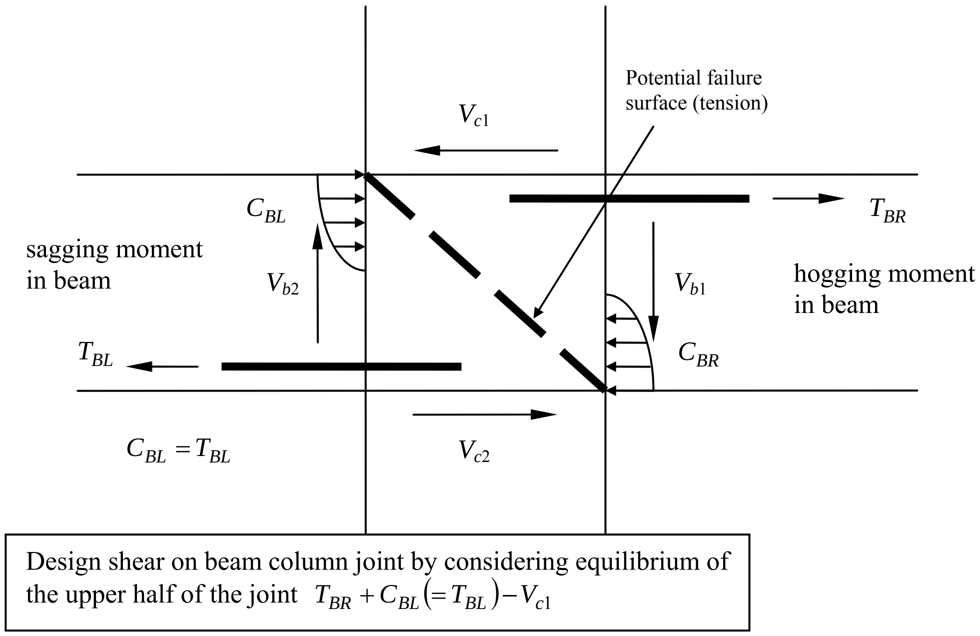


Figure 6.1(b) – Phenomenon of Diagonal Joint Splitting by Moments of Opposite Signs on Opposite Sides of Beam-Column Joint

In both cases, the unbalanced forces due to unbalanced flexural stresses by the adjoining beams on both sides of the joint tend to "tear" the joint off with a potential tension failure surface, producing "diagonal splitting". Though generally there are shears in the column (V_{c1} and V_{c2}) usually tend to act oppositely, the effects are usually small and therefore can be ignored in design. Reinforcements in form of links may therefore be necessary if the concrete alone is considered inadequate to resist the diagonal splitting.

In order to provide "ductility" in the design of the beam-column joint, the most stringent approach is to derive the design shear based on the scenario in Figure 6.1(b) with the provided reinforcement bars stressed to bar failure or at least at their yielding load. This will ensure that beam-column joint failure will not take place before plastic hinge formed in beam which is a preferred failure mode in seismic design. However, such a design is considered too stringent in Hong Kong of low to moderate seismicity, the Code defines 3 scenarios in the determination of the "design force" acting on the beam-column as summarized in 6.3 (i).

6.3 Design Procedures :

(i) Work out the Design Force, V_{jh} across the joint in X and Y directions generally according to the 3 scenarios as schematically shown in Figure 6.2(a), (b) and (c). The 3 scenarios are :
   (a) Where the frame in which the beam-column joint situates is not contributing to lateral load resisting system;
   (b) Where the frame in which the beam-column joint situates is contributing to lateral load resisting system but the gravity load is so dominant that reverse of moments (turning the supporting moment

from hogging to sagging) will not occur;

(c) Where the frame in which the beam-column joint situates is contributing to lateral load resisting system and the lateral load is so dominant that reverse of moments (turning the supporting moment from hogging to sagging) will occur;

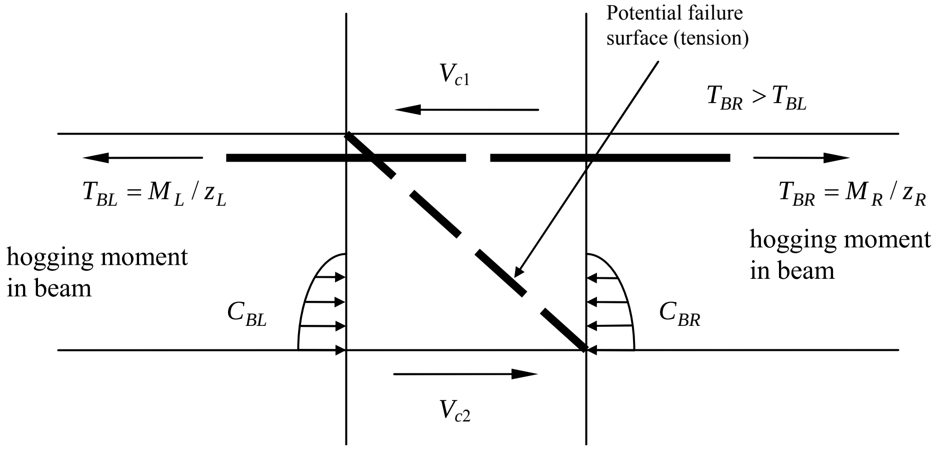


Figure 6.2(a) – Scenario 1 $V_{jh} = T_{BR} - T_{BL} - V_{c1}$ under the worst Combination of Load

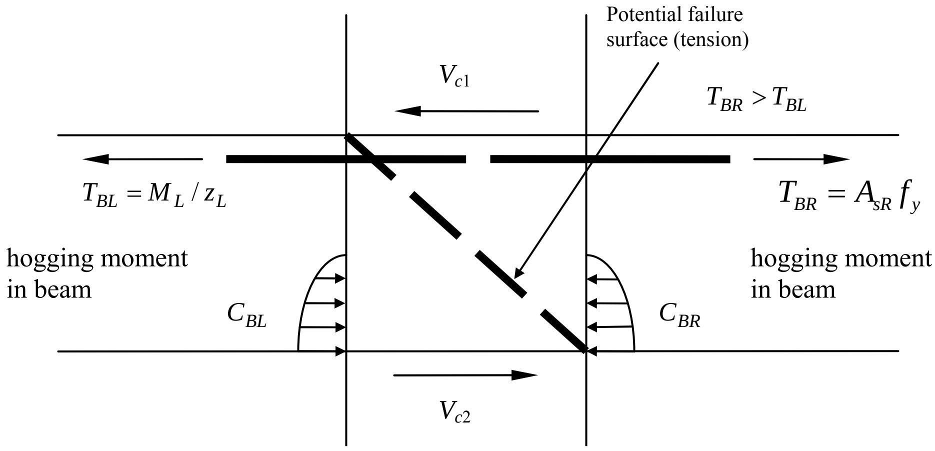


Figure 6.2(b) – Scenario 2 $V_{jh} = T_{BR} - T_{BL} - V_{c1}$ under the worst Combination of Load

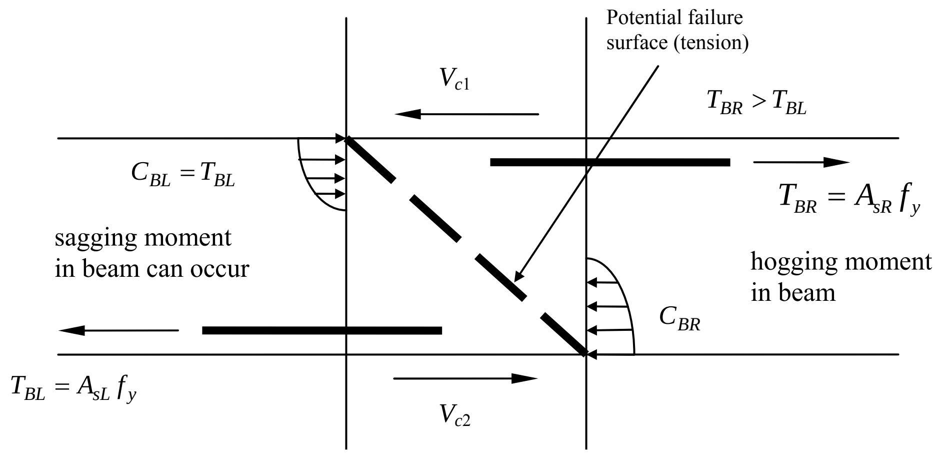


Figure 6.2(c) – Scenario 3 $V_{jh} = T_{BR} + T_{BL} - V_{c1}$ under the worst Combination of Load

It should be noted that Figures 6.2(a), (b) and (c) illustrate the principle of determination of design shear force by assuming higher "pull" from the right side. Checking should, however, be carried out on all possible load combinations on both sides to identify the greatest design shear, under the application of partial load factors as shown in Table 2.1 of the Code. For example, the most critical load case or pattern load for a beam column joint due to load on beams on both sides of the joint is that as shown in Figure 6.3.

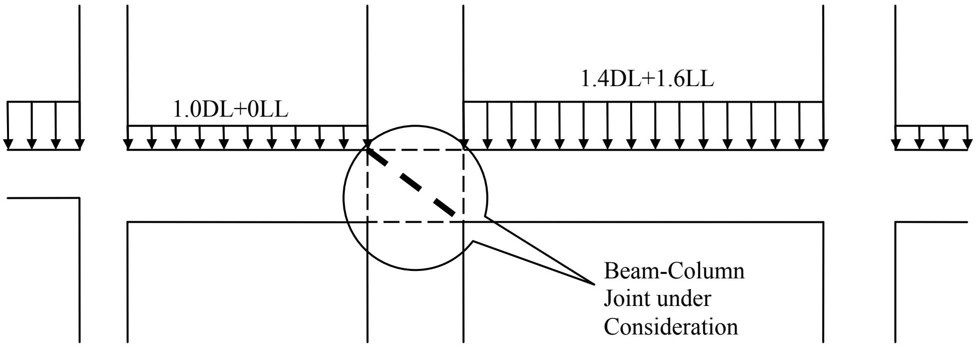


Figure 6.3 – Pattern Load that likely Produce the Greatest Shear on a Beam-Column Joint (Gravity Load Case)

However, as it is generally tedious to solve for the many combination of pattern loads in a continuous beam especially when the number of spans is great, we may take a conservative approach by carrying out analysis of the continuous beam according to the characteristic load and determine the design shear on a beam-column joint by taking 1.4DL+1.6LL on the adjacent span producing larger moments (where the live load moment is maximum due to the pattern load based on characteristic load) and 1.0DL on the opposite span.

(ii) With the design shear V_{jh} determined, the nominal shear stress is determined by (Ceqn 6.71) in the Code. $v_{jh} = \frac{V_{jh}}{b_j h_c}$

where h_c is the overall depth of the column in the direction of shear

$b_j = b_c$ or $b_j = b_w + 0.5h_c$ whichever is the smaller when $b_c \geq b_w$

$b_j = b_w$ or $b_j = b_c + 0.5h_c$ whichever is the smaller when $b_c < b_w$

where b_c is the width of column and b_w is the width of the beam.

Cl. 6.8.1.2 of the Code specifies that "At column of two-way frames, where beams frame into joints from two directions, these forces need be considered in each direction independently." So v_{jh} should be calculated independently for both directions even if they exist simultaneously and both be checked that they do not exceed $0.2 f_{cu}$ as per Cl. 6.8.1.3 of the Code.

(iii) By Cl. 6.8.1.5 of the Code, horizontal reinforcements based on Ceqn 6.72

reading $A_{jh} = \frac{V_{jh}}{0.87 f_{yh}}\left(0.5 - \frac{C_j N}{0.8 A_g f_{cu}}\right)$ should be worked out in the X and Y directions and be provided in the joint as horizontal links. In Ceqn 6.72, V_{jh} should be the design shear force in the direction (X or Y) under consideration and N be the minimum column axial load (thus creating least enhancing effect). If the numerical values of A_{jh} arrived at is positive, shear reinforcements of total cross sectional area A_{jh} should be provided in the joint. It may be more convenient to use close links which can serve as horizontal reinforcements in both directions and confinements as required by (v). There are 4 aspects warrant attention :

(a) If the numerical values arrived by (Ceqn 6.72) becomes negative, no horizontal shear reinforcements will be required;

(b) If N is negative implying uplift force by the column, C_j will be one, i.e. without distribution. In addition, the expression $-\frac{C_j N}{0.8 A_g f_{cu}}$ will become positive which will increase the value of A_{jh};

(c) Horizontal shear reinforcements be evenly distributed but not immediately adjacent to the inner most main bars as illustrated in Figure 6.4.

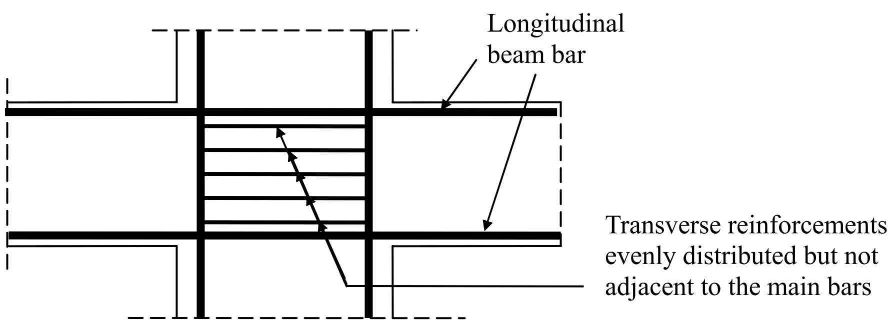


Figure 6.4 – Transverse reinforcements in Column Beam Joint

(iv) Similarly by Cl. 6.8.1.6 of the Code, vertical reinforcements based on (Ceqn 6.73) reading $A_{jv} = \frac{0.4(h_b / h_c)V_{jh} - C_j N}{0.87 f_{yv}}$ should be worked out in X and Y directions and be provided in the joint as vertical links. Again if the numerical values arrived by (Ceqn 6.73) is negative, no vertical shear reinforcements will be required. The vertical shear reinforcements, if required, shall be spaced not more than 200mm and 1/4 of the lateral dimension of the joint in the orthogonal direction. Each vertical face should be provided with at least one vertical bar as per Cl. 6.8.1.6.

(v) Notwithstanding the provisions arrived at in (iii) for the horizontal reinforcements, confinements in form of minimum closed links within the joint should be provided as per Cl. 6.8.1.7 of the Code as :

(a) Not less than that in the column shaft as required by Cl. 9.5.2 of the Code, i.e. Section 5.5 (i) to (iv) of this Manual if the joint has one or more than one free faces;

(b) Reduced by half to that provisions required in (a) if the joint is connected to beams in all its 4 faces;

(c) Link spacing ≤ 10Ø (diameter of smallest column bar) and 200 mm as illustrated in Figure 6.5.

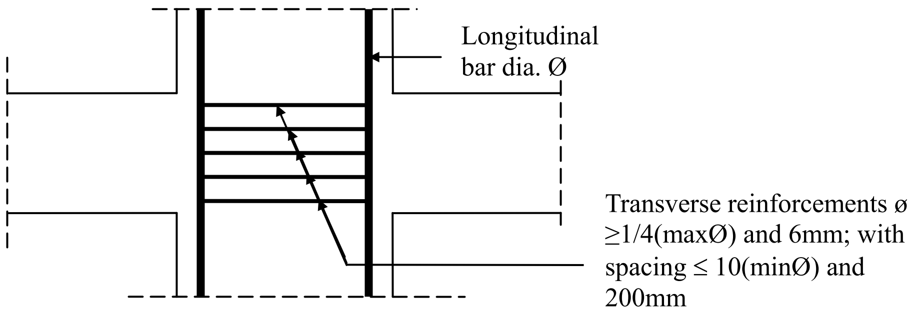
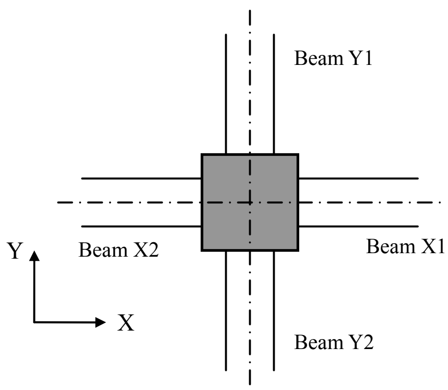


Figure 6.5 – Minimum transverse reinforcements in Column Beam Joint

6.4 <u>Worked Example 6.1</u>:

Consider the beam-column joint with columns and beams adjoining as indicated in Figure 6.6 in the X-direction and Y-directions. The joint is in a frame not contributing to the lateral resisting system. Concrete grade is C45. The design is as follows :



Concrete Grade of column : C45
Column Plan Size : 900 mm(along X) × 800mm (along Y)
Beam sizes : All are 700mm (deep) × 500mm (wide)
Effective depths are all 630mm
Minimum Axial Load on Column from above 9000kN

Summary of Beam Moments in kNm (characteristic values) at Column Faces :

Beam	Dead Load	Live Load	
		Maximum	Minimum
X1	400	120	40
X2	200	60	20
Y1	450	150	50
Y2	180	60	40

Figure 6.6 – Design Data for Worked Example 6.1 (Not Lateral Load Resisting)

In the absence of lateral load, the design should fall into that of Scenario 1 for both directions.

(1) Design Shear Force and Check Shear Stress in X-direction

By inspection, Beam X1 will exert a stronger moment. So the greatest shear should be resulted from Beam X1 exerting a maximum moment of $1.4 \times 400 + 1.6 \times 120 = 752$ kNm while Beam X2 exerts a minimum moment of $1.0 \times 200 = 200$ kNm.

By Figure 6.2(a), $K_{X1} = \dfrac{M_{X1}}{bd^2 f_{cu}} = \dfrac{752 \times 10^6}{500 \times 630^2 \times 45} = 0.0842$

$$z_{X1} = \left(0.5 + \sqrt{0.25 - \frac{K_{X1}}{0.9}}\right) d = 0.896 \times 630 = 564 \text{ mm}$$

$$T_{X1} = M_{X1} / z_{X1} = 752 \times 10^6 / 564 \times 10^{-3} = 1333 \text{ kN}$$

$$K_{X2} = \frac{M_{X2}}{bd^2 f_{cu}} = \frac{200 \times 10^6}{500 \times 630^2 \times 45} = 0.0224$$

$$\frac{z_{X2}}{d} = \left(0.5 + \sqrt{0.25 - \frac{K_{X2}}{0.9}}\right) = 0.974; \quad z_{X2} = 0.95 \times 630 = 598.5 \text{ mm}$$

$$T_{X2} = M_{X2} / z_{X2} = 200 \times 10^6 / 598.5 \times 10^{-3} = 334 \text{ kN}$$

Net design shear is thus $V_{jx} = T_{X1} - T_{X2} = 999$ kN

Check adequacy of structural size, $h_c = 900$

As $b_c = 800 > b_w = 500$, the effective joint width is the smaller of $b_c = 800$ and $b_w + 0.5h_c = 500 + 0.5 \times 900 = 950$, so $b_j = 800$

So, checking against Cl. 6.8.1.3 of the Code,

$$v_{jx} = \frac{V_{jx}}{b_j h_c} = \frac{999000}{800 \times 900} = 1.39 \text{ MPa} < 0.2 f_{cu} = 9 \text{ MPa.}$$

(2) Design Shear Force and Check Shear Stress in Y-direction

By inspection, Beam Y1 will exert a stronger moment. So the greatest shear should be resulted from Beam Y1 exerting a maximum moment of $1.4 \times 450 + 1.6 \times 150 = 870$ kNm while Beam Y2 exerts a minimum moment of 180kNm.

By Figure 6.2(a), $K_{Y1} = \dfrac{M_{Y1}}{bd^2 f_{cu}} = \dfrac{870 \times 10^6}{500 \times 630^2 \times 45} = 0.0974$

$$z_{Y1} = \left(0.5 + \sqrt{0.25 - \frac{K_{Y1}}{0.9}}\right) d = 0.877 \times 630 = 552 \text{ mm}$$

$$T_{Y1} = M_{Y1} / z_{Y1} = 870 \times 10^6 / 552 \times 10^{-3} = 1576 \text{ kN}$$

$$K_{Y2} = \frac{M_{Y2}}{bd^2 f_{cu}} = \frac{180 \times 10^6}{500 \times 630^2 \times 45} = 0.0202$$

$$\frac{z_{Y2}}{d} = \left(0.5 + \sqrt{0.25 - \frac{K_{Y2}}{0.9}}\right) = 0.977\,; \quad z_{Y2} = 0.95 \times 630 = 598.5\text{ mm}$$

$$T_{Y2} = M_{Y2}/z_{Y2} = 180 \times 10^6 / 598.5 \times 10^{-3} = 301\text{kN}$$

Net design shear is thus $V_{jy} = T_{Y1} - T_{Y2} = 1275\text{ kN}$

Check structural adequacy in the Y-direction $h_c = 800$ mm, $b_c = 900$ mm and $b_w + 0.5h_c = 500 + 0.5 \times 800 = 900$ mm, so $b_j = 900$ mm
So, checking against Cl. 6.8.1.3 of the Code,

$$v_{jy} = \frac{V_{jy}}{b_j h_c} = \frac{1275000}{900 \times 800} = 1.77\text{ MPa} < 0.2 f_{cu} = 9\text{ MPa}$$

(3) Design Shear Reinforcement in X-Direction

To calculate the horizontal shear reinforcement by Ceqn 6.72, reading

$$A_{jh} = \frac{V_{jh}}{0.87 f_{yh}}\left(0.5 - \frac{C_j N}{0.8 A_g f_{cu}}\right)$$

where $C_j = \dfrac{V_{jx}}{V_{jx} + V_{jy}} = \dfrac{999}{999 + 1275} = 0.439$

$$A_{jhx} = \frac{V_{jx}}{0.87 f_{yh}}\left(0.5 - \frac{C_j N}{0.8 A_g f_{cu}}\right) = \frac{999 \times 10^3}{0.87 \times 500}\left(0.5 - \frac{0.439 \times 9000 \times 10^3}{0.8 \times 900 \times 800 \times 45}\right)$$

$= 798\text{mm}^2$

To calculate the vertical shear reinforcement by (Ceqn 6.73 of the Code), reading $A_{jv} = \dfrac{0.4(h_b / h_c)V_{jh} - C_j N}{0.87 f_{yh}}$

$$A_{jvx} = \frac{0.4(h_b/h_c)V_{jx} - C_j N}{0.87 f_{yv}} = \frac{0.4(700/900) \times 999 \times 10^3 - 0.439 \times 9000 \times 10^3}{0.87 \times 500}$$

$= -8369\text{mm}^2$.
No vertical reinforcement required.

(4) Design Shear Reinforcement in Y-Direction

$$A_{jhy} = \frac{V_{jy}}{0.87 f_{yh}}\left(0.5 - \frac{C_j N}{0.8 A_g f_{cu}}\right) = \frac{1275 \times 10^3}{0.87 \times 500}\left(0.5 - \frac{0.561 \times 9000 \times 10^3}{0.8 \times 900 \times 800 \times 45}\right)$$

$= 895\text{mm}^2$

$$A_{jvy} = \frac{0.4(h_b / h_c)V_{jy} - C_j N}{0.87 f_{yv}} = \frac{0.4(700/800)1275 \times 10^3 - 0.561 \times 9000 \times 10^3}{0.87 \times 500}$$

$= -10581\text{mm}^2$.
No vertical reinforcement required.

The shear reinforcements are shown in Figure 6.7.

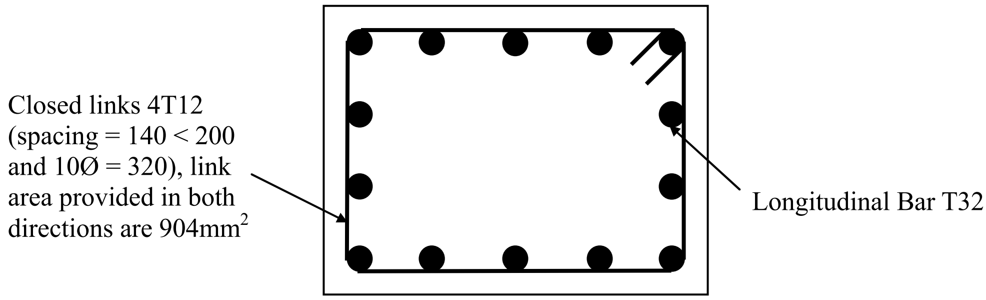
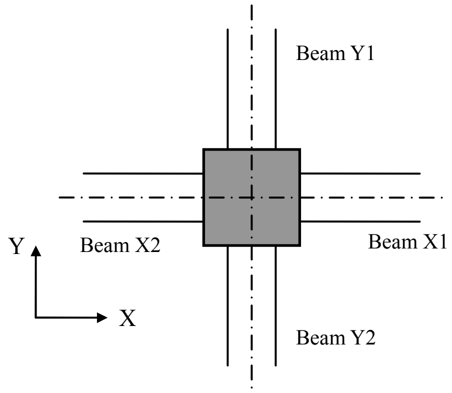


Figure 6.7 – Required Horizontal Shear Links of Beam-Column Joint (Plan) for Design Worked Example 6.1 – Other details Omitted for Clarity

6.5 Worked Example 6.2

Consider the same beam-column joint in Figure 6.8 in the X-direction and Y-directions with additional wind loads



Concrete Grade of column : C45
Column Plan Size : 900mm (along X) × 800mm (along Y)
Beam sizes : All are 700mm(deep) × 500mm (wide)
Effective depths are all 630mm
Minimum Axial Load on Column from above 9000kN

Summary of Beam Moments in kNm (characteristic values) at Column Faces :

Beam	Dead Load	Live Load		Wind Load
		Maximum	Minimum	
X1	400	120	40	±150
X2	200	60	20	±130
Y1	450	150	50	±200
Y2	180	60	40	±250

Figure 6.8 – Design Data for Worked Example 6.2 (Lateral Load Resisting)

It can be seen that there is no reverse of moment in the X-direction due to wind load and thus the design falls into Scenario 2 while that in the Y-direction will fall into Scenario 3 as there shall be reverse of moment due to wind load on Beam Y2.

(1) Design Shear Force and Check Shear Stress in X-direction

The provided top reinforcement of Beam X1 is 2T40 + 1T32, giving 3318mm^2 (against design hogging moment of 1.2(D+L+W) = 804kNm > 1.4(D+W) = 770kNm). So the “pull” force is

$$T_{X1} = f_y \times A_s = 500 \times 3318 \times 10^{-3} = 1659\,\text{kN}$$

The minimum hogging moment by Beam X2 is (1.0D – 1.4W)

$$1.0 \times 200 - 1.4 \times 130 = 18\,\text{kNm};$$

$$T_{X2} = \frac{18 \times 10^6 \times 10^{-3}}{0.95 \times 630} = 30\,\text{kN}$$

So $V_{jx} = 1659 - 30 = 1629\,\text{kN}$

Check adequacy of structural size, $h_c = 900\,\text{mm}$

As $b_c = 800 > b_w = 500$, the effective joint width is the smaller of $b_c = 800$ and $b_w + 0.5h_c = 500 + 0.5 \times 900 = 950\,\text{mm}$, so $b_j = 800\,\text{mm}$

So, checking against Cl. 6.8.1.3 of the Code,

$$v_{jx} = \frac{V_{jx}}{b_j h_c} = \frac{1629000}{800 \times 900} = 2.26\,\text{MPa} < 0.2 f_{cu} = 9\,\text{MPa}.$$

(2) Design Shear Force and Check Shear Stress in Y-direction

The provided top reinforcement of Beam Y1 is 3T40, giving 3770mm^2 (against design hogging moment of 1.2(D+L+W) = 960kNm > 1.4(D+W) = 910kNm). So the “pull” force is

$T_{Y1} = f_y A_s = 500 \times 3770 \times 10^{-3} = 1885\,\text{kN}$

The provided bottom reinforcement of Beam Y2 is 2T32+1T25, giving 2099mm^2. So the “pull” force is $T_{Y2} = f_y A_s = 500 \times 2099 \times 10^{-3} = 1050\,\text{kN}$

So $V_{jy} = 1885 + 1050 = 2935\,\text{kN}$

Check structural adequacy in the Y-direction $h_c = 800\,\text{mm}$, $b_c = 900\,\text{mm}$ and $b_w + 0.5h_c = 500 + 0.5 \times 800 = 900\,\text{mm}$, so $b_j = 900\,\text{mm}$

So, checking against Cl. 6.8.1.3 of the Code,

$$v_{jy} = \frac{V_{jy}}{b_j h_c} = \frac{2935000}{900 \times 800} = 4.08\,\text{MPa} < 0.2 f_{cu} = 9\,\text{MPa}$$

(3) Design Shear Reinforcement in X-Direction

To calculate the horizontal shear reinforcement by Ceqn 6.72, reading

$$A_{jh} = \frac{V_{jh}}{0.87 f_{yh}}\left(0.5 - \frac{C_j N}{0.8 A_g f_{cu}}\right)$$

where $C_j = \dfrac{V_{jx}}{V_{jx} + V_{jy}} = \dfrac{1629}{1629 + 2935} = 0.357$

$$A_{jhx} = \frac{V_{jx}}{0.87 f_{yh}}\left(0.5 - \frac{C_j N}{0.8 A_g f_{cu}}\right) = \frac{1629 \times 10^3}{0.87 \times 500}\left(0.5 - \frac{0.357 \times 9000 \times 10^3}{0.8 \times 900 \times 800 \times 45}\right)$$

= 1408mm^2

To calculate the vertical shear reinforcement by (Ceqn 6.73 of the Code), reading $A_{jv} = \dfrac{0.4(h_b / h_c)V_{jh} - C_j N}{0.87 f_{yv}}$

$$A_{jvx} = \frac{0.4(h_b / h_c)V_{jx} - C_j N}{0.87 f_{yv}} = \frac{0.4(700/900) \times 1629 \times 10^3 - 0.357 \times 9000 \times 10^3}{0.87 \times 500}$$

$= -6221\text{mm}^2$.

No vertical reinforcement required.

(4) Design Shear Reinforcement in Y-Direction

$$A_{jhy} = \frac{V_{jy}}{0.87 f_{yh}}\left(0.5 - \frac{C_j N}{0.8 A_g f_{cu}}\right) = \frac{2935\times10^3}{0.87\times500}\left(0.5 - \frac{0.643\times9000\times10^3}{0.8\times900\times800\times45}\right)$$

$= 1867\text{mm}^2$

$$A_{jvy} = \frac{0.4(h_b/h_c)V_{jy} - C_j N}{0.87 f_{yv}} = \frac{0.4(700/800)2935\times10^3 - 0.643\times9000\times10^3}{0.87\times500}$$

$= -10942\text{mm}^2$.

No vertical reinforcement required.

The shear reinforcements are shown in Figure 6.9.

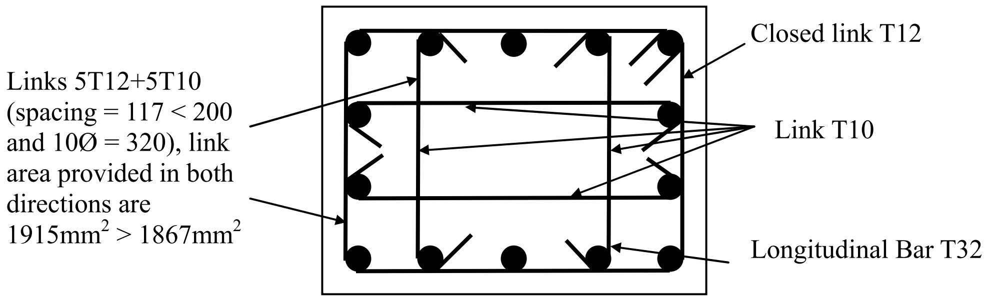


Figure 6.9 – Required Horizontal Shear Links of Beam-Column Joint (Plan) for Design Worked Example 6.2 – Other details Omitted for Clarity

7.0 Walls

7.1 Design Generally

7.1.1 The structural design of wall is similar to that of column which is designed to resist axial loads and moments (unaxial or biaxial). Detailed derivations and design charts for design of wall as an integral unit are enclosed in Appendix F.

7.1.2 The design ultimate axial force may be calculated on the assumption that the beams and slabs transmitting force to it are simply supported. (Re Cl. 6.2.2.2(a) and Cl. 6.2.2.3(a) of the Code).

7.1.3 Minimum eccentricity for transverse moment design is the lesser of 20 mm or $h/20$, where h is the wall thickness as similar to columns. (Re Cl. 6.2.2.2(c)).

7.1.4 Maximum and minimum ratios for vertical reinforcement are 4% and 0.4% (Re Cl. 9.6.2) respectively. The maximum percentage can be doubled at lap.

7.2 Categorization of Walls

Concrete walls can be categorized into "slender walls" and "stocky walls" according to their slenderness and "reinforced concrete walls" and "plain walls" according to the reinforcement contents.

7.3 Slender Wall Section Design

7.3.1 Determination of effective height l_e (of minor axis generally which controls) (Re Cl. 6.2.2.2(b)) –
(i) in case of monolithic construction, same as that for column; and
(ii) in case of simply supported construction, same as that for plain wall.

7.3.2 Limits of slenderness ratio l_e/h (Re Table 6.15 of the Code) –
(i) 40 for braced wall with reinforcements < 1%;
(ii) 45 for braced wall with reinforcements ≥ 1%;
(iii) 30 for unbraced wall.

7.3.3 Other than 7.3.1 and 7.3.2, reinforced concrete design is similar to that of columns, including the allowance of M_{add}.

7.4 Stocky Wall

7.4.1 Stocky walls are walls with slenderness ratio ≤ 15 for braced walls and slenderness ratio ≤ 10 for unbraced walls (Re Cl. 1.4.4).

7.4.2 Stocky reinforced wall may be designed for axial load n_w only by (Ceqn 6.59) of the Code provided that the walls support approximately symmetrical arrangement of slabs with uniformly distributed loads and the spans on either side do not differ by more than 15% and $n_w \le 0.35 f_{cu} A_c + 0.67 f_y A_{sc}$

7.4.3 Other than 7.4.2 and the design for deflection induced moment M_{add}, design of stocky wall is similar to slender walls.

7.5 Reinforced Concrete Walls design is similar to that of columns with categorization into slender walls and stocky walls.

7.6 Plain Wall – Plain wall are walls the design of which is without consideration of the presence of the reinforcements.

7.6.1 Effective height of unbraced plain wall, where l_0 is the clear height of the wall between support, is determined by (Re Cl. 6.2.2.3(b)):

(a) $l_e = 1.5l_0$ when it is supporting a floor slab spanning at right angles to it;

(b) $l_e = 2.0l_0$ for other cases.

Effective height ratio for braced plain wall is determined by (Re Cl. 6.2.2.3(c)):

(a) $l_e = 0.75l_0$ when the two end supports restraint movements and rotations;

(b) $l_e = 2.0l_0$ when one end support restraint movements and rotations and the other is free;

(c) $l_e = l_0'$ when the two end supports restraint movements only;

(d) $l_e = 2.5l_0'$ when one end support restraint movements only and the other is free; where l_0' in (c) is the height between centres of supports and l_0' in (d) is the height between the centre of the lower support and the upper free end.

7.6.2 For detailed design criteria including check for concentrated load, shear, load carrying capacities etc, refer to Cl. 6.2.2.3 of the Code.

7.7 Sectional Design

The sectional design of wall section is similar to that of column by utilizing stress strain relationship of concrete and steel as indicated in Figure 3.8 and 3.9 of the Code. Alternatively, the simplified stress block of concrete as indicated in Figure 6.1 can also be used. Nevertheless, the Code has additional requirements in case both in-plane and transverse moments are "significant" and such requirements are not identical for stocky wall and slender wall.

7.7.1 Wall with Axial Load and In-plane Moment

Conventionally, walls with uniformly distributed reinforcements along its length can be treated as if the steel bars on each side of the centroidal axis are lumped into 2 pair of bars each pair carrying half of the steel areas as shown in Figure 7.1 and design can then be carried out as if it is a 4 bar column.

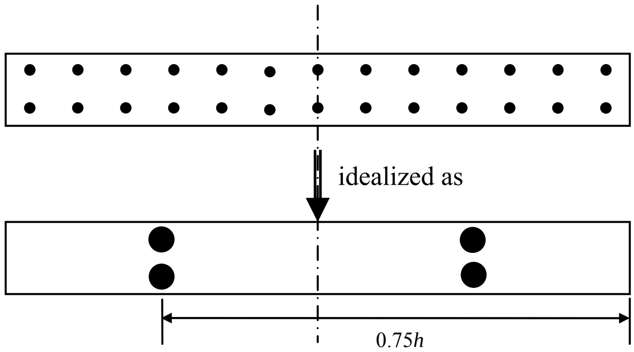


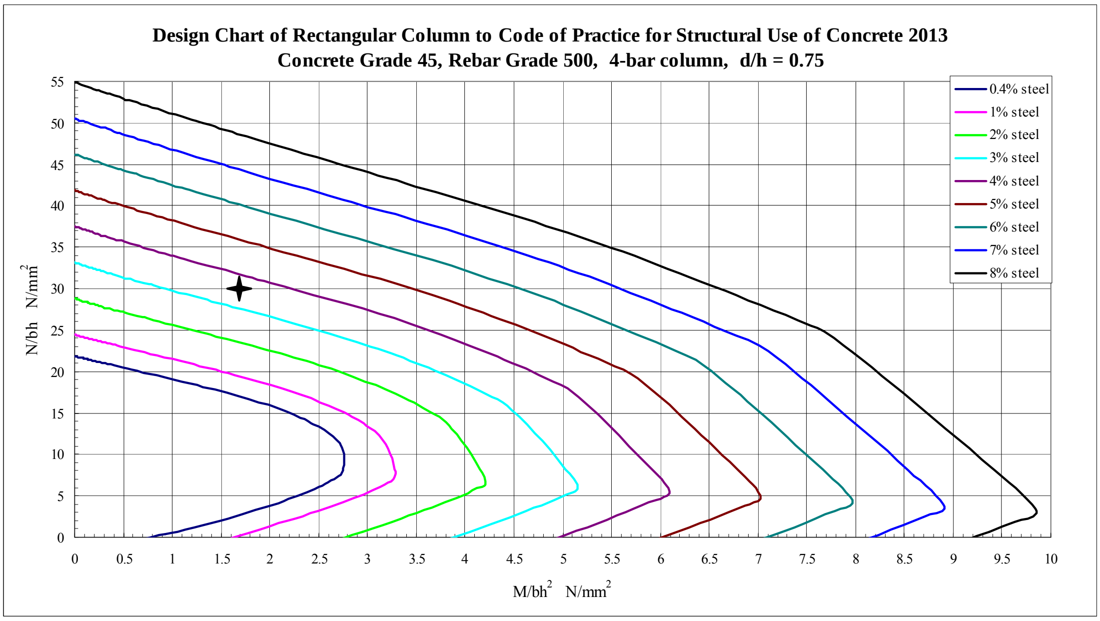
Figure 7.1 – Idealization of Wall into 4-Bar Column for Design

Worked Example 7.1

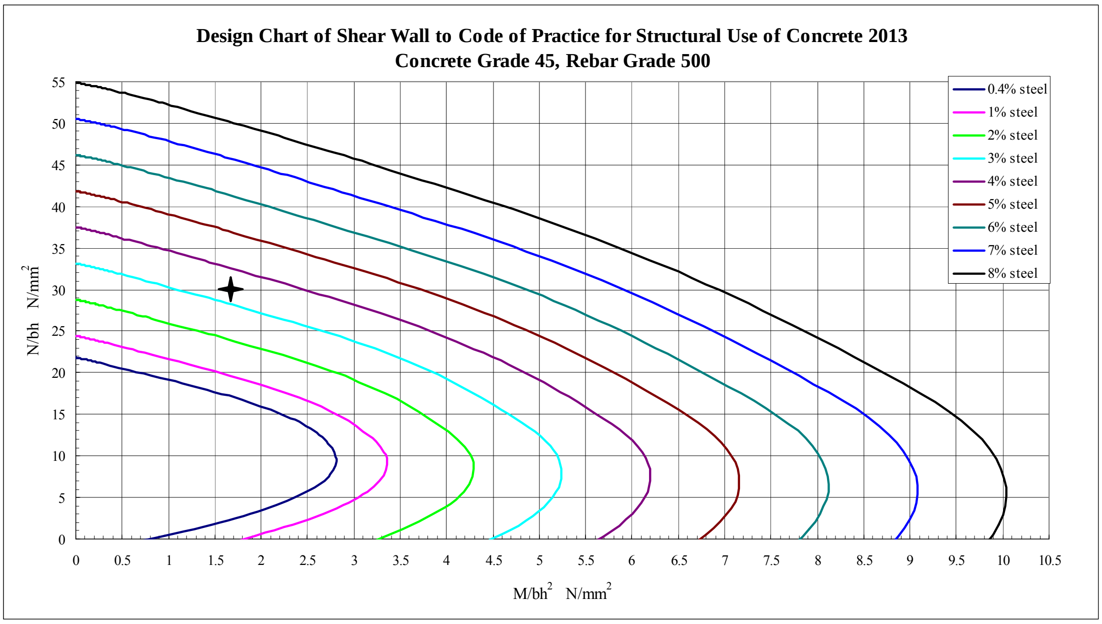
Consider a wall of thickness 300 mm, plan length 3000 mm and under an axial load $P = 27000$ kN and in-plane control design moment $M_x = 4500$ kNm (after consideration of the minor axis moment which does not control). Concrete grade is C45. The problem is a uniaxial bending problem. Then

$\frac{P}{bh} = \frac{27000\times10^3}{300\times3000} = 30$ and $\frac{M}{bh^2} = \frac{4500\times10^6}{300\times3000^2} = 1.67$.

If based on the 4-bar column chart with $d/h = 0.75$, $p = 3.5\%$, requiring 31500mm^2 or T32 – 152 (B.F.)



If use chart based on evenly distributed bars, the reinforcement ratio can be slightly reduced to 3.4% as follows



By superimposing the two design charts as in Figure 7.2, it can be seen that the idealization of steel re-bars as continuum is generally more conservative.

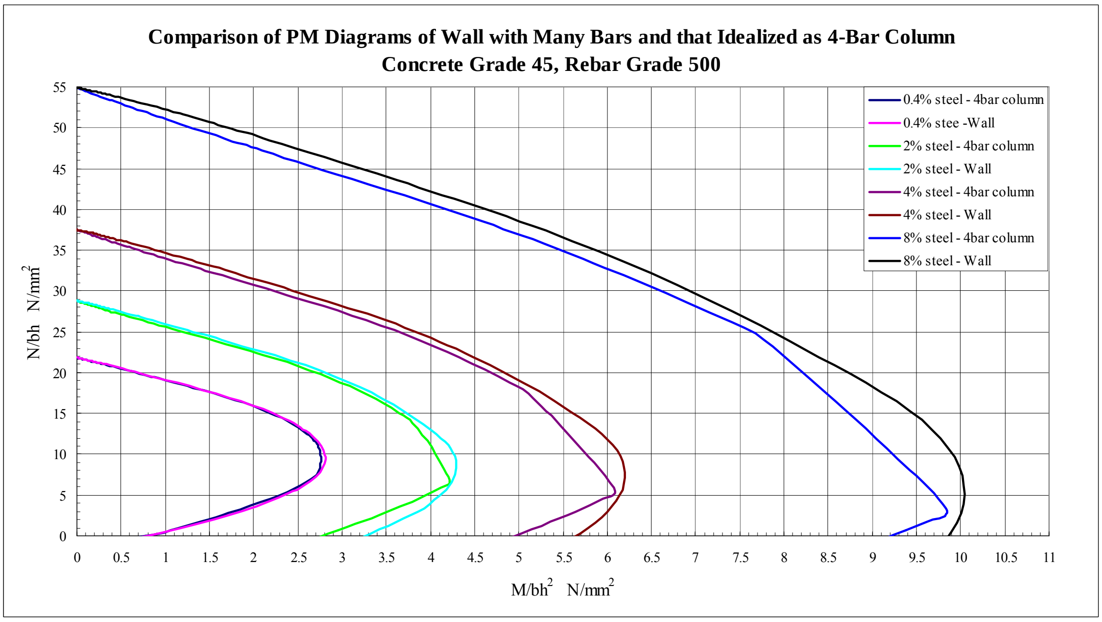


Figure 7.2 – Comparison of design curve between idealization of steel bars as 4 bar column and continuum

7.7.2 Wall with Axial Load and Transverse Moment

The design will also be similar to that of column with the two layers of longitudinal bars represented by the bars in the 4-bar column charts as shown in Figure 7.3.

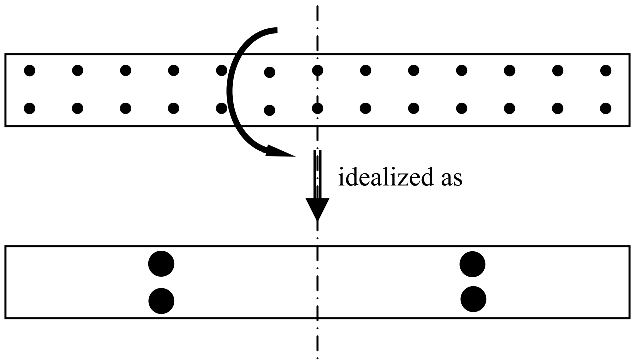


Figure 7.3 – Sectional Design for Wall with Axial Load and Transverse Moment

7.7.3 Wall with Significant In-plane and Transverse Moments

The Code has not defined the extent of the moment being "significant". Nevertheless, if significant in-plane and transverse moments exist, the Code effectively requires the wall section be examined at various points (for stocky wall) and unit lengths (for slender wall) along the length of the wall at the splitting up of the axial load and in-plane moment by elastic analysis as per Cl. 6.2.2.2(f)(iv)(1) and Cl. 6.2.2.2 (g)(i)(1) which are demonstrated in Figures 7.4(a) and (b). The checking of the "points" and "unit lengths" should follow the stress strain relationship of concrete and steel which are identical to that of column, taking into account of the transverse moments.

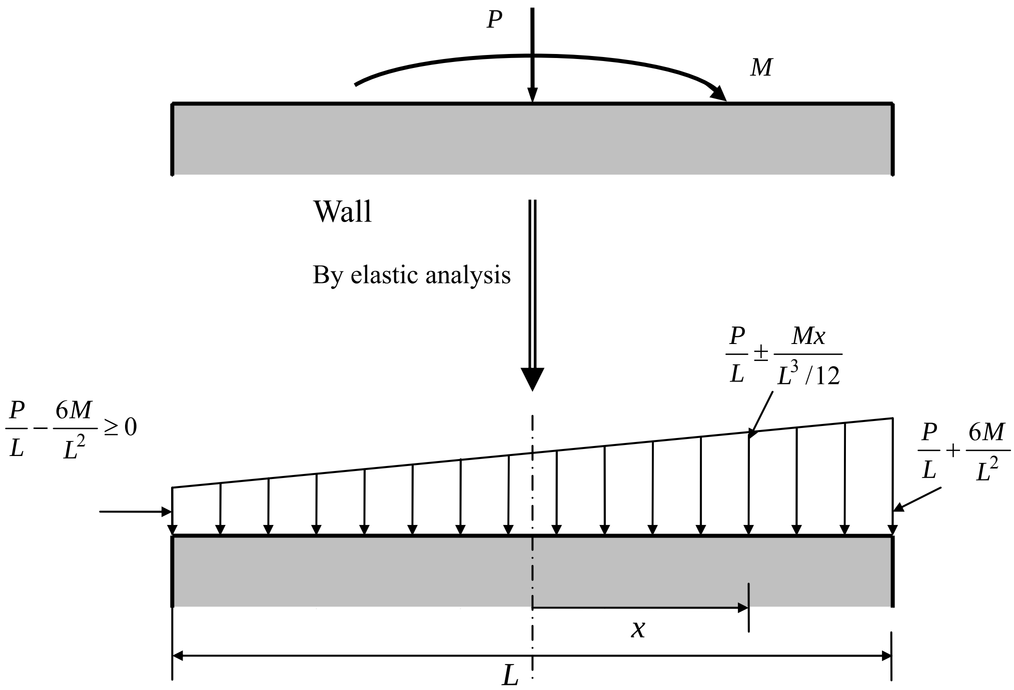


Figure 7.4(a) – Conversion of Axial load (kN) and In-plane Moment (kNm) into Linear Varying Load (kN/m) along Wall Section under No Tension

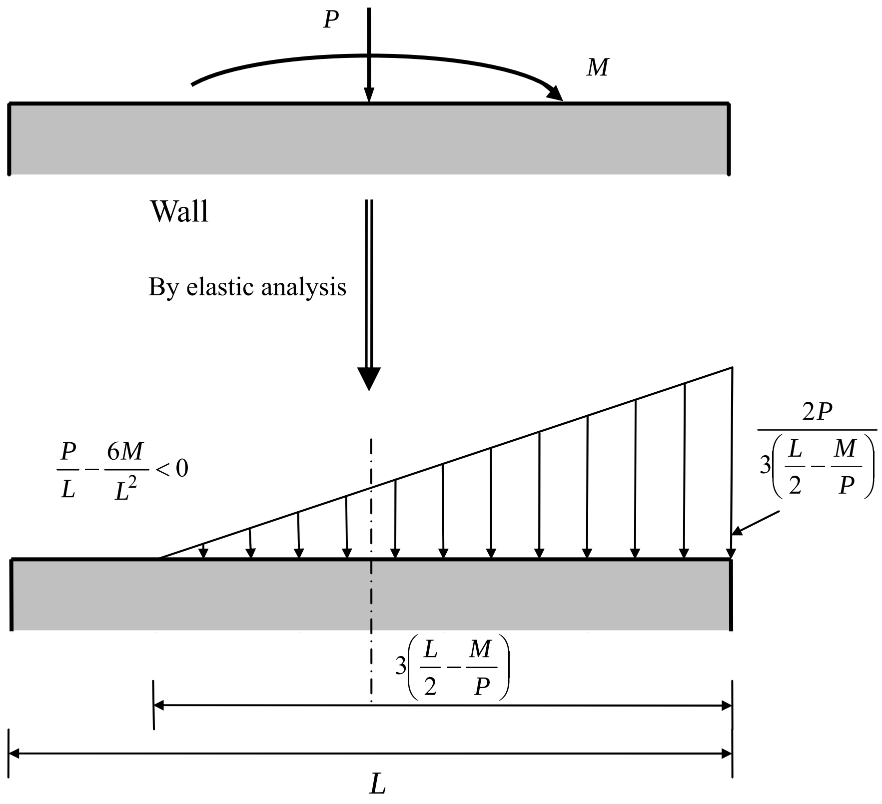


Figure 7.4(b) – Conversion of Axial load (kN) and In-plane Moment (kNm) into Linear Varying Load (kN/m) along Wall Section with Tension in Section

Worked Example 7.2 (Stocky Wall)

Consider a wall of thickness 300 mm, plan length 3000 mm and under an axial load $P = 27000$ kN and in-plane moment $M_x = 4500$ kNm and transverse moment $M_y = 300$ kNm uniformly along its length. Concrete grade is C45. By elastic analysis, the load intensities at the 4 points resolved are :

A : $\frac{27000}{3}+\frac{6\times 4500}{3^2}=12000$ kN/m;

B : $\frac{27000}{3}+\frac{4500\times 0.5}{3^3/12}=10000$ kN/m;

C : $\frac{27000}{3}-\frac{4500\times 0.5}{3^3/12}=8000$ kN/m;

D : $\frac{27000}{3}-\frac{6\times 4500}{3^2}=6000$ kN/m

The varying load intensities are as indicated in Figure 7.5.

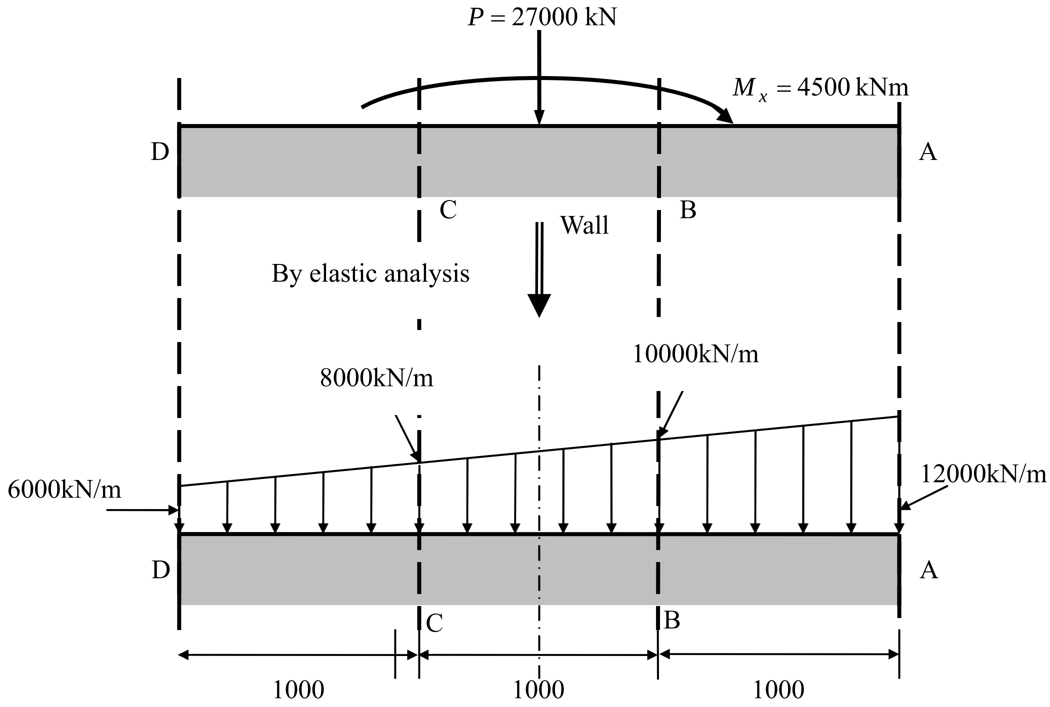
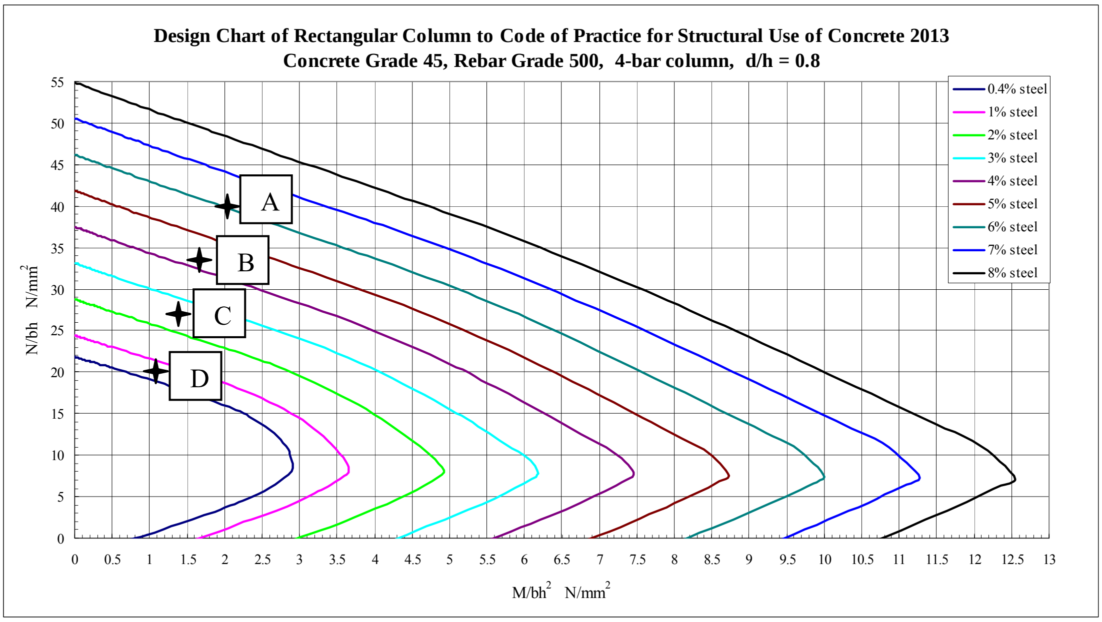


Figure 7.5 – Conversion of axial load (kN) and in-plane moment (kNm) into linear varying load (kN/m) along wall section for Worked Example 7.2

(i) If the wall is considered stocky, each of the points with load intensities as determined shall be designed for the load intensities as derived from the elastic analysis and a transverse moment of $300 \div 3 = 100$ kNm/m by Clause 6.2.2.2(f)(iv) of the Code. However, to fulfill the requirement of minimum eccentricity in the transverse direction of $h/20 = 15$ mm, the eccentric moments so created will be used for design if in excess of 100kN/m. Consider one metre length for each point, the 4 points shall be designed for the following loads with section 1000 mm by 300 mm as tabulated in Table 7.1, i.e. all the points are undergoing uniaxial bending (transverse) and the sectional design are done in the same Table 7.1 in accordance with the chart extracted from Appendix F, *d*/*h* = 0.8:

Point	A	B	C	D
Axial Load	12000	10000	8000	6000
In-plane Mt	0	0	0	0
Transverse Mt	180	150	120	100
N/bh	40	33.33	26.67	20
M/bh^2	2.00	1.67	1.33	1.11
p (%)	6.0	4.2	2.5	0.6
Re-bars (BF)	T40 – 140	T40 – 200	T32 – 214	T20 – 350

Table 7.1 – Summary of Design for Worked Example 7.2 as a Stocky Wall



The Code is not clear in the assignment of reinforcements at various segments of the section based on reinforcements worked out at various points. The assignment can be based on the tributary length principle, i.e. the reinforcement derived from A shall be extended from A to mid-way between A and B; the reinforcement derived from B be extended from mid-way between A and B to mid-way between B and C etc. As such, the average reinforcement ratio is 3.33%. Nevertheless, as a more conservative approach, the assignment of reinforcement design between A and B should be based on A and that of B and C be based on B etc. As such the reinforcement ratio of the whole section will be increased to 4.23% and the reinforcement ratio at D is not used. The exercise is for illustration only. The designer should bear in mind total steel percentage not to exceed 4%.

(ii) If the design approach of the wall follows Cl. 6.2.2.2(g)(i) of the Code as a slender wall, the wall should be divided into "unit lengths" with summing up of loads. Consider the three units AB, BC and CD. The loads and in-plane moments summed from the trapezoidal distribution of loads are as follows, with the transverse moments also dictated by the minimum eccentricity :

For Unit Length AB :

Summed axial load = $\dfrac{12000+10000}{2}\times 1 = 11000\,\text{kN}$

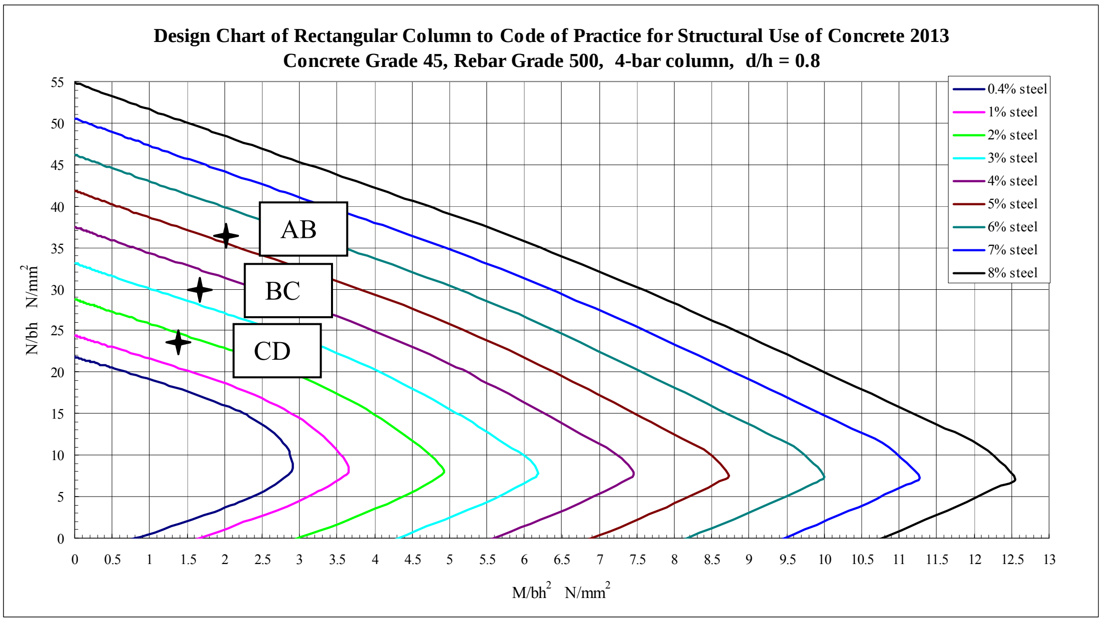
Summed in-plane moment $\dfrac{12000-10000}{2}\times 1\times\left(\dfrac{2}{3}-\dfrac{1}{2}\right)\times 1 = 167\,\text{kNm}$.

The summed axial loads and moments on the unit lengths BC and CD are similarly determined and design is summarized in Table 7.2, with reference to the design chart extracted from Appendix E. The transverse moments are again governed by the minimum eccentricity. In the computation of M_x/h' and M_y/b', h' and b' are taken as

750mm and 225mm respectively.

Unit Length	AB	BC	CD
Axial Load	11000	9000	7000
In-plane Mt (M_x)	167	167	167
Transverse Mt (M_y)	165	135	105
M_x / h' ($h' = 750$)	0.223	0.223	0.223
M_y / b' ($b' = 225$)	0.733	0.6	0.467
$N / f_{cu}bh$	0.815	0.667	0.519
β	0.3	0.3	0.397
$M_y' = M_y + \beta(b'/h')M_x$	180.03	150.03	124.89
N / bh	36.67	30	23.33
M_y' / hb^2	2.00	1.67	1.39
p (%)	5.2	3.4	1.8
Re-bars (BF)	T40 – 161	T32 – 158	T25 – 182

Table 7.2 – Summary of Design for Worked Example 7.2 as a Slender Wall



The average steel percentage is 3.47%. So the reinforcement worked out by Clause 6.2.2.2(g)(i) of the Code by the slender wall approach is between the results of the two methods of reinforcement ratios assignments as described in sub-section (i) based on Clause 6.2.2.2(f)(iv) of the Code.

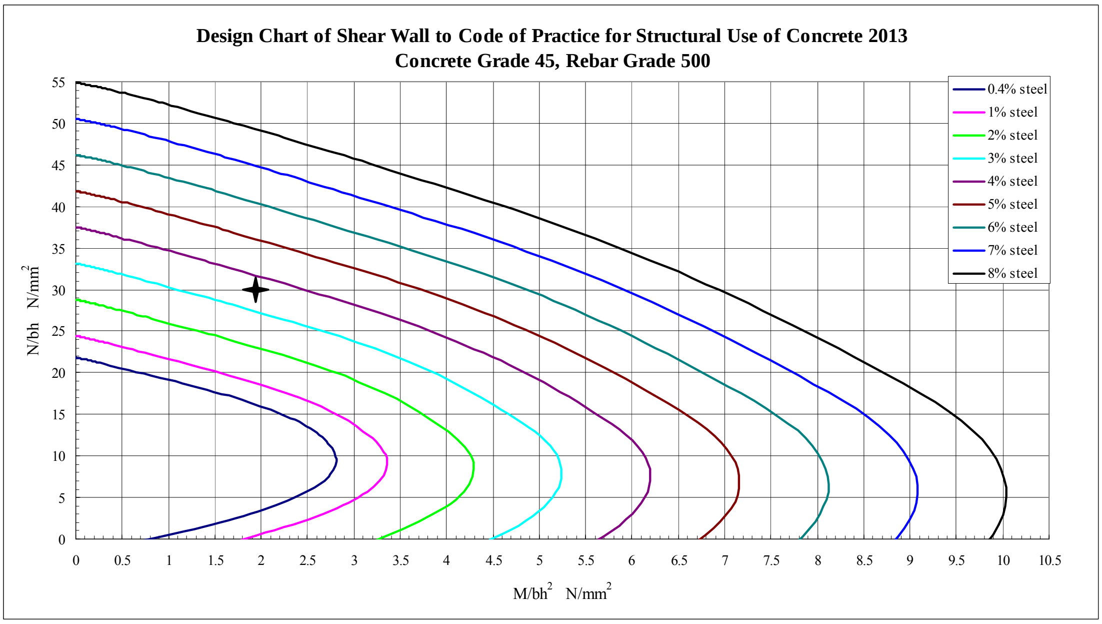
(iii) Nevertheless, if the wall is designed as an integral section, i.e. $N / bh = 27000 \times 10^3 / (300 \times 3000) = 30$; $N / f_{cu}bh = 0.67$; $\beta = 0.3$
$M_x / h' = 4500 \times 10^6 / 3000 = 1500000$ ($h' = h$ as the formulation of the wall design chart is based on the full length of wall)

$$M_y / b' = 405 \times 10^6 / 225 = 1800000 \qquad (Ne_{\min} = 405)$$

$$M_y' = M_y + \beta(b'/h')M_x = 405 + 0.3 \times (225/3000) \times 4500 = 506.25$$

$$M_y'/b^2h = 1.875$$

From the design chart extracted from Appendix F, the reinforcement percentage estimated is 3.6% or T32 – 149 (BF)



(iv) Summary of the reinforcements design of the four approaches are in Figure 7.6

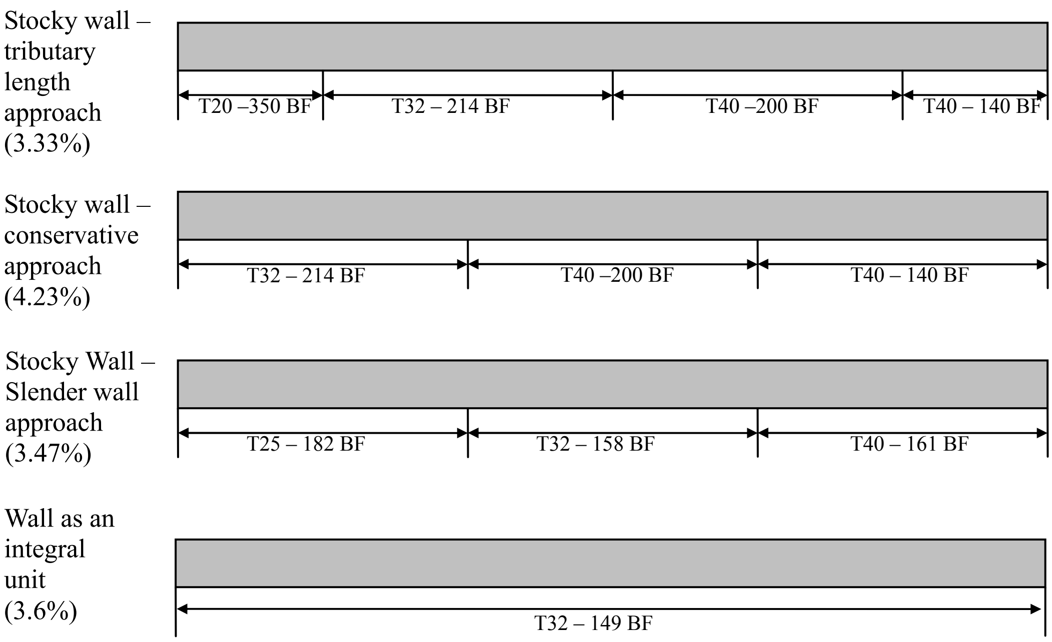


Figure 7.6 – Summary of reinforcement details of Worked Example 7.2

(v) So the approach recommended in the Code appears to be reasonable and probably economical as higher reinforcement ratios will be in region of high stresses. However, it should be noted that if moment arises from wind loads where the direction can reverse, design for the reversed direction may result in almost same provisions of reinforcements at the other end.

As the division of segments or points as recommended by the Code for design of wall with significant transverse and in-plane moments is due to the inaccurate account by the biaxial bending formula used for design of column, more accurate analysis can be done by true biaxial bending analysis as discussed in Section 5.3.5 and Figure 5.8 of this Manual, so long the "plane remain plane" assumption is valid, though the design can only be conveniently done by computer methods. The sections with reinforcement ratios arrived at in (i) and (ii) have been checked against by the software ADSEC, the section in (ii) has yielded an applied moment / moment capacity ratio of 0.8 showing there is room for slight economy. Nevertheless, the first reinforcement ratio in (i) is inadequate as checked by ADSEC whilst the second one yielded an over design with applied moment / moment capacity ratio up to 0.68.

7.8 The following Worked Example 7.3 serves to demonstrate the determination of design moment for a slender wall section, taking into account of additional moment due to slenderness.

Worked Example 7.3 – Braced Wall

Wall Section : thickness : 200 mm, plan length : 2000 mm;
Clear Wall Height : 3.6 m,
Concrete grade : C35
Connection conditions at both ends of the wall : connected monolithically with floor structures shallower than the wall thickness.

Check for slenderness

Generally only necessary about the minor axis.
End conditions are 2 for both ends, $\beta = 0.85$ (by Table 6.11 of the Code);
$l_e = 0.85 \times 3.6 = 3.06$ m
Taken $K = 1$ (Ceqn 6.49 of the Code) in the initial trial.

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 = \frac{1}{2000}\left(\frac{3060}{200}\right)^2 = 0.117$$

$$a_u = \beta_a K h = 0.117 \times 1 \times 0.2 = 0.0234$$

Axial Load : $N = 7200$ kN, $M_x = 1800$ kNm at top and 1200 kNm at bottom , $M_y = 25$ kNm at top and 24 kNm at bottom.
Determination of final design moment M_t about the major and minor axes is similar to that of column.

For bending about the major axis, $l_e / h = 3060/2000 = 1.53 < 15$, so not a slender wall, M_x will be the greater of

(1) $M_2 = 1800$;

(2) $N \times e_{\min} = 7200 \times 0.02 = 144$.

So $M_x = 1800$ kNm for design.

For bending about the minor axis, $l_e / b = 3060/200 = 15.3 > 15$, so slender.

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 = \frac{1}{2000}\left(\frac{3060}{200}\right)^2 = 0.117$$

$a_u = \beta_a K h = 0.117 \times 1 \times 0.2 = 0.0234$

$M_{add} = N a_u = 7200 \times 0.0234 = 168.48$ kNm,

$M_i = 0.4M_1 + 0.6M_2 = 0.4 \times (-24) + 0.6 \times 25 = 5.4 < 0.4M_2 = 0.4 \times 25 = 10$

So $M_i = 10$ kNm

$e_{\min} = 0.05 \times 200 = 10$ mm < 20mm

M_y will be the greatest of

(1) $M_2 = 25$ kNm;

(2) $M_i + M_{add} = 10 + 168.48 = 178.48$ kNm;

(3) $M_1 + M_{add}/2 = 24 + 168.48/2 = 108.3$ kNm; and

(4) $N \times e_{\min} = 7200 \times 0.01 = 72$ kNm. So $M_y = 178.48$ kNm for design.

So the factored axial load and moments for design are

$N = 7200$ kN; $M_x = 1800$ kNm; $M_y = 178.48$ kNm

Design can be performed in accordance with Cl. 6.2.2.2(g) of the Code as demonstrated in Worked Examples 7.2 and by calculations with the formulae derived in Appendices E and F. However, the calculations are too tedious and cases to try are too many without the use of computer methods. Spread sheets have been devised to solve the problem with a sample enclosed in Appendix F.

7.9 Design of Wall against Shear

Like the British Standard BS8110, the Code has no provisions for design against shear. However, as shear is also an important design parameter which sometimes cannot be disregarded, suggested methods based on analytical approach and that modified from the ACI 318-11 are enclosed in Appendix G.

7.10 Detailing Requirements

Ductility requirements for walls contributing to the lateral load resisting system, the Code has also ductility requirements in Cl. 9.9.3. The requirements comprise restriction of the "axial compression ratio" (N_{cr}) to 0.75 by Ceqn 9.8, i.e. $N_{cr} = \dfrac{1.4G_k + 1.6Q_k}{0.45 f_{cu} A_c} \le 0.75$. The restriction serves to ensure certain level of ductility as, like column, a high axial stress will result in low ductility level as demonstrated in Figure 7.7. From the figure, it is obvious that the curves of variation moment of resistance of the wall with curvature for high axial stresses are of "peak plateau" shape, indicating low ductility levels. In the figure, $f_{co} = 0.72f_{cu}$.

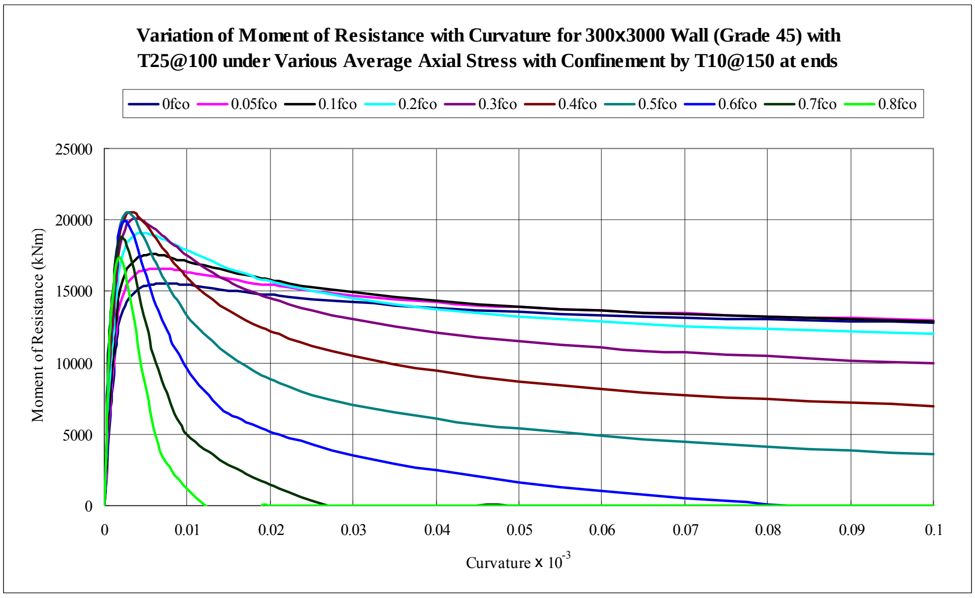


Figure 7.7 – Ductility Behaviour of Wall under Various Axial Stress Levels

To ensure high ductility levels (and also strengths), the Code also requires the end regions of the wall (called "confined boundary elements") to have higher longitudinal and transverse steel levels. The transverse steel serves to effect higher confining stress to the concrete which increases the concrete strength (Mander et al. 1988) and subsequently ductility of the wall. Similar to column and beam, critical zones are also defined at the base of the wall, the N_{cr} within which determines the required extent of different types of confined boundary elements as summarized in Figure 7.8(a) and (b).

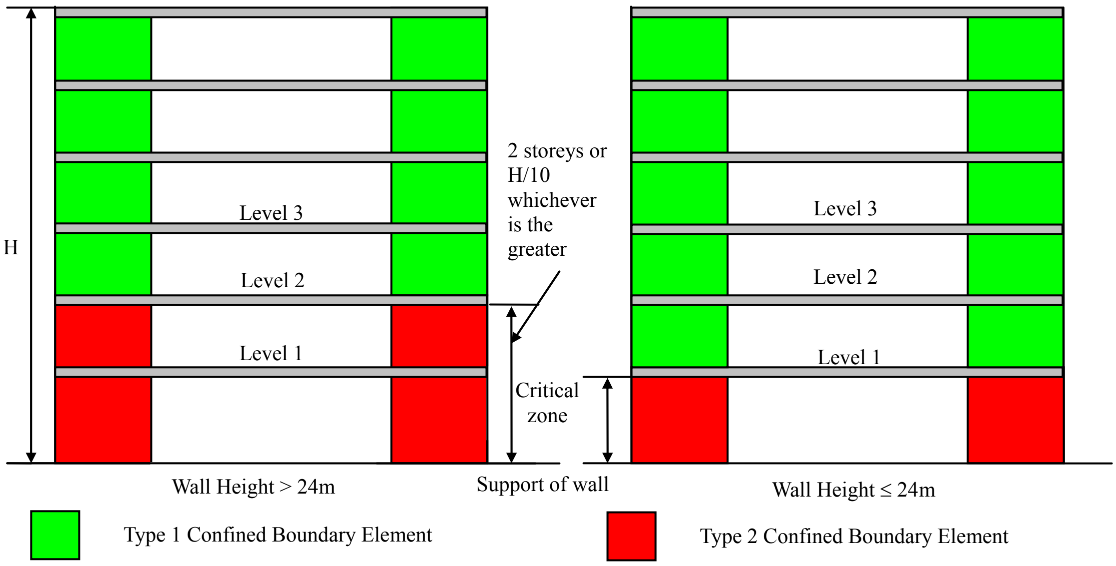


Figure 7.8(a) – Layout of Critical Zone and Confined Boundary Element for $0 < N_{cr} \leq 0.38$ within Critical Zone

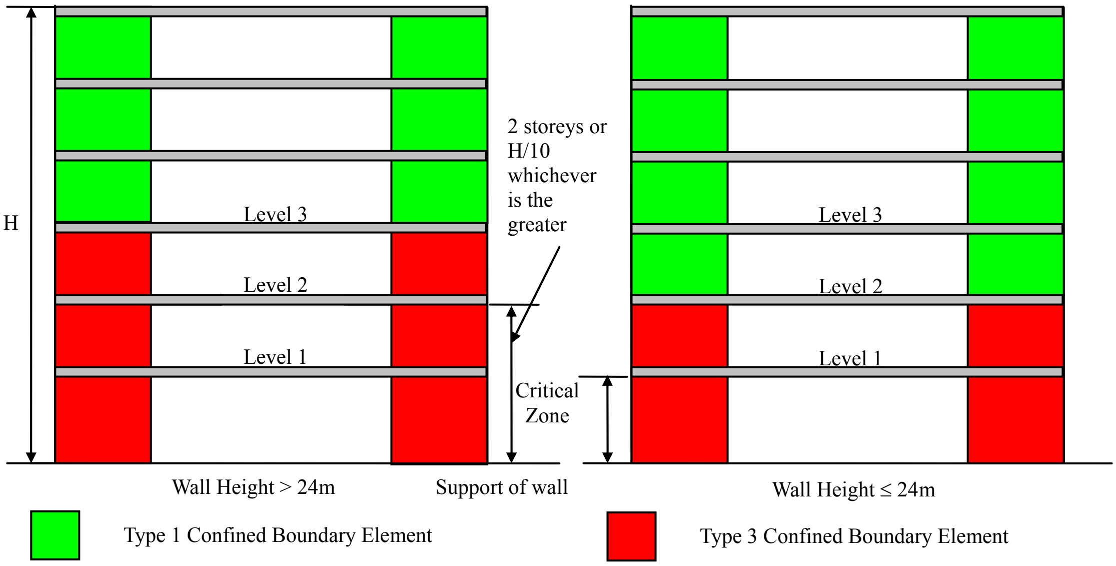
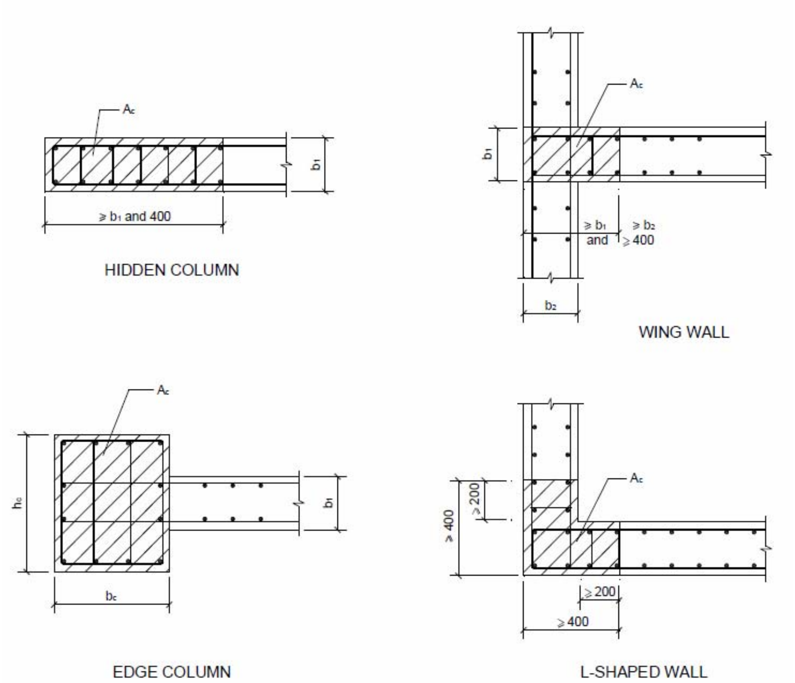


Figure 7.8(b) – Layout of Critical Zone and Confined Boundary Element for $0.38 < N_{cr} \leq 0.75$ within Critical Zone

7.10.1 <u>In Confined Boundary Elements – For Walls Contributing to the Lateral Load Resisting System</u>

Minimum requirements of longitudinal and transverse reinforcements for the three types of confined boundary elements are shown in Figure 9.11 of the Code extracted as follows :

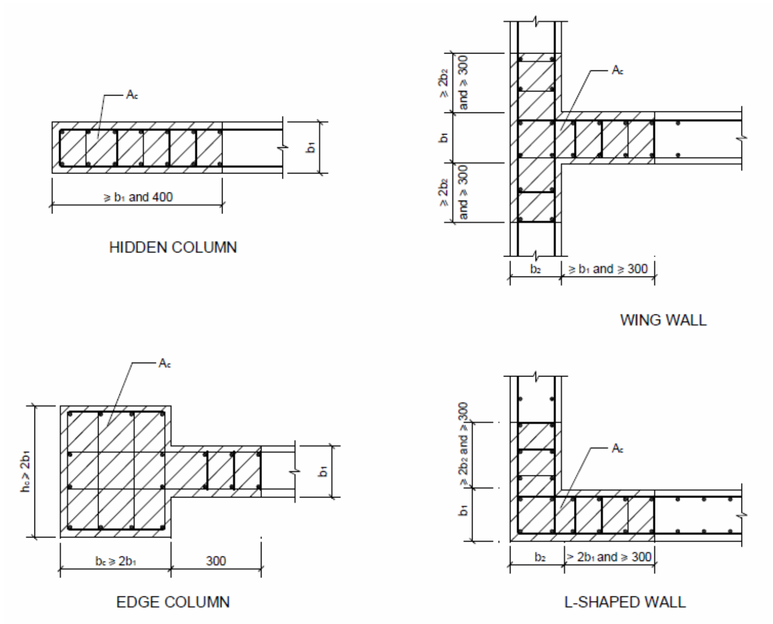


Reinforcements in Types 1 and 2 Confined Boundary Elements – Shaded regions:

<u>Longitudinal</u>:
(i) Rebar percentage ≥ 0.6% for Type 1 and ≥ 0.8% for Type 2;
(ii) Nos. of bars ≥ 6;
(iii) Dia. of bars ≥ 12mm for Type 1 and ≥ 16mm for Type 2;
(iv) All bars tied by links

<u>Transverse</u>
(i) Dia. of bars ≥ 10mm;
(ii) Vertical spacing ≤ 250mm for Type 1 and 200mm for Type 2

Figure 7.9(a) – Reinforcement Requirements of Types 1 and 2 Confined Boundary Elements in Walls



Reinforcements in Type 3 Confining Boundary Elements – Shaded regions:

<u>Longitudinal</u>:
(i) Rebar percentage ≥ 1%
(ii) Nos. of bars ≥ 6;
(iii) Dia. of bars ≥ 16mm;
(iv) Bar spacing ≤ 150mm
(v) All bars tied by links

<u>Transverse</u>
(i) Dia. of bars ≥ 12mm;
(ii) Vertical spacing ≤ 150mm

Figure 7.9(b) – Reinforcement Requirements of Type 3 Confined Boundary Elements in Walls

7.10.2 Locations other than Confined Boundary Elements

The detailing requirements in locations other than the confined boundary elements are summarized from Cl. 9.6 of the Code summarized as follows :

Vertical reinforcements for reinforced concrete walls

(i) Minimum steel percentage : 0.4%. When this reinforcement controls the design, half of the steel area be on each side;
(ii) Maximum steel percentage : 4% which can be doubled at laps;
(iii) All vertical compression reinforcements should be enclosed by a link as shown in Figure 7.12 if vertical reinforcements > 2%;
(iv) Maximum distance between bars : the lesser of 3 times the wall thickness or 400 mm as shown in Figure 7.10.

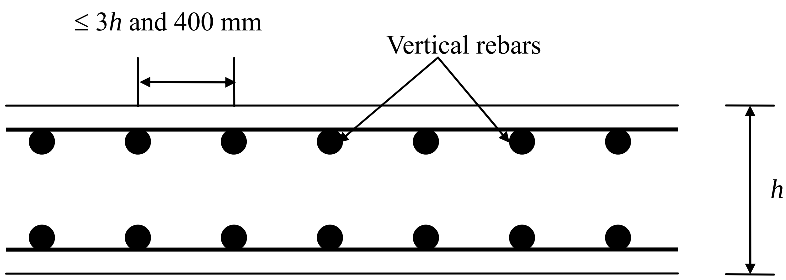


Figure 7.10 – Vertical Reinforcements of Walls for Locations of Walls other than Confined Boundary Elements

Horizontal and transverse reinforcements for reinforced concrete walls

(i) If the required vertical compression reinforcement does not exceed 2%, horizontal reinforcements be provided as follows and in accordance with Figure 7.11 :

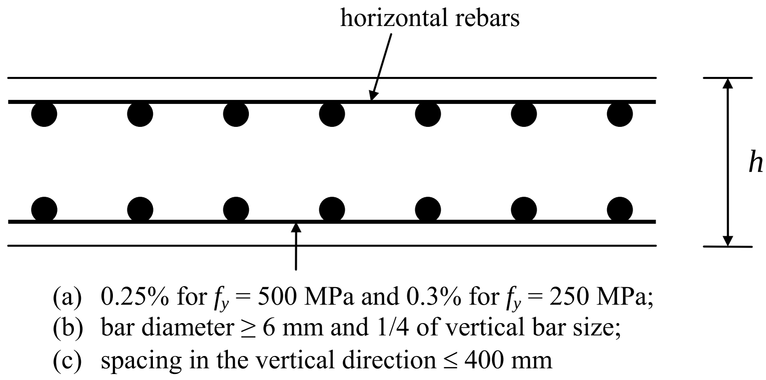


Figure 7.11 – Horizontal Reinforcements of Walls in Locations other than Confined Boundary Element with Longitudinal Reinforcement ≤ 2%

(ii) If the required vertical compression reinforcement > 2%, links be provided as follows as shown in Figure 7.12 :
   (a) to enclose every vertical compression longitudinal bar;
   (b) no bar be at a distance further than 200 mm from a restrained bar at which a link passes round at included angle ≤ 90°;
   (c) minimum diameter : the greater of 6 mm and 1/4 of the largest compression bar;
   (d) maximum spacing : twice the wall thickness in both the horizontal and vertical directions. In addition, maximum spacing not to exceed 16 times the vertical bar diameter in the vertical direction.

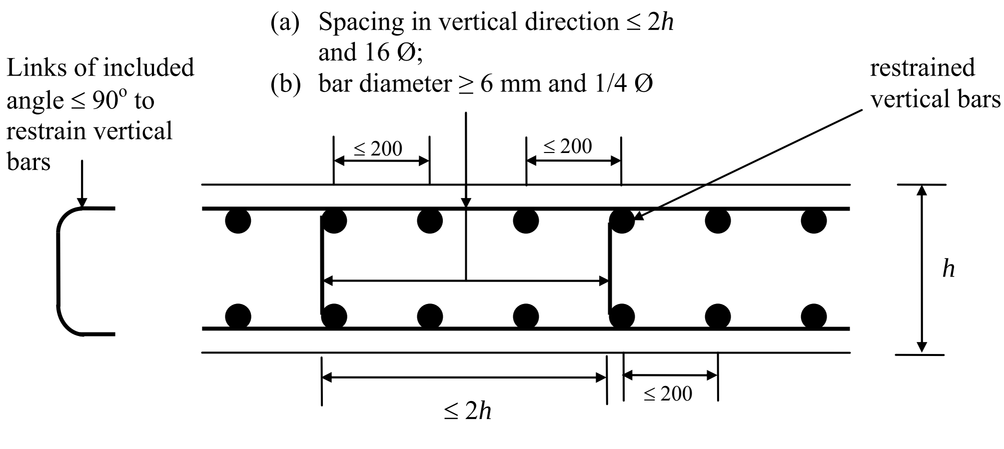


Figure 7.12 – Anchorage by links on Longitudinal Reinforcements of more than 2% in Walls in Locations other than Confined Boundary Elements

8.0 Corbels

8.1 General – A corbel is a short cantilever projection supporting a load-bearing member with dimensions as shown :

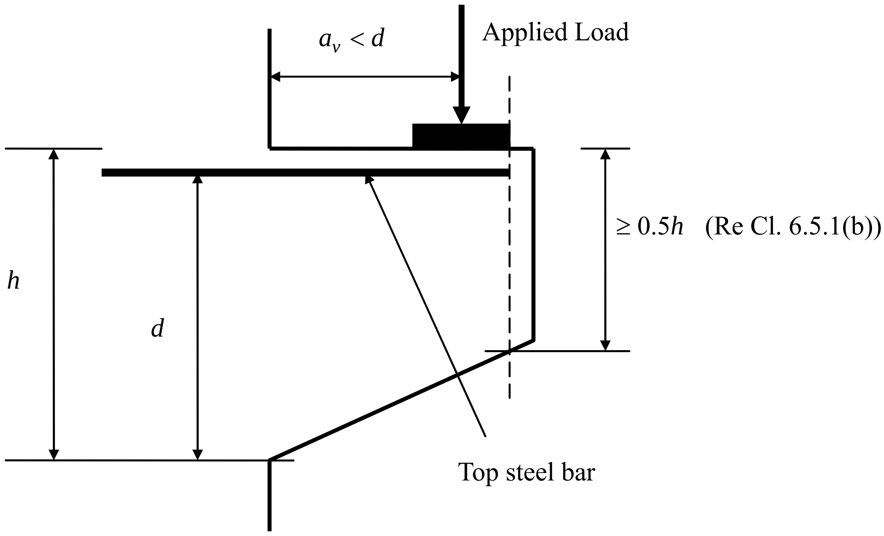


Figure 8.1 – Dimension Requirement for a Corbel

8.2 Basis of Design (Cl. 6.5.2 of the Code)

8.2.1 According to Cl. 6.5.2.1 of the Code, the basis of design method of a corbel is that it behaves as a "Strut-and-Tie" model as illustrated in Figure 8.2. The strut action (compressive) is carried out by concrete and the tensile force at top is carried by the top steel.

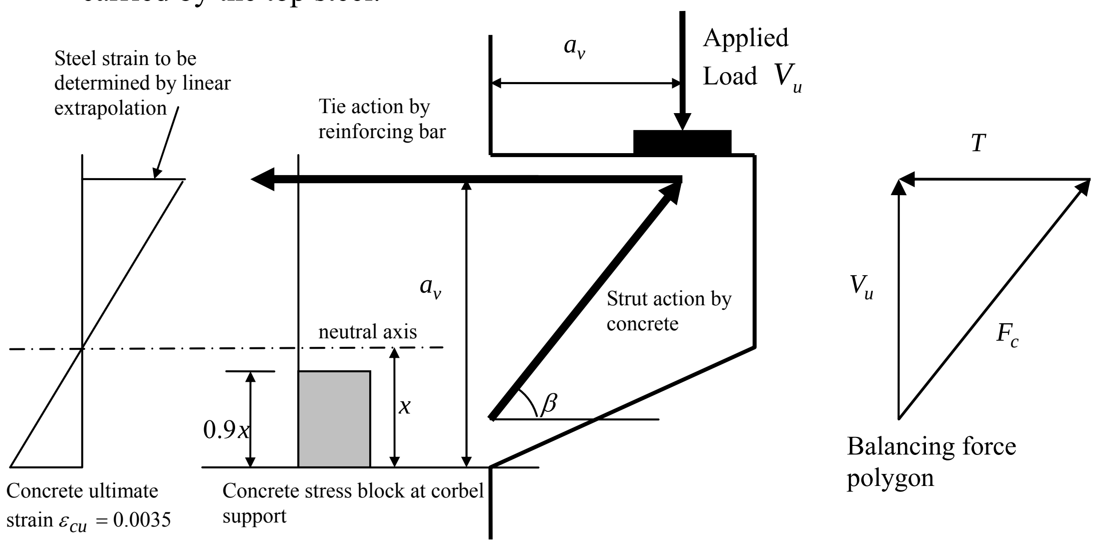


Figure 8.2 – Strut-and-Tie Action of a Corbel

8.2.2 As per Cl. 6.5.2.1(a), magnitude of resistance provided to the horizontal force should be not less than one half of the design vertical load, even the value of the angle β is large in Figure 8.2 or in turn, that the value of a_v is small.

8.2.3 Strain compatibility be ensured (Re Cl. 6.5.2.1(b) of the Code) by which steel stress may not reach $0.87 f_y$.

8.2.4 In addition to the strut-and tie model for the determination of the top steel bars, shear reinforcements should be provided in form of horizontal links in the upper two thirds of the effective depth of the corbel. The horizontal links should not be less than one half of the steel area of the top steel. (Re Cl. 6.5.2.3 of the Code).

8.2.5 Bearing pressure from the bearing pad on the corbel should be checked and properly designed in accordance with "Code of Practice for Precast Concrete Construction 2003" Cl. 2.7.9. In short, the design ultimate bearing pressure to ultimate loads should not exceed

(i) $0.4 f_{cu}$ for dry bearing;

(ii) $0.6 f_{cu}$ for bedded bearing on concrete;

(iii) $0.8 f_{cu}$ for contact face of a steel bearing plate cast on the corbel with each of the bearing width and length not exceeding 40% of the width and length of the corbel.

The net bearing width is obtained by

$$\frac{ultimate\ \ load}{effective\ \ bearing\ \ length \times ultimate\ \ bearing\ \ stress}$$

The Precast Concrete Code 2003 (in Cl. 2.7.9.3 of the Precast Concrete Code) has specified that the effective bearing length of a bearing be the least of

(i) physical bearing length;

(ii) one half of the physical bearing length plus 100 mm;

(iii) 600 mm.

8.3 Design Formulae for the Upper Steel Tie

The capacity of concrete in providing lateral force as per Figure 8.2 is $0.45 f_{cu} \times b \times 0.9x = 0.405 f_{cu} bx$ where b is the length of the corbel

The force in the compressive strut is therefore $F_c = 0.405 f_{cu} bx \cos\beta$

By the force polygon, $F_c \sin\beta = V_u \Rightarrow 0.405 f_{cu} bx \sin\beta \cos\beta = V_u$

As $\tan\beta = \dfrac{d - 0.45x}{a_v}$; $\cos\beta = \dfrac{a_v}{\sqrt{a_v^2 + (d - 0.45x)^2}}$

$$\sin\beta = \frac{(d - 0.45x)}{\sqrt{a_v^2 + (d - 0.45x)^2}}$$

So $0.405 f_{cu} bx \dfrac{a_v (d - 0.45x)}{a_v^2 + (d - 0.45x)^2} = V_u \Rightarrow V_u = \dfrac{0.405 f_{cu} bx a_v (d - 0.45x)}{a_v^2 + (d - 0.45x)^2}$

Expanding and re-arranging

$$(0.2025V_u + 0.18225 f_{cu} b a_v) x^2 - 0.9d(V_u + 0.45 f_{cu} b a_v) x + V_u (a_v^2 + d^2) = 0$$

Putting $A = 0.2025V_u + 0.18225 f_{cu} b a_v$; $B = -0.9d(V_u + 0.45 f_{cu} b a_v)$

$C = V_u (a_v^2 + d^2)$

$$x = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \qquad \text{(Eqn 8-1)}$$

By the equilibrium of force, the top steel force is $T = V_u \cot\beta = \frac{V_u a_v}{d - 0.45x}$ (Eqn 8-2)

The strain at the steel level is, by extrapolation of the strain diagram in Figure 8.2 is $\varepsilon_s = \frac{d-x}{x}\varepsilon_{cu} = \frac{d-x}{x} \times 0.0035$ (Eqn 8-3)

8.4 Design Procedure (in accordance with Allen 1988):

(i) Based on the design ultimate load and a_v, estimate the size of the corbel and check that the estimated dimensions comply with Figure 8.1;
(ii) Check bearing pressures;
(iii) Solve the neutral axis depth x by the equation (Eqn 8-1).
(iv) By the assumption plane remains plane and that the linear strain at the base of the corbel is the ultimate strain of concrete $\varepsilon_{cu} = 0.0035$, work out the strain at the top steel level as ε_s;
(v) Obtain the steel stress as $\sigma_s = E_s \varepsilon_s$ where $E_s = 200 \times 10^6$ kPa. However, the stress should be limited to $0.87 f_y$ even $\varepsilon_s \geq 0.002175$;
(vi) Obtain the force in the top steel bar T by (Eqn 8-2)
(vii) Check that $T \geq 0.5V_u$;
(viii) Obtain the required steel area of the top steel bars A_{st} by $A_{st} = \frac{T}{\sigma_s}$
(ix) Check the shear stress by $v = \frac{V_u}{bd}$. If $v > v_c$ (after enhancement as applicable), provide shear reinforcements by $\frac{A_{sv}}{s_v} = \frac{b(v - v_c)}{0.87 f_y}$ over the upper $(2/3)d$ where A_{sv} is the cross sectional area of the link and s_v is the link spacing.
(x) Check that the total shear area provided which is $\frac{A_{sv}}{s_v}d$ is not less than half of the top steel area, i.e. $A_{sv} \times \frac{d}{s_v} \geq \frac{1}{2}A_{st}$ even if $v \leq v_c$.

8.5 Detailing Requirements

(i) By Cl. 6.5.2.2 of the Code, anchorage of the top reinforcing bar should either
 (a) be welded to a transverse bar of equivalent strength and diameter. The bearing area of the load should stop short of the transverse bar

by a distance equal to the cover of the tie reinforcement as shown in Figure 8.3(a); or

(b) bent back to form a closed loop. The bearing area of the load should not project beyond the straight portion of the bars forming the tension reinforcements as shown in Figure 8.3(b).

(ii) By Cl. 6.5.2.3 of the Code, shear reinforcements be provided in the upper two thirds of the effective depth and total area not less than half of the top bars as shown in Figure 8.3(a) and 8.3(b).

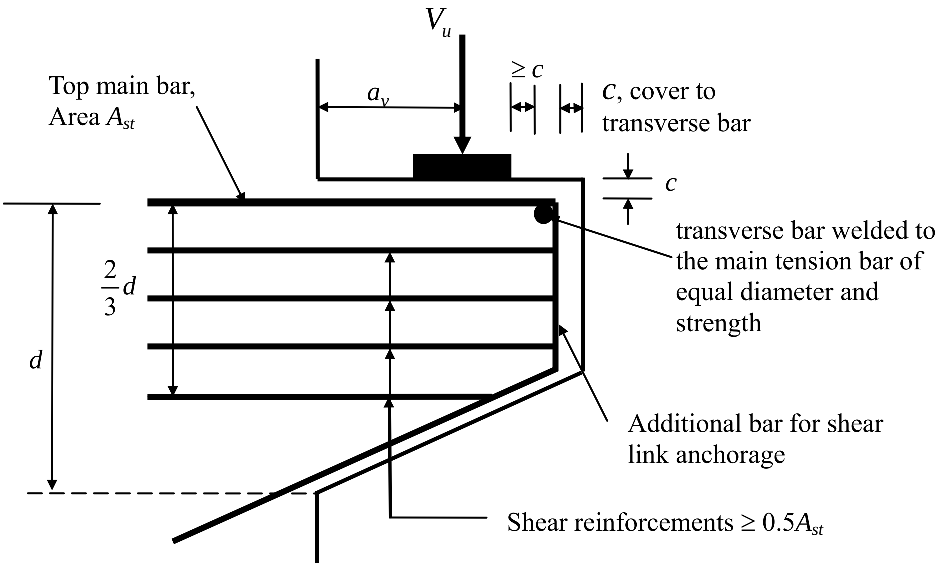


Figure 8.3(a) – Typical Detailing of a Corbel

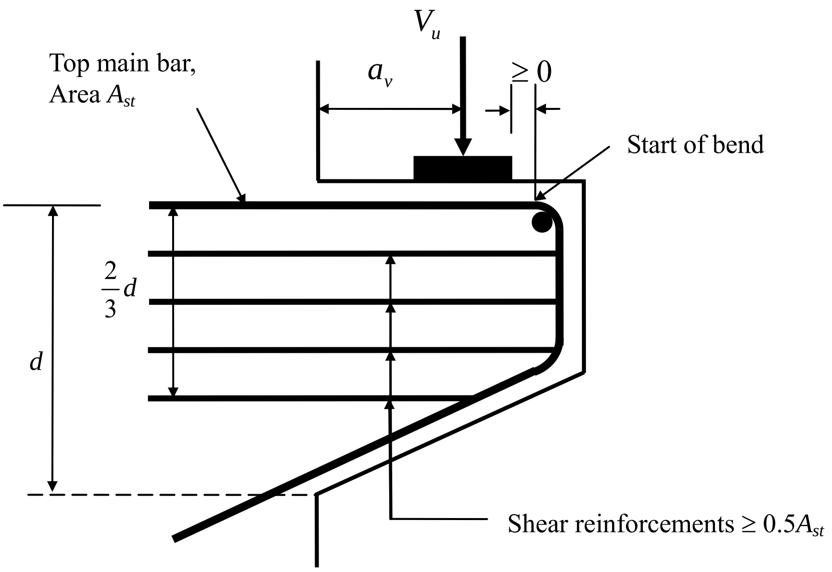


Figure 8.3(b) – Typical Detailing of a Corbel

8.6 Worked Example 8.1

Design a corbel to support a factored load of 600 kN at a distance 200 mm from a wall support, i.e. $V_u = 600$ kN, $a_v = 200$ mm. The load is transmitted from a bearing pad. Concrete grade is C40. The bearing plate is cast into the support and each dimension does not exceed 40% of the support. So the design ultimate stress is $0.8 f_{cu}$ in accordance with the Code of Practice for Precast Concrete Construction 2003 Cl. 2.7.9.4.

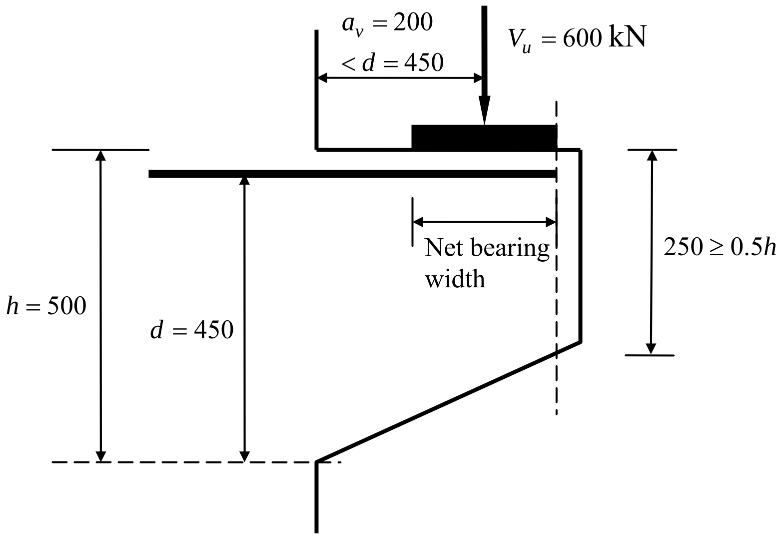


Figure 8.4 – Worked Example. 8.1

1. The dimensions of the corbel are detailed as shown which comply with the requirement of Cl. 6.5.1 of the Code with length of the corbel $b = 300$ mm. As the length of the bearing plate should not exceed 40% of the corresponding dimension, the length of the bearing pad is 120mm
2. Check bearing stress :
   Design ultimate bearing stress is $0.8 f_{cu} = 0.8 \times 40 = 32$ MPa.
   Net bearing width is $\dfrac{600 \times 10^3}{120 \times 32} = 156.25$ mm.
   So use net bearing width of bearing pad 160 mm.
3. With the following parameters :
   $V_u = 600$ kN; $f_{cu} = 40$ MPa; $b = 300$ mm;
   $a_v = 200$ mm $d = 450$ mm
   substituted into (Eqn 8-1)
   $$(0.2025 V_u + 0.18225 f_{cu} b a_v) x^2 - 0.9d(V_u + 0.45 f_{cu} b a_v) x + V_u (a_v^2 + d^2) = 0$$
   Solving $x = 276.77$ mm.
4. The strain at steel level,
   $$\varepsilon_s = \frac{d - x}{x} \varepsilon_{cu} = \frac{450 - 276.77}{276.77} \times 0.0035 = 0.00219 > 0.002175$$
5. The stress in the top steel is $0.87 f_y$ as $\varepsilon_s > 0.002175$;
   (if $\varepsilon_s \leq 0.002175$, $f_s = E_s \times \varepsilon_s$ where $E_s = 200$ GPa)
6. The force in the top steel is

$$T = \frac{V_u a_v}{d - 0.45x} = \frac{600 \times 200}{450 - 0.45 \times 276.77} = 368.71\,\text{kN} > \; 0.5 \times 600 = 300\,\text{kN};$$

7. Steel area required is $\frac{368710}{0.87 \times 500} = 848\,\text{mm}^2$, provide 3T20; (0.7%)

8. $v_c = 0.556 \times (40/25)^{1/3} = 0.65$ MPa without enhancement. With enhancement, it becomes $\frac{2d}{a_v} \times 0.65 = 2.925\,\text{MPa}$.

9. Check shear stress

   $\frac{600000}{450 \times 300} = 4.444\,\text{MPa} > \; v_c = 2.925\,\text{MPa}$.

   So shear reinforcement is

   $\frac{A_{sv}}{s_v} = \frac{b(v - v_c)}{0.87 f_y} = \frac{300(4.444 - 2.925)}{0.87 \times 500} = 1.048\,\text{mm};$

   $A_{sv} = 1.048 \times 450 = 472\,\text{mm}^2$. So use <u>3T12 closed links</u> over the top 300 mm.

10. Area of 3T12 closed link is 678 mm^2 > half of area of tensile top steel = 0.5×3×314=471 mm^2. So O.K.

The details of the Corbel is finally as shown in Figure 8.5.

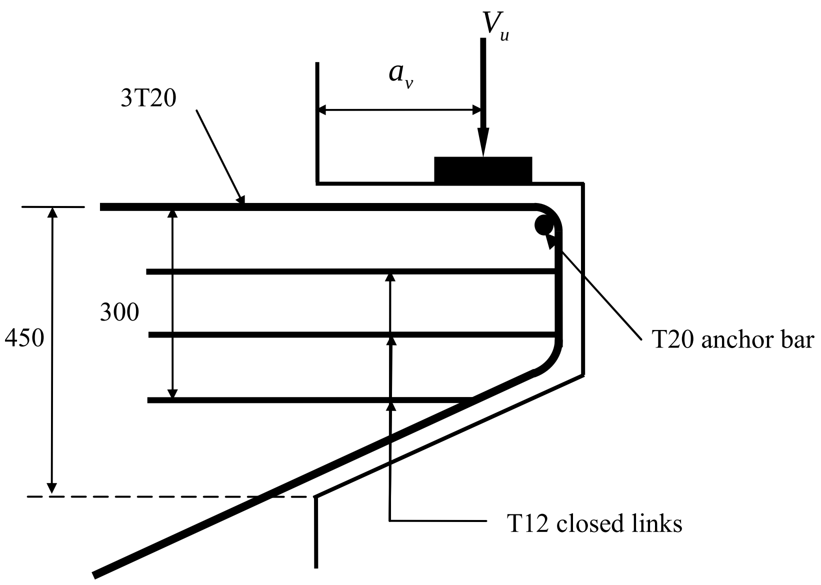


Figure 8.5 – Detailing of Worked Example 8.1

8.7 Resistance to Horizontal Forces

Cl. 6.5.2.4 and Cl. 9.8.4 requires additional reinforcement connected to the supported member to transmit external horizontal force exerted to the corbel in its entirety. It would be a simple task to add in extra reinforcements to resist the extra horizontal force. However, it should be more reasonable if strain compatibility is also considered in designing the corbel to resist also this horizontal factored force N_c as in addition to the vertical factored load V_u

which is also in consistency with the Code requirement. The force polygon as modified from Figure 8.2 becomes

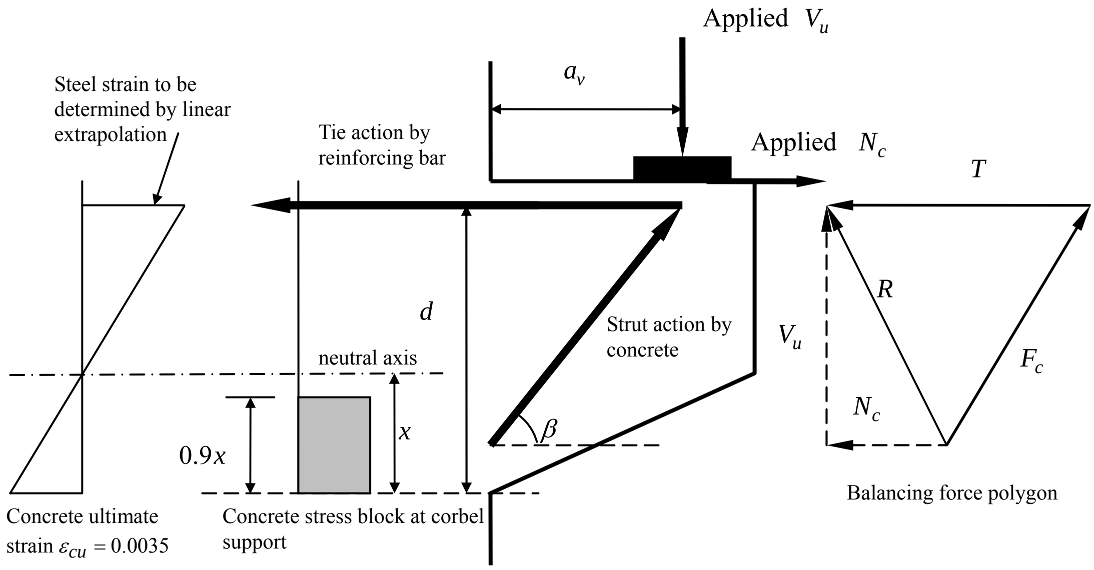


Figure 8.6 – Strut-and-Tie Action of a Corbel with inclusion of horizontal force

From Figure 8.6 and formulae derivation in Section 8.3 of this Manual, it can be seen that the determination of the neutral axis depth x and subsequently the strain profile of the root of the corbel is independent of N_c. Thus the steps (i) to (v) in Section 8.4 of this Manual can be followed in calculation of x, ε_s and σ_s as if N_c does not exist.

However, the tension in the top bar will be $T = N_c + \dfrac{V_u a_v}{d - 0.45x}$ (Eqn 8-4)

And the steel area of the top bar can be worked out as $A_{st} = \dfrac{T}{\sigma_s}$

If an addition horizontal force of 200kN is exerted on the corbel in Example 8.1, tending to pull away from the root of the corbel, the total tensile force to be resisted by the top bars will be $T = 368.71 + 200 = 568.71$ kN and the top bar area required is $\dfrac{568.71 \times 10^3}{0.87 \times 500} = 1307\,\text{mm}^2$, as the strain at the steel level has exceeded 0.002175. The top reinforcement has to be increased to 3T25.

Nevertheless, it may also be acceptable by simple addition of horizontal reinforcements of $\dfrac{N_c}{0.87 f_y}$ over the original design horizontal reinforcements as per the ultimate limit design state concept. As discussed above, such an approach will be conservative.

9.0 Cantilever Projecting Structures

9.1 Cl. 1.4 of the Code defines "Cantilever Projecting Structure" as "a structural element that cantilevers from the main structure, for example, canopies, balconies, bay windows, air conditioning platforms."

9.2 General Design Considerations

Design considerations for a cantilevered structure from the Code (Table 7.3, Cl. 9.4 etc. of the Code) are summarized as follows :

(i) Pure cantilever slab should not be used for span exceeding 750mm and beam-and-slab arrangement should be used for span exceeding 1000mm;

(ii) The span to effective depth of cantilever slabs or rectangular beams should not be greater than 7 while that of flanged beam with ratio of width of web to width of flange ≤ 0.3 should not exceed 5.5;

(iii) In addition to (ii), cantilever beam should have depth ≥ 300mm at support and the minimum thickness of short cantilever slabs are as tabulated in Table 9.1.

Span of Cantilever Slab L (mm)	L≤ 500	500<L≤ 750	L>750
Minimum Thickness (mm)	110	125	150

Table 9.1 – Minimum Thickness of Short Cantilever Slab

(iv) Cantilever slab exposed to weathering should be designed for exposure condition 2 for nominal cover as described in Table 4.2 of the Code, i.e. the concrete covers should follow Table 9.2.

Concrete Grade	C30	C35, C40	C45 and above
Nominal Cover (mm)	40	35	30

Table 9.2 – Nominal Cover to All Reinforcement for Cantilever Slab Exposed to Weathering

(v) The estimated crack width for cantilever slab exposed to weathering should not exceed 0.1mm under serviceability limit;

(vi) A cantilever slab should have ribbed steel reinforcing bars in both faces and both directions.

9.3 Detailing Considerations

(i) The minimum percentage of top tension longitudinal reinforcement based on gross sectional area should be 0.25% for all concrete grades with minimum diameter of 10mm and maximum centre to centre spacing of 150mm (Re Cl. 9.4.2 of the Code);

(ii) When full rotational restraint is provided at the near face of the supporting member, the anchorage of the top tension longitudinal bars shall deem to commence at the lesser of 1/2 width of the supporting member or 1/2 of the effective depth of the cantilever from the near face of the supporting member as illustrated in Figure 9.1.

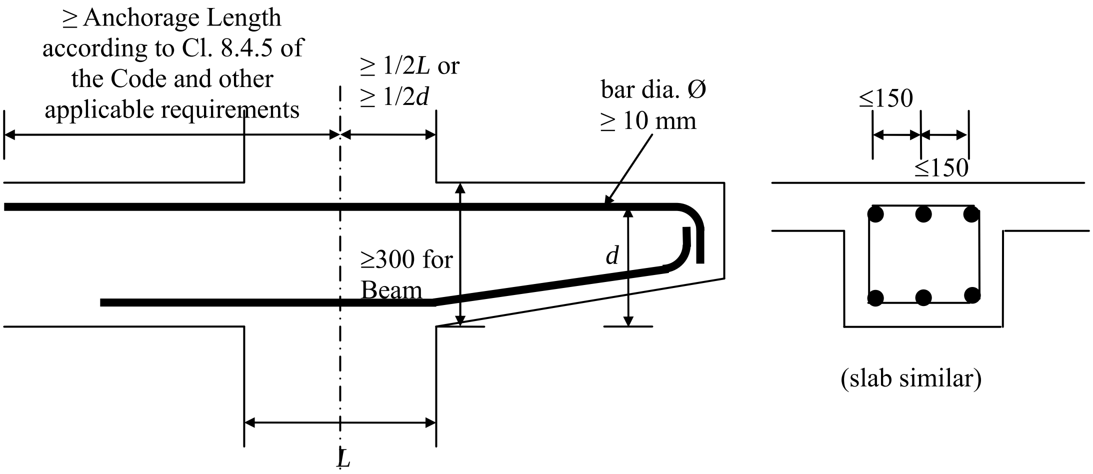


Figure 9.1 – Detailing Requirements of Cantilever Projecting Structure Supported on Members Providing Rotational Restraint

(iii) Where the cantilever projecting structure is a continuous slab and the supporting member is not designed to provide rotational restraint, the anchorage shall deem to commence at the far face of the supporting member and the top bar shall not terminate before the nearest point of contraflexure in the adjacent spans.

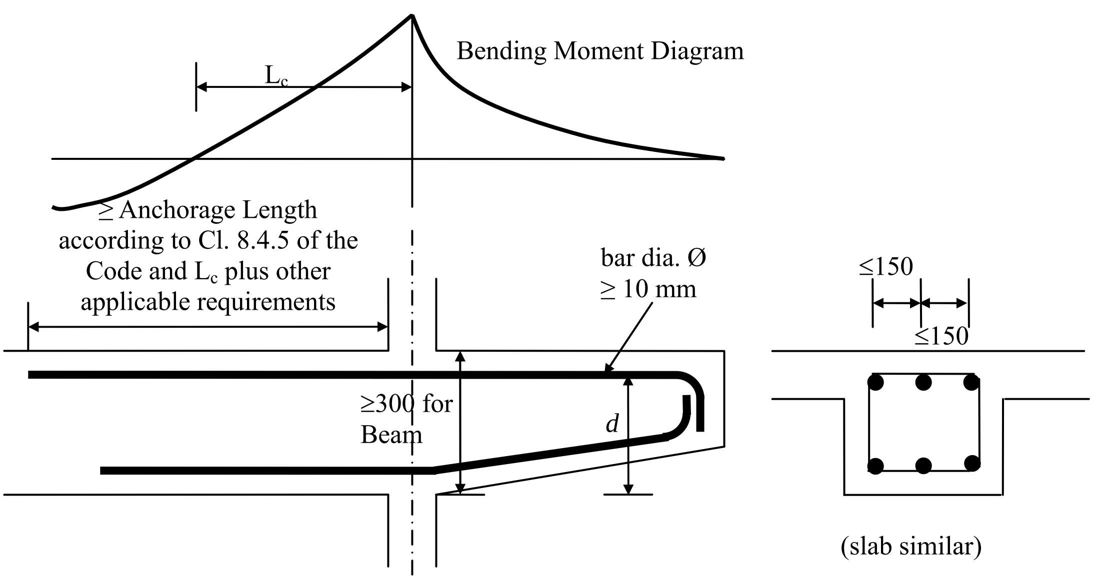


Figure 9.2 – Detailing Requirements of Cantilever Projecting Structure Supported on Members Not Providing Rotational Restraint

9.4 Particular Design Consideration

Cl. 9.4.4 of the Code requires cantilever structures shall be detailed in such a manner that they may be demolished or replaced without affecting the safety and integrity of the main structure of the building. This is demonstrated in Figure 9.3. A Worked Example 9.2 is included in Section 9.7 of this Manual.

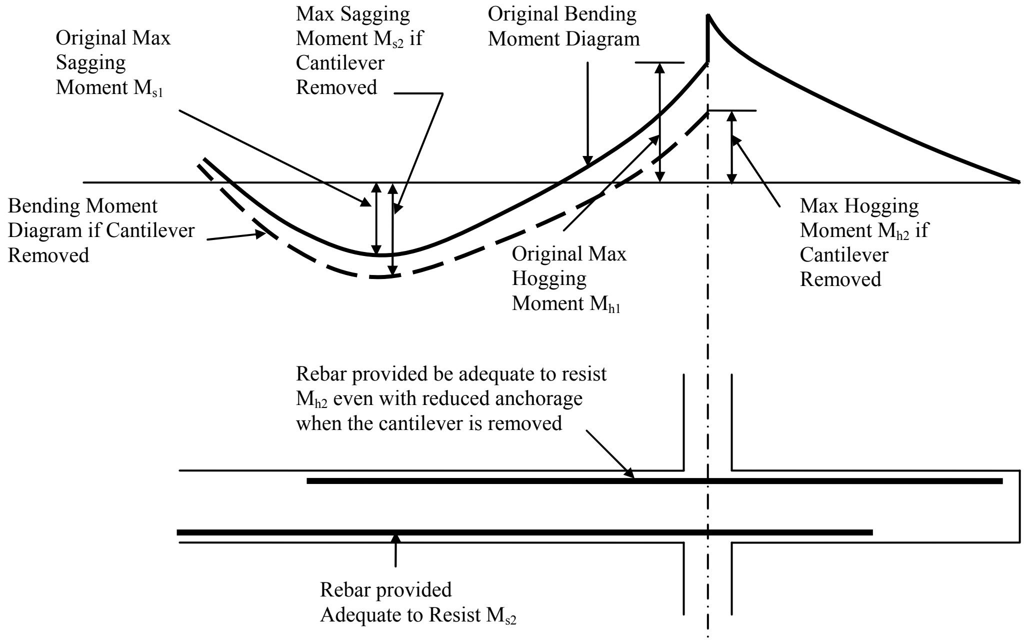


Figure 9.3 – Illustration of Required Structural Provisions for Probable Removal of Cantilever

9.5 Worked Example 9.1

R.C. design of a cantilevered slab as shown in Figure 9.4 is subject to weathering. Concrete grade is C35.

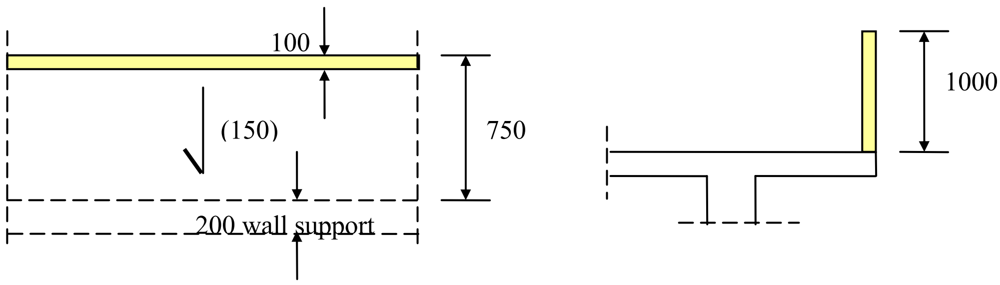


Figure 9.4 – Cantilever slab in Worked Example 9.1

Loading	D.L.	O.W.	$0.15 \times 24.5 = 3.675$ kN/m^2
		Fin	2.0 kN/m^2
			5.675 kN/m^2
		Para.	$0.1 \times 1.0 \times 24.5 = 2.45$ kN/m
	L.L.		1.5 kN/m^2

Effective span is taken to be $750 + 0.5 \times 150 = 825$

Moment $= (1.4 \times 5.675 + 1.6 \times 1.5) \times 0.825 \times 0.825 / 2 + 1.4 \times 2.45 \times 0.775$

$= 6.18$ kNm/m

Design for ultimate state,

$d = 150 - 40 - 5 = 105$

$$\frac{M}{bd^2} = \frac{6.18 \times 10^6}{1000 \times 105^2} = 0.561$$

$$A_{st} = \frac{6.18 \times 10^6}{0.87 \times 500 \times 0.95 \times 105} = 143 \text{ mm}^2\text{/m.}$$

Use T10 – 150 (Area provided is 523 mm^2/m, 0.35%>0.25%)

Crack width is checked by (Ceqn 7.1) and (Ceqn 7.2)

To calculate crack width, it is first necessary to assess the neutral axis depth x by the elastic theory in accordance with the cracked section of Figure 7.1 of the Code.

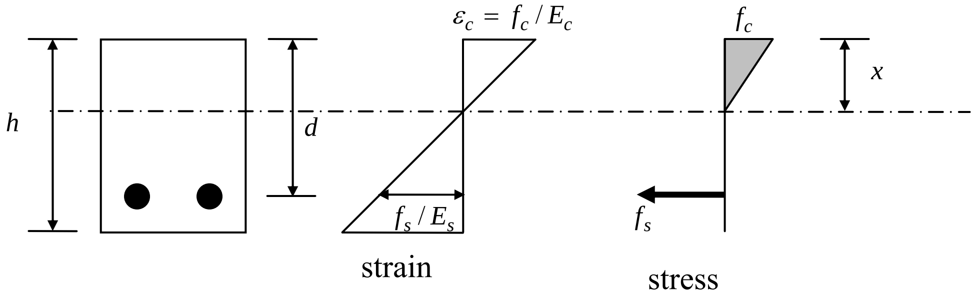


Figure 9.5 – Stress/strain relation of a cracked R.C. section

E_c is the long term value which, by Cl. 7.3.6(a) of the Code is modified from the short term value listed in Table 3.2 of the Code by the formula $E_c = 23.7/(1+\phi) = 23.7/2.656 = 8.92$ kN/mm^2 where ϕ, the creep coefficient is arrived at by $\phi = K_L K_m K_c K_e K_j = 2.3 \times 1 \times 0.8 \times 0.9 \times 1 = 1.656$ in accordance with Cl. 3.1.7 of the Code.

$E_s = 200$ kN/mm^2

Consider equilibrium of the section

$$\frac{1}{2} f_c bx = f_s A_{st} \Rightarrow \frac{1}{2} E_c \varepsilon_c bx = E_s \frac{\varepsilon_c (d-x)}{x} A_{st}$$

$$\Rightarrow \frac{1}{2} E_c bx^2 + E_s A_{st} x - E_s d A_{st} = 0 \qquad \text{(Eqn 9.1)}$$

Consider 1 m width of the section in Worked Example 9.1, $b = 1000$

(Eqn 9.1) becomes

$$\frac{1}{2} \times 8.92 \times 1000 x^2 + 200 \times 523 x - 200 \times 105 \times 523 = 0$$

Solving $x = 39.26$ mm

Taking moment about the centroid of the triangular concrete stress block (the moment should be the unfactored moment which is

4.34kNm/m), the steel tensile stress can be worked out as

$$M = f_s A_{st}\left(d - \frac{x}{3}\right) \Rightarrow f_s = \frac{M}{A_{st}(d - x/3)} = \frac{4.34\times10^6}{523(105 - 39.26/3)}$$

$$= 90.28\ \text{N/mm}^2 \qquad \text{(Eqn 9.2)}$$

So the strain of the steel is $\varepsilon_s = \dfrac{90.28}{200\times10^3} = 0.0004514$

At the level of the concrete tension side, the strain is

$$\varepsilon_1 = \varepsilon_s \frac{(h-x)}{(d-x)} = 0.0004514\times\frac{150-39.26}{105-39.26} = 0.0007604$$

By (CEqn 7.2),

$$\varepsilon_m = \varepsilon_1 - \frac{b_t(h-x)(a'-x)}{3E_s A_s(d-x)} = 0.0007604 - \frac{1000(150-39.26)(150-39.26)}{3\times200\times10^3\times523\times(105-39.26)}$$

$$= 0.00016593\,.$$

The cracked width should be the greatest at the concrete surface mid-way between steel bars as illustrated in Figure 9.6;

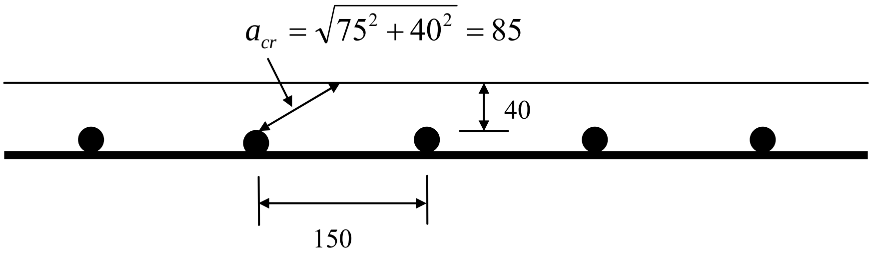


Figure 9.6 – Illustration of a_{cr} in Worked Example 9.1

By (CEqn 7.1) the cracked width is

$$\omega = \frac{3a_{cr}\varepsilon_m}{1+2\left(\dfrac{a_{cr}-c_{min}}{h-x}\right)} = \frac{3\times85\times0.0001659}{1+2\left(\dfrac{85-40}{150-39.26}\right)} = 0.023\ \text{mm} \le 0.1\ \text{mm O.K.}$$

Summing up, reinforcement details is as shown in Figure 9.7:

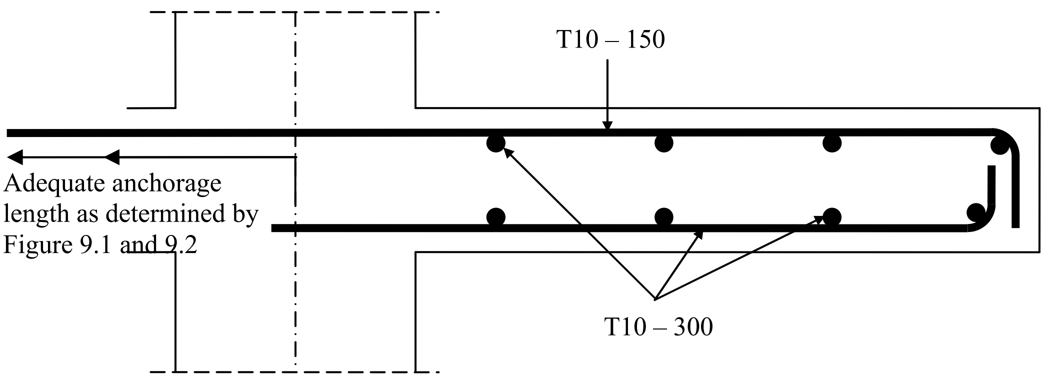


Figure 9.7 – Reinforcement Details for Worked Example 9.1

9.6 Procedure for Checking Crack Widths

The procedure for checking crack width is summarized as follows :

(i) With the assumed reinforcement area A_{st}, solve the neutral axis ratio as

$$\frac{x}{d} = -E_r\rho_{st} + \sqrt{E_r^{\,2}\rho_{st}^{\,2} + 2E_r\rho_{st}}\ ; \quad \text{where } E_r = E_s/E_c\ ;\ \rho_{st} = A_{st}/bd$$

(ii) Calculate the stress in steel by

$$f_s = \frac{M/bd^2}{\rho_{st}\left[1-(x/d)/3\right]}$$

(iii) Calculate steel strain by $\varepsilon_s = f_s/E_s$

(iv) Calculate strain of concrete at the tensile face $\varepsilon_1 = \varepsilon_s\dfrac{(h-x)}{(d-x)}$

(v) Calculate the average strain by $\varepsilon_m = \varepsilon_1 - \dfrac{b_t(h-x)(a'-x)}{3E_sA_{st}(d-x)}$

(vi) Calculate the crack width by $\omega = \dfrac{3a_{cr}\varepsilon_m}{1+2\left(\dfrac{a_{cr}-c_{\min}}{h-x}\right)}$

(vii) If ω exceeds permissible, try a higher value of A_{st}/bd with smaller bar spacing.

9.7 Worked Example 9.2 – Probable Removal of Cantilever

Consider a single bay frame with a cantilever at one end as shown in Figure 9.10. The size of the internal beam and the cantilever are 600(D)×400(B) and support a total factored load of 60kN/m and the column supports are 600×600 columns. The beams and the columns are grade C35. The frame is also assumed to be restrained from sidesway.

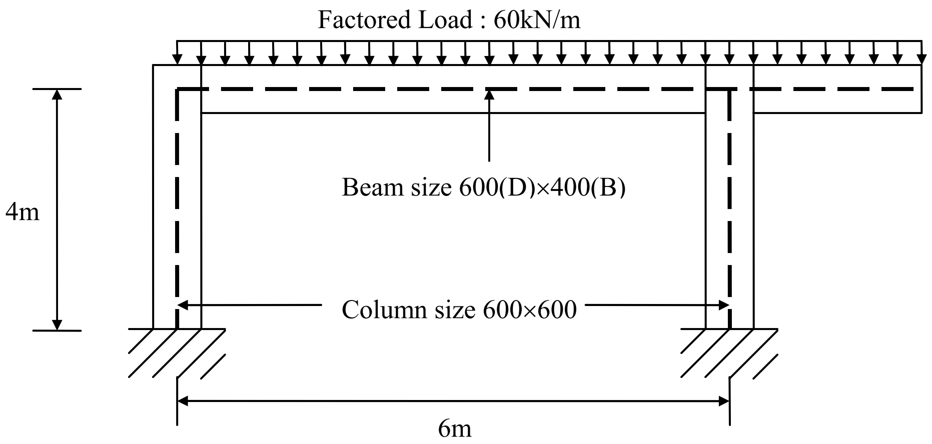


Figure 9.10 – Frame for Worked Example 9.2

By analysis, the original bending moment diagram is as shown in Figure 9.11. In the same figure, the revised bending moment diagram of the internal beam upon the removal of the cantilever is also shown.

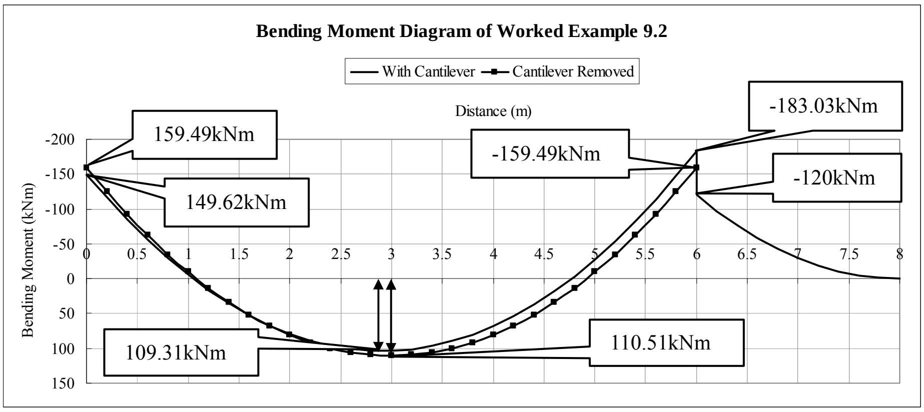


Figure 9.11 – Bending Moment Diagram for Worked Example 9.2

The internal beam should be designed for the envelope of the two bending moment diagrams. That is, the span moment should be 110.51kNm sagging and the left support moment should be 159.49kNm hogging for design.

At the right support of the internal beam, the design hogging moment of 183.03kNm requires 822mm^2 or 3T20 by the simplified stress block approach. However, when the cantilever is removed, the right support moment reduces to 159.49kNm requiring lever arm $0.95 \times 540 = 513$ mm. The total force on the bars required to provide the moment is $159.49 \times 10^6 / 513 = 310858$ N.

Upon removal of the cantilever, the anchorage length of the reinforcing bar will probably be confined by the depth of the column as shown in Figure 9.12. If the frame is not contributing in the lateral load resisting system, the available embedment length can be $600 - 40 = 560$ mm where the 600mm is the column depth and 40mm is the cover as shown in Figure 9.12. The total circumferential areas for the 3T20 bars available for bonding will then be $3 \times 20\pi \times 560 = 105558$ mm^2. With the ultimate bond strength of $f_{bu} = \beta\sqrt{f_{cu}} = 2.958$ MPa (Re Cl. 8.4.4 and Table 8.3 of the Code by which $\beta = 0.5$ for grade C35 concrete), the ultimate bond capacity that can be provided will be $105558 \times 2.958 = 312241$ N slightly greater than 310858N, the required force as calculated in the previous paragraph. So it is marginally adequate. The mobilization of bond capacity by bond length less than the ultimate anchorage bond length is allowed as per Cl. 8.4 of the Code.

In order to provide greater margin for the limited bond length in this Worked Example, moment redistribution may be carried out by reducing the right support moment of the internal span by 10% as shown in Figure 9.13. Greater span moment and left support moment have to be designed for accordingly. The limitation of 10% moment redistribution is taken from Cl. 5.2.9.2 of the Code as applied to frames over 4 storeys and providing lateral stability.

However, if the frame is contributing to lateral resisting system, the problem becomes a "ductility" problem required to comply with Cl. 9.9.1.2(c) with full anchorage bond length of the reinforcing bars as shown in Figure 9.12. As such, the lapping bonding of the top reinforcing bars of the end internal span into the cantilever cannot be carried out. Instead these top bars will have to bonded into the column as shown in Figure 9.12.

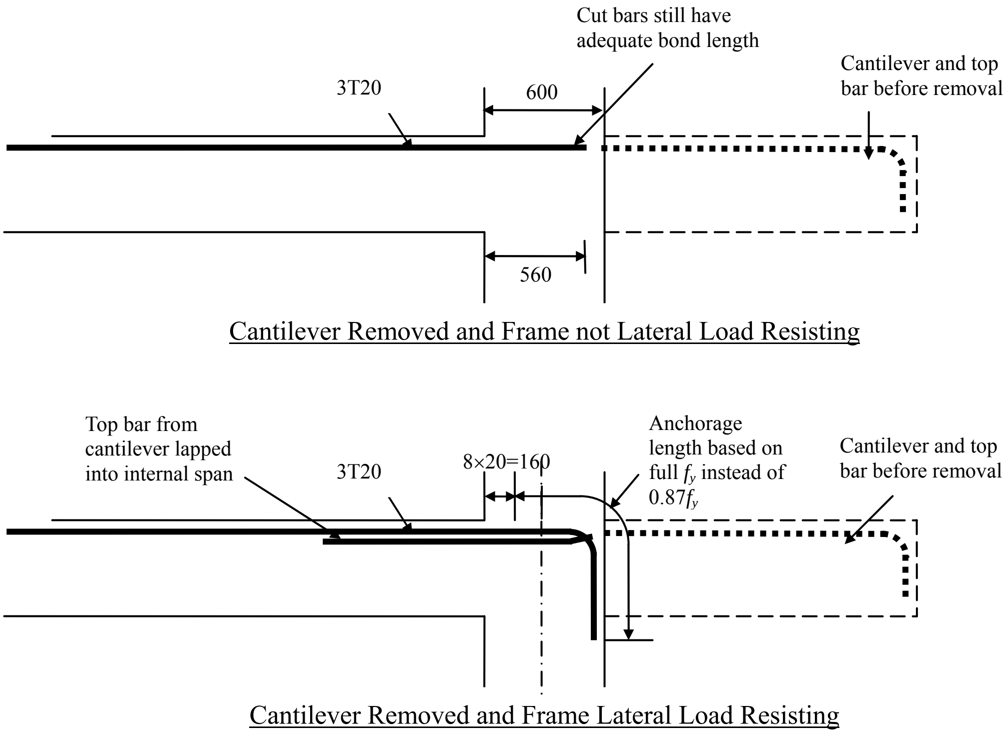


Figure 9.12 – Removal of Cantilever for Worked Example 9.2

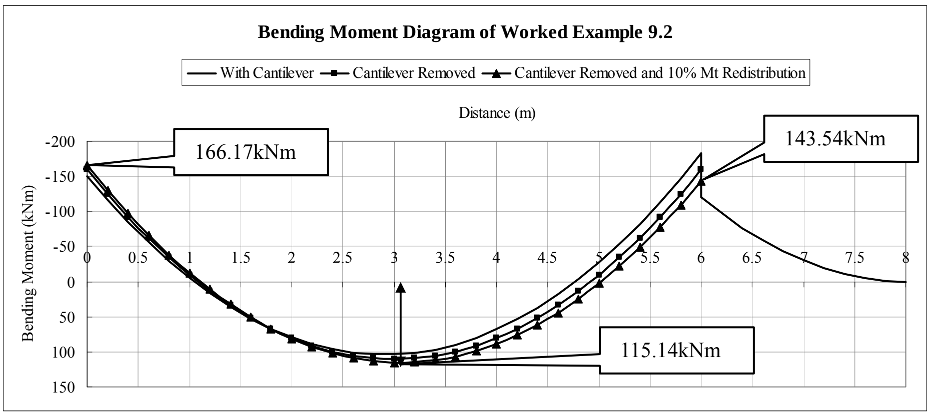


Figure 9.13 – Bending Moment Diagram with Moment Redistribution for Worked Example 9.2

10.0 Transfer Structures

10.1 According to Cl. 5.5 of the Code, transfer structures are horizontal elements which redistribute vertical loads where there is a discontinuity between the vertical structural elements above and below.

10.2 In the analysis of transfer structures, consideration should be given to the followings as per Cl. 5.5 of the Code :

(i) Construction and pouring sequence – the effects of construction sequence can be important in design of transfer structures due to the comparatively large stiffness of the transfer structure and sequential built up of stiffness of structures above the transfer structure as illustrated in Figure 10.1;

(ii) Temporary and permanent loading conditions – especially important when it is planned to cast the transfer structures in two shifts and use the lower shift to support the upper shift as temporary conditions which will create locked-in stresses in the lower shift;

(iii) Varying axial shortening of elements supporting the transfer structures – which leads to redistribution of loads. The phenomenon is more serious as the transfer structure usually possesses large flexural stiffness in comparison with the supporting structural members, behaving somewhat between (a) flexible floor structures on hard columns; and (b) rigid structures (like rigid cap) on flexible columns;

(iv) Local effects of shear walls on transfer structures – shear walls will stiffen up transfer structures considerably and the effects should be taken into account in more accurate assessment of structural behaviour;

(v) Deflection of the transfer structures – will lead to redistribution of loads of the superstructure. Care should be taken if the structural model above the transfer structure is analyzed separately with the assumption that the supports offered by the transfer structures are rigid. Re-examination of the load redistribution should be carried out if the deflections of the transfer structures are found to be significant;

(vi) Lateral shear forces on the transfer structures – though the shear is lateral, it will nevertheless create out-of-plane loads in the transfer structures which needs be taken into account;

(vii) Sidesway of the transfer structures under unbalanced gravity loads should also be taken into account. The effects should be considered if the transfer structure is analyzed as a 2-D model;

(viii) The relative lateral deflection at the transfer structure level with respect to the storey below should not exceed $Hs/700$ where Hs is the height of storey below the transfer structure. (The requirement is a common one in seismic design to avoid “soft storey”).

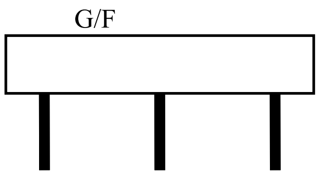
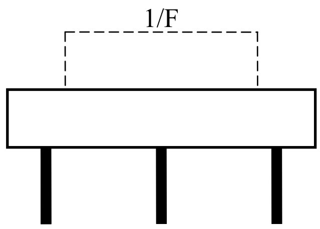
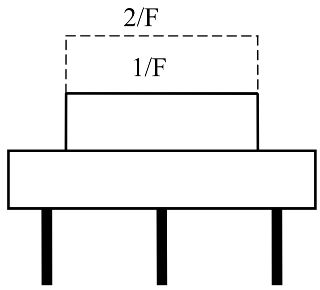
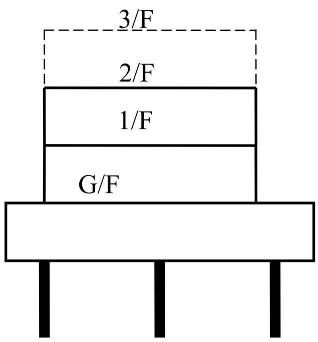
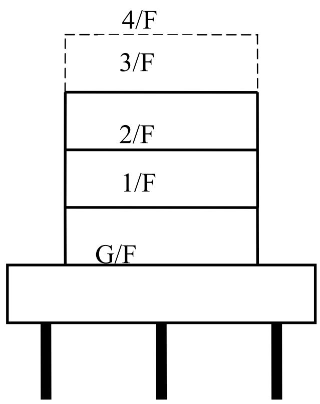
Stage (1) : Transfer Structure (T.S.) just hardened  Stress/force in T.S. being $\{F_1\}$ due to own weight of T.S. and stiffness of the T.S.	Stage (2) : Wet concrete of 1/F just poured  Stress/force in T.S. being $\{F_1\} + \{F_2\}$, $\{F_2\}$ being force induced in transfer structure due to weight of 1/F structure and stiffness of the T.S. only	Stage (3) : 1/F hardened and 2/F wet concrete just poured  Stress/force in T.S. being $\{F_1\} + \{F_2\} + \{F_3\}$, $\{F_3\}$ being force induced in transfer structure due to weight of 2/F structure and stiffness of the T.S. + 1/F
Stage (4) : 2/F hardened and 3/F wet concrete just poured  Stress/force in T.S. being $\{F_1\} + \{F_2\} + \{F_3\} + \{F_4\}$, $\{F_4\}$ being force induced in T.S. due to weight of 3/F structure and stiffness of the T.S. + 1/F + 2/F	Stage (5) : 3/F hardened and 4/F wet concrete just poured  Stress/force in T.S. being $\{F_1\} + \{F_2\} + \{F_3\} + \{F_4\} + \{F_5\}$, $\{F_5\}$ being force induced in T.S. due to weight of 4/F structure and and stiffness of the T.S. + 1/F + 2/F + 3/F	Stage (6) and onwards Structure above transfer structure continues to be built. Final force induced on T.S. becomes $\{F_n\} + \{F_{n-1}\} + \{F_{n-2}\} + ... + \{F_2\} + \{F_1\}$

Figure 10.1 – Diagrammatic Illustration of the Effects of Construction Sequence of Loads Induced on Transfer Structure

10.3 Mathematical Modeling of Transfer Structures as 2-D model (by SAFE) :

The general comments in mathematical modeling of transfer structures as 2-D model to be analyzed by computer methods are listed :

(i) The 2-D model can only be analyzed against out-of-plane loads, i.e. vertical loads and out-of-plane moments. Lateral loads have to be analyzed separately;

(ii) It is a basic requirement that the transfer structure must be adequately stiff so that detrimental effects due to settlements of the columns and walls being supported on the transfer structure are tolerable. In view of the relatively large spans by comparing with pile cap, such settlements should be checked. Effects of construction sequence may be taken into account in checking;

(iii) The vertical settlement support stiffness should take the length of the column/wall support down to level of adequate restraint against further settlement such as pile cap level. Reference can be made to Appendix H discussing the method of "Compounding" of vertical stiffness and the underlying assumption;

(iv) Care should be taken in assigning support rotational stiffness to the transfer structures. It should be noted that the conventional use of either $4EI/L$ or $3EI/L$ have taken the basic assumption of no lateral movements at the transfer structure level. Correction to allow for sidesway effects is necessary, especially under unbalanced applied moments such as wind moment. Fuller discussion and means to assess such effects are incorporated in Appendix H;

(v) Walls which are constructed monolithically with the supporting transfer structures may help to stiffen up the transfer structures considerably. However, care should be taken to incorporate such stiffening effect in the mathematical modeling of the transfer structures which is usually done by adding a stiff beam in the mathematical model. It is not advisable to take the full height of the wall in the estimation of the stiffening effect if it is of many storeys as the stiffness can only be gradually built up in the storey by storey construction so that the full stiffness can only be effected in supporting the upper floors. Four or five storeys of walls may be used for multi-storey buildings. Furthermore, loads induced in these stiffening structures (the stiff beams) have to be properly catered for which should be resisted by the wall forming the stiff beams;

10.4 Modeling of the transfer structure as a 3-dimensional mathematical model can eliminate most of the shortcomings of 2-dimensional analysis discussed in section 10.3, including the effects of construction sequence if the software has provisions for such effects. However, as most of these softwares may not have the sub-routines for detailed design, the designer may need to "transport" the 3-D model into the 2-D model for detailed design. For such "transportation", two approaches can be adopted :

(i) Transport the structure with the calculated displacements by the 3-D software (after omission of the in-plane displacements) into the 2-D software for re-analysis and design. Only the displacements of the nodes with external loads (applied loads and reactions) should be transported. A 2-D structure will be re-formulated in the 2-D software for re-analysis by which the structure is re-analyzed by forced displacements (the transported displacements) with recovery of the external loads (out-of-plane components only) and subsequently recovery of the internal forces in the structure. Theoretically results of the two models should be identical if the finite element meshing and the shape functions adopted in the 2 models are identical. However, as the finite element meshing of the 2-D model is usually finer than that of the 3-D one, there are differences incurred between the 2 models, as indicated by the differences in recovery of nodal forces in the 2-D model. The designer should check consistencies in reactions acting on the 2 models. If large differences occur, especially when lesser loads are revealed in the 2-D model, the designer should review his approach;

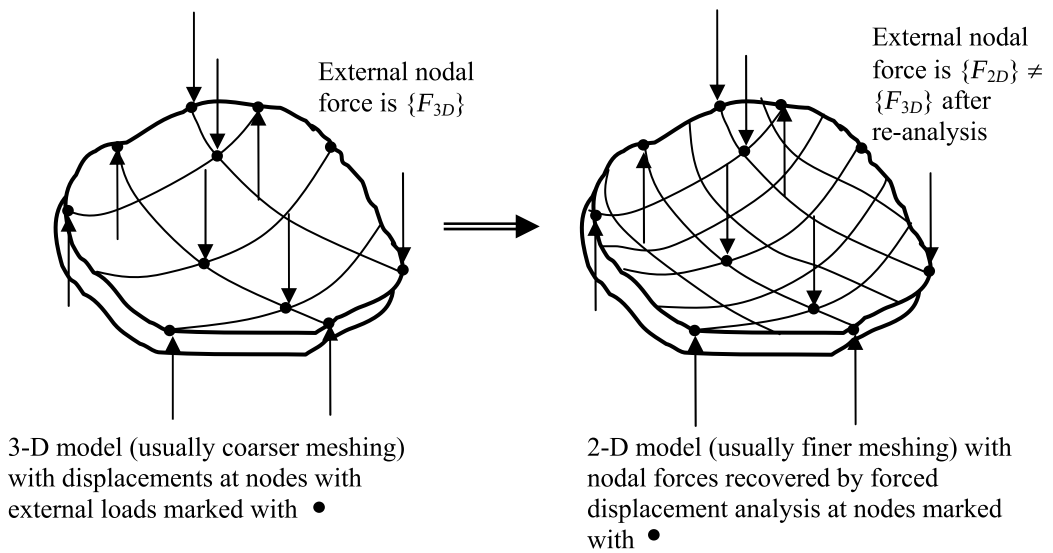
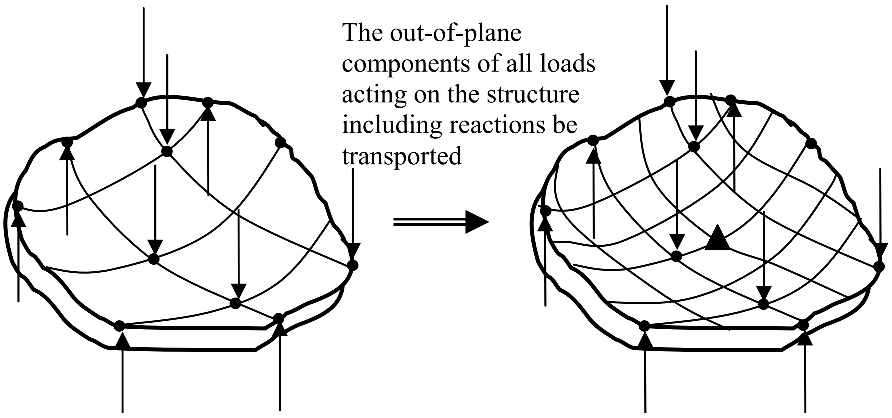


Figure 10.2 – 3-D model to 2-D with transportation of nodal displacements

(ii) Transport the out-of-plane components of the external loads (applied loads and reactions) acting on the 3-D model to the 2-D model for further analysis. This type of transportation is simpler and more reliable as full recovery of loads acting on the structure is ensured. However, in the re-analysis of the 2-D structure, a fixed support has to be added on any point of the structure for analysis as without which the structure will be unstable. Nevertheless, no effects due to this support will be incurred by this support because the support reactions should be zero as the transported loads from the 3-D model are in equilibrium.



3-D model with external loads obtained by analysis

2-D model with out-of-plane components of external forces transported from 3-D model and re-analyzed with a fixed support ▲ at any point

Figure 10.3 – 3-D model to 2-D with transportation of nodal forces

10.5 Structural Sectional Design and r.c. Detailing

The structural sectional design and r.c. detailing of a transfer structure member should be in accordance with the structural element it simulates, i.e. it should be designed and detailed as a beam if simulated as a beam and be designed and detailed as a plate structure if simulated as a plate structure. Though not so common in Hong Kong, if simulation as a "strut-and-tie" model is employed, the sectional design and r.c. detailing should accordingly be based on the tie and strut forces so analyzed.

The commonest structural simulation of a transfer plate structure is as an assembly of plate bending elements analyzed by the finite element method. As such, the analytical results comprising bending, twisting moments and out-of-plane shears should be designed for. Reference to Appendix D can be made for the principles and design approach of the plate bending elements.

10.6 Effects on the Superstructure

Effects on the superstructure due to deflections of the transfer structures should be examined, especially at the lower storeys. If a full 3-dimensional mathematical model (the better model would be those with simulation of construction sequence) has been constructed, the stresses as resulted should be studied. Experience has shown that large shear stresses can be resulted especially near the interfaces of the shear walls with the transfer structures. Reference can be made to Appendix G for the checking and design for shear. If the detailed 3-dimensional mathematical model is not available for examination of such stresses, the superstructures should also be examined with prescribed displacements of the transfer structures, if the effects are considered significant.

11.0 Footings

11.1 Analysis and Design of Footing Based on the Assumption of Rigid Footing

Cl. 6.7.1 of the Code allows a footing be analyzed as a "rigid footing" provided it is of sufficient rigidity with uniform or linearly varying pressures beneath. As suggested by the Code, the critical section for design is at column or wall face as marked in Figure 11.1, though in case of circular columns, the critical section may need be shifted into 0.2 times the diameter of the column, as in consistency with Cl. 5.2.1.2(b) of the Code.

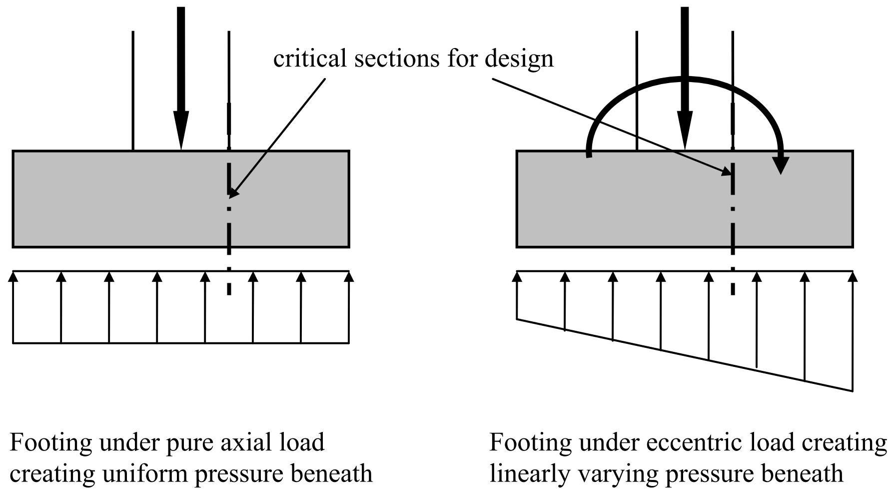


Figure 11.1 – Assumed Reaction Pressure on Rigid Footing

As it is a usual practice of treating the rigid footing as a beam in the analysis of its internal forces, Cl. 6.7.2.2 of the Code requires concentration of steel bars in areas with high stress concentrations as illustrated in Figure 11.2.

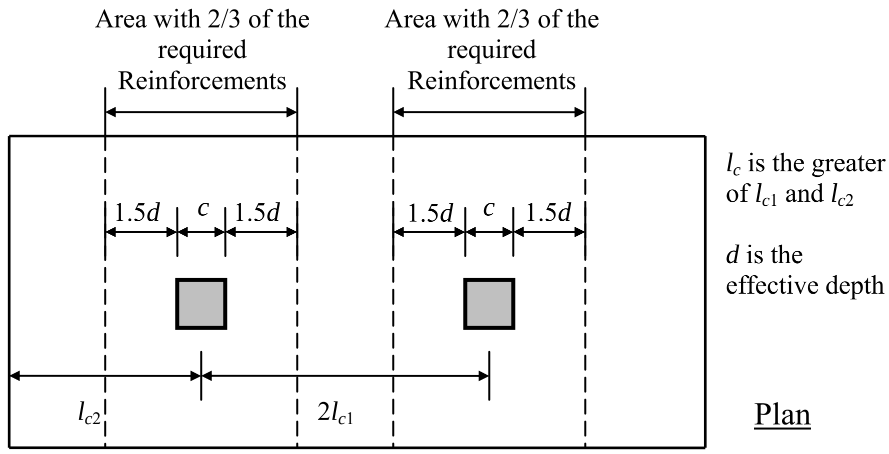


Figure 11.2 – Distribution of Reinforcing Bars when $l_c > (3c/4 + 9d/4)$

Cl. 6.7.2.4 of the Code requires checking of shear be based on (i) section through the whole width of the footing (as a slab); and (ii) local punching shear check as if it is a flat slab. (Re Worked Example 4.5 in Section 4).

11.2 Worked Example 11.1

Consider a raft footing under two column loads as shown in Figure 11.3. Design data are as follows :

Column Loads (for each): Axial Load: D.L. 800 kN L.L. 200 kN
Moment D.L. 100kNm L.L. 20 kNm

Overburden soil : 1.5 m deep

Footing dimensions : plan dimensions as shown, structural depth 400 mm, cover = 75 mm; Concrete grade of footing : grade C35

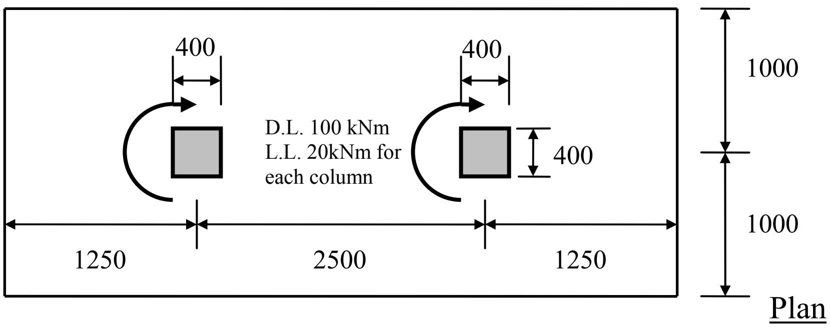


Figure 11.3 – Footing Layout for Worked Example 11.1

(i) Loading Summary :

D.L. Column: $2\times800=$ 1600 kN;
O.W. $5.0\times2.0\times0.4\times24.5=98$ kN
Overburden Soil $5.0\times2\times1.5\times20=$ 300 kN
Total 1998 kN
Moment (bending upwards as shown in Figure 11.3)
$2\times100=200\,\text{kNm}$

L.L. Column $2\times200=400\,\text{kN}$.
Moment (bending upwards as shown in Figure 11.3)
$2\times20=40\,\text{kNm}$

Factored load : Vertical load $1.4\times1998+1.6\times400=3437.2\,\text{kN}$
Moment $1.4\times200+1.6\times40=344\,\text{kNm}$

(ii) The pressure beneath the footing is first worked out as :

At the upper end : $\dfrac{3437.2}{5\times2}+\dfrac{6\times344}{5\times2^2}=343.72+103.2=446.92\,\text{kN/m}^2$

At the lower end : $\dfrac{3437.2}{5\times2}-\dfrac{6\times344}{5\times2^2}=343.72-103.2=240.52\,\text{kN/m}^2$

Critical section $\dfrac{3437.2}{5\times2}+\dfrac{344\times0.2}{5\times2^3/12}=343.72+20.64=364.36\,\text{kN/m}^2$

The pressures are indicated in Figure 11.3(a)

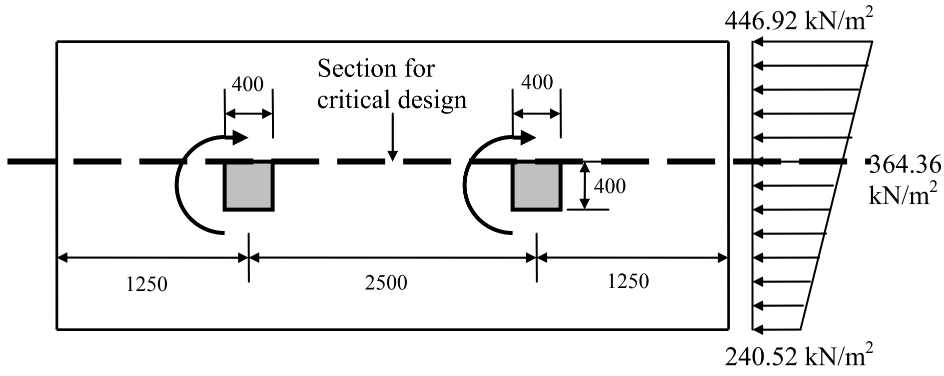


Figure 11.3(a) – Bearing Pressure for Worked Example 11.1

(iv) At the critical section for design as marked in Figure 11.3(a), the total shear is due to the upward ground pressure minus the weight of the footing and overburden soil ($1.4(0.4\times24.5+1.5\times20)=55.72$ kN/m^2) which is $\left(\dfrac{446.92+364.36}{2}\right)\times0.8\times5-55.72\times0.8\times5=1399.68\,\text{kN}$

The total bending moment is

$$(364.36-55.72)\times\frac{0.8^2}{2}\times5+\left(\frac{446.92-364.36}{2}\right)\times0.8^2\times\frac{2}{3}\times5$$
$$=581.89\,\text{kNm}$$

(v) Design for bending : Moment per m width is :

$\dfrac{581.89}{5}=116.38\,\text{kNm/m};$

$d=400-75-8=317$ mm, assume T16 bars

$K=\dfrac{M}{bd^2}=\dfrac{116.38\times10^6}{1000\times317^2}=1.158$,

By the formulae in Section 3 for Rigorous Stress Approach,

$p_0=0.282\,\%$; $A_{st}=894\,\text{mm}^2/\text{m}$

As $l_c=1250>3c/4+9d/4=3\times400/4+9\times317/4=1013$, two thirds of the reinforcements have to be distributed within a zone of $c+2\times1.5d$ from the centre and on both sides of the column, i.e. a total width of $400+1.5\times317\times2=1.351$ m about the centre line of the columns.

Total flexural reinforcements over the entire width is $894\times5=4470\,\text{mm}^2$, 2/3 of which in $1.351\times2=2.702$ m.

So $4470\times2/3/2.702=1103\,\text{mm}^2/\text{m}$ within the critical zone.

Provide T16 – 175.

Other than the critical zone, reinforcements per metre width is $4470/3/(5-2.702)=648$ mm^2/m. Provide T16 – 300 (Area = 670 mm^2/m > min. of $0.13\%\times1000\times400=520\,\text{mm}^2/\text{m}$).

(vi) Design for Strip Shear : Total shear along the critical section is 1399.86 kN, thus shear stress is

$$v = \frac{1399.68 \times 10^3}{5000 \times 317} = 0.883\,\text{N/mm}^2$$

$> v_c = 0.79 \times 0.282^{1/3} \left(\frac{400}{317}\right)^{1/4} \frac{1}{1.25} \times \left(\frac{35}{25}\right)^{1/3} = 0.492\,\text{N/mm}^2$ as per Table 6.3 of the Code.

So shear reinforcement required is

$$\frac{A_{sv}}{s_v} = \frac{b(v - v_c)}{0.87 f_{yv}} = \frac{5000(0.883 - 0.492)}{0.87 \times 500} = 4.494 < \frac{5000 \times 0.4}{0.87 \times 500} = 4.598$$

Two-thirds of the total shear reinforcements be placed within the width 2.702m :

$4.598 \times \frac{2}{3} \div 2.702 = 1.1347\,\text{mm}^2\text{/m}$. Use T12 – 300 S.W. and – 300 L.W.

For the rest of the footing,

$4.598 \times \frac{1}{3} \div 2.298 = 0.667\,\text{mm}^2\text{/m}$. Use T10 – 300 BWs.

(vii) Check punching shear along perimeter of column

Factored load by a column is $1.4 \times 800 + 1.6 \times 200 = 1440\,\text{kN}$. By Cl. 6.1.5.6(d), along the column perimeter,

$$\frac{V_{eff}}{ud} = \frac{1440 \times 10^3}{4 \times 400 \times 317} = 2.84 < 0.8\sqrt{f_{cu}} = 4.7\,\text{MPa. O.K.}$$

Locate the next critical perimeter for punching shear checking as shown in Figure 11.3(b) which is at 1.5*d* from the column face.

Weight of overburden soil and weight of footing is

$1.4 \times 1.351^2 \times 39.8 - 1.4 \times 0.4^2 \times 1.5 \times 20 = 94.98\,\text{kN}$

Upthrust by ground pressure is $\frac{3437.2}{5 \times 2} \times 1.351^2 = 627.36\,\text{kN}$

Net load along the critical perimeter is

$1440 + 94.98 - 627.36 = 907.62\,\text{kN}$

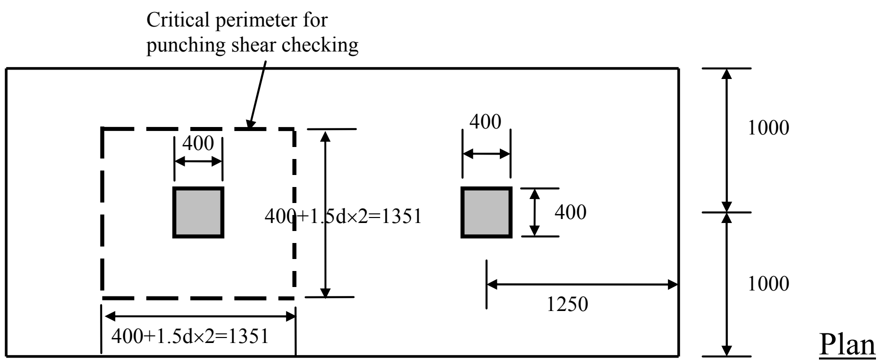


Figure 11.3(b) – checking punching shear for Worked Example 10.1

By (Ceqn 6.40)

$$V_{eff}=V_t\left(1+\frac{1.5M_t}{V_t x_{sp}}\right)=907.62\left(1+\frac{1.5\times 172}{907.62\times 1.351}\right)=1098.59\,\text{kN}$$

Punching shear stress is $v=\dfrac{1098.59\times 10^3}{1351\times 4\times 317}=0.641\,\text{N/mm}^2$

As $v<1.6v_c=0.808$, use (Ceqn 6.44) in determining punching shear reinforcement,

$$\frac{(v-v_c)ud}{0.87f_{yv}}=\frac{(0.641-0.489)\times 1351\times 4\times 317}{0.87\times 500}<\frac{0.4\times 1351\times 4\times 317}{0.87\times 500}$$

$=1575\text{m}^2$. The reinforcement should be distributed in the manner as that of flat slab, i.e. with 40%, 630mm^2 (i.e. 9 nos. of T10) at $0.5d$ (158.5mm) and others 945 mm^2 (i.e. 13 nos. of T10) at $1.25d$ (396.25mm) away from the surface of the column as per the advice in Figure 6.13 of the Code.

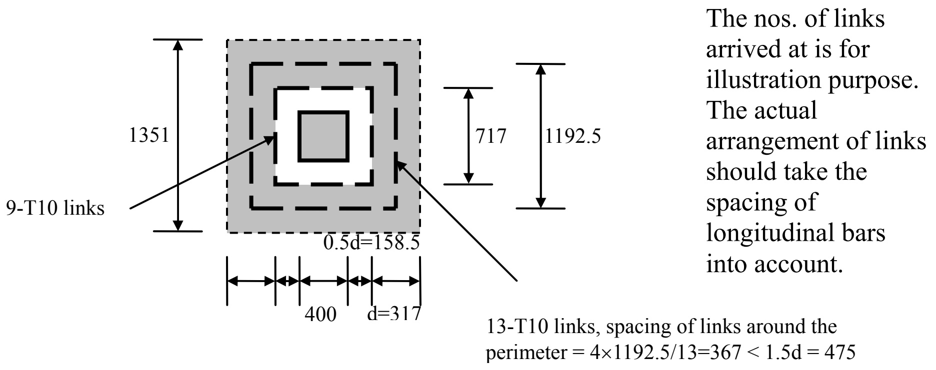


Figure 11.3(c) – Area for punching shear reinforcement

So the provision by the strip shear obtained in (v) which is greater is adopted as per Cl. 6.7.2.4 of the Code which requires the more "severe" provision for checking of strip and punching shears.

(viii) Checking of bending and shear in the direction parallel to the line joining the columns can be carried out similarly. However, it should be noted that there is a net "torsion" acting on any section perpendicular to the line joining the two columns due to linearly varying ground pressure. To be on the conservative side, shear arising due to this torsion should be checked and designed accordingly as a beam as necessary. Nevertheless, one can raise a comment that the design has to some extent be duplicated as checking of bending has been carried out in the perpendicular direction. Furthermore, for full torsion to be developed for design in accordance with (Ceqn 6.65) to (Ceqn 6.68) of the Code, the "beam" should have a free length of beam stirrup width + depth to develop the torsion (as illustrated in Figure 3.31 in Section 3) which is generally not possible for footing of considerable width. As unlike vertical shear where enhancement can be adopted with "shear span" less than $2d$ or $1.5d$, no similar strength enhancement is allowed in Code, though by the same phenomenon there should be some shear strength enhancement. So full design for bending in both ways together with torsion will likely result in over-design.

(ix) The flexural and shear reinforcements provisions for the direction perpendicular to the line joining the columns is

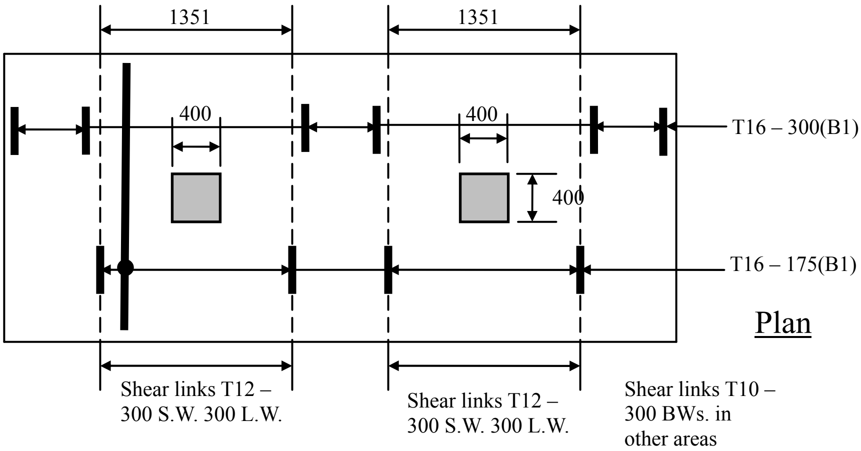


Figure 11.3(c) – Reinforcement Details for Worked Example 11.1 (in the direction perpendicular to the line joining the two columns only)

11.3 Flexible Footing Analysis and Design by Computer Methods

The "flexible footing" analysis takes the stiffness of the structure and the supporting ground into account by which the deformations of the structure itself are analyzed, which is in contrast to the rigid footing assumption that the footing is perfectly rigid. As the deformations will affect the distribution of the internal forces of the structure, the reactions are generally significantly different from that by the rigid footing analysis. While the analysis involves high degrees of structural indeterminacy and often with the employment of the finite element method, computer methods have to be used in the analysis. In the flexible footing analysis, though it is comparatively easy to model the footing structure, the modeling of the subgrade is much more difficult because subgrade (soil or rock) is often not truly an elastic medium with Young's modulus easily identified. Nevertheless, there are approaches as follows that can be used in analysis :

(i) The "Winkler spring" support by which the ground is modeled as a series of "independent elastic springs" supporting the footing structure. By this approach, the settlement of each of the springs depends only the load directly transmitted to it without consideration of loads exerted by the footing in other locations. This is not correct as the settlement of a point of the ground is due to all loads of the footing. By this approach, if the reactions of the springs so analyzed are exerted onto the ground as an elastic medium, the settlements of the ground are not compatible with that of the footing generally. So the approach does not reflect truly the structural behaviour of the footing. Yet, due to its relative simplicity in analysis, this approach is popularly employed currently by such software as "SAFE";

(ii) The "Subgrade Structure Interaction Approach" as formulated in details by Lam et al (2009) by which the supports are interacting with one another instead of being independent "Winkler springs" supports is a relatively new and more accurate approach. Full compatibility of settlements of the subgrade and the footing structure is achieved, resulting in more realistic structural behaviour. Cheng and the Housing Department (2013), based on the underlying theory of Lam et al (2009) has developed a software "PLATE" which has gained Approval by the Buildings Department. The software can take into full account of soil structure interactions which ensures compatibility of settlement of the structure and the subgrade.

As the out-of-plane deformations and forces are most important in footing analysis and design, flexible footings are often modeled as plate bending elements analyzed by the finite element method as discussed in 11.4 in more details.

11.4 Reinforced Concrete Design by the Computer Method

The followings are highlighted for design of footing modeled as 2-D model (idealized as assembly of plate bending elements) on surface supports:

(i) The analytical results comprise bending, twisting moments and out-of-plane shears for consideration in design;

(ii) As local "stresses" within the footing are revealed in details, the rules governing distribution of reinforcements in footing analyzed as a beam need not be applied. The design at any location in the footing can be based the calculated stresses directly. However, if "peak stresses" (high stresses dropping off rapidly within short distance) occur at certain locations as illustrated in Figure 11.4 which are often results of finite element analysis at points with heavy loads or point supports, it would be reasonable to "spread" the stresses over certain width for design. Nevertheless, care must be taken not to adopt widths too wide for "spreading" as local effects may not be well captured. A fuller discussion can be found in Lam and Law (2009).

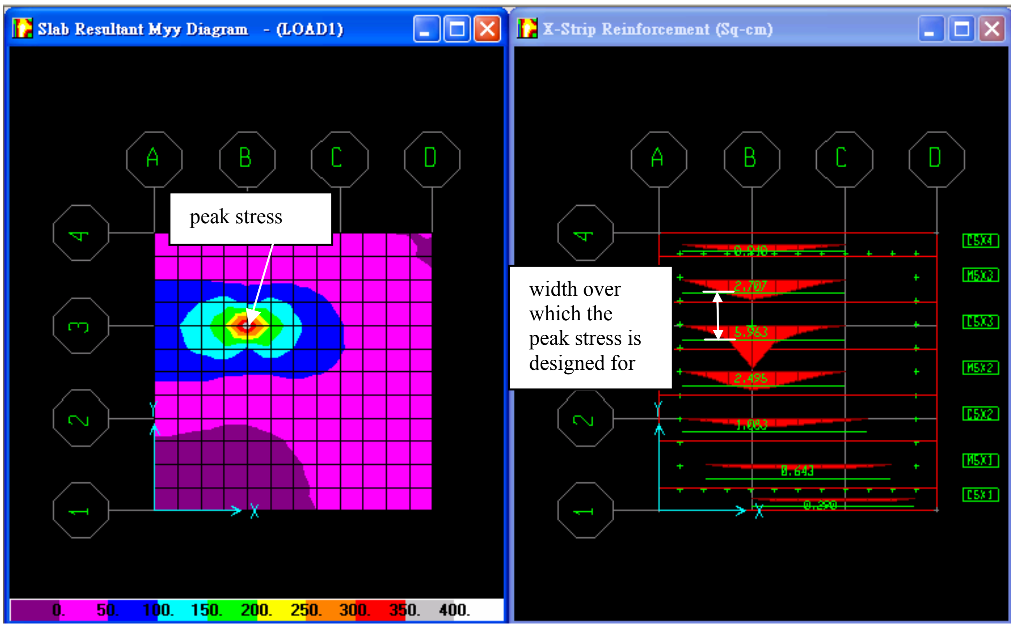


Figure 11.4 – Spreading of peak stress over certain width for design

(iii) The design against flexure should be done by the "Wood Armer Equations" listed in Appendix D, together with discussion of its underlying principles. As the finite element mesh of the mathematical model is often very fine, it is a practice of "lumping" the design reinforcements of a number of nodes over certain widths and evenly distributing the total reinforcements over the widths, as is done by the popular software "SAFE". Again, care must be taken in not taking widths too wide for "lumping" as local effects may not be well captured. The design of reinforcements by SAFE is illustrated on the right portion of Figure 11.4;

(iv) The principle together with a worked example for design against shear is included in Appendix D, as illustrated in Figure D-5a to D-5c. It should be noted that as the finite element analysis give detailed distribution of shear stresses on the structure, it is not necessary to calculate shear stress as done for flat slab under empirical analysis in accordance with the Code. The checking of shear and design of shear reinforcements can be based directly on the shear stresses revealed by the finite element analysis.

12.0 Pile Caps

12.1 Rigid Cap Analysis

Cl. 6.7.3 of the Code allows a pile cap be analyzed and designed as a "rigid cap" by which the cap is considered as a perfectly rigid structure so that the supporting piles deform in a co-planar manner at their junctions with the cap. As the deformations of the piles are governed, the reactions by the piles can be found with their assigned (or assumed) stiffnesses. If it is assumed that the piles are identical (in stiffnesses), the reactions of the piles follow a linearly varying pattern. Appendix I contains derivation of formulae for solution of pile loads under rigid cap assumption.

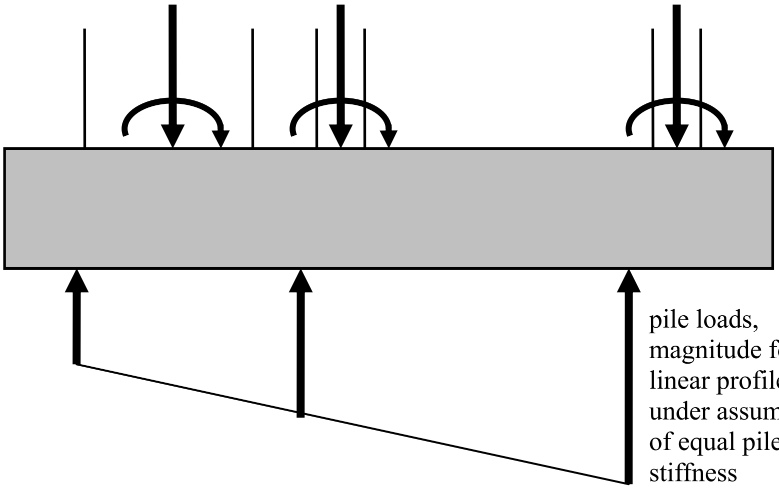


Figure 12.1 – Pile Load Profile under Rigid Cap Assumption

Upon solution of the pile loads, the internal forces of the pile cap structure can be obtained with the applied loads and reactions acting on it as a free body. The conventional assumption is to consider the cap as a beam structure spanning in two directions and perform analysis and design separately. It is also a requirement under certain circumstances that some net torsions acting on the cap structure (being idealized as a beam) need be checked. As the designer can only obtain a total moment and shear force in any section of full cap width, there may be under-design against heavy local effects in areas having heavy point loads or pile reactions. The Code (Cl. 6.7.3.3(b)) therefore imposes a condition that when shear distribution across section has not been considered, shear enhancement of concrete cannot be applied.

Cl. 6.7.3.5 of the Code requires checking of torsion based on rigid body theory which is similar to discussion in Section 11.2 (vii).

12.2 <u>Worked Example 12.1 (Rigid Cap Design)</u>

The small cap as shown in Figure 12.2 is analyzed by the rigid cap assumption and will then undergo conventional design as a beam spanning in two directions.

Design data : Pile cap plan dimensions : as shown
Pile cap structural depth : 2m

Depth of Overburden Soil : 1.5m
Pile diameter : 2m
Concrete grade of Cap : C40
Cover to main reinforcements : 75mm
Column dimension : 2 m square
Factored Load from the central column :
$P = 50000$ kN
$M_x = 2000$ kNm (along X-axis)
$M_y = 1000$ kNm (along Y-axis)

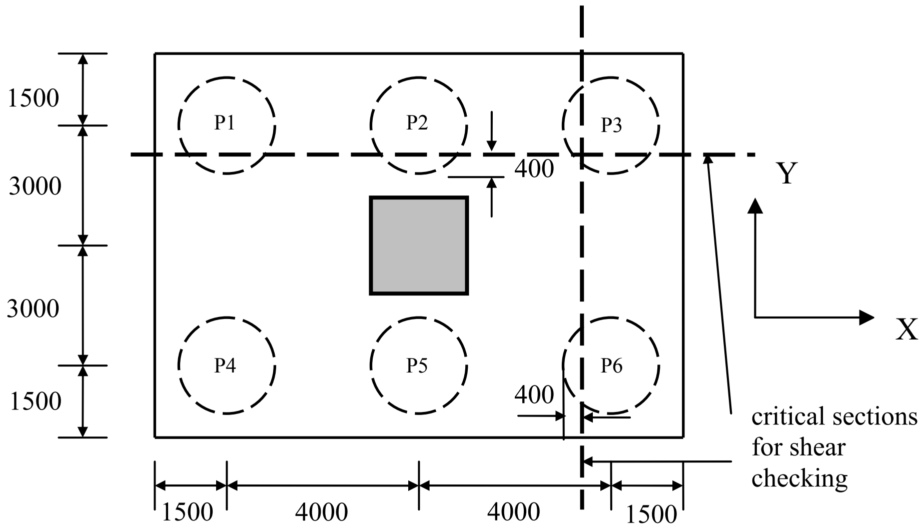


Figure 12.2 – Pile Cap Layout of Worked Example 12.1

(i) Factored Loads from the Column :

$P = 50000$ kN

$M_x = 2000$ kNm (along X-axis)

$M_y = 1000$ kNm (along Y-axis)

O.W. of Cap $11 \times 9 \times 2 \times 24.5 = 4851$ kN

Weight of overburden soil $11 \times 9 \times 1.5 \times 20 = 2970$ kN

Factored load due to O.W. of Cap and soil is

$$1.4 \times (4851 + 2970) = 10949.4 \text{ kN}$$

So total vertical load is $50000 + 10949.4 = 60949.4$ kN

(ii) <u>Analysis of Pile Loads – Assume all Piles are Identical</u>

(Reference to Appendix I for general analysis formulae)

I_x of pile group = $\sum x^2 = 6 \times 3^2 = 54$

I_y of pile group = $\sum y^2 = 4 \times 4^2 + 2 \times 0 = 64$

Pile Loads on P1 : $\dfrac{60949.4}{6} - \dfrac{2000 \times 4}{64} + \dfrac{1000 \times 3}{54} = 10088.79$ kN

P2: $\dfrac{60949.4}{6} - \dfrac{2000 \times 0}{64} + \dfrac{1000 \times 3}{54} = 10213.79$ kN

P3: $\dfrac{60949.4}{6} + \dfrac{2000 \times 4}{64} + \dfrac{1000 \times 3}{54} = 10338.79$ kN

P4: $\frac{60949.4}{6} - \frac{2000 \times 4}{64} - \frac{1000 \times 3}{54} = 9977.68\,\text{kN}$

P5: $\frac{60949.4}{6} - \frac{2000 \times 0}{64} - \frac{1000 \times 3}{54} = 10102.68\,\text{kN}$

P6: $\frac{60949.4}{6} + \frac{2000 \times 4}{64} - \frac{1000 \times 3}{54} = 10227.68\,\text{kN}$

(iii) Design for Bending along the X-direction

The most critical section is at the column face

Moment created by Piles P3 and P6 is

$(10338.79 + 10227.68) \times 3 = 61699.41\,\text{kNm}$

Counter moment by O.W. of cap and soil is

$10949.4 \times 4.5 \div 11 \times 2.25 = 10078.43\,\text{kNm}$

The net moment acting on the section is

$61699.41 - 10078.43 = 51620.99\,\text{kNm}$

$d = 2000 - 75 - 60 = 1865$ (assume 2 layers of T40); $b = 9000$

$\frac{M}{f_{cu}bd^2} = \frac{51620.99 \times 10^6}{40 \times 9000 \times 1865^2} = 0.0412$; $\frac{z}{d} = 0.95$

$A_{st} = 66978\,\text{mm}^2$, provide T40 – 200 (B1) + T25 – 200 (B3)

(A_{st} provided is 78638mm^2, i.e. $p = 0.469\,\%$, $v_c = 0.574\,\text{N/mm}^2$ by Table 6.3 of the Code.)

(iv) Design for Shear in the X-direction

By Cl. 6.7.3.2 of the Code, the critical section for shear checking is at 20% of the diameter of the pile inside the face of the pile as shown in Figure 12.2.

Total shear at the critical section is :

Upward shear by P3 and P6 is $10338.79 + 10227.68 = 20566.47\,\text{kN}$

Downward shear by cap's O.W. and soil is

$10949.4 \times \frac{2.1}{11} = 2090.34\,\text{kN}$

Net shear on the critical section is $20566.47 - 2090.34 = 18476.13\,\text{kN}$

$v = \frac{18476.13 \times 10^3}{9000 \times 1865} = 1.10\,\text{N/mm}^2 > v_c = 0.574\,\text{N/mm}^2$.

No shear enhancement in concrete strength can be effected as per Cl. 6.7.3.3(b) of the Code because no shear distribution across section has been considered.

Shear reinforcements in form of links per metre width is

$\frac{A_{sv}}{s_v} = \frac{b(v - v_c)}{0.87 f_{yv}} = \frac{1000(1.10 - 0.574)}{0.87 \times 500} = 1.209$

Use T12 links – 200 in X-direction and 400 in Y-direction by which $\frac{A_{sv}}{s_v}$ provided is 1.41.

(v) Design for Bending along the Y-direction

The most critical section is at the column face
Moment created by Piles P1, P2 and P3
$(10088.79+10123.79+10338.79)\times 2 = 30551.37\times 2 = 61102.74$ kNm
Counter moment by O.W. of cap and soil is
$10949.4\times 3.5\div 9\times 1.75 = 7451.68$ kNm
The net moment acting on the section is
$61102.74 - 7451.68 = 53651.06$ kNm
$d = 2000-75-60-40 = 1825$; (assume 2 layers of T40) $b = 11000$

$$\frac{M}{f_{cu}bd^2} = \frac{53651.06\times 10^6}{40\times 11000\times 1825^2} = 0.0366\ ;\ \frac{z}{d} = 0.95$$

$A_{st} = 71138$ mm^2, provide T40 – 200 (B2) + T25 – 200 (B4)
(A_{st} provided is 96113mm^2, i.e. $p = 0.479\%$, $v_c = 0.578$ N/mm^2 by Table 6.3 of the Code.)

(vi) Checking for Shear in the Y-direction
By Cl. 6.7.3.2 of the Code, the critical section for shear checking is at 20% of the diameter of the pile inside the face of the pile as shown in Figure 12.2
Total shear at the critical section is :
Upward shear by P1, P2 and P3 is 30551.37 kN
Downward shear by cap's O.W. and soil is

$$10949.4\times\frac{2.1}{9} = 2554.86\ \text{kN}$$

Net shear on the critical section is $30551.37 - 2554.86 = 27996.51$ kN

$$v = \frac{27996.51\times 10^3}{11000\times 1825} = 1.39\ \text{N/mm}^2 > v_c = 0.578\ \text{N/mm}^2$$ by Table 6.3 of the Code.

Similar to checking of shear checking in X-direction, no shear enhancement of concrete strength can be effected.

Shear reinforcements in form of links per metre width is

$$\frac{A_{sv}}{s_v} = \frac{b(v-v_c)}{0.87f_{yv}} = \frac{1000(1.39-0.578)}{0.87\times 500} = 1.867$$

As $\frac{A_{sv}}{s_v}$ in Y-direction is greater than that in X-direction, so adopt this for shear reinforcement provision.

Use T10 links – 200 BWs by which $\frac{A_{sv}}{s_v}$ provided is 1.96.

(vii) Punching Shear :
Punching shear check for the column and the heaviest loaded piles at their perimeters in accordance with Cl. 6.1.5.6(b) and (c) of the Code :

Column : $\frac{1.15\times 50000\times 10^3}{4\times 2000\times 1825} = 3.94\ \text{MPa} < 0.8\sqrt{f_{cu}} = 5.06\ \text{MPa}.$

Pile P3 : $\dfrac{1.25\times 10338.79\times 10^3}{2000\pi\times 1825}=1.13\,\text{MPa} < 0.8\sqrt{f_{cu}}=5.06\,\text{MPa}$.

Not necessary to check punching shear at the next critical perimeters as the piles and column overlap with each other to very appreciable extents;

(viii) Checking for Torsion : There are unbalanced torsions in any full width sections at X-Y directions due to differences in the pile reactions. However, as discussed in sub-section 11.2(vii) of this Manual for footing, it may not be necessary to design the torsion as for that for beams. Anyhow, the net torsion is this example is small, being $250\times 4=1000$ kNm along X-X (250kN is the difference in pile loads between P1 and P3), creating torsional shear stress in the order of $v_t=\dfrac{2T}{h_{min}^2\left(h_{max}-\dfrac{h_{min}}{3}\right)}=\dfrac{2\times 1000\times 10^6}{2000^2\left(11000-\dfrac{2000}{3}\right)}=0.048\,\text{N/mm}^2$. So the torsional shear effects should be negligible;

(ix) Finally reinforcement details are as shown in Figure 12.3,

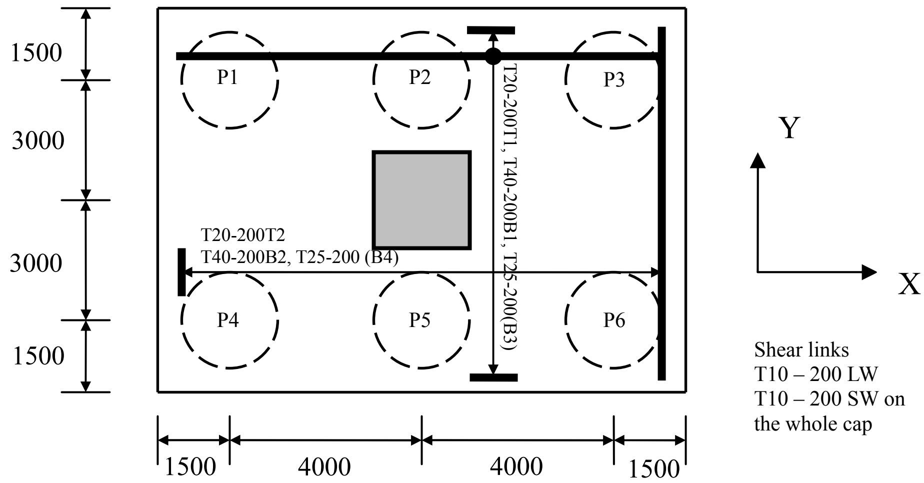


Figure 12.3 – Reinforcement Design of Worked Example 12.1

12.3 Strut-and-Tie Model

Cl. 6.7.3.1 of the Code allows pile cap be designed by the truss analogy, or more commonly known as "Strut-and-Tie Model" (S&T Model) in which a concrete structure is divided into a series of struts and ties which are beam-like members along which the stress are anticipated to follow. In a S&T model, a strut is a compression member whose strength is provided by concrete compression and a tie is a tension member whose strength is provided by added reinforcements. In the analysis of a S&T model, the following basic requirements must be met (Re ACI 318-11):

(i) Equilibrium must be achieved;
(ii) The strength of a strut or a tie member must exceed the stress induced on it;
(iii) Strut members cannot cross each other while a tie member can cross another tie member;
(iv) The smallest angle between a tie and a strut joined at a node should exceed 25°.

The Code has specified the following requirements (Cl. 6.7.3.1 of the Code):
(i) Truss be of triangular shape;
(ii) Nodes be at centre of loads and reinforcements;
(iii) For widely spaced piles (pile spacing exceeding 3 times the pile diameter), only the reinforcements within 1.5 times the pile diameter from the centre of pile can be considered to constitute a tension member of the truss.

12.4 Worked Example 12.2 (Strut-and-Tie Model)

Consider the pile cap supporting a column factored load of 6000kN supported by two piles with a column of size 1m by 1 m. The dimension of the cap is as shown in Figure 12.4, with the width of cap equal to 1.5 m.

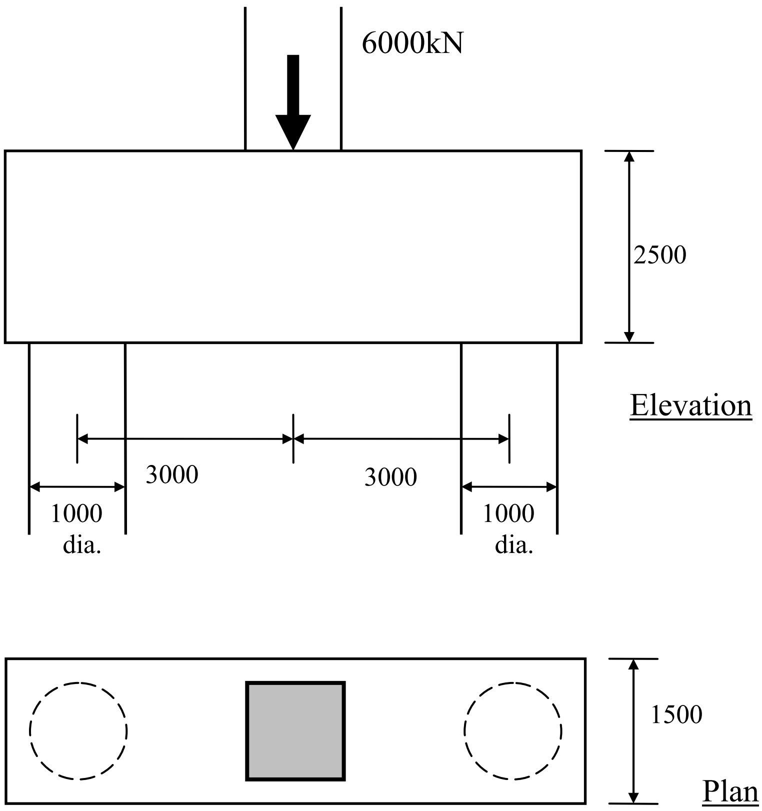


Figure 12.4 – Pile Cap Layout of Worked Example 12.2

(i) Determine the dimension of the strut-and-tie model
Assume two layers of steel at the bottom of the cap, the centroid of both

layers is at $75+40+20=135$ mm from the base of the cap. So the effective width of the tension tie is $135\times 2=270$ mm. The dimensions and arrangement of the ties and struts are drawn in Figure 12.5.

(ii) A simple force polygon is drawn and the compression in the strut can be simply worked out as
$2C\sin 38.25^0=6000 \Rightarrow C=4845.8$ kN;
And the tension in the bottom tie is $T=C\cos 38.25^0=3805.49$ kN.

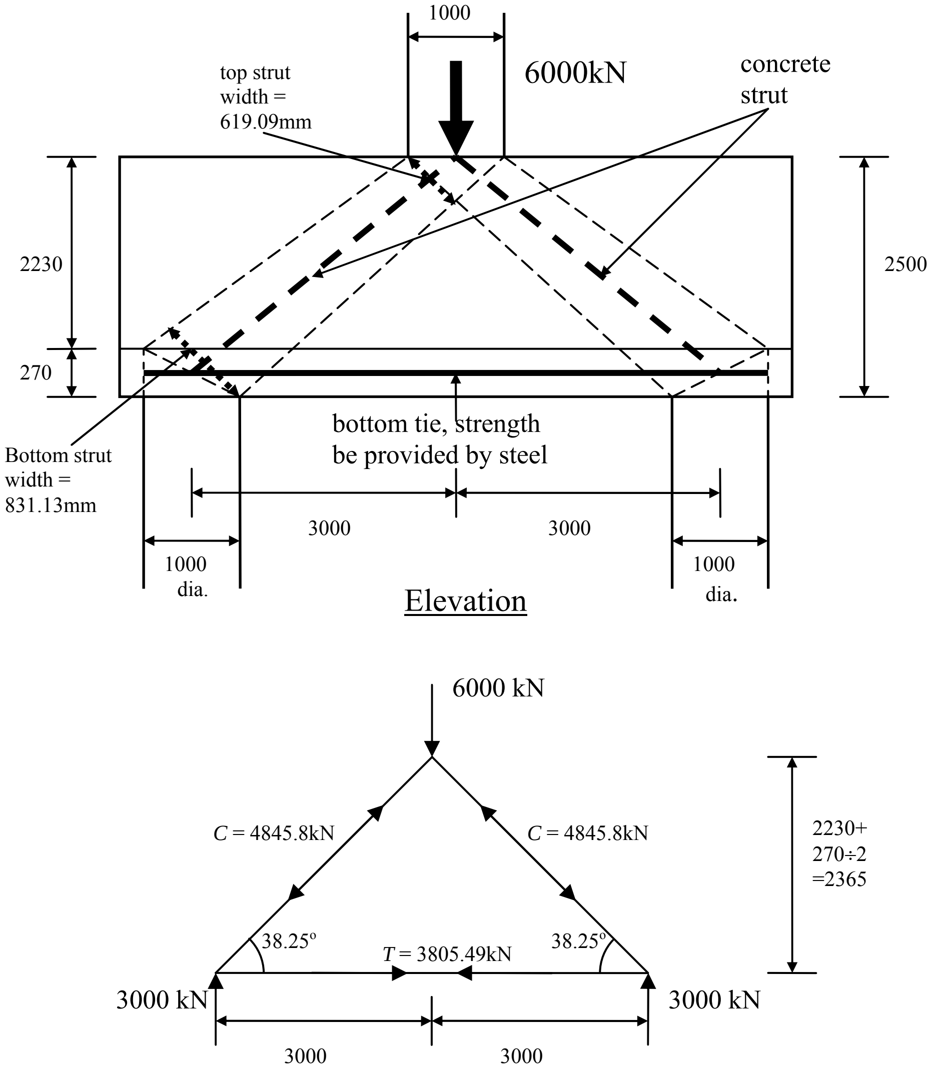


Figure 12.5 – Analysis of strut and tie forces in Worked Example 12.2

(iii) To provide the bottom tension of 3805.49 kN, the reinforcement steel required is $\dfrac{3805.49\times 10^3}{0.87f_y}=\dfrac{3805.49\times 10^3}{0.87\times 500}=8748\,\text{mm}^2$.
Use 6–T40 + 2–T32 (A_{st} provided is 9148mm^2);

(iv) Check stresses in the struts :
Bottom section of the strut, the strut width at bottom is
$1000\sin 38.25^0+270\cos 38.25^0=831.13$ mm
As the bottom part is in tension, there is a reduction of compressive

strength of concrete to $1.8\sqrt{f_{cu}} = 1.8\sqrt{40} = 11.38$ MPa as suggested by OAP, which is an implied value of the ultimate concrete shear strength of $0.8\sqrt{f_{cu}}$ as stated in the Code and BS8110.

As a conservative approach, assuming a circular section at the base of the strut since the pile is circular, the stress at the base of the strut is $\frac{4845.8\times10^3}{831^2\pi/4} = 8.93$ MPa < 11.38MPa

For the top section of the strut, the sectional width is $2\times500\sin 38.25^0 = 619.09$ mm

As the sectional length of the column is 1 m, it is conservative to assume a sectional area of 1000mm × 619.09 mm.

The compressive stress of the strut at top section is

$$\frac{4845.8\times10^3}{1000\times619.09} = 7.83\text{ MPa} < 0.45f_{cu} = 18\text{ MPa}$$

(v) The reinforcement details are indicated in Figure 12.6. Side bars are omitted for clarity.

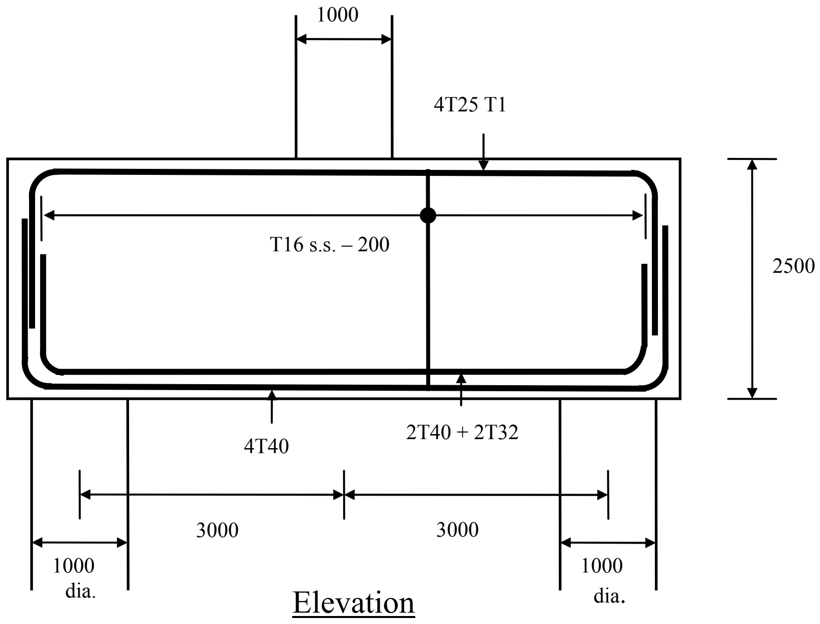


Figure 12.6 – Reinforcement Details of Worked Example 12.2

12.5 Flexible Cap Analysis

A pile cap can be analyzed by treating it as a flexible structure, i.e., as in contrast to the rigid cap assumption in which the cap is a perfectly rigid body undergoing rigid body movement upon the application of loads, the flexible pile cap structure will deform and the deformations will affect the distribution of internal forces of the structure and the reactions. Analysis of the flexible cap structure will require input of the stiffness of the structure which is comparatively easy. However, as similar to that of footing, the support stiffness of the pile cap which is mainly offered by the supporting pile is often difficult, especially for the friction pile which will interact significantly with each other through the embedding soil. Effects by soil restraints on the piles can be considered as less significant in end-bearing piles such as large diameter bored piles.

Similar to the flexible footing, as the out-of-plane loads and deformation are most important in pile cap structures, most of the flexible cap structures are modeled as plate structures and analyzed by the finite element method.

12.6 Analysis and Design by Computer Method

Analysis and design by computer method for pile cap are similar to Section 11.3 for footing. Nevertheless, as analysis by computer methods can often account for load distribution within the pile cap structure, Cl. 6.7.3.3 of the Code has specified the followings which are particularly applicable for pile cap design :

(i) shear strength enhancement of concrete may be applied to a width of 3ϕ for circular pile, or pile width plus 2 × least dimension of pile as shown in Figure 12.7 as shear distribution across section has generally been considered in flexible cap analysis (Cl. 6.7.3.3(c) of the Code);

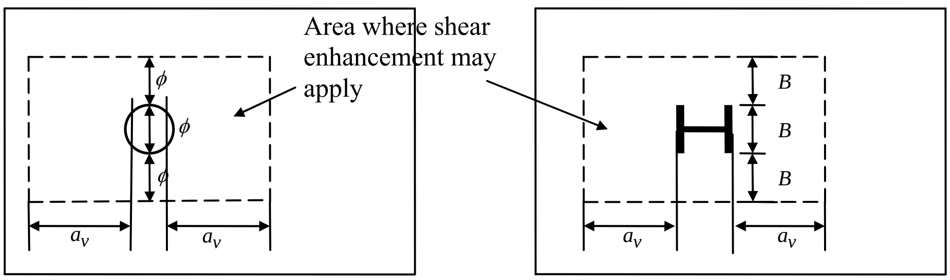


Figure 12.7 – Effective width for shear enhancement in pile cap around a pile

(ii) averaging of shear force shall not be based on a width > the effective depth on either side of the centre of a pile, or as limited by the actual dimension of the cap (cl. 6.7.3.3(d) of the Code).

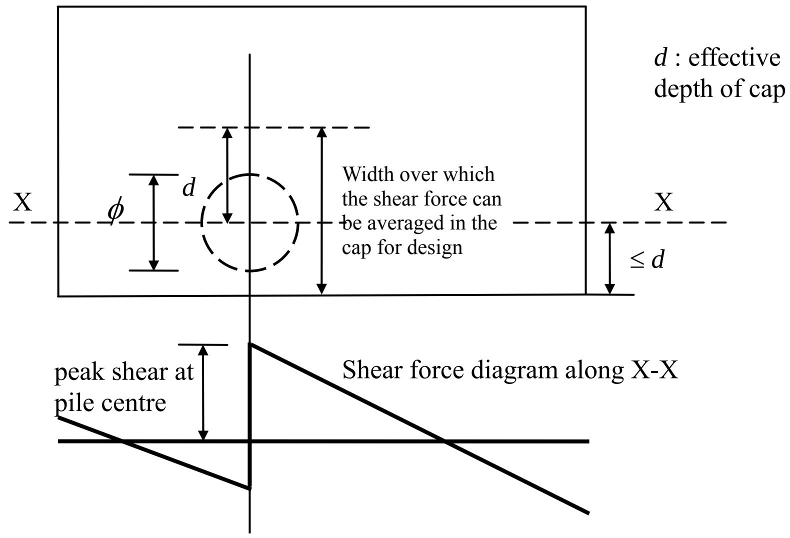


Figure 12.8 – Width in cap over which shear force at pile can be averaged for Design

Illustration in Figure 12.8 can be a guideline for determination of "effective widths" adopted in averaging "peak stresses" as will often be encountered in finite element analysis for pile cap structure modeled as an assembly of plate bending elements under point loads and point supports, as in the same manner as that for footing discussed in 11.4(ii) of this Manual.

13.0 General Detailing

13.1 In this section, the provisions of detailing requirements are general ones applicable to all types of structural members. They are mainly taken from Section 8 of the Code which are mostly quoted from the Eurocode BSEN1992:1:2004 Section 8. Again requirements marked with (D) in this Section of the Manual are ductility requirements.

13.2 Minimum spacing of reinforcements (Cl. 8.2 of the Code) – clear distance (horizontal and vertical) is the greatest of
(i) maximum bar diameter;
(ii) maximum aggregate size (h_{agg}) + 5 mm;
(iii) 20 mm.

13.3 Permissible bent radii of bars. The purpose of requiring minimum bend radii for bars are to
(i) avoid damage of bar;
(ii) avoid overstress by bearing on concrete in the bend.

Table 8.2 of the Code requires the minimum bend radii to be 2ϕ for $\phi \le 12$ mm; 3ϕ for $\phi < 20$ mm and 4ϕ for $\phi \ge 20$ mm (for both mild steel and ribbed steel reinforcing bar) and can be adopted without causing concrete failures if any of the conditions shown in Figure 13.1 is satisfied as per Cl. 8.3 of the Code.

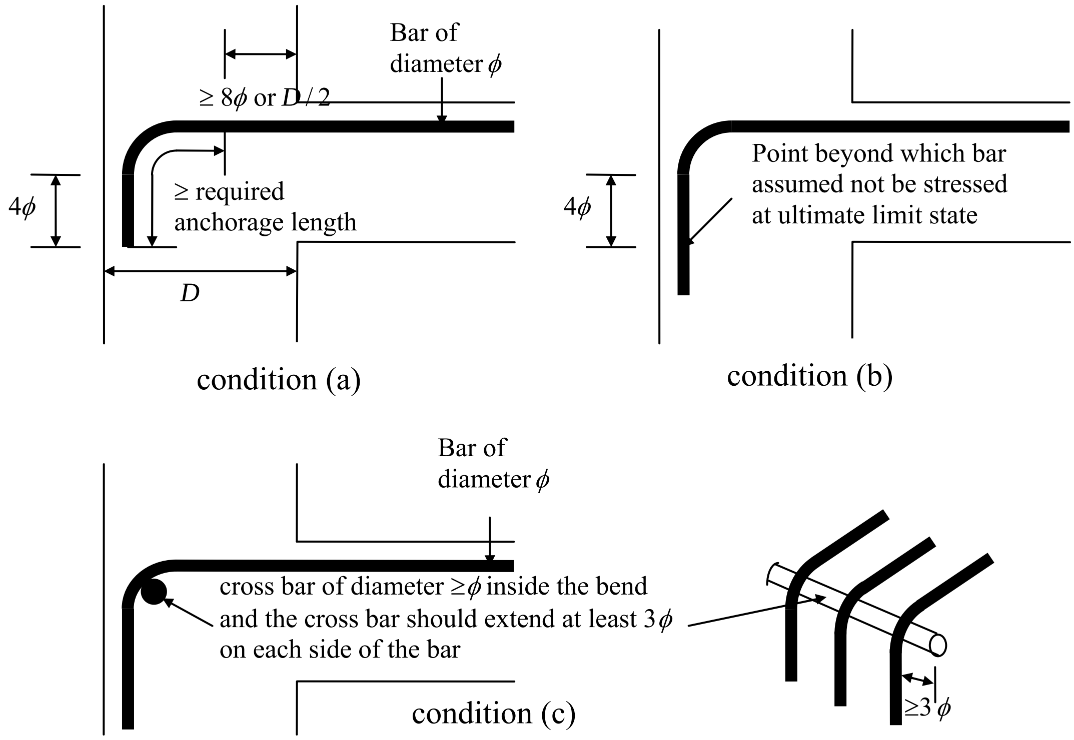


Figure 13.1 – Conditions by which Concrete Failure can be Avoided at Bend of Bars

If none of the conditions in Figure 13.1 is fulfilled, (Ceqn 8.1) of the Code, reproduced as (Eqn 13.1) in this Manual should be checked to ensure that bearing pressure inside the bend is not excessive.

bearing stress = $\dfrac{F_{bt}}{r\phi} \leq \dfrac{2f_{cu}}{\left(1+2\dfrac{\phi}{a_b}\right)}$ (Eqn 13.1)

In (Eqn 13.1), F_{bt} is the tensile force in the bar at the start of the bend; r the internal bend radius of the bar; ϕ is the bar diameter, a_b is centre to centre distance between bars perpendicular to the plane of the bend and in case the bars are adjacent to the face of the member, $a_b = \phi +$ cover.

Take an example of a layer of T40 bars of centre to centre separation of 100 mm and internal bend radii of 160mm in grade C35 concrete.

$F_{bt} = 0.87 \times 500 \times 1257 = 546795\text{ N}$

$$\frac{F_{bt}}{r\phi} = \frac{546795}{160 \times 40} = 85.44 > \frac{2f_{cu}}{\left(1+2\dfrac{\phi}{a_b}\right)} = \frac{2\times 35}{\left(1+2\times\dfrac{40}{100}\right)} = 38.89$$

So (Ceqn 8.1) is not fulfilled. Practically a cross bar should be added as in Figure 13.1(c) as conditions in Figure 13.1(a) and 13.1(b) can unlikely be satisfied.

13.4 Anchorage of longitudinal reinforcements

(i) Anchorage is derived from ultimate anchorage bond stress with concrete assessed by the (Ceqn 8.3) of the Code.
$f_{bu} = \beta\sqrt{f_{cu}}$ where for ribbed steel reinforcing bars $\beta = 0.5$ for tension and $\beta = 0.63$ for compression. For example, $f_{bu} = 0.5\sqrt{35} = 2.96\text{ MPa}$ for tension bar in grade C35 concrete. For a bar of diameter ϕ, the total force up to $0.87f_y$ is $0.87f_y\left(\dfrac{\phi^2\pi}{4}\right)$. The required bond length L will then be related by $0.87f_y\left(\dfrac{\phi^2\pi}{4}\right) = \beta\sqrt{f_{cu}}\pi\phi L \Rightarrow L = \dfrac{0.87f_y\phi}{4\beta\sqrt{f_{cu}}} = 36.8\phi \approx 38\phi$ which agrees with Table 8.4 of the Code;

(ii) Notwithstanding provision in (i), it has been stated in 9.9.1.2(c) of the Code which contains ductility requirements for longitudinal bars of beams anchoring into exterior column requiring anchorage length to be increased by 15% as discussed in Section 3.6 (v); (D)

(iii) With the minimum support width requirements as stated in Cl. 8.4.8 of the Code, bends of bars in end supports of slabs or beams will start beyond the centre line of supports offered by beams, columns and walls. By the same clause the requirement can be considered as not confining to simply supported beam as stated in Cl. 9.2.1.7 of the Code (originated from BS8110-1:1997 Cl. 3.12.9.4) as illustrated in Figure 13.2.

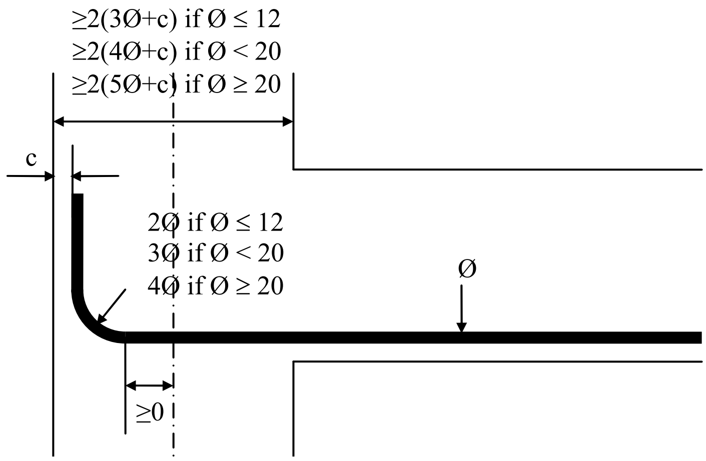


Figure 13.2 – Support width requirement

13.5 Anchorage of Links – Figure 8.2 of the Code displays bend of links of bend angles from 90° to 180°. However, it should be noted that the Code requires anchorage links in beams and columns to have bend angles not less than 135° as ductility requirements. But where adequate confinement to prevent end anchorage from "kick-off", the 135° hook may be replaced by other standard hooks as per Cl. 9.9.1.3(b), Cl. 9.9.2.2(c) of the Code; (D)

13.6 Laps Arrangement – Cl. 8.7.2 of the Code Requires Laps be "normally" staggered with the followings requirement for alternate lapping:

(i) Sum of reinforcement sizes in a particular layer must not exceed 40% of the breadth of the section at that level, otherwise the laps must be staggered;

(ii) Laps be arranged symmetrically;

(iii) Details of requirements in bar lapping are indicated in Figure 8.4 of the Code reproduced in Figure 13.3 for ease of reference;

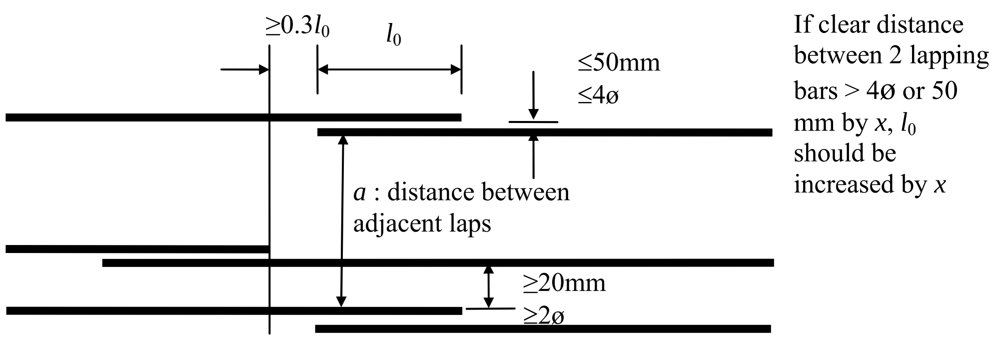


Figure 13.3 – Adjacent Laps

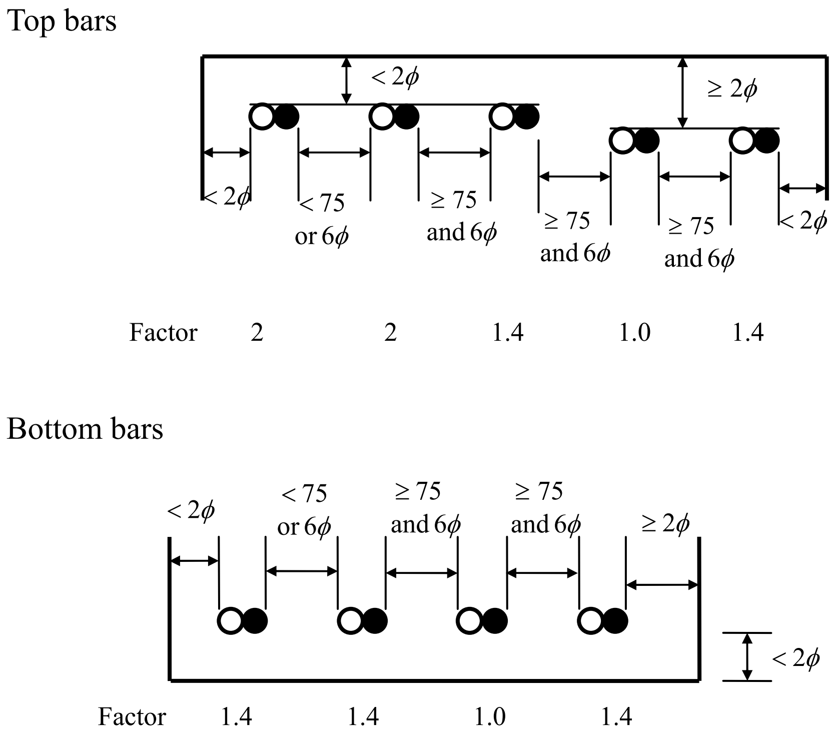
(iv) However, tension bars in one layer are allowed to have 100% lapping at any section (alternate lapping not required) while 50% lapping (alternate lapping) is allowed for bars in several layers;

(v) Compression and secondary reinforcements can be lapped in one section.

13.7 Lap Lengths (Cl. 8.7.3 of the Code)

The followings should be noted for tension lap lengths:

(i) Absolute minimum lap length is the greater of 15ϕ and 300 mm;
(ii) Tension lap length should be at least equal to the design tension anchorage length and be based on the diameter of the smaller bar;
(iii) Lap length be increased by a factor of 1.4 or 2.0 as indicated in Figure 13.4 which is reproduced from Figure 8.5 of the Code.



Note

Condition 1: Lap at top as cast and cover $< 2\phi$;

Condition 2 : Lap at corner and cover to either face $< 2\phi$

Condition 3 : Clear distance between adjacent laps < 75 or 6ϕ

Any one of the 3 conditions, factor is 1.4.

Condition 1 + 2 or conditions 1 + 3 : factor is 2.0

Figure 13.4 – Factors for tension lapping bars

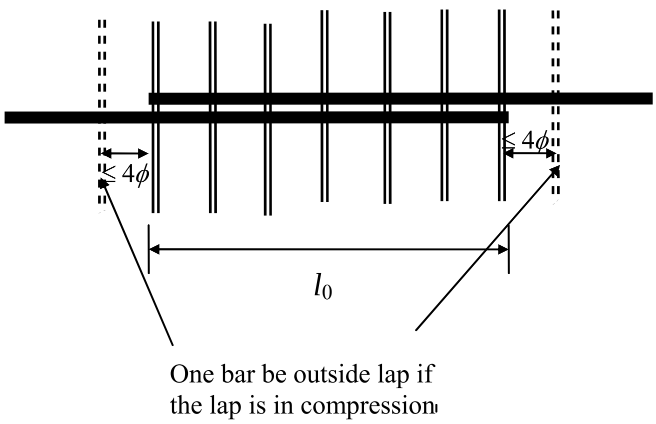
The compression lap length should be at least 25% greater than the design compression anchorage length as listed in Table 8.4 of the Code (Re Cl. 8.7.3.3).

13.8 Transverse Reinforcement in the Tension Lap Zone (Cl. 8.7.4 of the Code)

For lapped longitudinal bars in <u>tension</u>, the transverse reinforcement is used to resist transverse tension force which is originated from the lateral component of the force exerted by the lugs of the bars. The lateral force "pushes" the embedding concrete outwards and creates circumferential tension as explained in Law and Mak (2013) and extracted at Appendix J. Transverse reinforcements are thus used to resist the circumferential tension which serve to reduce concrete cracking so as to (1) enhance bonding of rebars with concrete; (2) enhance durability of concrete by greater resistance to rebar rusting. 3 cases should be considered as :

(i) No additional transverse reinforcement is required (existing transverse reinforcement for other purpose can be regarded as sufficient to resist the transverse tension forces) when the longitudinal bar diameter $\phi < 20$ mm or percentage of lapping in any section < 25%;

(ii) When $\phi \geq 20$ mm, the transverse reinforcement should have area $\sum A_{st} \geq A_s$ where A_s is the area of one spiced bar and be placed between the longitudinal bar and the concrete surface as shown in Figure 13.5;



For T40 bars in tension lap in concrete grade 35 with spacing 200 mm with lap length

l_0=1.4×standard lap = 2080mm. (52ϕ)

Transverse reinforcement area required is $\sum A_{st} = 1257$ mm^2.

Use 12T12, spacing is 180 mm along the lapped length.

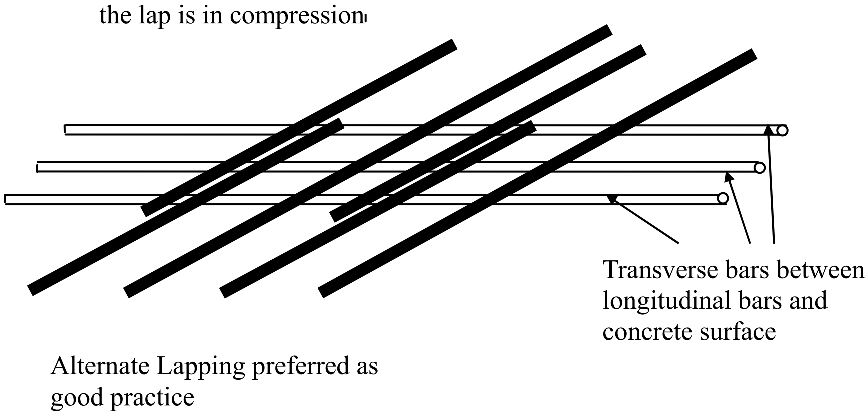


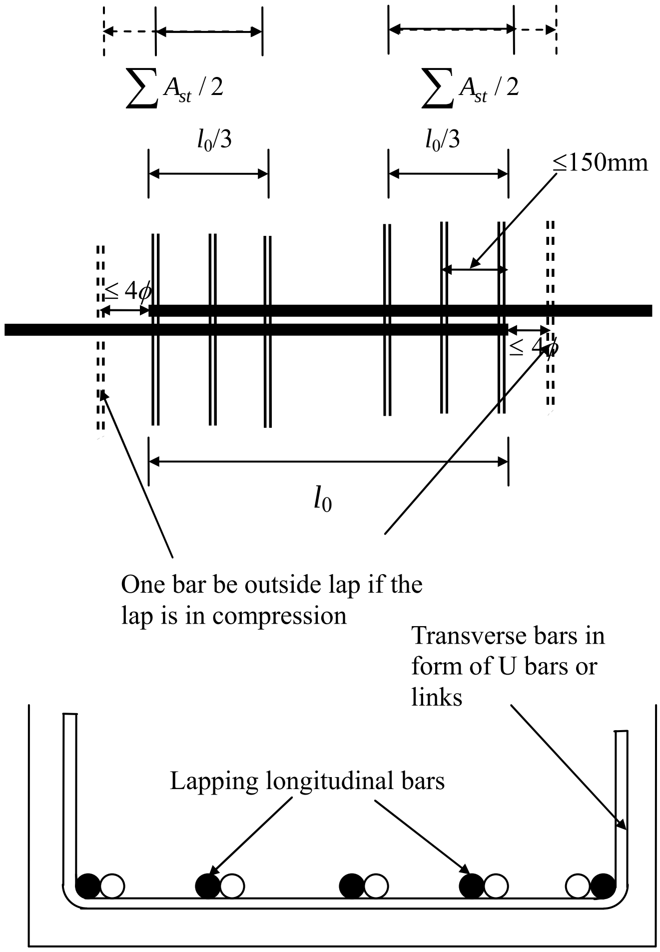
Figure 13.5 – Transverse Reinforcement for Lapped Splices – not greater than 50% of Reinforcement is Lapped at One section and $\phi \geq 20$ mm

(iii) If more than 50% of the reinforcement is lapped at one point and the distance between adjacent laps is $\leq 10\phi$, the transverse reinforcement should be formed by links or U bars anchored into the body of the section. The transverse reinforcement should be positioned at the outer sections of the lap as shown in Figure 13.6;

It should be noted that effectively condition (ii) requires area of transverse reinforcement identical to that of (iii), except that the bars need not be concentrated at the ends of the laps and the transverse reinforcements be in form of links or U bars.

13.9 Transverse Reinforcement in the Permanent Compression Lap Zone (Re Cl. 8.7.4.2)

The requirement will be identical to that of tension lap except that the placing of the transverse bars should extend by 4ϕ beyond each end of the lap as shown in Figures 13.5 and 13.6 (dotted lines shown for compression lap). The end transverse bar for the compression lap is used to reinforce concrete in resisting stresses created by the end-thrust of the compression bar.



For T40 bars in tension lap in concrete grade 35 with spacing 200 mm with lap length l_0=1.4×standard lap = 2080 mm.
$l_0/3$=693mm

Transverse reinforcement area required is $\sum A_{st} = 1257$ mm^2.

For tension lap, on each $l_0/3$=693mm, 1257/2 = 629mm^2 is required. So use 6T12, area provided is 678mm^2 over 2080/3=693 mm zone, i.e. spacing is 138mm < 150.mm.

For compression lap, also use 7T12, (629mm^2=1257/2) with 6T12 within 693mm (equal spacing = 138mm) and the 7[th] T12 at 150mm from the outermost transverse bar but ≤4∅ from the end of lap.

Figure 13.6 – Transverse Reinforcement for Lapped Splices – more than 50% is Lapped at One section and Clear Distance between Adjacent Laps ≤ 10∅

14.0 Design against Robustness

14.1 The Code defines the requirement for robustness in Clause 2.1.4 as "a structure should be designed and constructed so that it is inherently robust and not unreasonably susceptible to the effects of accidents or misuse, and disproportionate collapse." By disproportionate collapse, we refer to the situation in which damage to small areas of a structure or failure of single elements may lead to collapse of large parts of the structure.

14.2 By Cl. 2.2.2.3, design requirements comprises :

(i) building layouts checked to avoid inherent weakness;

(ii) capable to resist notional loads simultaneously at floor levels and roof as shown in Figure 14.1. (Re Cl. 2.3.1.4(a) of the Code)

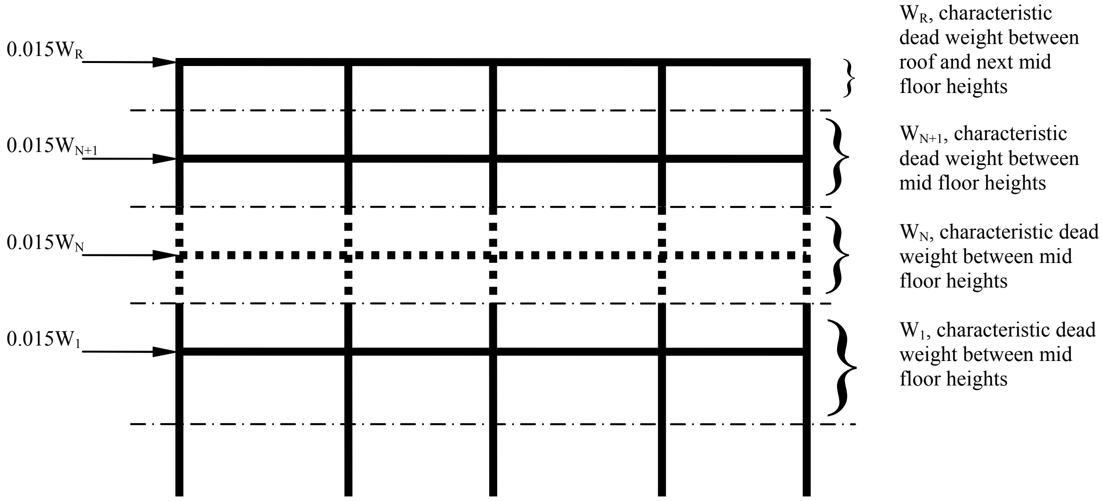


Figure 14.1 – Illustration of notional loads for robustness design

(iii) provide effective horizontal ties (in form of reinforcements embedded in concrete) (a) around the periphery; (b) internally; (c) to external columns and walls; and (d) vertical ties as per Cl. 6.4.1 of the Code.

14.3 Principles in Design of ties (Cl. 6.4.1.2 and Cl 6.4.1.3 of the Code)

(i) The reinforcements are assumed to be acting at f_y instead of $0.87f_y$;

(ii) To resist only the tying forces specified, not any others;

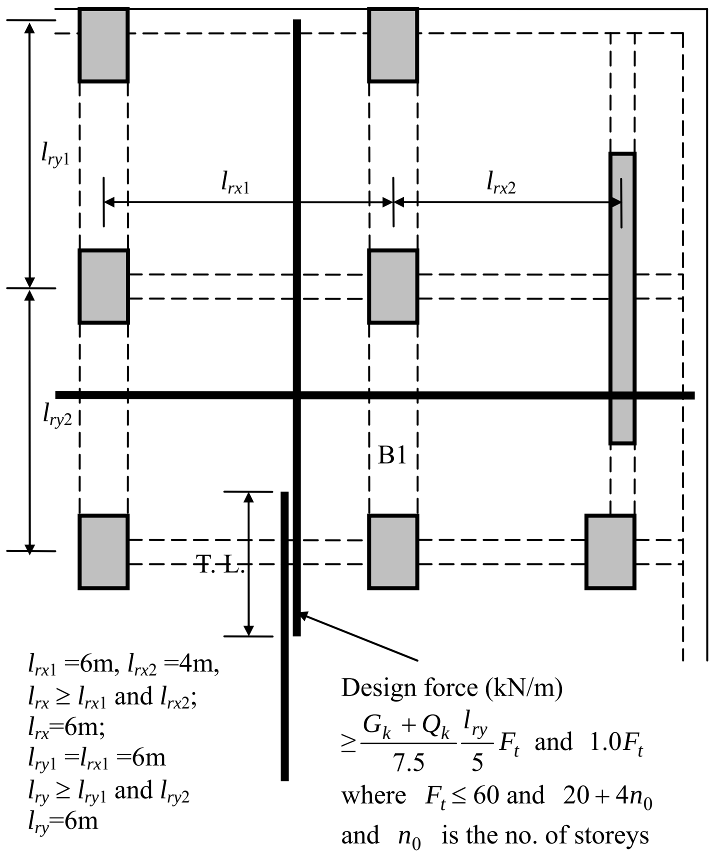
(iii) Reinforcements provided for other purpose can also act as ties;

(iv) Laps and anchorage of bars as ties similar to other reinforcements;

(v) Independent sections of a building divided by expansion joints have appropriate tying system.

14.4 Design of ties

(i) Internal Ties be provided evenly distributed in two directions in slabs – design force is illustrated in Figure 14.2 with example. The tie reinforcements can be grouped and provided in beam or wall.



<u>40 storeys building with average floor dead load as 9 kPa and average floor live load as 3 kPa</u>
<u>Internal ties in Y-direction</u>

$n_0 = 40$

$G_k = 9$ kPa; $Q_k = 3$ kPa

$l_{ry1} = l_{ry2} = 6$ m

$F_t \leq 60$ and $20 + 4n_0 = 180$

$\therefore F_t = 60$

$\frac{G_k + Q_k}{7.5}\frac{l_r}{5}F_t = 115.2$

$1.0F_t = 60$

Design tensile force in ties is 115.2 kN/m

Required A_{st} for internal ties is

$\frac{F}{f_y} = \frac{115.2 \times 10^3}{500} = 230$ mm²/m

Use T10 – 300
(can likely be met by DB or rebars provided for strength purpose)

If tying bars grouped in beam, total rebars in the middle beam B1 may be $230 \times (4 + 6)/2 = 1150$ mm², likely can be met by the longitudinal bars provided for strength purpose.

Figure 14.2 – Derivation of Internal Tie Reinforcement Bars in Slabs (Evenly Distributed)

(ii) Peripheral ties – Continuous tie capable resisting $1.0F_t$, located within 1.2 m of the edge of the building or within perimeter wall;

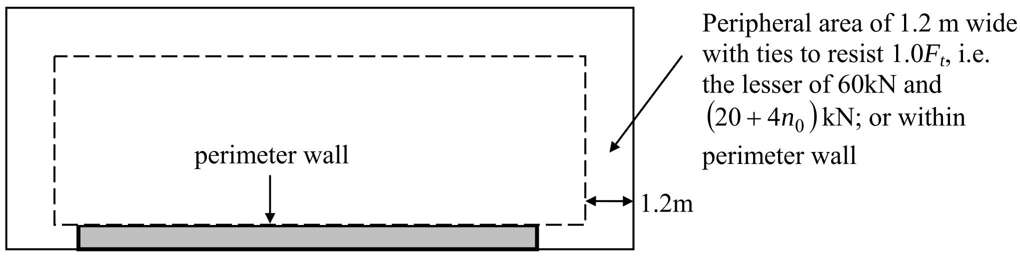
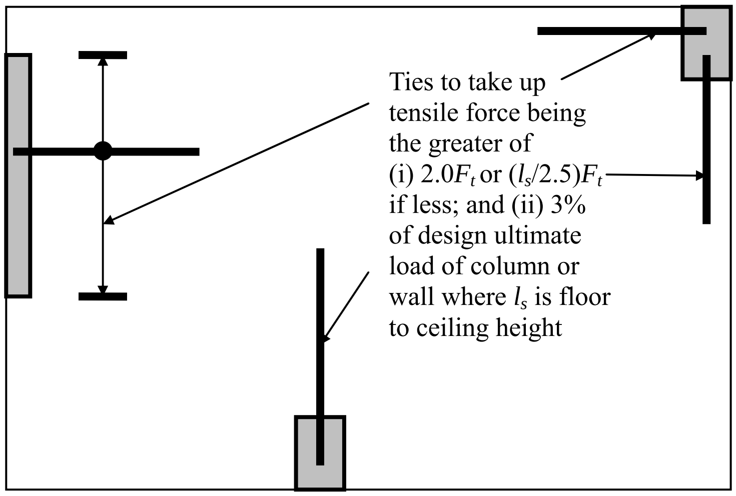


Figure 14.3 – Location and determination of Peripheral ties

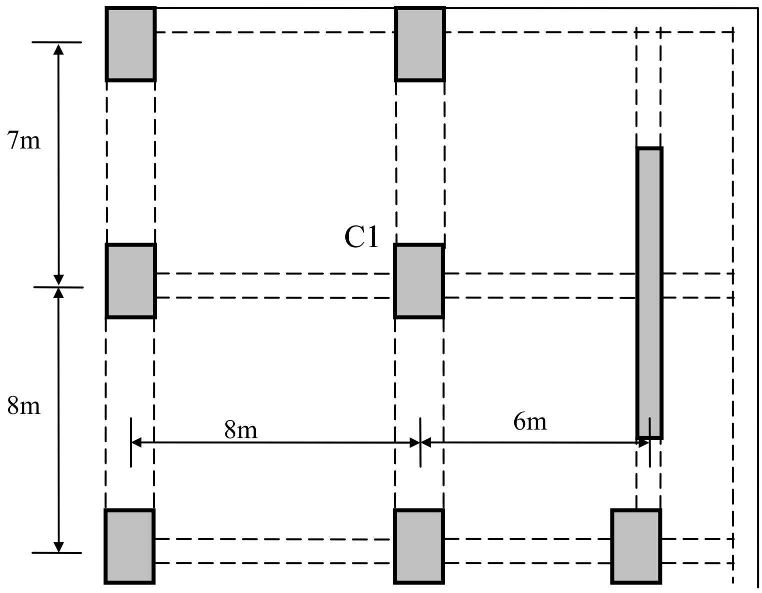
(iii) External columns and wall to have ties capable of developing forces as indicated in Figure 14.4;



Corner column in 40 storey building, tie design force >
(i) $2.0F_t$=2×60=120kN; $(l_s/2.5)F_t$ =(3/2.5)×60=72kN; smaller is 72kN
(ii) 400×600, grade C40 with 8T32 steel, design ultimate load is (400×600×0.45×40+ 0.87×500×6434)×10^{-3} =7119kN; 3% is 214kN
So design tie force is 214kN. Provide F/f_y = 214×10^3/500 = 428mm^2; can likely be provided by beam steel anchoring into the column.

Figure 14.4 – Ties to External Column and Wall with Example

(iv) Vertical ties provided to wall and column should be continuous and be capable of carrying exceptional load. Use γ_f ×[dead load + 1/3 imposed load + 1/3 wind load] of one floor to determine the design load for the vertical ties where γ_f = 1.05.

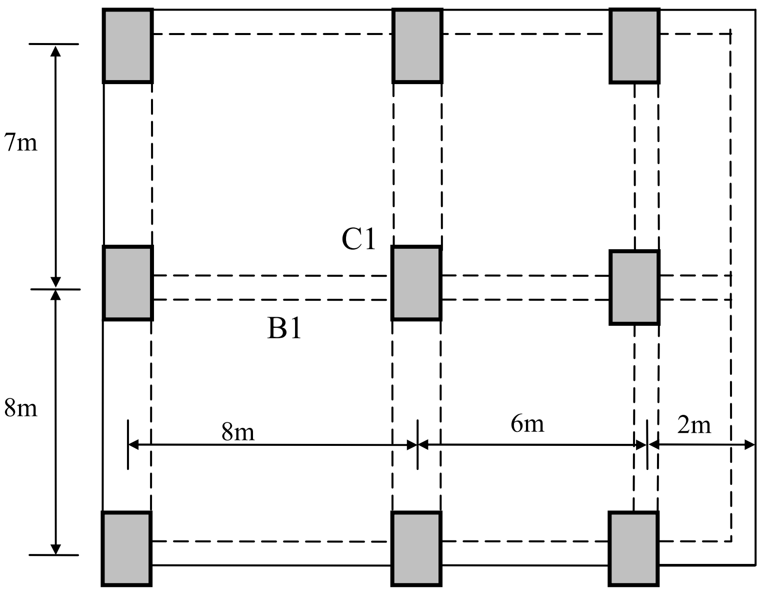


Design load for the vertical tie of the interior column C1 is
(i) Dead load from one storey 800kN;
(ii) One third of imposed load from one storey 1/3×240 = 80 kN;
(iii) One third of wind load = 1/3×90=30kN
Designed tensile load is 1.05(800+80+30) = 956kN.
Requiring 956000/500 = 1912mm^2

Figure 14.5 – Design of Vertical Ties in Columns and Walls

14.5 Design of "Key Elements"

When for some reasons it is not possible to introduce ties as described above, key elements (usually columns or walls), the failure of which will cause disproportionate collapse shall be identified. These elements and the supporting building components should be designed for an ultimate load of 34 kN/m^2. In the design, no partial safety factor should be applied in any direction as per Cl. 2.3.1.4(b) of the Code. In addition, the Code has not defined the extent of "disproportionate collapse" for the element to be qualified as a "key element". However, reference can be made to the "Code of Practice for the Structural Use of Steel 2011" Cl. 2.3.4.3 by which an element will be considered a key element if the removal of it will cause collapse of 15% of the floor area or 70m^2, whichever is the greater. The design is illustrated in Figure 14.6.



Examples

C1 is identified as a key element as the collapse of which will lead to disproportionate collapse of area around it (more than 15% collapse of the floor).
The tributary area is $7.5\times7=52.5\text{m}^2$;
The design load is $34\times52.5=1785\text{kN}$.

Similarly, beam B1 is also required as the removal of which will cause more than 15% collapse of the floor. So B1 needs be designed for a u.d.l. of 34kPa on the linking slabs.

Figure 14.6 – Design of Key Elements

14.6 Nevertheless, it should be noted that requirements in the Code for robustness design often poses no additional requirements in monolithic reinforced concrete design in comparison with the criteria listed in 14.2 as per the followings:

(i) Normally no inherent weakness in the structure for a reasonable structural layout;

(ii) Ultimate wind loads normally applied to the structure according to the local Wind Code can usually cover the notional loads (1.5% characteristic dead weight) specified in 14.2(ii);

(iii) Requirements for various types of ties can normally be met by the reinforcements provided for other purposes. Nevertheless, continuity and end anchorage of the ties should be checked.

15.0 Shrinkage and Creep

15.1 Shrinkage

Shrinkage is the shortening movement of concrete as it dries after hardening. If the movement is restrained, stress and/or cracking will be created.

(Ceqn 3.5) gives estimate of drying shrinkage of plain concrete under un-restrained conditions. Together with the incorporation of the "reinforcement coefficient" K_s, the equation can be written as

$$\varepsilon_s = c_s K_L K_c K_e K_j K_s \qquad \text{(Eqn 15-1)}$$

where $c_s = 2.5$ and other coefficients can be found by Figures 3.3, 3.5, 3.6 and 3.7 and (Ceqn 3.4) of the Code, depending on atmospheric humidity, dimensions, compositions of the concrete, time and reinforcement content. It should be noted that K_j is a time dependent coefficient.

The equation and the figures giving values of the various coefficients are adopted from BS5400 which in turn are quoted from CEB-FIP *International Recommendation for the Design and Construction*, 1970 (CEB 1970) (MC-70). It should, however, be noted that the coefficient c_s is extra to CEB-FIP. The value accounts for the comparatively higher shrinkage value (2.5 times as high) found in Hong Kong. However, it should be noted that the Structural Design Manual of Highways (2013) has adopted the approach in Eurocode Code EC2.

Shrinkage is always in contraction.

15.2 Creep

Creep is the prolonged deformation of the structure under sustained stress. (Ceqn 3.2) and (Ceqn 3.3) give estimate of the creep strain :

$$\text{Creep strain} = \frac{stress}{E_{28}} \times \phi_c \qquad \text{(Eqn 15-2)}$$

where $\phi_c = K_L K_m K_c K_e K_j K_s$ (Eqn 15-3)

Again (Eqn 15-3) has incorporated the reinforcement coefficient K_s.

Thus creep strain depends on the *stress* in the concrete and various coefficients related to parameters similar to that of shrinkage (which can be read from Figures 3.1 to 3.5 of the Code) with K_m and K_j dependent on time. As stress and strain are inter-dependent, it will be shown that assessment of strain will require successive time staging in some cases.

Creep creates deformation in the direction of the stress. In case shrinkage which results in tensile stress under restrained condition such as a floor structure under lateral restraints, the creep strain will serve to relax the stress due to shrinkage. Both stress and strain due to shrinkage and creep vary with time, as can be shown in the analyses that follow. It should be noted that if the floor is un-restrained, no shrinkage stress will be induced as the floor is free to shrink.

15.3 The determination of the time dependent coefficients K_m and K_j as listed in (Ceqn 3.3) and (Ceqn 3.5) will be tedious in calculation of stress and strain of a structure in a specified time step which may involve reading the figures many times. Curves in Figures 3.2 and 3.5 are therefore simulated by polynomial equations as shown in Appendix K to facilitate determination of the coefficients by spreadsheets.

15.4 Worked Example 15.1

A grade C35 square column of size 800 × 800 in a 4 storey building with reinforcement ratio 2% is under an axial stress from the floors as follows :

Floor	Height (m)	Time of stress creation from floor (days)	Stress (MPa)
G	4	28	3.5
1st	3	56	2.1
2nd	3	84	2.1
3rd	3	120	3.5

Table 15.1 – Data for Worked Example 15.1

Strain and shortening of the G/F column due to shrinkage and creep at 360 days are determined as follows :

Shrinkage

The coefficients for determination of the free shrinkage strain are as follows :
$K_L = 0.000275$ for normal air from Figure 3.6;
Based on empirical formulae, for grade C35:
Water / Cement ratio = $-0.0054 f_{cu} + 0.662 = -0.0054 \times 35 + 0.662 = 0.473$
Cement content = $3.6 f_{cu} + 308 = 3.6 \times 35 + 308 = 434$ kg/m^3
From Figure 3.3 $K_c = 1.17$;
For the 800 × 800 column, the effective thickness h_e, defined as the ratio of the area of the section A, to the semi-perimeter, $u/2$ (defined in Cl. 3.1.7 of the Code) is $\frac{800 \times 800}{800 \times 4/2} = 400$ mm. So from Figure 3.7, $K_e = 0.55$;
From Figure 3.5 of the Code, time at 360 days $K_j = 0.51$;
$K_s = \frac{1}{1 + \rho\alpha_e} = 0.856$; where $\rho = 0.02$ (2% steel) and $\alpha_e = \frac{200}{23.7} = 8.44$
So the shrinkage strain under perfectly free condition is :
$\varepsilon_s = c_s K_L K_c K_e K_j K_s = 2.5 \times 0.000275 \times 1.17 \times 0.55 \times 0.51 \times 0.856 = 193.14 \times 10^{-6}$

Creep

For estimation of creep strain, Creep strain $\varepsilon_c = \frac{stress}{E_{28}} \times \phi_c$

where $\phi_c = K_L K_m K_c K_e K_j K_s$ and $E_{28} = 23.7$ GPa for grade C35 concrete. All coefficients are same as that for shrinkage except $K_L = 2.3$ (Figure 3.1), $K_m = 1.0$ (Figure 3.2 – loaded at 28 days) and $K_e = 0.72$ (Figure 3.4)

Load from Floor	Concrete age at time of load (Day)	K_m	Time since Loading (Day)	K_j	ϕ_c	Stress by Floor (MPa)	ε_c ($\times 10^{-6}$)
1/F	28	1.000	332	0.489	0.811	3.5	119.73
2/F	56	0.850	304	0.467	0.658	2.1	58.30
3/F	84	0.761	276	0.443	0.560	2.1	49.61
Roof	120	0.706	240	0.412	0.482	3.5	71.15
						$\sum \varepsilon_c =$	298.80

Table 15.2 – Summary of Results for Worked Example 15.1

So the creep strain at 360 days is $\varepsilon_c = 298.80 \times 10^{-6}$

Elastic strain

The elastic strain is simply $\varepsilon_e = \dfrac{\sigma}{E} = \dfrac{3.5 + 2.1 + 2.1 + 3.5}{23700} = 472.57 \times 10^{-6}$

So the total strain is

$\varepsilon = \varepsilon_s + \varepsilon_c + \varepsilon_e = 193.14 \times 10^{-6} + 298.80 \times 10^{-6} + 472.57 \times 10^{-6} = 964.51 \times 10^{-6}$.

Total shortening of the column at G/F is at 360 days is $\varepsilon \times H = 964.51 \times 10^{-6} \times 4000 = 3.86$ mm.

15.5 Estimation of Shrinkage and Creep Effect on Restrained Floor Structure

It is well known that shrinkage and creep effects of long concrete floor structures can be significant. The following derivations which are based on Lam and Law (2012) aim at providing a design approach to account for such effects based on recommendations by the Code.

Consider a floor structure of span L undergoing shrinkage and creep and is being restrained at its ends. The linear deformation δ with shrinkage, creep and elastic strains denoted respectively as ε_{cs}, ε_{cc} and ε_{ce} can be formulated as follows :

$$\delta = L(\varepsilon_{cs} - \varepsilon_{cc} - \varepsilon_{ce}) \qquad \text{(Eqn 15-4)}$$

As $\varepsilon_{ce} = \dfrac{\sigma}{E_{28}}$ and with (Eqn 15-2), we may list (dropping the suffix 28 for *E*)

$$\delta = L\left[\varepsilon_{cs} - \frac{\sigma}{E}(1 + \phi_c)\right] \qquad \text{(Eqn 15-5)}$$

If the restraint of the floor structure is by two vertical members of lateral support stiffness $K_{\sup 1}$ and $K_{\sup 2}$ as shown in Figure 15.1, let the lateral deflections at supports 1 and 2 be δ_1 and δ_2 respectively and the total shortening of the floor is δ. By (Eqn 15-5) at any time we may list

$$\delta = \delta_1 + \delta_2 = L\left[\varepsilon_{cs} - \frac{\sigma}{E}(1+\phi_c)\right]$$

Let F be the axial force induced in the floor structure.

As $\delta_1 = F \times f_{\sup 1} = \sigma A f_{\sup 1}$, $\delta_2 = F \times f_{\sup 2} = \sigma A f_{\sup 2}$, we may list

$$L\left[\varepsilon_{cs} - \frac{\sigma}{E}(1+\phi_c)\right] = \delta_1 + \delta_2 = f_{\sup 1}\sigma A + f_{\sup 2}\sigma A \Rightarrow \sigma\left(1+\phi_c + \frac{f_{\sup 1} + f_{\sup 2}}{f_b}\right) = E\varepsilon_{cs}$$

$$\Rightarrow \sigma = \frac{E\varepsilon_{cs}}{(1+\phi_c + f_r)} \qquad \text{(Eqn 15-6)}$$

by putting $f_b = L/AE$ and $f_r = (f_{\sup 1} + f_{\sup 2})/f_b$

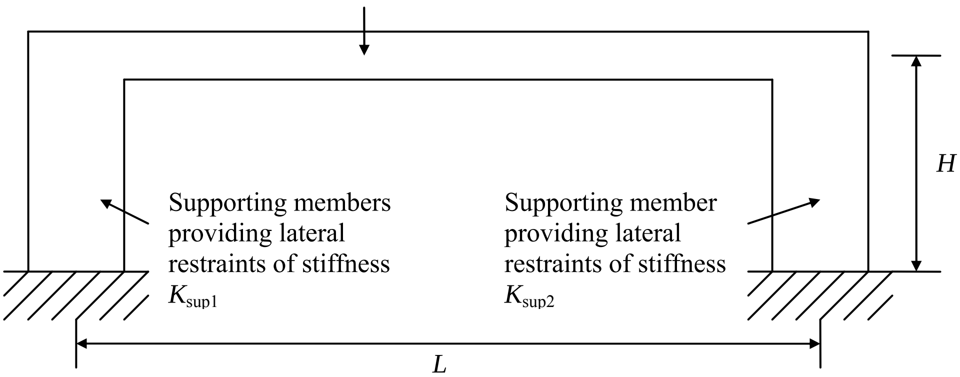


Figure 15.1 – Idealization of Floor Structure for Shrinkage and Creep Estimation

In the determination of stress due to shrinkage and creep, the main difficulty lies in the determination of ϕ_c which is time dependent. Stress in concrete has therefore to be determined in successive time steps and with numerical method as demonstrated in Figure 15.2 for calculation of the creep strains.

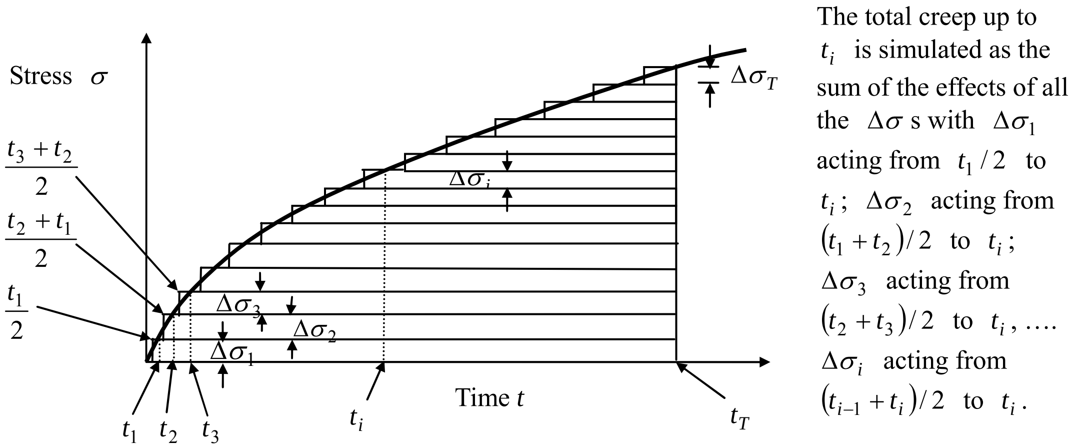


Figure 15.2 – Decomposition of Stress History into Stress Steps

In the figure, instead of being treated as continuously increasing, the stress is split up into various discrete values, each of which commences at pre-determined "station" of times. Fine divisions of time steps will create good simulation of the actual performance. By utilizing the principle of superposition, which assumes that the response of a sum of a series of stress (or strain) steps is the sum of the responses to each of them taken separately, we may apply (Eqn 15-6) to each of the stress steps with stresses $\Delta\sigma_1$, $\Delta\sigma_2$, $\Delta\sigma_3,\ldots\Delta\sigma_i$ at selected times t_1, t_2, $t_3,\ldots\ldots t_i$ and the total stress at time t_i is

$$\sum_{j=1}^{i}\Delta\sigma_j\left(1+\phi_{cij}+f_r\right)=\sum_{j=1}^{i}E\varepsilon_{csj}=E\sum_{j=1}^{i}\varepsilon_{csj}=E\varepsilon_{csi} \qquad \text{(Eqn 15-7)}$$

In (Eqn 15-7), E, f_r are constants. ε_{csi} is the total shrinkage strain up to time t_i which can be determined by (Eqn 15-1) and the creep coefficient ϕ_{cij} for individual time steps which is independent of the stress can be determined by (Eqn 15-2). It should be noted that in determining ϕ_{cij} for the j^{th} time step, K_m is at concrete age $\left(t_j+t_{j-1}\right)/2$ whilst the time duration of load for determining K_j is $t_i-\left(t_j+t_{j-1}\right)/2$ as illustrated in Figure 15-2. By assigning $j=1$, 2, ...i. i no. of equations can be formulated for solving $\Delta\sigma_1$, $\Delta\sigma_2$, $\Delta\sigma_3,\ldots\ldots\Delta\sigma_i$ and the total stress at time t_i

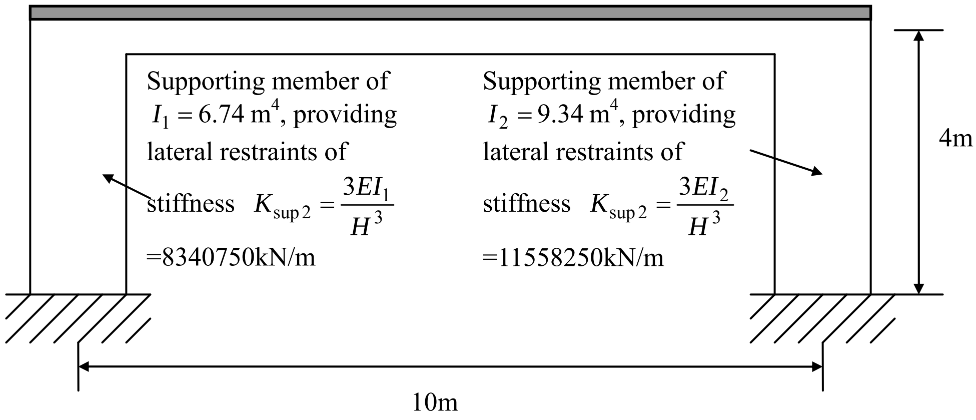
$$\sigma_i=\sum_{j=1}^{i}\Delta\sigma_j\ . \qquad \text{(Eqn 15-8)}$$

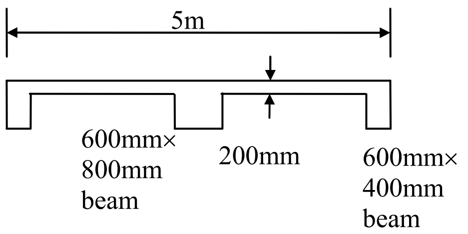
Or at the end of the time history t_T, $\sigma_T=\sum_{j=1}^{T}\Delta\sigma_j$ (Eqn 15-9)

A closer examination will reveal that the solution process is in fact a forward substitution of equations in solving the $\Delta\sigma_i$s. As it is more common in structural analysis to use stiffness instead of flexibility, the ratio $K_r=f_r$ where K_r is expressed in term of stiffness may be used instead. It is easy to prove that $K_r=f_r=K_b\left(1/K_{\sup 1}+1/K_{\sup 2}\right)$ where $K_b=AE/L$ is the axial stiffness of the floor structure (inverses to the floor flexibility) and $K_{\sup 1}$, $K_{\sup 2}$ as the support stiffness of supports 1 and 2 (again inverses of $f_{\sup 1}$ and $f_{\sup 2}$ respectively defined as the force required to produce unit displacement at supports 1 and 2 at the floor level).

15.6 Worked Example 15.2

Based on the derivations in 15.5, a single bay storey floor structure of width 5m, concrete grade C35 ($E_{28}=23.7$ GPa) represented by Figure 15.3 is analyzed for its stress due to shrinkage and creep. The concrete grade of the supporting corewalls is C45. The cross section of the floor structure is as shown in Figure 15.3 which is a beam slab structure. For analysis, the "effective thickness of the slab" is to be determined. The effective thickness is, in accordance with Cl. 3.1.7 of the Code, is the ratio of the area of the section to the semi-perimeter which is half of the perimeter length, in contact with the atmosphere. If one of the dimensions of the section under consideration is very large compared with the other, the effective thickness corresponds approximately to the actual thickness. This parameter governs the rate of loss of water, i.e. shrinkage and thus determines the coefficients K_j and K_e.





Cross Sectional Area of the Floor is $5\times0.2+2\times0.4\times0.4+0.4\times0.8=1.64$ m^2;
So axial stiffness
$$K_b=\frac{AE}{L}=\frac{23.7\times10^6\times1.64}{10}=3886800 \text{ kN/m}$$
Perimeter is $5\times2+0.6\times2+0.4\times4=12.8$ m
Effective thickness is $1.64/(12.8/2)=0.256$ m

Figure 15.2 – Structure for Worked Example 15.2

From the above, $K_r = K_b\left(1/K_{\sup 1}+1/K_{\sup 2}\right)=0.8023$.

Relative humidity is taken as normal at 70%. The time independent K coefficients as used in (Eqn 15-1) and (Eqn 15-3) are determined as :
For shrinkage, $K_L = 275\times10^{-6}$; $K_c = 1.17$; $K_e = 0.71$; $K_s = 1.0$;
For creep, $K_L = 2.3$; $K_c = 1.17$; $K_e = 0.79$; $K_s = 1.0$.

In the determination of K_c which is dependent on the cement content and water/cement ratio, it can be reduced to the simple formula of $K_c = -0.016D + 1.73$ where D is the concrete grade based on the usual cement content and water/cement ratio of the concrete grade. So in this example $K_c = -0.016\times35+1.73=1.17$. In addition, the coefficients K_m and K_j are determined as discussed in the foregoing. The successive "time points" in this example are taken as 3, 7, 14, 28, 40, ….., 480 days after the casting of the floor structure. So for the first time point where there is only one time step, the stress $\Delta\sigma_1$ is assumed to commence (0+3)/2 = 1.5days (average of 0 and 3) after casting of concrete and last to 3 days. For the second time point where the time history is considered to be made up of two time steps, the first one will commence at 1.5days (as in the previous time point) and last to 7 days whilst the second time step will start at (3+7)/2 = 5days and also last to 7 days. Timings and durations for the subsequent time points are worked out similarly for the determination of K_m and K_j as indicated in Table 15.3. The $\Delta\sigma$ s are solved by the application of (Eqn 15-7).

Time Point (Days)	Shrinkage			Creep					$\Delta\sigma$ (kPa) for time step	σ at end of time point (kPa)
	Time after casting (days)	K_j	ε_{cs} ($\times 10^{-6}$)	Time step	load starts (days)	K_m	K_j	ϕ_c		
1	3	0.05653	32.16	1	1.5	1.7438	0.0492	0.1826	*384.53*	*384.53*
2	7	0.07836	44.58	1	1.5	1.7438	0.0716	0.2659	384.53	
				2	5	1.4667	0.0513	0.1601	*133.88*	*518.42*
3	14	0.11767	66.94	1	1.5	1.7438	0.1083	0.4025	384.53	
				2	5	1.4667	0.0862	0.2694	133.88	
				3	10.5	1.2885	0.0596	0.1635	*235.93*	*754.35*
4	28	0.18981	107.98	1	1.5	1.7438	0.1833	0.6807	384.53	
				2	5	1.4667	0.1673	0.5227	133.88	
				3	10.5	1.2885	0.1394	0.3826	235.93	
				4	21	1.0617	0.0784	0.1772	*394.47*	*1148.82*

Table 15.3 – Successive Determination of Stress of Worked Example 15.2

The process is carried out until the time point of 480 days is reached and the increase in stresses due to shrinkage and creep of the floor structure with time are plotted in Figure 15.3. It can be seen that the stress increases are fast at the beginning and slow down as time passes which, as anticipated, become almost exponential. The 480 days stress is 3.03MPa.

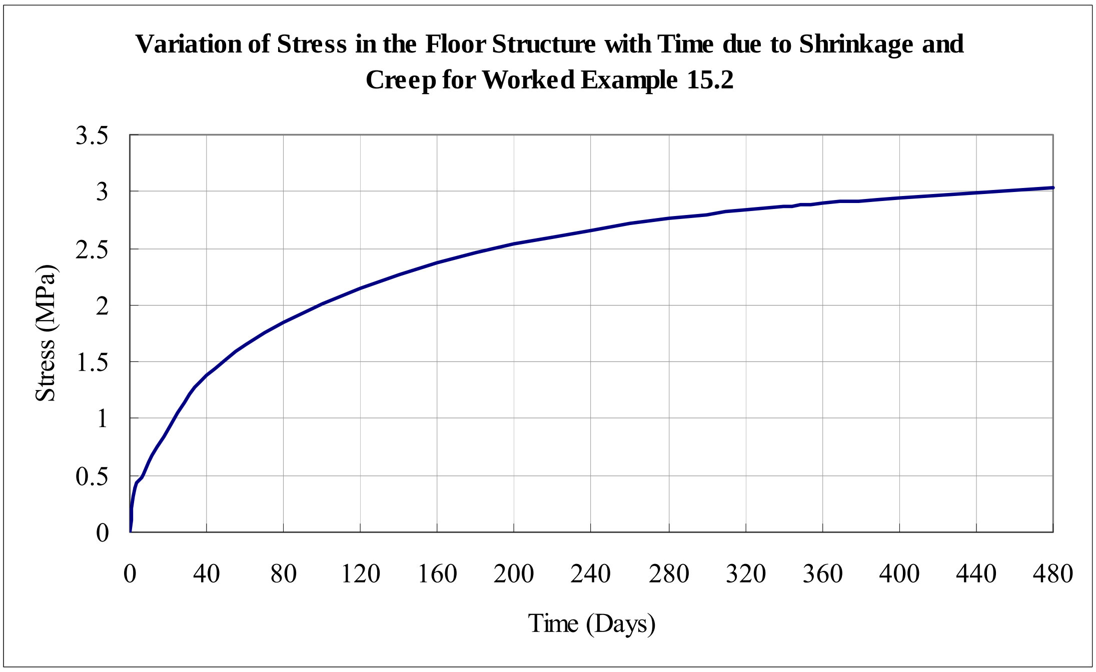


Figure 15.3 – Variation of Stress of Worked Example 15.2 with Time

15.7 Standard Design Charts

As can be seen from the foregoing analysis, the parameters for determination of the stresses due to shrinkage and creep in plain concrete are concrete grade, effective thickness of the floor structure and K_r. So the following design standard charts are prepared for quick determination of stresses at 480 days which is the time beyond which the stresses become fairly constant.

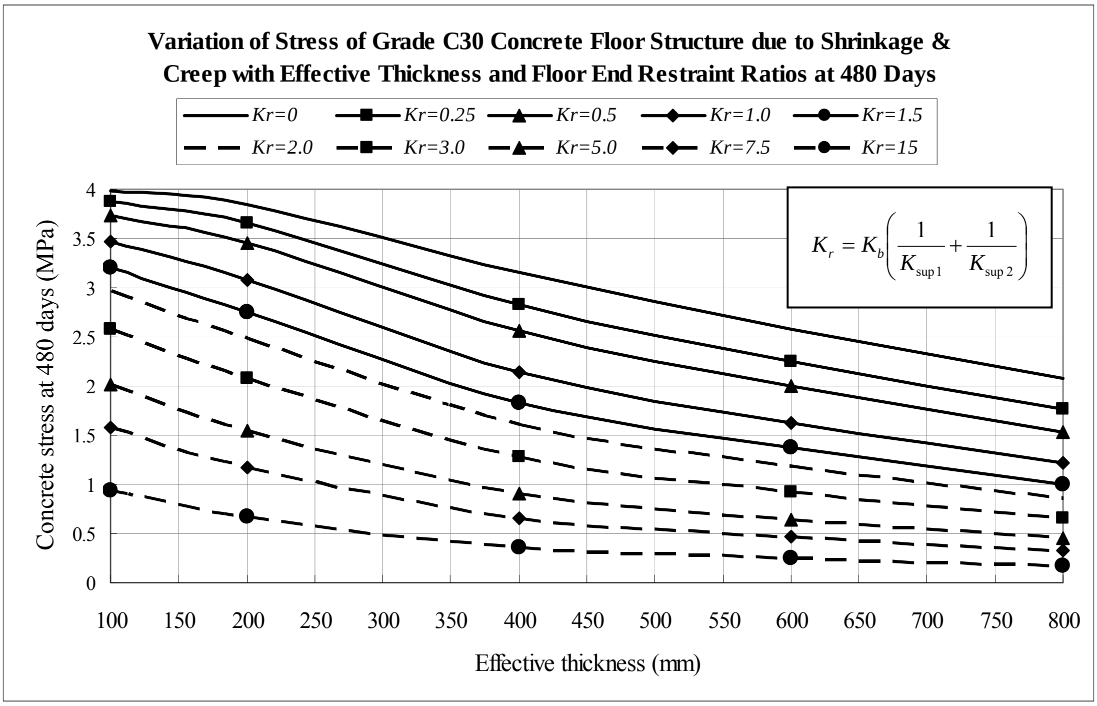


Figure 15.4(a) – Shrinkage and Creep Stresses of Floor Structure for Grade C30

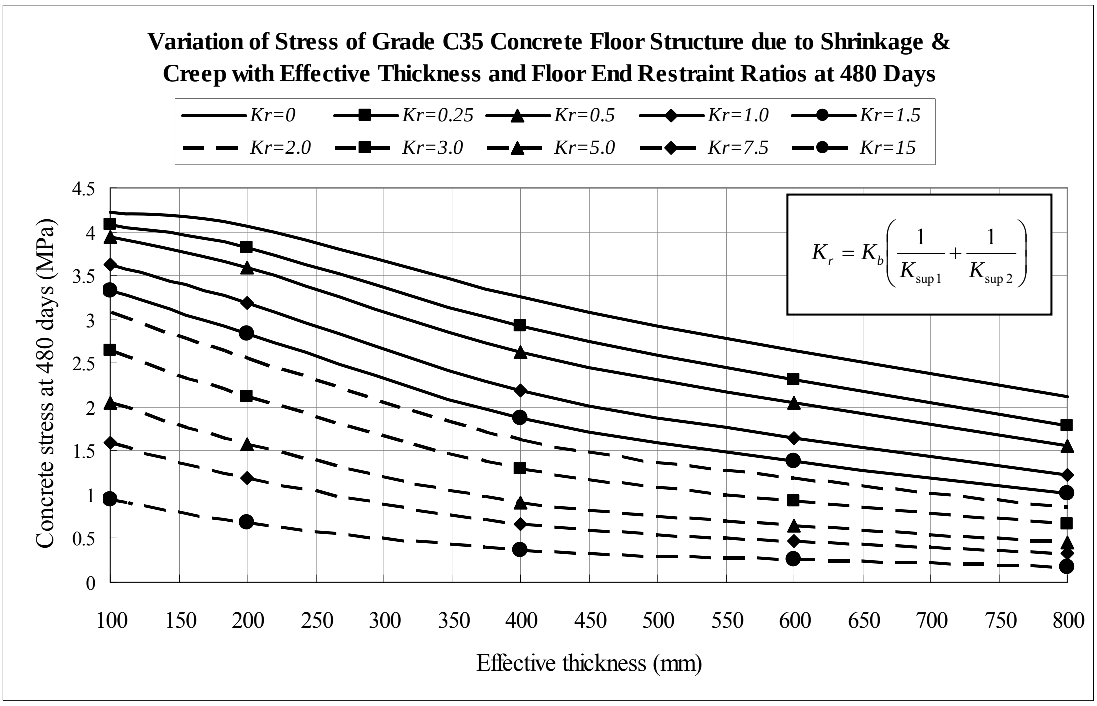


Figure 15.4(b) – Shrinkage and Creep Stresses of Floor Structure for Grade C35

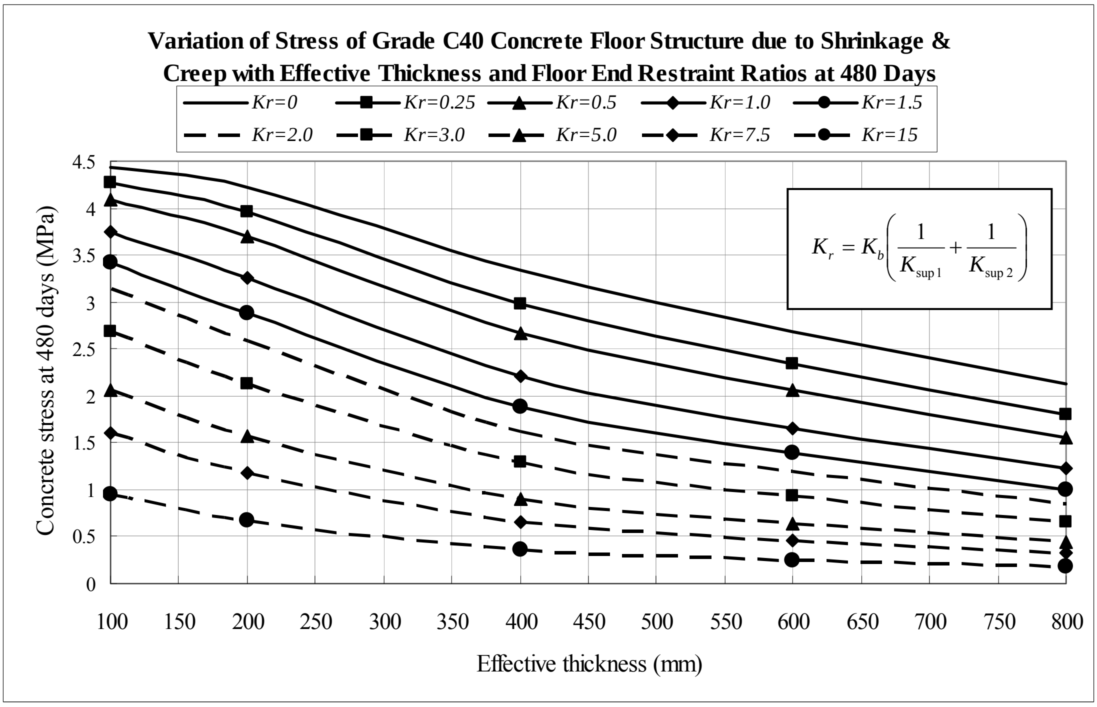


Figure 15.4(c) – Shrinkage and Creep Stresses of Floor Structure for Grade C40

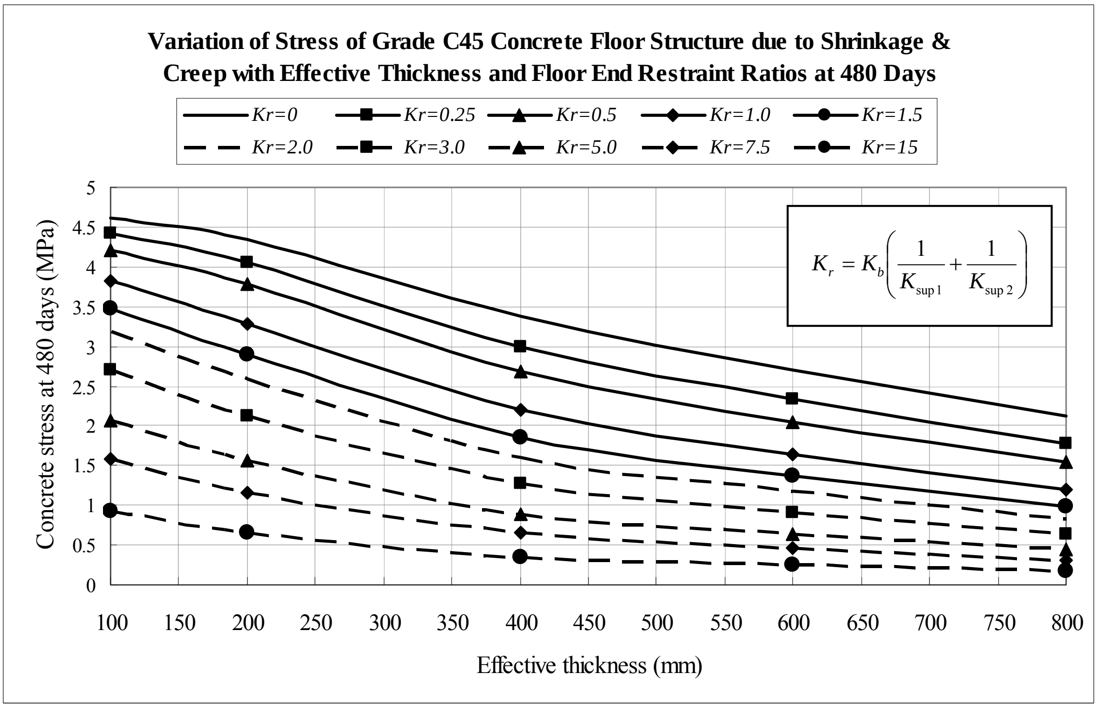


Figure 15.4(d) – Shrinkage and Creep Stresses of Floor Structure for Grade C45

Referring to Worked Example 15.2, concrete grade C35, effective thickness = 256.25mm, $K_r = 0.802$, the stress estimated from Figure 15.4(b) is 3MPa which agrees with the result of analysis in 15.6.

15.8 If steel reinforcement is considered, both shrinkage and creep will be reduced and subsequently the stress will be reduced through the reduction of the coefficient K_s. The floor structure in Worked Example 15.2 is re-analyzed with steel percentages varying from 1% to 4% and the variation of stress with time is plotted in Figure 15.5. It can be seen that the reduction of stress is not very significant.

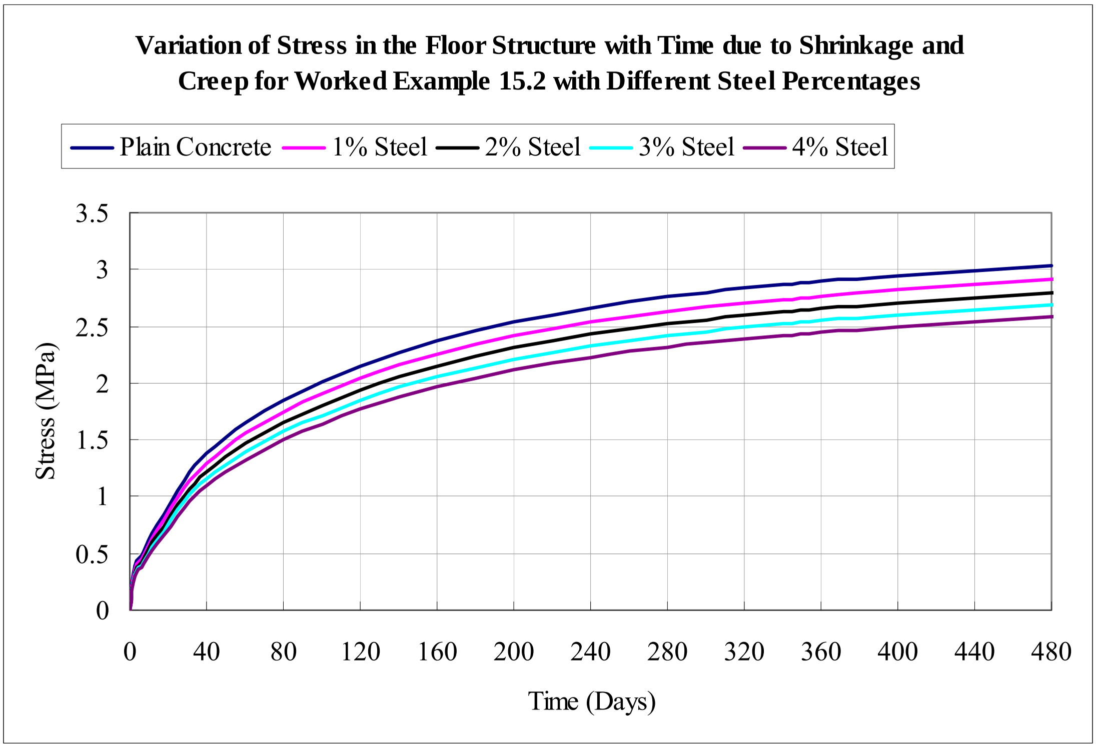


Figure 15.5 – Variation of Stress of Worked Example 15.2 with Time for Different Steel Ratios

15.9 The followings can be summarized from the analysis of a single floor structure under lateral restraints :

(i) Stress decreases with increase of effective thickness mainly because of the less shrinkage rate. So a thick floor structure will experience less stress due to shrinkage and creep;

(ii) Smaller value of K_r, i.e. longer span or smaller cross sectional area of the floor structure and stronger lateral restraint will give rise to higher stress due to shrinkage and creep. So floors at high levels will be subject to less stress because the lateral restraints are smaller due to the higher length of the restraining walls or columns. Fuller details are discussed in Lam and Law (2012). In fact, stress reduces quickly at the upper floors not only because of the smaller restraints, but also due to the "compression effects" by the floors below;

(iii) The presence of reinforcements will help reduce stress though the effects are not very significant.

15.10 Multi-bay and multi-storey structures

Multi-bay and multi-storey structures can be similarly analyzed by the time step analysis as for single bay single storey coupled with flexibility method of analysis, as carried out by Lam and Law (2012). Analytical result of a typical single bay multi-storey structure is extracted from Lam and Law (2012) for illustration.

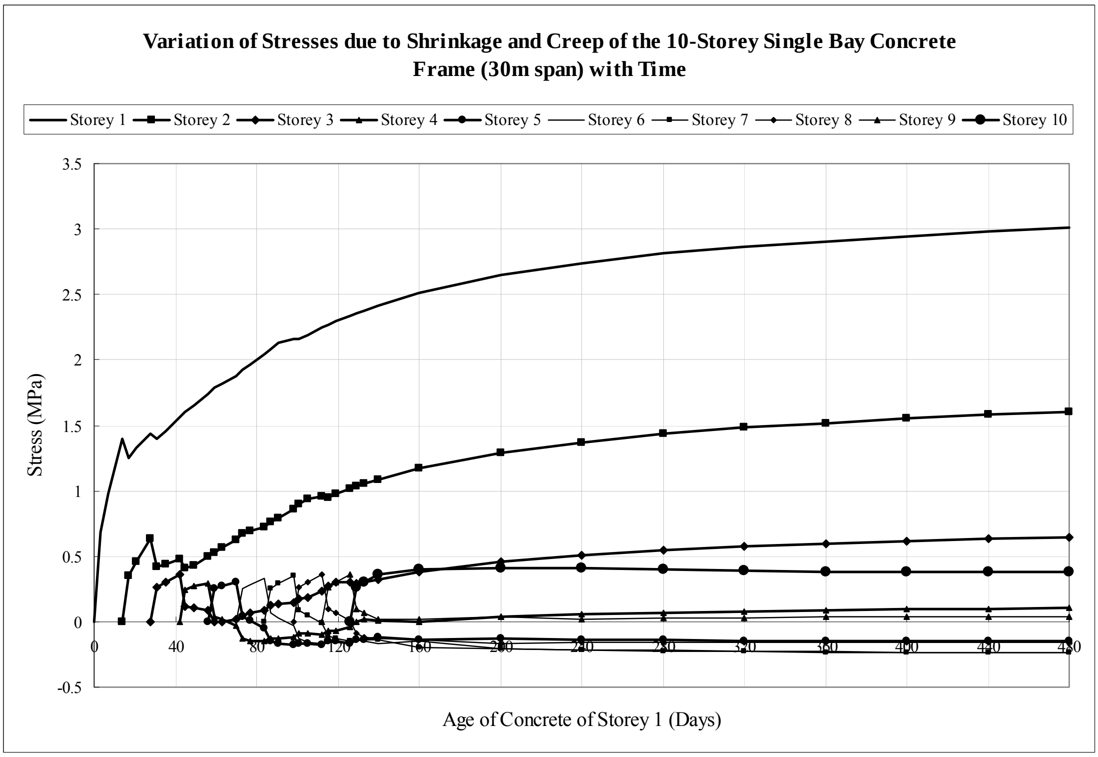


Figure 15.6 – Variation of Stress of Stresses due to Shrinkage and Creep with Time for a Typical Single Bay 10 Storey Structure

The followings can be deduced :

(i) The stresses drop rapidly in the higher floors because of the smaller lateral restraints, say controlled by $3EI/H^3$ and the "compression effects" by the floors below which shrink continuously;

(ii) Though the floor stress increases generally with time, it drops when the floor immediately below it starts to shrink which compresses it effectively at its early days of shrinkage.

15.11 Checking of Crack Widths to Stress

The induced stress in the concrete structure can be resisted by the tensile strength of concrete under no cracking conditions. Or if the tensile stress is excessive, it should be resisted by reinforcements with cracks limited to various widths according to exposure conditions.

<u>Worked Example 15.3</u>

Consider a grade C35 floor structure of unit width under restraints at ends of the following design parameters :
Stress induced is 3MPa;
Thickness $h = 160$ mm;
Longitudinal reinforcement content : T10@100 (B.F.) $\rho = 0.982$ %;
The floor structure is now checked for pure tension created due to shrinkage and creep alone :
Crack width is checked in accordance with Cl. 3.2.2 and Appendix B of BS8007:1987 with limiting crack width of 0.2mm;
Strain for coaxial tension :

$$\varepsilon_m = \varepsilon_1 - \varepsilon_2 = \frac{\sigma}{E_s \rho} - \frac{2b_t h}{3E_s A_s} = \frac{1}{200000}\left(\frac{3.0}{0.00982} - \frac{2\times1000\times160}{3\times1571}\right) = 0.001188,$$

$$< \frac{0.8 f_y}{E_s} = 0.002;$$

(ε_1 is the strain due to steel only without consideration of the tensile strength of the concrete and ε_2 represents the stiffening effect by the cracked concrete.)

Cover to reinforcement is $c_{\min} = 25$ mm;
So the greatest value a_{cr} (distance from the point under consideration to the nearest reinforcement) that will lead to greatest crack width is $\sqrt{50^2 + 25^2} = 55.9$ mm;

By equation 4 of Appendix B of BS8007, the crack width is
$\omega = 3a_{cr}\varepsilon_m = 3 \times 55.9 \times 0.001188 = 0.199$ mm < 0.2mm;
The crack width is acceptable for all exposure conditions as required by Table 7.1 of the Code.

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Appendices

List of Appendices

Assessment of "Along wind" Acceleration of Buildings (at Top Residential Floor)

The approach described in this Appendix is that listed in Australian Wind Code AS/NSZ 1170.2:2002 Appendix G2. The approach is chosen because the estimations of dynamic effects due to wind in the current wind code 2004 are largely based on the Australian Code.

A.1 Description of the Approach

The approach is that is described as follows :

(i) It is based on the simple formula $a = \frac{3}{m_0 h^2}\hat{M}_b$ where m_0 is the average mass per unit height of the building, h is the average roof height of the building above ground $\hat{M}_b$ is the "dynamic resonant component" of peak base bending moment;

(ii) Reference is made to the G factor in Appendix F of the Wind Code 2004 which is equal to $1 + 2I_h\sqrt{g_v^{\,2}B + \frac{g_f^{\,2}SE}{\zeta}}$ for the determination of total wind loads on a building structure. The expression comprises 3 components : (a) the static part which is 1 in the expression; (b) the dynamic background component which is $2I_h\sqrt{g_v^{\,2}B}$; and (c) the dynamic resonant component $2I_h\sqrt{\frac{g_f^{\,2}SE}{\zeta}}$, it is the last component that would create acceleration creating discomfort. So it is only necessary to calculate the displacement due to the dynamic resonant component by multiplying the total displacement by the factor $2I_h\sqrt{\frac{g_f^{\,2}SE}{\zeta}} \Bigg/ \left(1 + 2I_h\sqrt{g_v^{\,2}B + \frac{g_f^{\,2}SE}{\zeta}}\right)$. Alternatively, the same result can be obtained by multiplying the factor $2I_h\sqrt{\frac{g_f^{\,2}SE}{\zeta}}$ to the static wind pressure, i.e. Table 2 of the Hong Kong Wind Code 2004. By either method, $\hat{M}_b$ can be determined from the total wind moment acting on the building.

(iii) The parameters comprising m_0 and h are used for assessment of the dynamic properties of the building. In addition, there is a denominator of $1 + 2g_v I_h$ in the expression for $\hat{M}_b$ in the Australian Code as different from Hong Kong Wind Code, the reason being that the Australian Code is based on $V_{des,\theta}$ which is 3 second gust whilst Hong Kong Code is based on hourly mean wind speed. So this factor should be ignored when using Hong Kong Code which is based on hourly mean speed.

In addition, two aspects should also be noted :

(i) The Concrete Code requires the wind load for assessment of acceleration to be 1-in-10 year return period of 10 minutes duration whilst the wind load arrived for structural design in the Hong Kong Wind Code is based on 1-in-50 year return period of hourly duration. For conversion, the formula listed in Appendix B of the Wind Code can be used (as confirmed by some experts that the formula can be used for downward conversion from 1-in-50 year to 1-in-10 year return periods). The 10 minutes mean speed can also be taken as identical to that of hourly mean speed (also confirmed by the experts.) Or alternatively, as a conservative approach, the factor $1-0.62I^{1.27}\ln(t/3600)$ can be applied where I is the turbulence intensity $I=0.087\left(\frac{h}{500}\right)^{-0.11}$ taken at top of the building and $t=600\,\text{sec}$;

(ii) The damping ratio recommended in the Wind Code which is 2% is for ultimate design. A lower ratio may need to be considered for serviceability check including acceleration. Nevertheless, a 10-year return period at damping ratio 2% should be accepted which is the general practice by the Americans. The worked examples follow are therefore based on damping ratio of 2%, though the readers can easily work out the same for damping ratio of 1% under the same principle.

The procedures for estimation of acceleration are demonstrated by a worked example that follow :

A.2 Worked Example

Worked Example A-1

For the 40-storey building shown in Figure A-1 which has been analyzed by ETABS, the acceleration of the top residential floor for wind in X-direction is to be computed.

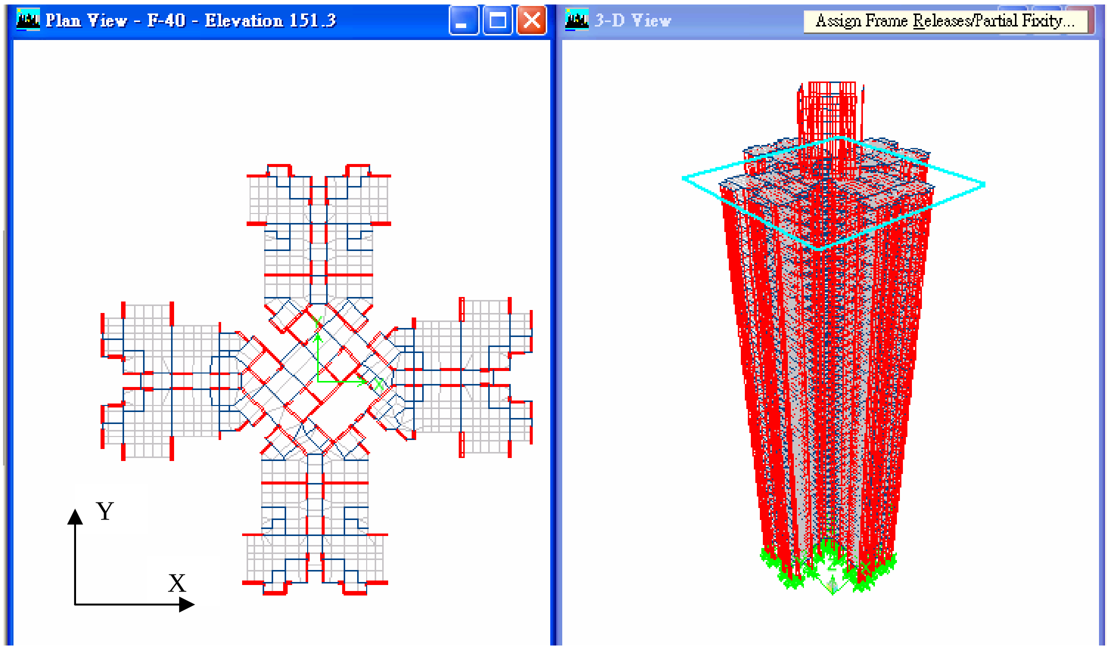


Figure A-1 – 40 storeys building for Worked Example A-1

Data : Building height $h = 121.05$ m;
Building plan width and depth are $b = d = 43$ m;
Lowest building natural frequencies for the respective motion can be obtained with reference to the modal participating mass ratios as revealed by dynamic analysis in ETABS or other softwares:

$n_{a1} = 0.297$ Hz for rotation about Z axis (torsional)

$n_{a2} = 0.3605$ Hz for translation along Y-direction

$n_{a1} = 0.3892$ Hz for translation along X-direction

Total dead load is 539693 kN and total live load is 160810 kN
Using full dead load and 40% live load for mass computation :
Mass per unit height is

$$m_0 = \frac{(539693 + 160810 \times 0.4) \div 9.8}{121.05} \times 10^3 = 509.165 \times 10^3 \text{kg/m}$$

Overturning moment at 50 years return period is 1114040 kNm when wind is blowing in the X-direction.

For wind in X-direction :

$$I_h = 0.1055\left(\frac{h}{90}\right)^{-0.11} = 0.1055\left(\frac{121.05}{90}\right)^{-0.11} = 0.1021; \quad g_v = 3.7$$

$$g_f = \sqrt{2\ln(3600 n_a)} = \sqrt{2\ln(3600 \times 0.3892)} = 3.8066;$$

$$L_h = 1000\left(\frac{h}{10}\right)^{0.25} = 1000\left(\frac{121.05}{10}\right)^{0.25} = 1865.35;$$

$$B = \frac{1}{1 + \frac{\sqrt{36h^2 + 64b^2}}{L_h}} = \frac{1}{1 + \frac{\sqrt{36 \times 121.05^2 + 64 \times 43^2}}{1865.35}} = 0.6989;$$

$$\overline{V}_h = \overline{V}_g\left(\frac{h}{500}\right)^{0.11} = 59.5\left(\frac{121.05}{500}\right)^{0.11} = 50.905\,\text{m/sec};$$

$$N = \frac{n_a L_h}{\overline{V}_h} = \frac{0.3892\times 1865.35}{50.905} = 14.262\,;$$

$$S = \frac{1}{\left[1+\dfrac{3.5n_a h}{\overline{V}_h}\right]\left[1+\dfrac{4n_a b}{\overline{V}_h}\right]} = \frac{1}{\left[1+\dfrac{3.5\times 0.3892\times 121.05}{50.905}\right]\left[1+\dfrac{4\times 0.3892\times 43}{50.905}\right]} = 0.1019$$

$$E = \frac{0.47N}{\left(2+N^2\right)^{5/6}} = \frac{0.47\times 14.262}{\left(2+14.262^2\right)^{5/6}} = 0.0793\,;$$

$$G = 1+2I_h\sqrt{g_v{}^2 B + \frac{g_f{}^2 SE}{\zeta}} = 1+2\times .1021\sqrt{3.7^2\times 0.6989+\frac{3.8066^2\times 0.1019\times 0.0793}{0.02}}$$

=1.8155;

$$G_{res} = 2I_h\sqrt{\frac{g_f{}^2 SE}{\zeta}} = 2\times .1021\sqrt{\frac{3.8066^2\times 0.1019\times 0.0793}{0.02}} = 0.494\,;$$

$$\therefore G_{res}/G = 0.272$$

Procedures :

(i) Conversion from 50 years return period to 10 years return period is by the factor listed in Appendix B of HKWC2004. The factor is

$$\left(\frac{5+\ln(R)}{5+\ln 50}\right)^2 = \left(\frac{5+\ln 10}{5+\ln 50}\right)^2 = 0.6714$$

(ii) Conversion from hourly mean wind speed to 10 minutes mean wind speed is by the factor

$$1-0.62I^{1.27}\ln(t/3600) = 1-0.62\times 0.1021^{1.27}\ln(600/3600) = 1.061$$

(iii) So the effect converted to contain only the dynamic resonant component and to 10 years return period, 10 minutes wind speed can be obtained by multiplying the deflections obtained in accordance with Appendix G of HKWC2004 by the aggregate factor of $0.272\times 0.6714\times 1.061 = 0.1938$

(iv) $\hat{M}_b = 0.1938\times 1114040 = 215857\,\text{kNm};$

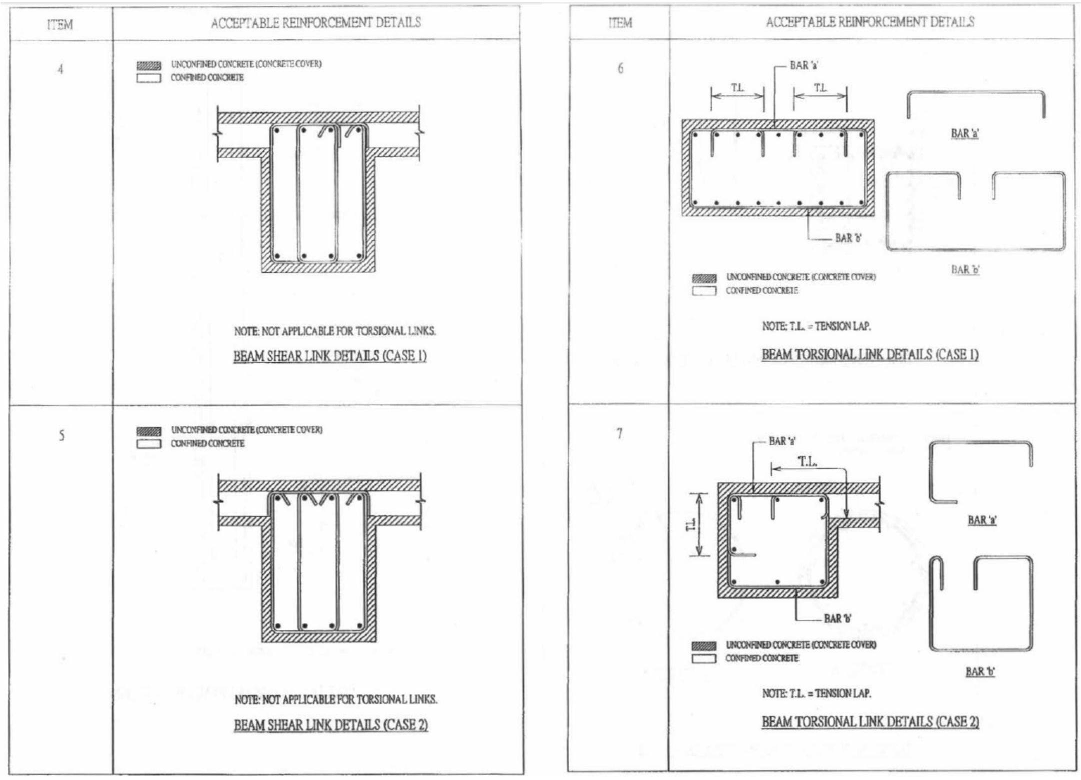
The acceleration in the X-direction is therefore

$$a = \frac{3}{m_0 h^2}\hat{M}_b = \frac{3\times 215857\times 10^3}{506.165\times 10^3\times 121.05^2} = 0.087\ \text{m/sec}^2$$

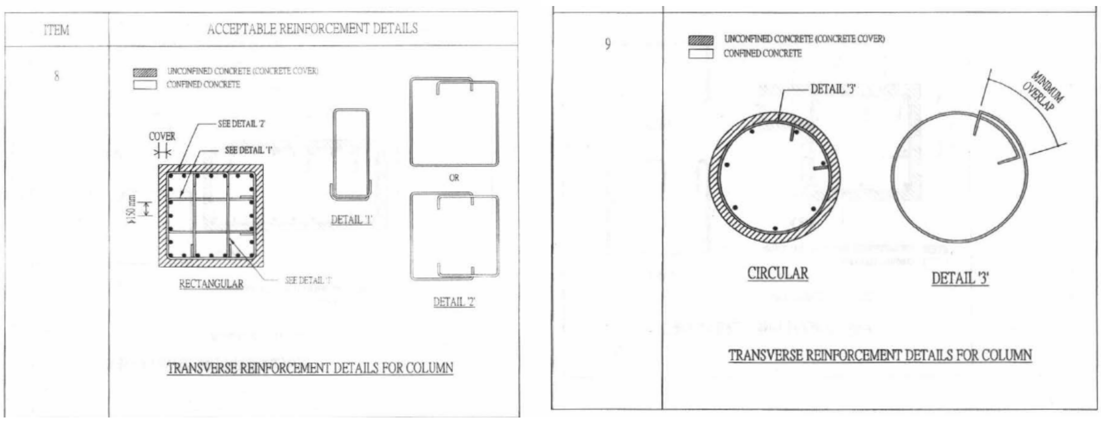
Reinforcing Bar Details Acceptable to the Buildings Department as Contained in the Annex of their Letter to the AP/RSE/RGE/RGBC/RSC dated 29 April 2011

The followings indicate conditions under which transverse reinforcements need not be anchored by bent angle exceeding 135^{o} :

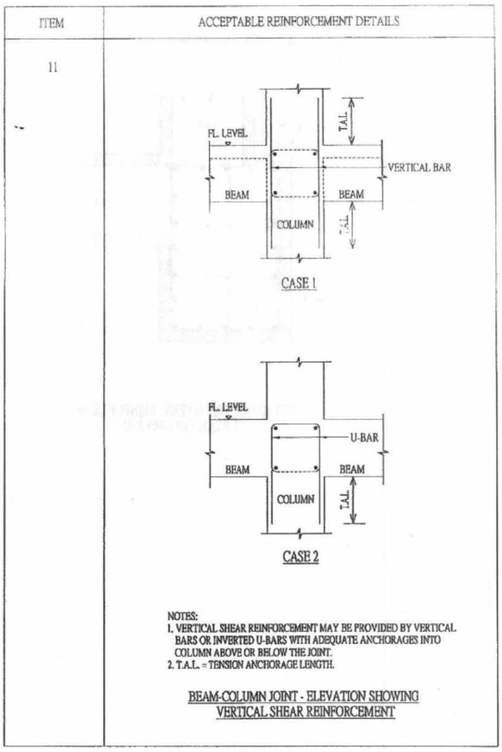
Beam Transverse Reinforcement Details

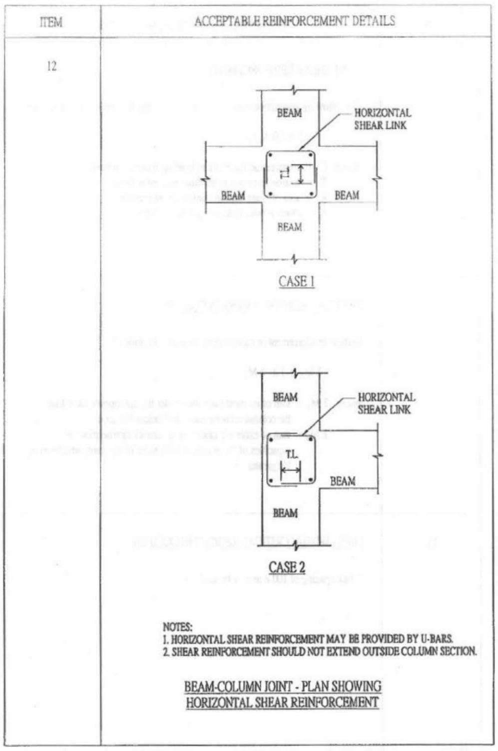


Column Transverse Reinforcement Details



Beam Column Joint Transverse Reinforcement Details





Derivation of Basic Design Formulae of R.C. Beam sections against Bending

C.1 Stress Strain Relationship of Concrete in R.C. Beam Section

The stress strain relationship of a R.C. beam section is illustrated in Figure C-1.

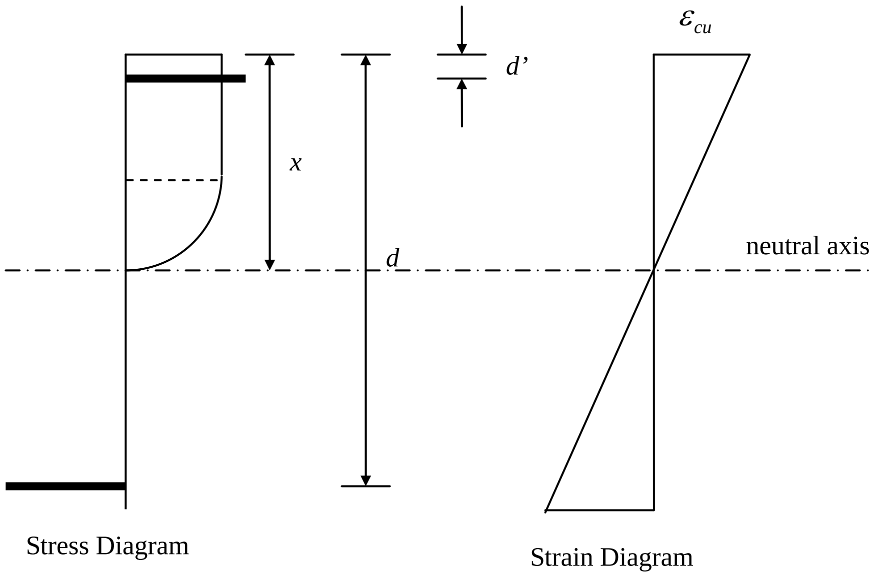


Figure C-1 – Stress Strain diagram for Beam

In Figure C-1 above, the symbols for the neutral axis depth, effective depth, cover to compressive reinforcements are *x*, *d*, and *d’*, as used in BS8110 and the Code.

To derive the contribution of force and moment by the concrete stress block, assume the parabolic portion of the concrete stress block be represented by the equation

$\sigma = A\varepsilon^2 + B\varepsilon$ (where A and B are constants) (Eqn C-1)

So $\dfrac{d\sigma}{d\varepsilon} = 2A\varepsilon + B$ (Eqn C-2)

As $\left.\dfrac{d\sigma}{d\varepsilon}\right|_{\varepsilon=0} = E_d \Rightarrow B = E_d$ where $E_d = 3.46\sqrt{f_{cu}/\gamma_m} + 3.21$ is the tangential Young’s Modulus of concrete, as distinguished from $E_c = 3.46\sqrt{f_{cu}} + 3.21$ in Cl. 3.15 of the Code.

Also $\left.\dfrac{d\sigma}{d\varepsilon}\right|_{\varepsilon=\varepsilon_0} = 0 \Rightarrow 2A\varepsilon_0 + B = 0 \Rightarrow A = -\dfrac{B}{2\varepsilon_0} = -\dfrac{E_d}{2\varepsilon_0}$ (Eqn C-3)

As $\sigma = 0.67\dfrac{f_{cu}}{\gamma_m}$ when $\varepsilon = \varepsilon_0$

$\therefore 0.67\dfrac{f_{cu}}{\gamma_m} = -\dfrac{E_d}{2\varepsilon_0}\varepsilon_0^{\,2} + E_d\varepsilon_0 = \dfrac{1}{2}E_d\varepsilon_0 \Rightarrow \varepsilon_0 = \dfrac{1.34 f_{cu}}{E_d\gamma_m}$ (Eqn C-4)

So the equation of the parabola is

$$\sigma = -\frac{E_d}{2\varepsilon_0}\varepsilon^2 + E_d\varepsilon \quad \text{for} \quad \varepsilon \le \varepsilon_0 \quad \text{where} \quad \varepsilon_0 = \frac{1.34 f_{cu}}{E_d \gamma_m}$$

Consider the linear strain distribution

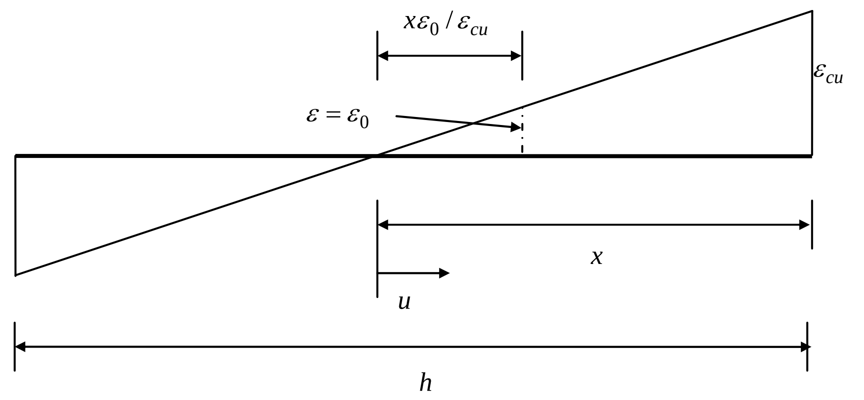


Figure C-2 – Strain Diagram across Concrete Section

At distance u from the neutral axis, $\varepsilon = \varepsilon_{cu}\dfrac{u}{x}$

So stress at u from the neutral axis up to $x\dfrac{\varepsilon_0}{\varepsilon_{cu}}$ is

$$\sigma = -\frac{E_d}{2\varepsilon_0}\varepsilon^2 + E_d\varepsilon = -\frac{E_d}{2\varepsilon_0}\left(\varepsilon_{cu}\frac{u}{x}\right)^2 + E_d\left(\varepsilon_{cu}\frac{u}{x}\right) = -\frac{E_d\varepsilon_{cu}{}^2}{2\varepsilon_0 x^2}u^2 + \frac{E_d\varepsilon_{cu}}{x}u$$

(Eqn C-5)

Based on (Eqn C-5), the stress strain profiles can be determined. A plot for grades C35, and C45 is included for illustration :

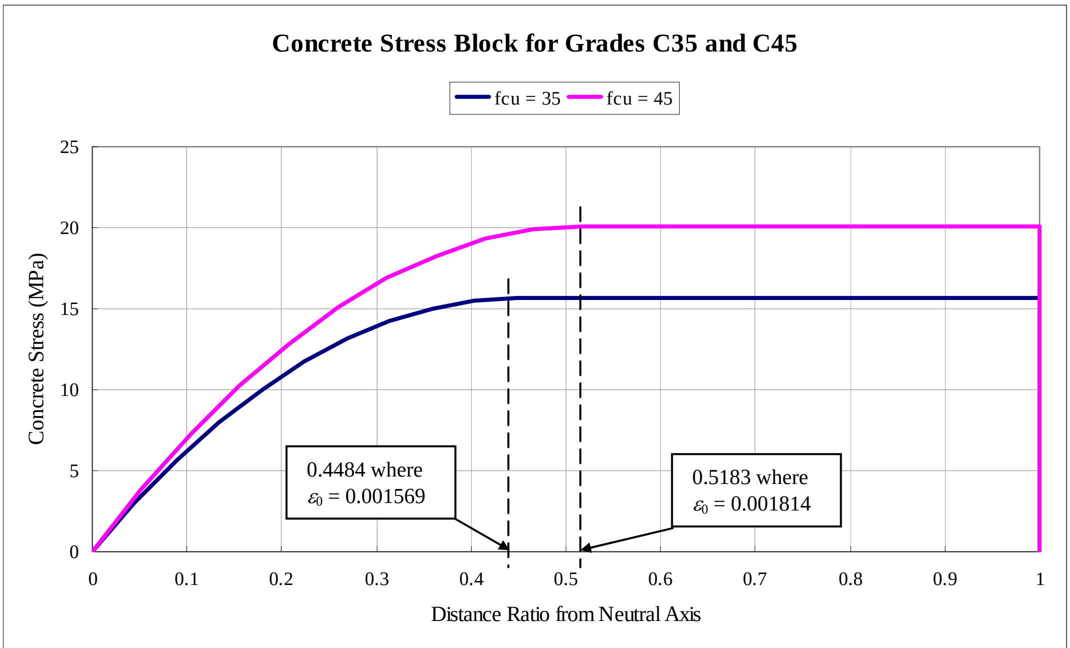


Figure C-3 – Stress Strain Profile of Grades C35 and C45

C.2 Sectional Design of Rectangular Section to Rigorous Stress Strain Profile

Making use of the properties of parabola in Figure C-4 offered by the parabolic section as F_{c1} given by

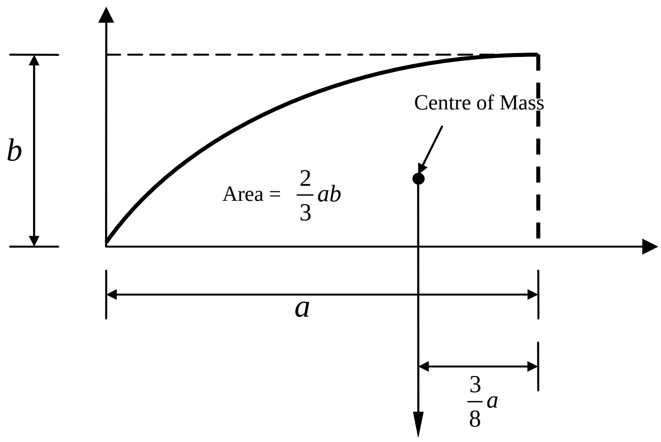


Figure C-4 – Geometrical Properties of Parabola

$$F_{c1} = b\frac{2}{3}\frac{\varepsilon_0}{\varepsilon_{cu}} x 0.67\frac{f_{cu}}{\gamma_m} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{cu}} bx \qquad \text{(Eqn C-6)}$$

and the moment exerted by F_{c1} about centre line of the tensile rebars is

$$M_{c1} = F_{c1}\left[d - x\left(1 - \frac{\varepsilon_0}{\varepsilon_{cu}}\right) - \frac{3}{8}x\frac{\varepsilon_0}{\varepsilon_{cu}}\right] = F_{c1}\left[\frac{h}{2} - x\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\right] \qquad \text{(Eqn C-7)}$$

The force by the straight portion is

$$F_{c2} = \frac{0.67 f_{cu}}{\gamma_m}\left(x - x\frac{\varepsilon_0}{\varepsilon_{cu}}\right)b = \frac{0.67 f_{cu} bx}{\gamma_m}\left(1 - \frac{\varepsilon_0}{\varepsilon_{cu}}\right) \qquad \text{(Eqn C-8)}$$

The moment offered by the constant part about the centre line of tensile rebars is

$$M_{c2} = F_{c2}\left[d - \left(1 - \frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{2}\right] \qquad \text{(Eqn C-9)}$$

The compressive force by concrete as stipulated in (Eqn C-6) and (Eqn C-8) is

$$F_c = F_{c1} + F_{c2} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{cu}} bx + \frac{0.67 f_{cu} bx}{\gamma_m}\left(1 - \frac{\varepsilon_0}{\varepsilon_{cu}}\right) = \frac{0.67 f_{cu} bx}{3\gamma_m}\left(3 - \frac{\varepsilon_0}{\varepsilon_{cu}}\right)$$

(Eqn C-10)

For singly reinforcing sections, moment by concrete about the level of the tensile steel is, by (Eqn C-7) and (Eqn C-9)

$$M = M_{c1} + M_{c2} = F_{c1}\left[d - x\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\right] + F_{c2}\left[d - \left(1 - \frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{2}\right]$$

$$= \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{cu}} bx\left[d - x\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\right] + \frac{0.67 f_{cu} bx}{\gamma_m}\left(1 - \frac{\varepsilon_0}{\varepsilon_{cu}}\right)\left[d - \left(1 - \frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{2}\right]$$

$$\Rightarrow \frac{M}{bd^2} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{cu}} \frac{x}{d}\left[1-\frac{x}{d}\left(1-\frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\right] + \frac{0.67 f_{cu}}{\gamma_m}\frac{x}{d}\left(1-\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\left[1-\frac{1}{2}\left(1-\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{d}\right]$$

$$\Rightarrow \frac{M}{bd^2} = \frac{0.67 f_{cu}}{\gamma_m}\frac{x}{d}\left\{\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right) + \left[-\frac{1}{2}+\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{cu}}\right)^2\right]\frac{x}{d}\right\}$$

$$\Rightarrow \frac{0.67}{\gamma_m}\left[-\frac{1}{2}+\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{cu}}\right)^2\right]\left(\frac{x}{d}\right)^2 + \frac{0.67}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{d} - \frac{M}{f_{cu}bd^2} = 0$$

(Eqn C-11)

which is a quadratic equation in $\frac{x}{d}$ that can be solved.

As $\frac{x}{d}$ is limited to 0.5 for singly reinforcing sections for grades up to 45 under moment distribution not greater than 10% (Clause 6.1.2.4 of the Code), by (Eqn C-11), $\frac{M}{bd^2 f_{cu}}$ will be limited to K' values listed as

$K' = 0.151$ for grade C30; $K' = 0.149$ for grade C35
$K' = 0.147$ for grade C40; $K' = 0.146$ for grade C45

which are all smaller than 0.156 under the simplified stress block.

So the design by the simplified stress block results in less amounts of rebars.

With the $\frac{x}{d}$ analyzed by (Eqn C-11), the forces in concrete

$$\frac{F_c}{bd} = \frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{d}$$

can be calculated which will be equal to the required force to be provided by steel, thus

$$0.87 f_y \frac{A_{st}}{bd} = \frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{d} \Rightarrow \frac{A_{st}}{bd} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{d}$$

(Eqn C-12)

When $\frac{M}{bd^2 f_{cu}}$ exceeds the limited value for single reinforcement. Compression reinforcements at d' from the surface of the compression side should be added. The compression reinforcements will take up the difference between the applied moment and $K'bd^2$

$$0.87 f_y \frac{A_{sc}}{bd}\left(1-\frac{d'}{d}\right) = \left(\frac{M}{bd^2 f_{cu}} - K'\right) \Rightarrow \frac{A_{sc}}{bd} = \frac{\left(\frac{M}{bd^2 f_{cu}} - K'\right) f_{cu}}{0.87 f_y\left(1-\frac{d'}{d}\right)}$$ (Eqn C-13)

Adding the same amount of steel will be added to the tensile steel,

$$\frac{A_{st}}{bd}=\frac{1}{0.87f_y}\frac{0.67f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\eta+\frac{A_{sc}}{bd} \qquad \text{(Eqn C-14)}$$

where η is the limit of $\frac{x}{d}$ ratio which is 0.5 for grade C40 and below and 0.4 for grades up to and including C70.

Furthermore, there is a limitation of lever arm ratio not to exceed 0.95 which requires

$$\frac{\frac{0.67f_{cu}}{\gamma_m}\left[-\frac{1}{2}+\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{cu}}\right)^2\right]\left(\frac{x}{d}\right)^2+\frac{0.67f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{d}}{\frac{0.67f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\frac{x}{d}}\leq 0.95$$

$$\Rightarrow \frac{x}{d}\geq 0.05\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}\right)\Bigg/\left[\frac{1}{2}-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{cu}}+\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{cu}}\right)^2\right] \qquad \text{(Eqn C-15)}$$

Thus the lower limits for the neutral axis depth ratios are 0.114, 0.116, 0.117 and 0.118 for grades C30, C35, C40 and C45 respectively. With such lower limits of neutral axis depth ratios, the tensile steel required will be 0.33%, 0.39%, 0.44% and 0.5% for grades C30, C35, C40 and C45 respectively which are considerably in excess of the minimum of 0.13% in accordance with Table 9.1 of the Code. However, it is not necessary to base on the “minimum neutral axis” for calculation of minimum steel reinforcements which will result in a minimum moment of resistance. The practice is, when the lever arm ratio exceeds 0.95, it is only necessary to calculate the steel reinforcement by

$$A_{st}=\frac{M}{0.95d\times 0.87f_{st}} \qquad \text{(Eqn C-16)}$$

As illustration for comparison between the rigorous and simplified stress block approaches, plots of $\frac{M}{bd^2}$ against steel percentages for grade C35 is plotted as

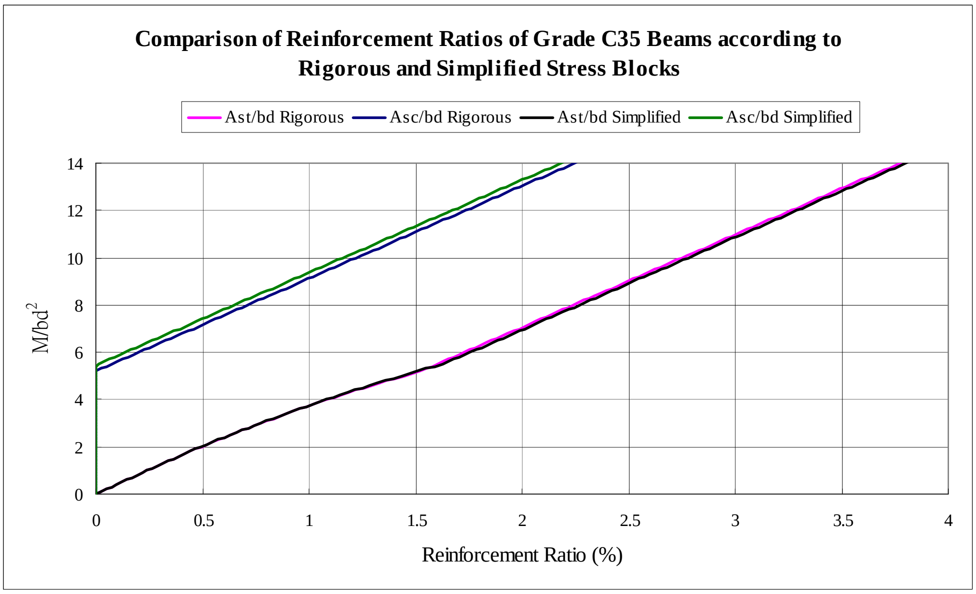


Figure C-5 – Comparison of Reinforcement Ratios of Grade C35 Beams according to Rigorous and Simplified Stress Blocks to CoP2013

It can be seen that the differences are very small for tension reinforcement while that of compression reinforcement are greater. Maximum error is 1.5%.

Design Charts based on the Rigorous Stress Block are presented at the end of this Appendix for Concrete grades varying from C30 to C45 which are commonly used in beams. Normally it will be uneconomical to use higher grades. It should also be noted that the Code limits the ratio of neutral axis depth to the effective depth to 0.4 for grade higher than C45 and up to C70, resulting in significantly more compressive steel reinforcements while that of tensile steel is not significant. The design chart with inclusion up to grade C50 is indicated in Figure C-6 for illustration.

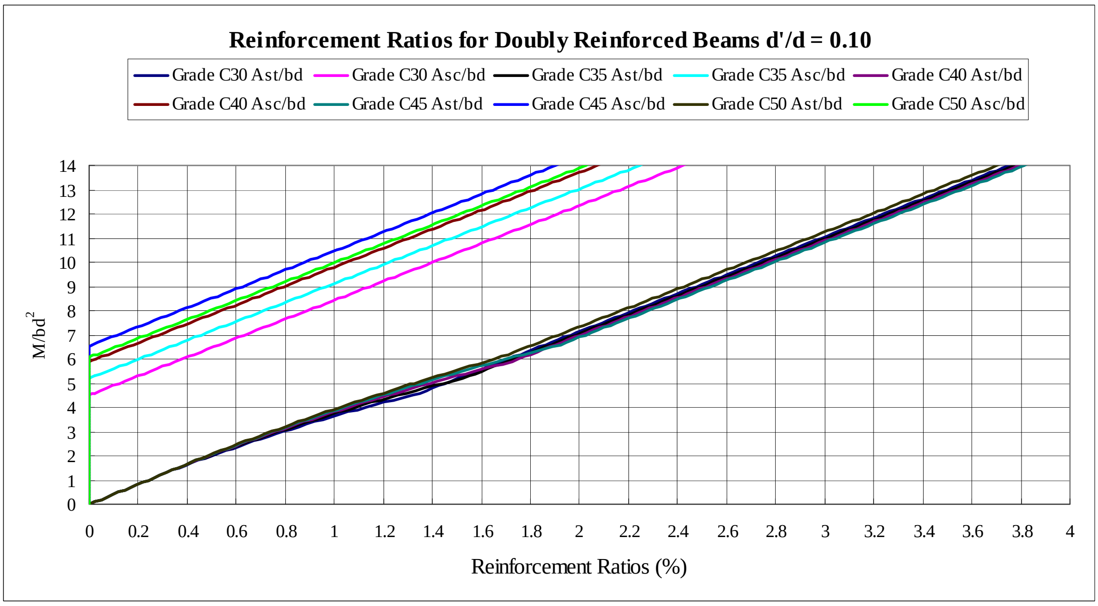


Figure C-6 – Design Chart for Beam Reinforcements to the Code

C.3 Beam Design by Simplified Stress Block

The simplified stress block for beam design is as indicated in Figure C-7.

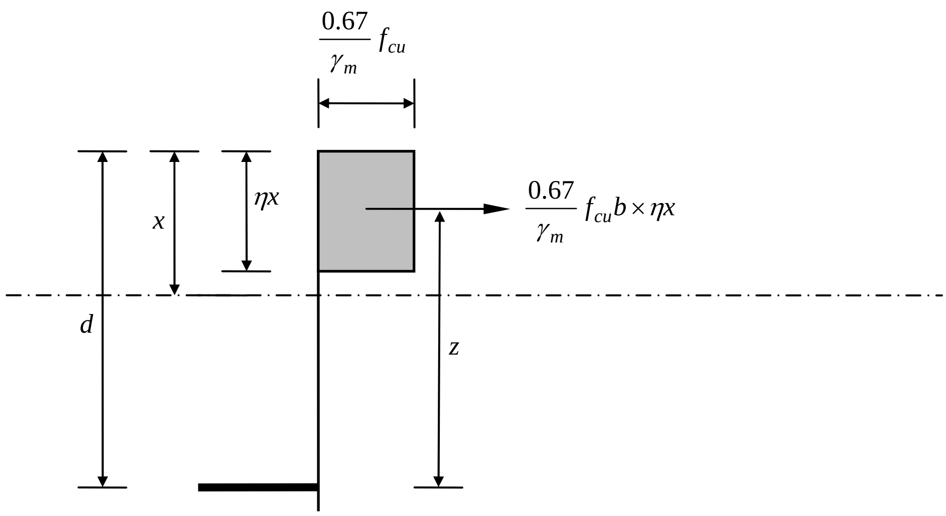


Figure C-7 – Stress Strain Profile of the Simplified Stress Block

In accordance with Figure C-7, moment of resistance of the beam section, taken at the steel level is

$$M = \frac{0.67}{\gamma_m} f_{cu} b \times \eta x \left(d - \frac{\eta}{2} x \right)$$

$$\Rightarrow \frac{M}{f_{cu} bd^2} = 0.45\eta \frac{x}{d}\left(1 - \frac{\eta}{2}\frac{x}{d}\right) = 0.45\eta \frac{x}{d} - 0.225\eta^2 \left(\frac{x}{d}\right)^2$$

Putting $K = \frac{M}{f_{cu} bd^2}$

$$\Rightarrow 0.225\eta^2 \left(\frac{x}{d}\right)^2 - 0.45\eta \frac{x}{d} + K = 0$$

The solution is

$$\frac{x}{d} = \frac{0.45\eta - \sqrt{0.45^2\eta^2 - 4 \times 0.225\eta^2 \times K}}{2 \times 0.225\eta^2} = \frac{0.45 - \sqrt{0.2025 - 0.9K}}{0.45\eta}$$

$$z = d - \frac{\eta}{2} x \Rightarrow \frac{z}{d} = 1 - \frac{\eta}{2}\left(\frac{0.45 - \sqrt{0.2025 - 0.9K}}{0.45\eta}\right) = 1 - \frac{0.45 - \sqrt{0.2025 - 0.9K}}{0.9}$$

$$\Rightarrow \frac{z}{d} = 0.5 + \frac{\sqrt{0.2025 - 0.9K}}{0.9} = 0.5 + \sqrt{\frac{0.2025 - 0.9K}{0.81}} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}}$$

(Eqn C-17)

(Eqn C-17) is that given in the Code as (Ceqn 6.10)

C.4 Determination of Reinforcements for Flanged Beam

For simplicity, only the simplified stress block in accordance with Figure 6.1 of the Code is adopted in the following derivation.

The exercise is first carried out by treating the width of the beam as b_f and analyze the beam as if it is a rectangular section. If the ηx (where η is the factor 0.9, 0.8 and 0.72 for simulating the equivalent concrete stress block by rectangular stress block in Figure 6.1 of the Code) is within the depth of the flange, i.e. $\eta\frac{x}{d} \le \frac{d_f}{d}$, the reinforcement so arrived is adequate for the section. The requirement for $\eta\frac{x}{d} \le \frac{d_f}{d}$ is

$$\eta\frac{x}{d} = 2\left(0.5 - \sqrt{0.25 - \frac{K}{0.9}}\right) \le \frac{d_f}{d} \qquad \text{(Eqn C-18)}$$

If, however, $\eta\frac{x}{d} > \frac{d_f}{d}$, the section has to be reconsidered with reference to Figure C-8.

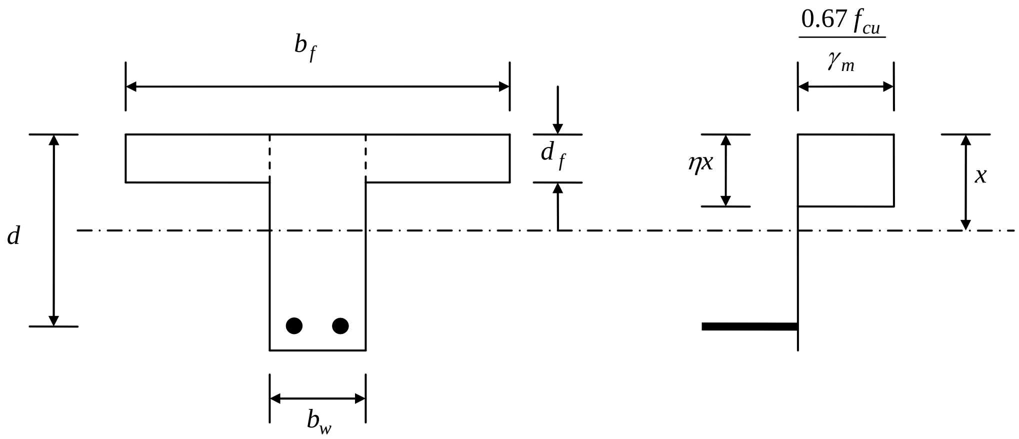


Figure C-8 – Analysis of a T or L Beam Section

For singly reinforced sections, taking moment about the level of the reinforcing steel,

$$M = \frac{0.67 f_{cu}}{\gamma_m}\left(b_f - b_w\right)d_f\left(d - \frac{d_f}{2}\right) + \frac{0.67 f_{cu}}{\gamma_m}b_w\left(\eta x\right)\left(d - 0.45x\right)$$

$$\Rightarrow \frac{M}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{b_f}{b_w} - 1\right)\frac{d_f}{d}\left(1 - \frac{1}{2}\frac{d_f}{d}\right) + \frac{0.67 f_{cu}}{\gamma_m}\left(\eta\frac{x}{d}\right)\left(1 - 0.45\frac{x}{d}\right) \qquad \text{(Eqn C-19)}$$

Putting $\frac{M_f}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m}\frac{d_f}{d}\left(\frac{b_f}{b_w} - 1\right)\left(1 - \frac{1}{2}\frac{d_f}{d}\right)$, the equation becomes

$$\Rightarrow \frac{0.67 \times 0.45 \times \eta \times f_{cu}}{\gamma_m}\left(\frac{x}{d}\right)^2 - \frac{0.67 \times \eta \times f_{cu}}{\gamma_m}\frac{x}{d} + \frac{M - M_f}{b_w d^2} = 0 \qquad \text{(Eqn C-20)}$$

For concrete grades not greater than Grade C45, $\eta = 0.9$, (Eqn C-20) can be simplified to

$$0.1809 f_{cu}\left(\frac{x}{d}\right)^2 - 0.402 f_{cu}\frac{x}{d} + \frac{M - M_f}{b_w d^2} = 0 \qquad \text{(Eqn C-21)}$$

As the neutral axis depth ratio is limited to 0.5 or 0.4 depends on concrete grade in accordance with (Ceqn 6.1) to (Ceqn 6.3) of the Code, the maximum moment of resistance is, by (Eqn C-19),

For grades C45 and below :

$$K_f' = \frac{M}{f_{cu} b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m f_{cu}}\frac{d_f}{d}\left(\frac{b_f}{b_w} - 1\right)\left(1 - \frac{1}{2}\frac{d_f}{d}\right) + \frac{0.67 f_{cu}}{\gamma_m f_{cu}}\left(0.9\frac{x}{d}\right)\left(1 - 0.45\frac{x}{d}\right)$$

$$= \frac{0.67}{\gamma_m}\frac{d_f}{d}\left(\frac{b_f}{b_w} - 1\right)\left(1 - \frac{1}{2}\frac{d_f}{d}\right) + 0.156$$

which is simply $\frac{M_f}{f_{cu} b_w d^2}$ plus K' of the rectangular section

Similarly for grade above C45 and up to C70

$$K_f' = \frac{0.67}{\gamma_m}\frac{d_f}{d}\left(\frac{b_f}{b_w} - 1\right)\left(1 - \frac{1}{2}\frac{d_f}{d}\right) + 0.132$$

For tensile steel ratio :

$$0.87 f_y \frac{A_{st}}{b_w d} = \frac{0.67 f_{cu}}{\gamma_m}\left(b_f - b_w\right)d_f + \frac{0.67 f_{cu}}{\gamma_m} b_w (\eta x)$$

$$\Rightarrow \frac{A_{st}}{b_w d} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left[\left(\frac{b_f}{b_w} - 1\right)\frac{d_f}{d} + \eta\frac{x}{d}\right] \qquad \text{(Eqn C-22)}$$

When $\frac{M}{b_w d^2 f_{cu}}$ exceeds the limited value for single reinforcement. Compression reinforcements at d' from the surface of the compression side should be added. The compression reinforcements will take up the difference between the applied moment and Kbd^2

$$0.87 f_y \frac{A_{sc}}{b_w d}\left(1 - \frac{d'}{d}\right) = \left(\frac{M}{bd^2 f_{cu}} - K\right) \Rightarrow \frac{A_{sc}}{b_w d} = \frac{\left(\frac{M}{b_w d^2 f_{cu}} - K_f'\right) f_{cu}}{0.87 f_y \left(1 - \frac{d'}{d}\right)} \qquad \text{(Eqn C-23)}$$

And the same amount of steel will be added to the tensile steel.

$$\frac{A_{st}}{b_w d} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left[\left(\frac{b_f}{b_w} - 1\right)\frac{d_f}{d} + 0.45\right] + \frac{A_{sc}}{b_w d} \qquad \text{(Eqn C-24)}$$

The formulae in (Eqn C-23) and (Eqn C-24) are different from (Ceqn 6.17) of the Code which is actually taken from BS8110. (Ceqn 6.17) is for determination of tensile reinforcement and has assumed that the effective depth is at the maximum value. Though conservative, it is less economical.

General Plate Bending Theory and Design Methods

D.1 Analysis by the Finite Element Method for Plate Bending Problem

By the finite element method, a plate bending structure is idealized as an assembly of discrete elements joined at nodes. Through the analysis, "node forces" at each node of an element, each of which comprises two bending moments and a shear force can be obtained, the summation of which will balance the applied load at the node. Figures D-1a and D-1b illustrates the phenomena.

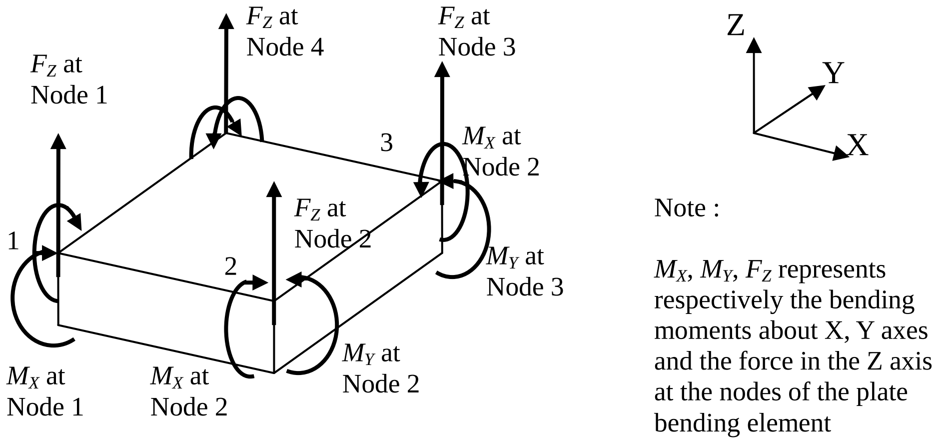


Figure D-1a – Diagrammatic Illustration of the Node Forces at the four Nodes of a Plate Bending Element 1234.

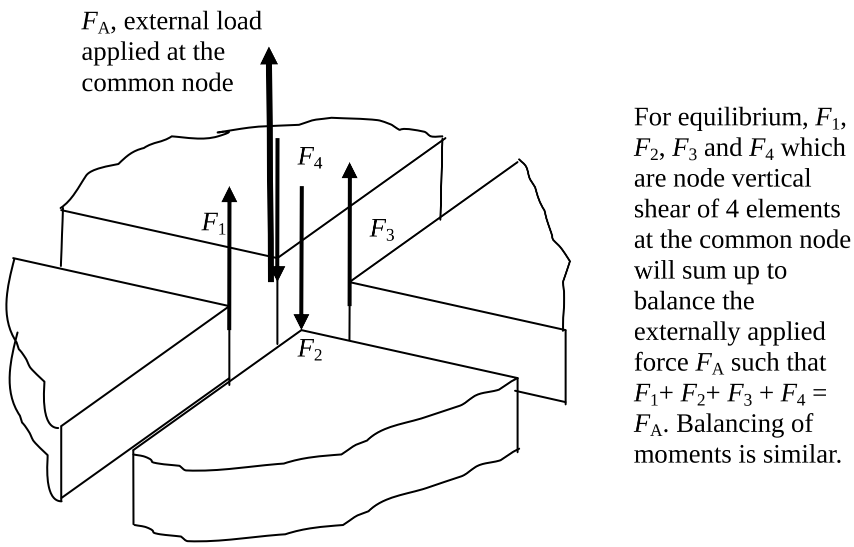


Figure D-1b – Diagrammatic Illustration of Balancing of Node Shear Forces at a Common Node to 2 or more Adjoining Elements. The Four Elements joined at the Common Node are Displaced Slightly Diagrammatically for Clarity.

The finite element method goes further to analyze the "stresses" within the discrete elements. It should be noted that "stress" is a terminology of the finite element method which refers to bending moments, twisting moments and shear forces per unit width in plate bending element. They represent the actual internal forces within the plate structure in accordance with the plate bending theory. Woods (1968) has developed the Wood-Armer Equations to convert the bending moments and twisting moments (both are moments per unit width) at any point to "design moments" in two directions for structural design purpose. The Wood-Armer Equations are listed at the latter part of this appendix.

D.2 Outline of the Plate Bending Theory

Apart from the bending moments in two mutually perpendicular directions as well known by engineers, a twisting moment can be proved to be in existence by the plate bending theory. The bending and twisting moments constitutes a "moment field" which represents the actual structural behaviour of a plate bending structure. The existence of the twisting moment and its nature are discussed in the followings. Consider a small triangular element in a plate bending structure with two of its sides aligning with the global X and Y directions as shown in Figure D-2 where moments M_X and M_Y (both in kNm per m width) are acting respectively about X and Y. A moment M_B will generally be acting about the hypotenuse making an angle of θ with the X-axis as shown to achieve equilibrium. However, as the resultant of M_X and M_Y does not necessarily align with M_B, so there will generally be a moment acting in the perpendicular direction of M_B to achieve equilibrium which is denoted as M_T. The vector direction of M_T is normal to the face of the hypotenuse. So instead of "bending" the element like M_X, M_Y and M_B which produces flexural stresses, it "twists" the element and produce shear stress in the in-plane direction. The shear stress will follow a triangular pattern as shown in Figure D-2 for stress-strain compatibility. M_T is therefore termed the "twisting moment". Furthermore, in order to achieve rotational equilibrium about an axis out of plane, the shear stress will have to be "complementary". As the hypotenuse can be in any directions of the plate structure, it follows that at any point in the plate bending structure, there will generally be two bending moments, say M_X and M_Y in two mutually perpendicular directions coupled with a complementary twisting moment M_{XY} as indicated in Figure D-3a. The phenomenon is in exact analogy to the in-plane stress problem where generally two direct stresses coupled with a shear stress exist and these components vary with directions. The equations relating M_B, M_T with M_X, M_Y, M_{XY} and θ derived from equilibrium conditions are stated as follows:

$$M_B = M_X \cos^2\theta + M_Y \sin^2\theta + 2M_{XY}\cos\theta\sin\theta \qquad \text{(Eqn D-1)}$$

$$M_T = (M_X - M_Y)\sin\theta\cos\theta - M_{XY}\cos 2\theta \qquad \text{(Eqn D-2)}$$

In addition, if θ is so varied that M_T vanishes when $\theta = \phi$, then the element will be having pure bending in the direction. The moments will be termed the "principal moments" and denoted as M_1, M_2, again in exact analogy with the in-plane stress problem having principal stresses at orientations where shear stresses are zero. The angle ϕ can be worked out by

$$\phi = \frac{1}{2}\tan^{-1}\frac{2M_{XY}}{(M_X - M_Y)} \qquad \text{(Eqn D-3)}$$

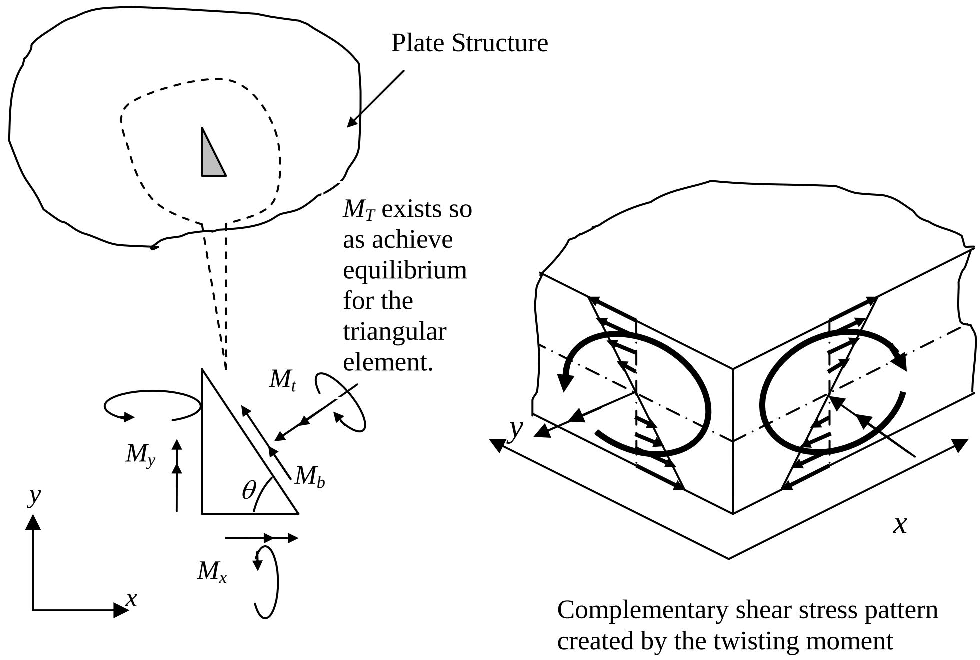


Figure D-2 – Derivation and Nature of the “Twisting Moment”

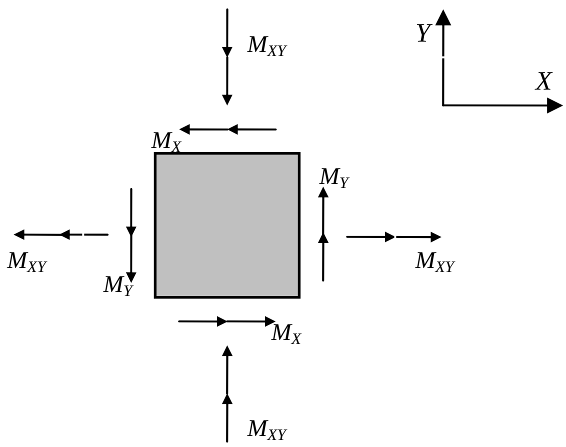


Figure D-3a – General co-existence of bending moments and twisting moment in a plate bending structure

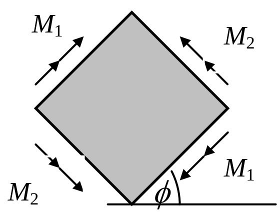


Figure D-3b – Principal moment in a plate bending structure

Again, as similar to the in-plane stress problem, one may view that the plate bending structure is actually having principal moments “bending” in the principal directions which are free of “twisting”. Theoretically, it will be adequate if the designer designs for these principal moments in the principal directions which generally vary from point to point. However, practically this is not achievable for reinforced concrete structures as we cannot vary the directions of the reinforcing steels from point to point and from load case to load case.

D.3 Structural Design for a "Moment Field" – Stress Approach

The "stress" approach for design against flexure would therefore involve formulae for providing reinforcing steels in two directions (mostly in orthogonal directions) adequate to resist the "moment field" comprising the bending moments and twisting moments. The most popular one is the "Wood Armer" Equations by Woods (1968), the derivation of which is based on the "normal yield criterion". The criterion requires the provided reinforcing steel bars at any point to be adequate to resist the normal moment which is the bending moment M_B in any directions as calculated from (Eqn D-1) which can be mathematically expressed as follows :

$$M_X^* \cos^2\theta + M_Y^* \sin^2\theta \geq M_X \cos^2\theta + M_Y \sin^2\theta + 2M_{XY}\cos\theta\sin\theta \quad \text{(Eqn D-4)}$$

The left side of the above inequality represents flexural strengths of the plate structure in the direction θ upon the provision of flexural strengths of M_X^* and M_Y^* in the X and Y directions as derived by the Johansen's Criterion (1962) whilst the right side represents the 'normal moment' M_B which is the bending moment in the direction θ. The effects of the twisting moments have been taken into account in the formulae in the calculation of M_B.

The Wood Armer Equations are listed as follows with sagging moment carrying positive sign and negative moment carrying negative sign.

For bottom steel reinforcement provisions:

$$M_X^* = M_X + |M_{XY}| \text{ if } M_X^* \geq 0; \qquad M_Y^* = M_Y + |M_{XY}| \text{ if } M_Y^* \geq 0$$

$$\text{If } M_X^* < 0 \text{, then } M_X^* = 0 \text{ and } M_Y^* = M_Y + \left|\frac{M_{XY}^2}{M_X}\right|$$

$$\text{If } M_Y^* < 0 \text{, then } M_Y^* = 0 \text{ and } M_X^* = M_X + \left|\frac{M_{XY}^2}{M_Y}\right|$$

For top steel reinforcement provisions:

$$M_X^* = M_X - |M_{XY}| \text{ if } M_X^* \leq 0; \qquad M_Y^* = M_Y - |M_{XY}| \text{ if } M_Y^* \leq 0$$

$$\text{If } M_X^* > 0 \text{, then } M_X^* = 0 \text{ and } M_Y^* = M_Y - \left|\frac{M_{XY}^2}{M_X}\right|$$

$$\text{If } M_Y^* < 0 \text{, then } M_Y^* = 0 \text{ and } M_X^* = M_X - \left|\frac{M_{XY}^2}{M_Y}\right|$$

(Eqn D-5)

The equations have been incorporated in the New Zealand Standard NZS 3101:Part 2:2006 as solution approach for a general moment field.

The following 2 examples demonstrate the use of the Wood-Armer Equations and their fulfillment of the "normal yield criterion".

<u>Worked Example D-1</u> – (bending moments of equal sign)

$M_X = 7$; $M_Y = 23$; $M_{XY} = 9$

For sagging :

$M_X^* = M_X + |M_{XY}| = 7 + 9 = 16 > 0$, so $M_X^* = 16$;

$M_Y^* = M_Y + |M_{XY}| = 23 + 9 = 32 > 0$, so $M_Y^* = 32$;

For hogging :

$M_X^* = M_X - |M_{XY}| = 7 - 9 = -2 < 0$, so $M_X^* = -2$;

$M_Y^* = M_Y - |M_{XY}| = 23 - 9 = 14 > 0$, so $M_Y^* = 0$;

and $M_X^* = M_X - \left|\dfrac{M_{XY}{}^2}{M_Y}\right| = 7 - \left|\dfrac{9^2}{23}\right| = 3.478$

So for sagging : $M_X^* = 16$; $M_Y^* = 32$;

for hogging : $M_X^* = 0$; $M_Y^* = 0$

Plot of the strengths provided by $M_X{}^*$ and $M_Y{}^*$ (sagging only) as determined by the left side of (Eqn D-4) and the normal moments worked out by $M_X = 7$; $M_Y = 23$; $M_{XY} = 9$ as determined by right side of (Eqn D-4) for orientations from 0^o to 360^o have been done and presented in Figure D-4. It can be seen that the moment capacity curve (only sagging) envelops the normal bending moment in all orientations.

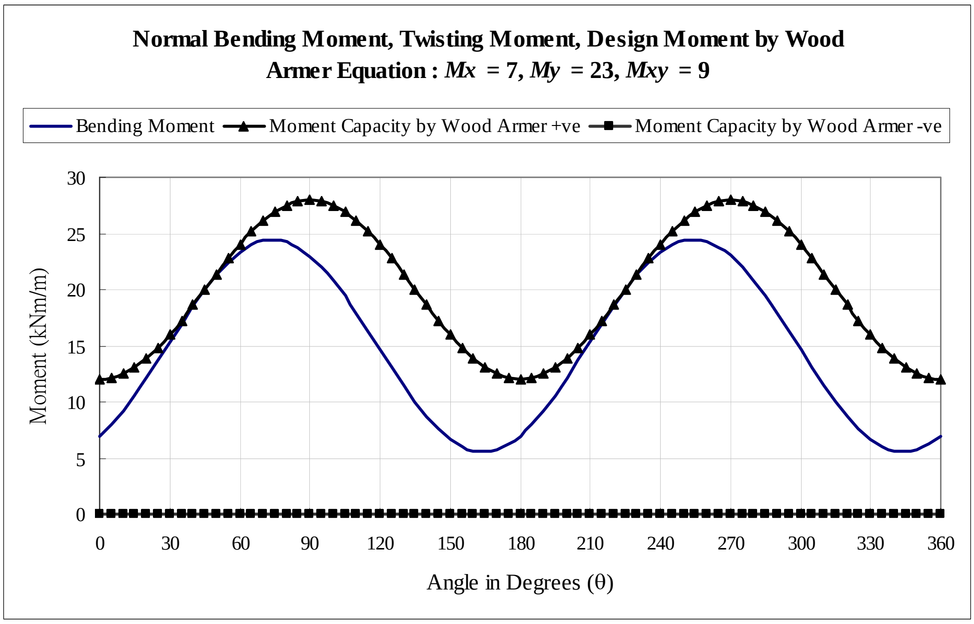


Figure D-4 – Plot of Moments and Strengths by Wood Armer Equation for Worked Example D-1

Worked Example D-2 – (bending moments of different signs)

$M_X = 7$; $M_Y = -23$; $M_{XY} = 9$

For sagging :

$M_X^* = M_X + |M_{XY}| = 7 + 9 = 16 > 0$, so $M_X^* = 16$;

$M_Y^* = M_Y + |M_{XY}| = -23 + 9 = -14 < 0$, so $M_Y^* = 0$ and

$$M_X^* = M_X + \left|\frac{M_{XY}{}^2}{M_Y}\right| = 7 + \left|\frac{9^2}{23}\right| = 10.522$$

For hogging :

$M_X^* = M_X - |M_{XY}| = 7 - 9 = -2 < 0$, so $M_X^* = -2$;

$M_Y^* = M_Y - |M_{XY}| = -23 - 9 = -32 < 0$,

So for sagging $M_X^* = 10.522$; $M_Y^* = 0$

for hogging $M_X^* = -2$ $M_Y^* = -32$

Plots of the strengths provided by $M_X{}^*$ and $M_Y{}^*$ (for both sagging and hogging) as determined by the left side of (Eqn D-4) and the normal moments worked out by $M_X = 7$; $M_Y = -23$; $M_{XY} = 9$ as determined by right side of (Eqn D-1) for orientations from 0^o to 360^o have been done and presented in Figure D-5. It can be seen that the moment capacity curves (including both sagging and hogging) envelop the normal bending moment in all orientations.

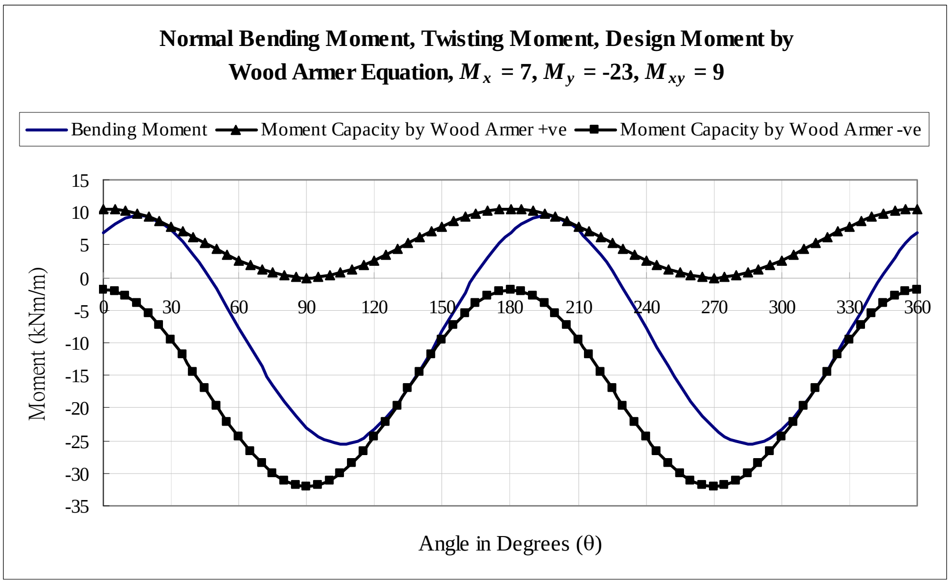


Figure D-5 – Plot of Moments and Strengths by Wood Armer Equation for Worked Example D-2

D.4 Shear in Slab (Plate) Structure

As an alternative to checking or designing punching shear in slab structures in accordance with 6.1.5.7 of the Code by which the punching shear load created by column (or pile in pile cap) is effectively averaged over a perimeter (with some account of the moment effects if any), more accurate design or checking can be carried out by the finite element analysis by which an accurate shear stress distribution in the slab structure can be obtained. The finite element analysis outputs the "shear stresses" (shear force per unit width) in accordance with the general plate bending theory at the "X-face" and "Y-face" of an element which are respectively $Q_{XZ} = -\frac{\partial M_Y}{\partial x} + \frac{\partial M_{XY}}{\partial y}$ and $Q_{YZ} = \frac{\partial M_X}{\partial y} - \frac{\partial M_{XY}}{\partial y}$, as diagrammatically illustrated in the upper part of Figure D-6.

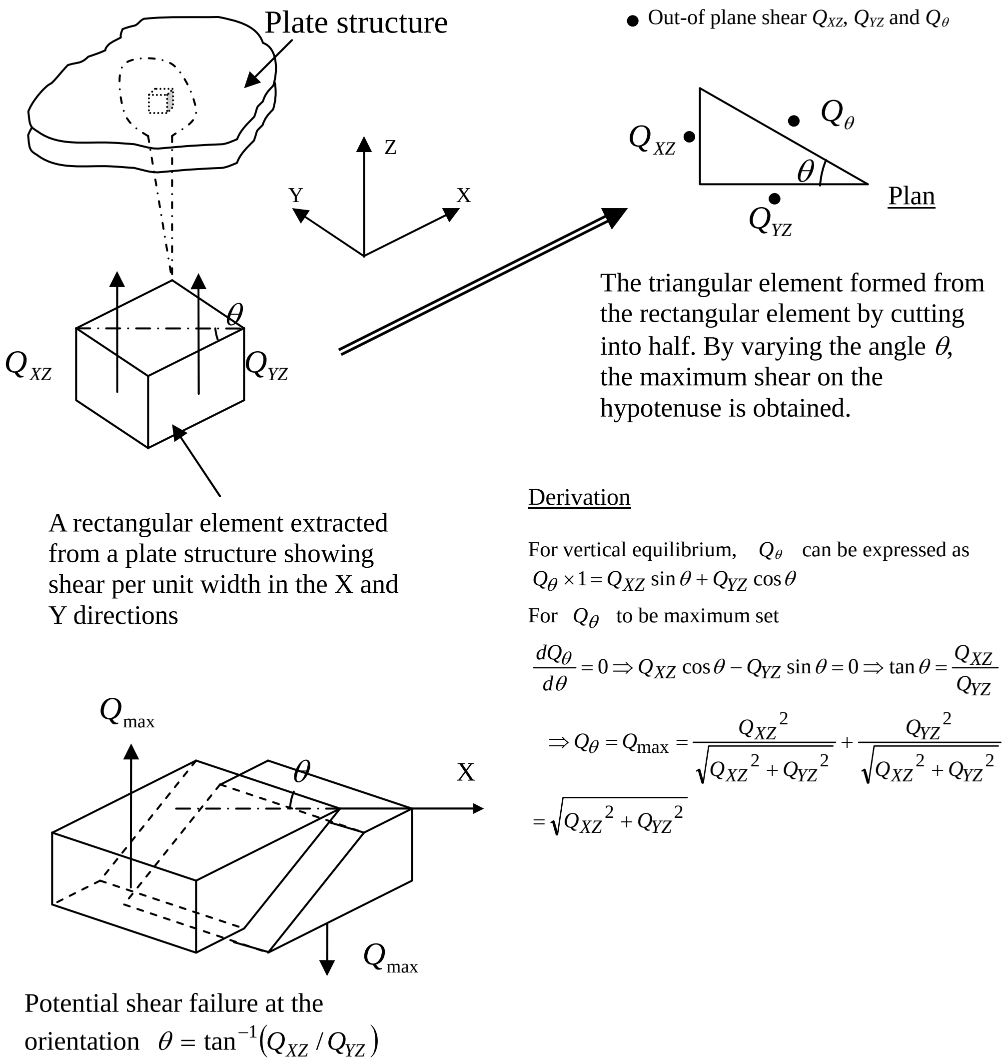


Figure D-6 – Diagrammatic Illustration of Shear "Stresses" in the X and Y Faces of an Element in Plate Bending Structure, Potential Shear Failure and Derivation of the Magnitude and Orientation of the Design Shear Stress

It is also shown in the Figure D-6 that the maximum shear after "compounding" these two components will occur in a plane at an orientation $\theta = \tan^{-1}\left(\frac{Q_{XZ}}{Q_{YZ}}\right)$

on plan and the value of the maximum shear is $Q_{\max} = \sqrt{Q_{XZ}^2 + Q_{YZ}^2}$ based on equilibrium. Thus one can view the actual shear stress in the plate is $Q_{\max}$, the action of which tends to produce shear failure at the angle θ on plan as shown in Figure D-6. So the designer needs to check or design for $Q_{\max}$ at the spot. There is no necessity to design for Q_{XZ} and Q_{YZ} separately.

D.5 Design against Shear in Slab (Plate) Structure

Following the usual practice of designing against shear in accordance with the Code, if the $Q_{\max}$ does not exceed allowable shear strength of concrete based on the design concrete shear stress v_c as defined in Cl. 6.1.2.5(c) and Table 6.3 of the Code (with enhancement as appropriate) no shear reinforcements will be required. Otherwise, reinforcements will be required to cater for the excess.

v_c can be modified in accordance with the amounts of flexural reinforcements in accordance with the formula given in Table 6.3 of the Code. In addition, an important enhancement due to close proximity to “loaded area” (supports and / or column wall loads) of footings (Re Cl. 6.7.2.4) and pile caps (Re Cl. 6.7.3.3) in accordance with Cl. 6.1.3.5(d) Table 6.8 Note 3 of the Code which make references to 6.1.3.5(b) and then 6.1.2.5(g) to (i) can be made use of. The clause allows an enhancement of v_c by a factor $2d/a_v$ where d is the slab effective depth and a_v is the distance from the edge of the loaded area to the point being considered as illustrated in Figure D-7.

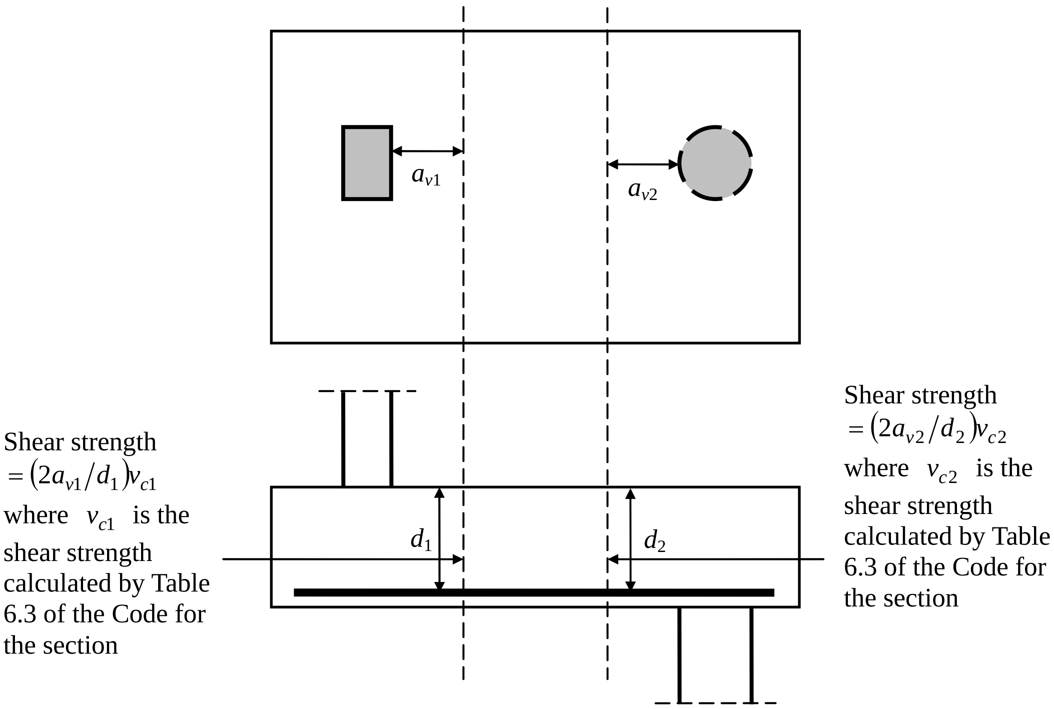


Figure D-7 – Illustration for Enhancement of Shear Strength in Slab Structure due to Closeness of “Loaded Area”

The "stress" approach for shear design based on $Q_{\max}$ can best be carried out by graphical method, so as to avoid handling the large quantity of data obtainable in the finite element analysis. An illustration of the method for a raft footing is indicated in Figure D-8 as :

(i) an enveloped shear stress (shear force per unit width) contour map of a structure due to applied loads is first plotted as shown in Figure D-8(a);

(ii) the concrete shear strength contour of the structure which is a contour map indicating the shear strength of the concrete structure after enhancement of the design concrete shear stress (v_c) to closeness of supports in accordance with the Code Cl. 6.1.2.5(g) is plotted as shown in Figure D-8(b);

(iii) locations where the stresses exceed the enhanced strengths be reinforced by shear links in accordance with requirements by the Cl. 6.1.2.5(h) and (Ceqn 6.21) of the Code. By modifying (Ceqn 6.21) the required shear reinforcements of total cross sectional area A_{sv} per unit width with a spacing s_v should be $\frac{A_{sv}}{s_v} = \frac{[v-(2d/a_v)v_c]b_v}{0.87f_{yv}} \geq \frac{0.4b_v}{0.87f_{yv}}$ where b_v is the unit length;

(iv) The resulting shear reinforcement layout is as shown in Figure D-8(c).

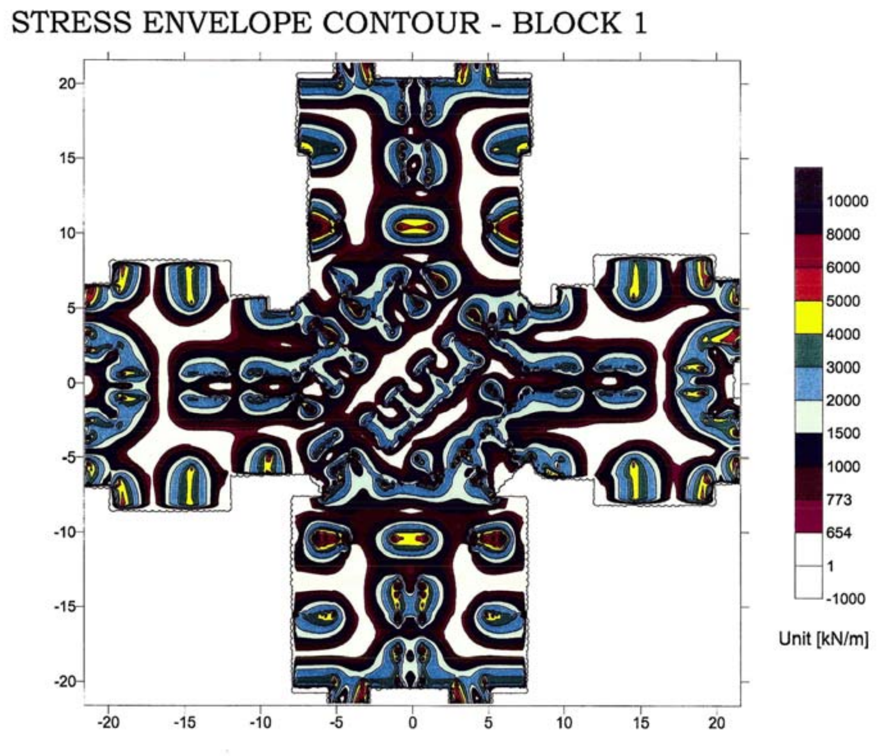


Figure D-8(a) – Stress Contour of Enveloped Shear "Stresses" of a Raft Footing due to Applied Loads

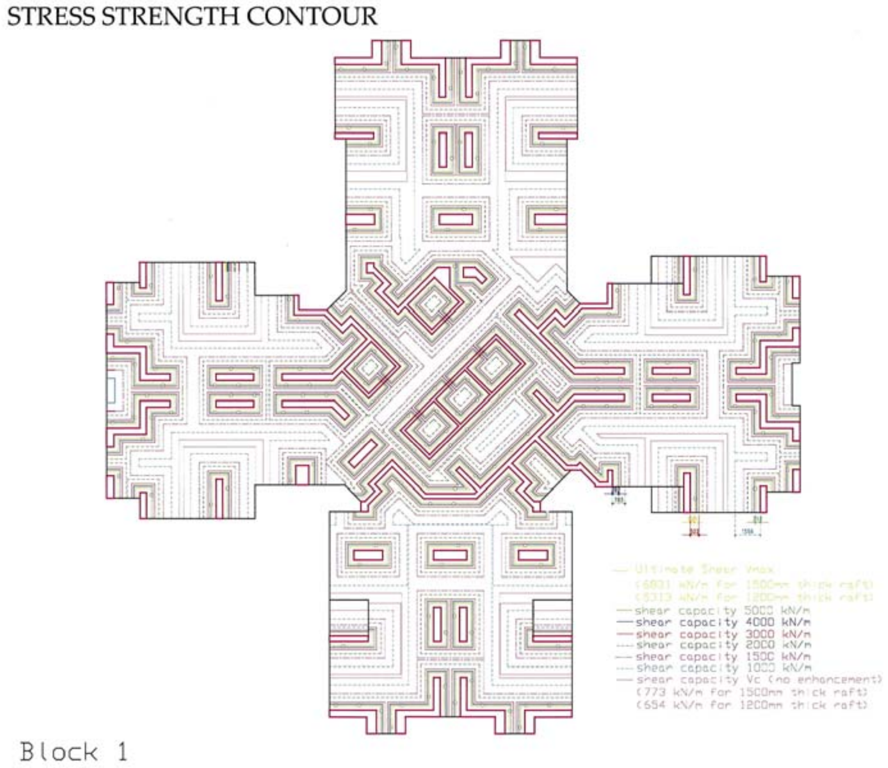


Figure D-8(b) – Strength Contour of the Raft Footing with Enhancement

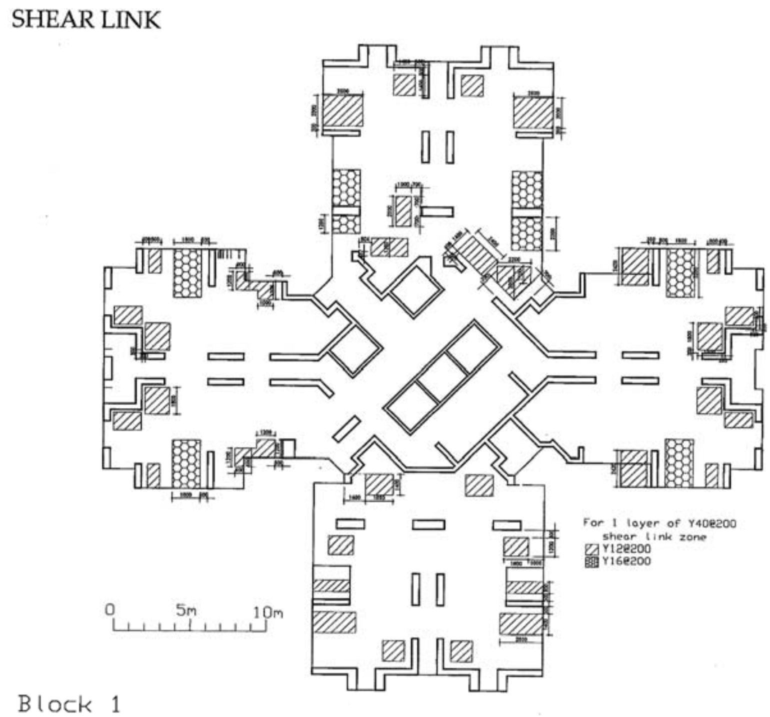


Figure D-8(c) – Arrangement of Shear Reinforcements

Derivation of Formulae for Structural Design of Column

E.1 Computing Stress / Strain Relationship of the Concrete Stress Block

Assuming the parabolic portion of the concrete stress block as indicated in Fig. 3.8 of the Code be represented by the equation

$\sigma = A\varepsilon^2 + B\varepsilon$ (where A and B are constants) (Eqn E-1)

So $\dfrac{d\sigma}{d\varepsilon} = 2A\varepsilon + B$ (Eqn E-2)

As $\left.\dfrac{d\sigma}{d\varepsilon}\right|_{\varepsilon=0} = E_d \Rightarrow B = E_d$ where $E_d = 3.46\sqrt{\dfrac{f_{cu}}{\gamma_m}} + 3.21$ (in kN/mm^2).

Also $\left.\dfrac{d\sigma}{d\varepsilon}\right|_{\varepsilon=\varepsilon_0} = 0 \Rightarrow 2A\varepsilon_0 + B = 0 \Rightarrow A = -\dfrac{B}{2\varepsilon_0} = -\dfrac{E_d}{2\varepsilon_0}$ (Eqn E-3)

As $\sigma = 0.67\dfrac{f_{cu}}{\gamma_m}$ when $\varepsilon = \varepsilon_0$

$\therefore 0.67\dfrac{f_{cu}}{\gamma_m \varepsilon_0^2} - \dfrac{E_d}{\varepsilon_0} = -\dfrac{E_d}{2\varepsilon_0} \Rightarrow \dfrac{0.67 f_{cu}}{\gamma_m \varepsilon_0} = \dfrac{E_d}{2} \Rightarrow \varepsilon_0 = \dfrac{1.34 f_{cu}}{E_d \gamma_m}$ (Eqn E-4)

So the equation of the parabola is $\sigma = -\dfrac{E_d}{2\varepsilon_0}\varepsilon^2 + E_d\varepsilon$ for $\varepsilon \le \varepsilon_0$

Consider the linear strain distribution across the column section. It should be noted that the strain at the extreme fibre is always pre-set at the concrete's ultimate strain ε_{cu} which is a constant for a specified concrete grade. By Note 3 of Figure 3.8 of the Code, $\varepsilon_{cu} = 0.0035$ for concrete grade less than and equal to C60 and becomes $\varepsilon_{cu} = 0.0035 - 0.00006\sqrt{f_{cu} - 60}$ for grade greater than C60.

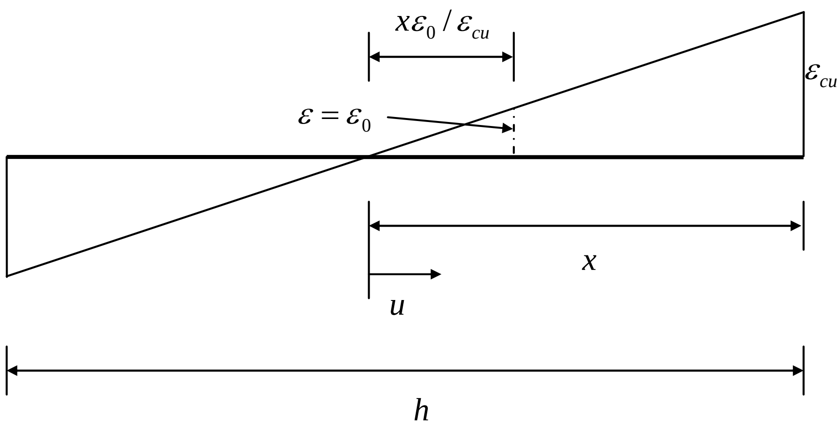


Figure E-1 – Strain Diagram across Concrete Section

At distance u from the neutral axis, $\varepsilon = \varepsilon_{cu}\dfrac{u}{x}$

So stress at u from the neutral axis up to $x\dfrac{\varepsilon_0}{\varepsilon_{cu}}$ is

$$\sigma = -\frac{E_d}{2\varepsilon_0}\varepsilon^2 + E_d\varepsilon = -\frac{E_d}{2\varepsilon_0}\left(\varepsilon_{cu}\frac{u}{x}\right)^2 + E_d\left(\varepsilon_{cu}\frac{u}{x}\right) = -\frac{E_d\varepsilon_{cu}^{\ 2}}{2\varepsilon_0}\left(\frac{u}{x}\right)^2 + E_d\varepsilon_{cu}\left(\frac{u}{x}\right)$$

As $\varepsilon_0 = \dfrac{1.34 f_{cu}}{E_d\gamma_m} \Rightarrow E_d = \dfrac{1.34 f_{cu}}{\varepsilon_0\gamma_m}$ and putting $\varepsilon_r = \dfrac{\varepsilon_0}{\varepsilon_{cu}}$

$$\sigma = -\frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r^{\ 2}}\left(\frac{u}{x}\right)^2 + \frac{2\times 0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left(\frac{u}{x}\right) = \frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[-\frac{1}{\varepsilon_r}\left(\frac{u}{x}\right)^2 + 2\left(\frac{u}{x}\right)\right] \qquad \text{(Eqn E-5)}$$

Based on (Eqn E-5), the stress strain profiles for grades C35, C60 and C100 within the concrete compression section are plotted in Figure E-2 for illustration. It should, however, be noted that ε_{cu} for grade 100 is $\varepsilon_{cu} = 0.0035 - 0.00006\sqrt{f_{cu} - 60} = 0.003121$ instead of 0.0035 for grade C35 and C60.

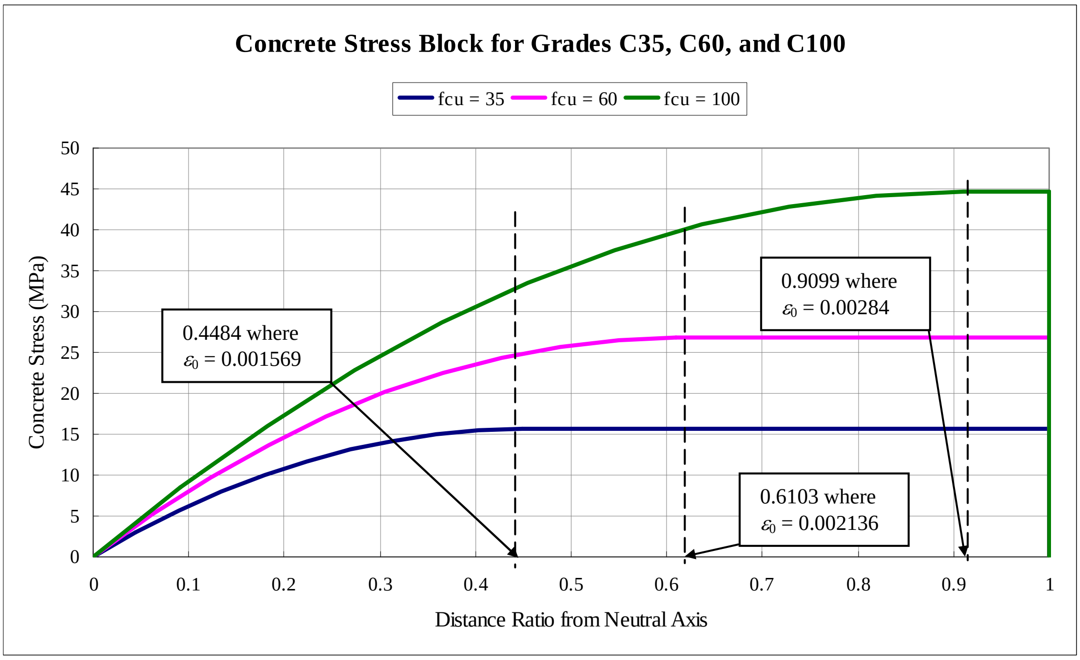


Figure E-2 – Stress Strain Profiles of Grades 35, 60 and 100

E.2 Computing Force and Moments of the Concrete Stress Block for Rectangular Column Section where the Neutral Axis Depth is within the Depth of Section $x/h \le 1.0$

Case 1 : For $x/h \le 1.0$, i.e. the neutral axis is entirely within the depth of the section, by the geometric properties of parabola as shown in Figure E-3 and the strain-stress profile for this case as shown in Figure E-4, we can formulate total force offered by the parabolic section for rectangular column as F_{c1} given by the followings :

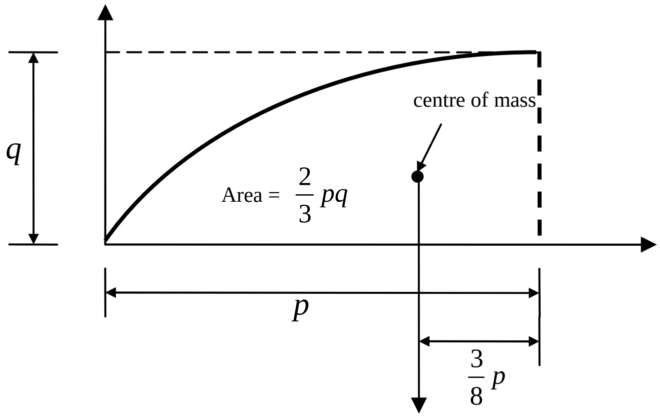


Figure E-3 – Geometrical Properties of Parabola

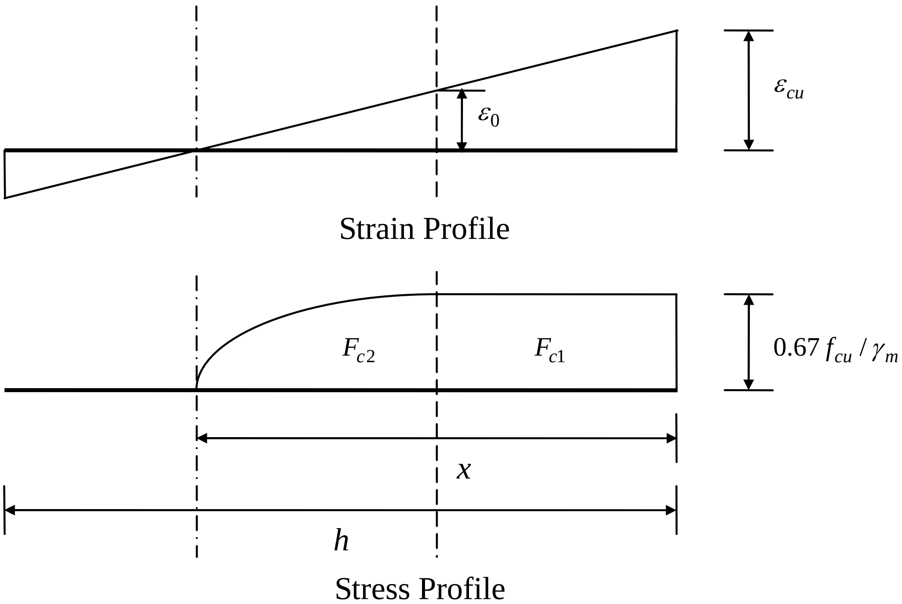


Figure E-4 – Stress Strain Diagram for $x/h \le 1.0$

Let b be the width and h be the depth of the column section, F_{c1}, F_{c2} be the force exerted by the straight and parabolic portions and M_{c1}, M_{c2} be the moments about the centre of the column section of the two portions.

$$\therefore F_{c1} = \frac{0.67 f_{cu}}{\gamma_m}(x - x\varepsilon_r)b = \frac{0.67 f_{cu} bx}{\gamma_m}(1-\varepsilon_r) \qquad \text{(Eqn E-6)}$$

$$M_{c1} = F_{c2}\left[\frac{h}{2} - (1-\varepsilon_r)\frac{x}{2}\right] = \frac{0.67 f_{cu} bx}{\gamma_m}(1-\varepsilon_r)\left[\frac{h}{2} - (1-\varepsilon_r)\frac{x}{2}\right] \qquad \text{(Eqn E-7)}$$

$$F_{c2} = b\frac{2}{3}\varepsilon_r x 0.67\frac{f_{cu}}{\gamma_m} = \frac{0.67 f_{cu} bx}{\gamma_m}\frac{2}{3}\varepsilon_r \qquad \text{(Eqn E-8)}$$

$$M_{c2} = F_{c1}\left[\frac{h}{2} - x(1-\varepsilon_r) - \frac{3}{8}x\varepsilon_r\right] = \frac{0.67 f_{cu} bx}{\gamma_m}\frac{2}{3}\varepsilon_r\left[\frac{h}{2} - x\left(1-\frac{5}{8}\varepsilon_r\right)\right] \qquad \text{(Eqn E-9)}$$

Adding (Eqn E-6) and (Eqn E-8) for the total concrete forces and simplifying

$$\frac{F_c}{bh} = \frac{F_{c1}}{bh} + \frac{F_{c2}}{bh} = \frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\varepsilon_r\right)\frac{x}{h} \qquad \text{(Eqn E-10)}$$

For (Eqn E-7) and (Eqn E-9)

$$\frac{M_{c1}}{bh^2} = \frac{0.67 f_{cu} bx}{\gamma_m}(1-\varepsilon_r)\left[\frac{h}{2} - (1-\varepsilon_r)\frac{x}{2}\right]\frac{1}{bh^2} = \frac{0.67 f_{cu}}{2\gamma_m}\left(\frac{x}{h}\right)(1-\varepsilon_r)\left[1-(1-\varepsilon_r)\frac{x}{h}\right]$$

$$\frac{M_{c2}}{bh^2} = \frac{0.67 f_{cu} bx}{\gamma_m}\frac{2}{3}\varepsilon_r\left[\frac{h}{2} - x\left(1-\frac{5}{8}\varepsilon_r\right)\right]\frac{1}{bh^2} = \frac{1.34 f_{cu}}{3\gamma_m}\varepsilon_r\left(\frac{x}{h}\right)\left[\frac{1}{2} - \left(\frac{x}{h}\right)\left(1-\frac{5}{8}\varepsilon_r\right)\right]$$

$$\frac{M_c}{bh^2} = \frac{M_{c1}+M_{c2}}{bh^2} = \frac{0.67 f_{cu}}{2\gamma_m}\left(\frac{x}{h}\right)(1-\varepsilon_r)\left[1-(1-\varepsilon_r)\frac{x}{h}\right] + \frac{1.34 f_{cu}}{3\gamma_m}\varepsilon_r\left(\frac{x}{h}\right)\left[\frac{1}{2} - \left(\frac{x}{h}\right)\left(1-\frac{5}{8}\varepsilon_r\right)\right]$$

$$= \frac{0.67 f_{cu}}{\gamma_m}\left[\left(\frac{1}{2} - \frac{1}{6}\varepsilon_r\right)\left(\frac{x}{h}\right) + \left[-\frac{1}{2} + \frac{1}{3}\varepsilon_r - \frac{1}{12}\varepsilon_r^{\,2}\right]\left(\frac{x}{h}\right)^2\right] \qquad \text{(Eqn E-11)}$$

Case 2 : For $x/h > 1.0$ and $(1-\varepsilon_r)x/h \le 1.0$, i.e. the neutral axis depth is outside the depth of the Section but the rectangular portion of the stress block is within the depth of the section. The stress strain profiles are shown in Figure E-5.

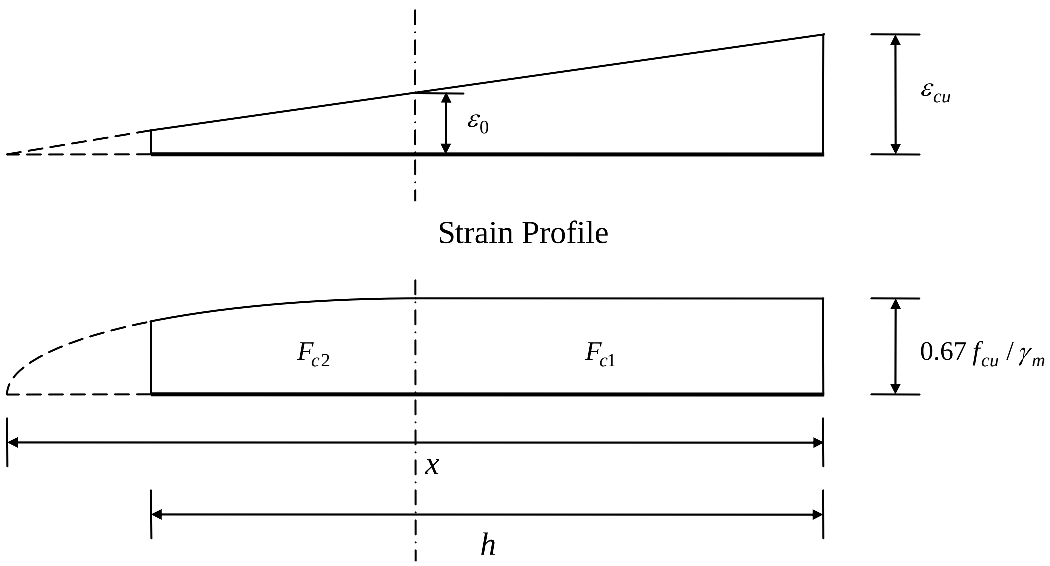


Figure E-5 – Stress Strain Diagram for $x/h > 1.0$ and $(1 - \varepsilon_0/\varepsilon_{ult})x/h \le 1.0$

The followings are formulated :

$$F_{c1} = \frac{0.67 f_{cu}}{\gamma_m}(1-\varepsilon_r)\frac{x}{h} \quad \text{(same as the previous cases)}$$

$$F_{c2} = \int_{x-h}^{x\varepsilon_0/\varepsilon_{cu}} \sigma b du \quad \text{where} \quad \sigma = \frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[-\frac{1}{\varepsilon_r}\left(\frac{u}{x}\right)^2 + 2\left(\frac{u}{x}\right)\right] \qquad \text{(Eqn E-12)}$$

$$F_{c2} = \frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[\int_{x-h}^{x\varepsilon_r}\left[-\frac{1}{\varepsilon_r}\frac{1}{x^2}u^2\right]bdu + \int_{x-h}^{x\varepsilon_r}\left[\frac{2}{x}u\right]bdu\right]$$

$$\Rightarrow \frac{F_{c2}}{bh} = \frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[\int_{x-h}^{x\varepsilon_r}\left[-\frac{1}{\varepsilon_r}\frac{1}{x^2 h}u^2\right]du + \int_{x-h}^{x\varepsilon_r}\left[\frac{2}{xh}u\right]du\right]$$

$$\Rightarrow \frac{F_{c2}}{bh} = \frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[-\frac{1}{3}\frac{1}{\varepsilon_r}\frac{1}{x^2 h}\left[\varepsilon_r^{\ 3}x^3 - (x-h)^3\right] + \frac{1}{xh}\left[\varepsilon_r^{\ 2}x^2 - (x-h)^2\right]\right]$$

$$\Rightarrow \frac{F_{c2}}{bh} = \frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[\frac{2}{3}\varepsilon_r^{\ 2}\frac{x}{h} + \frac{1}{\varepsilon_r}\left(\frac{1}{3}\frac{x}{h} - 1 + \frac{h}{x} - \frac{1}{3}\left(\frac{h}{x}\right)^2\right) - \left(\frac{x}{h} - 2 + \frac{h}{x}\right)\right] \quad \text{(Eqn E-13)}$$

$$\frac{F_c}{bh} = \frac{F_{c1}}{bh} + \frac{F_{c2}}{bh} = \frac{0.67 f_{cu}}{\gamma_m}(1-\varepsilon_r)\frac{x}{h}$$

$$+\frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[\frac{2}{3}\varepsilon_r^{\ 2}\frac{x}{h} + \frac{1}{\varepsilon_r}\left(\frac{1}{3}\frac{x}{h} - 1 + \frac{h}{x} - \frac{1}{3}\left(\frac{h}{x}\right)^2\right) - \left(\frac{x}{h} - 2 + \frac{h}{x}\right)\right]$$

$$= \frac{0.67 f_{cu}}{\gamma_m}\left[\left[1 - \frac{1}{3}\varepsilon_r - \frac{1}{\varepsilon_r} + \frac{1}{3}\frac{1}{\varepsilon_r^{\ 2}}\right]\frac{x}{h} + \frac{1}{\varepsilon_r}\left[2 - \frac{1}{\varepsilon_r}\right] + \frac{1}{\varepsilon_r}\left[\frac{1}{\varepsilon_r} - 1\right]\frac{h}{x} - \frac{1}{3}\frac{1}{\varepsilon_r^{\ 2}}\left(\frac{h}{x}\right)^2\right] \quad \text{(Eqn E-14)}$$

The determination of M_{c1} is same as (Eqn E-7)

$$\frac{M_{c1}}{bh^2} = \frac{0.67 f_{cu}}{2\gamma_m}\frac{x}{h}(1-\varepsilon_r)\left[1-(1-\varepsilon_r)\frac{x}{h}\right]$$

$$M_{c2} = \int_{x-h}^{x\varepsilon_r}\sigma b du\left(\frac{h}{2} - x + u\right) \quad \text{where} \quad \sigma = \frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[-\frac{1}{\varepsilon_r}\left(\frac{u}{x}\right)^2 + 2\left(\frac{u}{x}\right)\right]$$

$$M_{c2} = \int_{x-h}^{x\varepsilon_r}\left[\frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[-\frac{1}{\varepsilon_r}\left(\frac{u}{x}\right)^2 + 2\left(\frac{u}{x}\right)\right]\right]b\left(\frac{h}{2} - x + u\right)du$$

$$\frac{M_{c2}}{bh^2} = \frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\int_{x-h}^{x\varepsilon_r}\left[\left[-\frac{1}{\varepsilon_r}\left(\frac{u}{x}\right)^2 + 2\left(\frac{u}{x}\right)\right]\right]\left(\frac{1}{2} - \frac{x}{h} + \frac{u}{h}\right)\frac{1}{h}du \quad \text{(Eqn E-15)}$$

Integrating and expanding

$$\frac{M_{c2}}{bh^2} = \frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[\begin{array}{l}\frac{1}{12}\frac{1}{\varepsilon_r}\left[2\frac{x}{h} - 2\frac{h}{x} + \left(\frac{h}{x}\right)^2 - \left(\frac{x}{h}\right)^2\right] + \frac{1}{3}\varepsilon_r^{\ 2}\frac{x}{h}\left(1 - 2\frac{x}{h}\right) + \frac{5}{12}\varepsilon_r^{\ 3}\left(\frac{x}{h}\right)^2 \\ -\frac{1}{2}\frac{x}{h} + \frac{1}{6}\frac{h}{x} + \frac{1}{3}\left(\frac{x}{h}\right)^2\end{array}\right]$$

(Eqn E-16)

$$\therefore \frac{M_c}{bh^2} = \frac{M_{c1}}{bh^2} + \frac{M_{c2}}{bh^2}$$

$$= \frac{0.67 f_{cu}}{\gamma_m}\frac{1}{\varepsilon_r}\left[\begin{array}{l}\frac{1}{2}\frac{x}{h}\varepsilon_r(1-\varepsilon_r)\left[1-(1-\varepsilon_r)\frac{x}{h}\right] + \frac{1}{12}\frac{1}{\varepsilon_r}\left[2\frac{x}{h} - 2\frac{h}{x} + \left(\frac{h}{x}\right)^2 - \left(\frac{x}{h}\right)^2\right] \\ +\frac{1}{3}\varepsilon_r^{\ 2}\frac{x}{h}\left(1 - 2\frac{x}{h}\right) + \frac{5}{12}\varepsilon_r^{\ 3}\left(\frac{x}{h}\right)^2 - \frac{1}{2}\frac{x}{h} + \frac{1}{6}\frac{h}{x} + \frac{1}{3}\left(\frac{x}{h}\right)^2\end{array}\right]$$

$$= \frac{0.67 f_{cu}}{\gamma_m} \frac{1}{\varepsilon_r} \left[\begin{array}{l} \left[\frac{-1}{12}\varepsilon_r^{\ 3} + \frac{1}{3}\varepsilon_r^{\ 2} - \frac{1}{2}\varepsilon_r + \frac{1}{3} - \frac{1}{12}\frac{1}{\varepsilon_r} \right] \left(\frac{x}{h} \right)^2 \\ + \left[\frac{-1}{6}\varepsilon_r^{\ 2} + \frac{1}{2}\frac{1}{\varepsilon_r} - \frac{1}{2} + \frac{1}{6}\varepsilon_r \right] \frac{x}{h} + \frac{1}{6}\left(1 - \frac{1}{\varepsilon_r} \right) \frac{h}{x} + \frac{1}{12}\frac{1}{\varepsilon_r}\left(\frac{h}{x} \right)^2 \end{array} \right] \qquad \text{(Eqn E-17)}$$

Case 3 : For $(1-\varepsilon_r)x/h > 1.0$, i.e. the parabolic portion of the stress block is also outside the depth of the Section. The stress strain profiles are shown in Figure E-6.

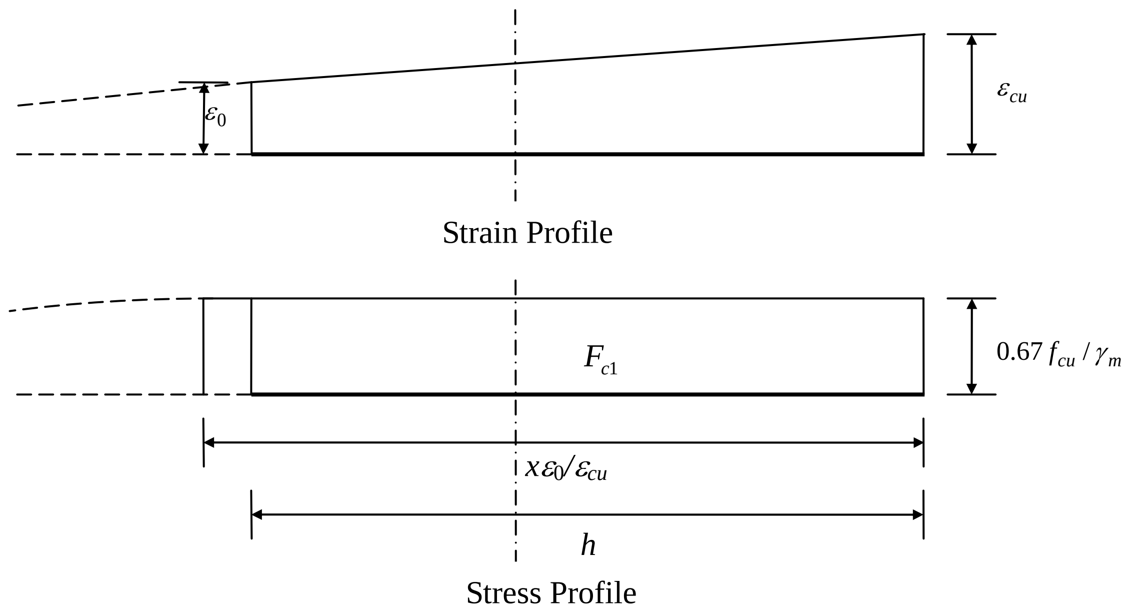


Figure E-6 – Stress Strain Diagram for $(1 - \varepsilon_0/\varepsilon_{cu})x/h > 1.0$

The followings are formulated :

$$F_{c1} = \frac{0.67 f_{cu}}{\gamma_m} bh \Rightarrow \frac{F_{c2}}{bh} = \frac{0.67 f_{cu}}{\gamma_m}$$

$$F_{c2} = 0$$

$$\therefore \frac{F_c}{bh} = \frac{0.67 f_{cu}}{\gamma_m} \qquad \text{(Eqn E-18)}$$

The moment by concrete is also zero as the concrete stress is uniform throughout the section. So $\frac{M_c}{bh^2} = 0$ (Eqn E-19)

E.3 Computing Force and Moments of the Concrete Stress Block for Circular Column Section

Consider a circular column section of radius R and neutral axis at coordinate $u = x$ as shown in Figure E-7. The coordinate system is that the coordinate is zero at the centre of the circular section. It should be noted that x can be negative.

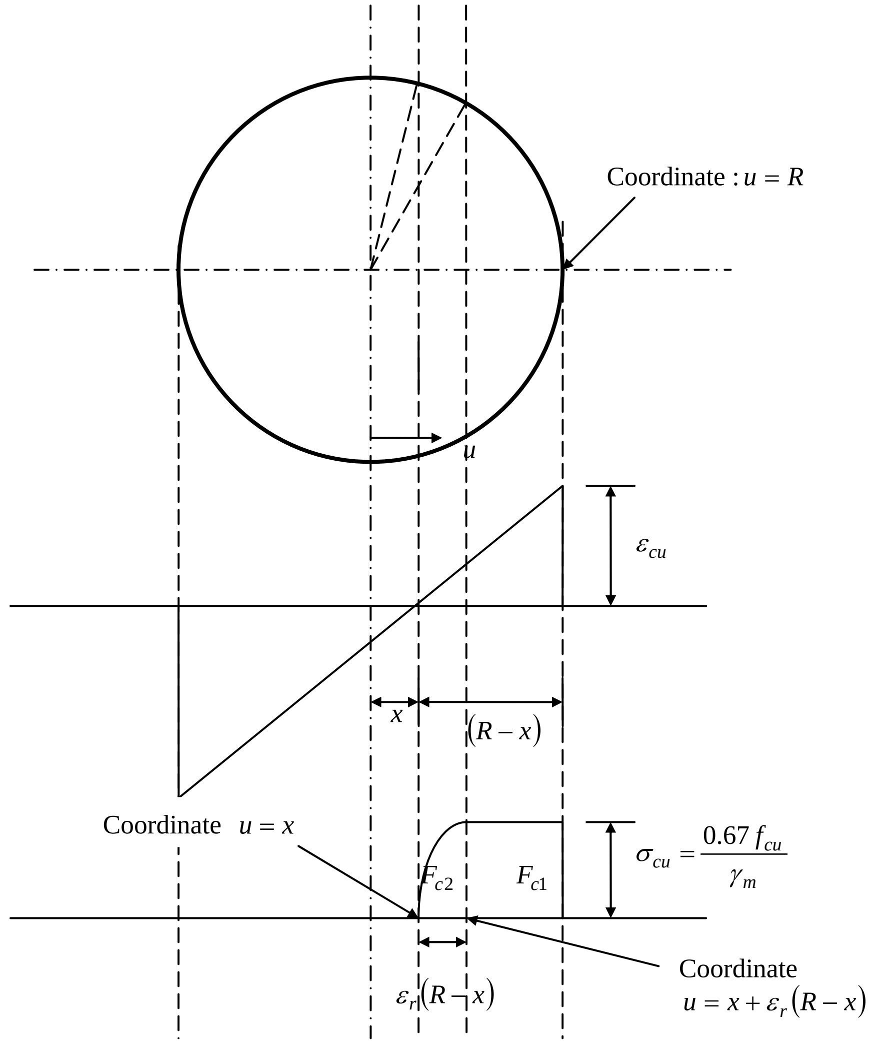


Figure E-7 – Concrete Stress Relationship in Circular Column (Case 1)

Case 1 : For $x+\varepsilon_r(R-x)\geq -R \Rightarrow \dfrac{x}{R}\geq -\left(\dfrac{1+\varepsilon_r}{1-\varepsilon_r}\right)$, i.e. the neutral axis entirely within the circular section as shown in Figure E-7,

$$F_{c1}=\int_{x+\varepsilon_r(R-x)}^{R}\frac{0.67f_{cu}}{\gamma_m}2\sqrt{R^2-u^2}\,du=\frac{0.67f_{cu}}{\gamma_m}R^2\left[\frac{u}{R}\sqrt{1-\left(\frac{u}{R}\right)^2}-\cos^{-1}\left(\frac{u}{R}\right)\right]_{x+\varepsilon_r(R-x)}^{R}$$

$$\Rightarrow\frac{F_{c1}}{R^2}=\frac{0.67f_{cu}}{\gamma_m}\left[\cos^{-1}\left[\varepsilon_r+(1-\varepsilon_r)\frac{x}{R}\right]-\left[\varepsilon_r+(1-\varepsilon_r)\frac{x}{R}\right]\sqrt{1-\left[\varepsilon_r+(1-\varepsilon_r)\frac{x}{R}\right]^2}\right] \qquad \text{(Eqn E-20)}$$

$$M_{c1}=\int_{x+\varepsilon_r(R-x)}^{R}\frac{0.67f_{cu}}{\gamma_m}2\sqrt{R^2-u^2}\,u\,du=-\frac{0.67f_{cu}}{\gamma_m}R^3\times\frac{2}{3}\left[\left[1-\left(\frac{u}{R}\right)^2\right]^{\frac{3}{2}}\right]_{\varepsilon_rR+(1-\varepsilon_r)x}^{R}$$

$$=\frac{1.34f_{cu}}{3\gamma_m}R^3\left[1-\left(\frac{\varepsilon_rR+(1-\varepsilon_r)x}{R}\right)^2\right]^{\frac{3}{2}}=\frac{1.34f_{cu}}{3\gamma_m}R^3\left[1-\left[\varepsilon_r+(1-\varepsilon_r)\frac{x}{R}\right]^2\right]^{\frac{3}{2}}$$

$$\frac{M_{c1}}{R^3}=\frac{1.34f_{cu}}{3\gamma_m}R^3\left[1-\left[\varepsilon_r+(1-\varepsilon_r)\frac{x}{R}\right]^2\right]^{\frac{3}{2}} \quad \text{(Eqn E-21)}$$

For the parabolic portion, stress at u from the neutral axis up to $x+\varepsilon_r(R-x)$ is

$$\sigma=A(u-x)^2+B(u-x) \quad \text{(Eqn E-22)}$$

where $A=-\frac{0.67f_{cu}\varepsilon_{cu}{}^2}{\gamma_m(R-x)^2\varepsilon_0{}^2}=-\frac{0.67f_{cu}\varepsilon_{cu}{}^2}{\gamma_m\left(1-\frac{x}{R}\right)^2R^2\varepsilon_0{}^2}$; $B=\frac{1.34f_{cu}\varepsilon_{cu}}{\gamma_m\left(1-\frac{x}{R}\right)R\varepsilon_0}$

or $\sigma=Au^2+(B-2Ax)u+Ax^2-Bx=Xu^2+Yu+Z$

where $X=A$, $Y=B-2Ax$, $Z=Ax^2-Bx$

For $x>-R$, force by the parabolic portion is

$$F_{c2}=\int_x^{\varepsilon_rR+(1-\varepsilon_r)x}\left(Xu^2+Yu+Z\right)\sqrt{R^2-u^2}\,du$$

$$=X\int_x^{\varepsilon_rR+(1-\varepsilon_r)x}u^2\sqrt{R^2-u^2}\,du+Y\int_x^{\varepsilon_rR+(1-\varepsilon_r)x}u\sqrt{R^2-u^2}\,du+Z\int_x^{\varepsilon_rR+(1-\varepsilon_r)x}\sqrt{R^2-u^2}\,du$$

$$=XR^4\left[-\frac{1}{4}\left(\frac{u}{R}\right)^3\sqrt{1-\left(\frac{u}{R}\right)^2}+\frac{1}{8}\left(\frac{u}{R}\right)\sqrt{1-\left(\frac{u}{R}\right)^2}+\frac{1}{8}\cos^{-1}\left(\frac{u}{R}\right)\right]_x^{\varepsilon_rR+(1-\varepsilon_r)x}$$

$$-\left[\frac{YR^3}{3}\left[1-\left(\frac{u}{R}\right)^2\right]\sqrt{1-\left(\frac{u}{R}\right)^2}\right]_x^{\varepsilon_rR+(1-\varepsilon_r)x}+\frac{ZR^2}{2}\left[\frac{u}{R}\sqrt{1-\left(\frac{u}{R}\right)^2}-\cos^{-1}\left(\frac{u}{R}\right)\right]_x^{\varepsilon_rR+(1-\varepsilon_r)x}$$

$$\Rightarrow\frac{F_{c2}}{R^2}=XR^2\left[-\frac{1}{4}\left(\frac{u}{R}\right)^3\sqrt{1-\left(\frac{u}{R}\right)^2}+\frac{1}{8}\left(\frac{u}{R}\right)\sqrt{1-\left(\frac{u}{R}\right)^2}+\frac{1}{8}\cos^{-1}\left(\frac{u}{R}\right)\right]_x^{\varepsilon_rR+(1-\varepsilon_r)x}$$

$$-\frac{YR}{3}\left[\left[1-\left(\frac{u}{R}\right)^2\right]\sqrt{1-\left(\frac{u}{R}\right)^2}\right]_x^{\varepsilon_rR+(1-\varepsilon_r)x}+\frac{Z}{2}\left[\frac{u}{R}\sqrt{1-\left(\frac{u}{R}\right)^2}-\cos^{-1}\left(\frac{u}{R}\right)\right]_x^{\varepsilon_rR+(1-\varepsilon_r)x} \quad \text{(Eqn E-23)}$$

Moment by the parabolic portion is

$$M_{c2}=\int_x^{\varepsilon_rR+(1-\varepsilon_r)x}\left(Xu^2+Yu+Z\right)u\sqrt{R^2-u^2}\,du=\int_x^{\varepsilon_rR+(1-\varepsilon_r)x}\left(Xu^3+Yu^2+Zu\right)\sqrt{R^2-u^2}\,du$$

$$\Rightarrow\frac{M_{c2}}{R^3}=-XR^2\left[\left[1-\left(\frac{u}{R}\right)^2\right]^{\frac{3}{2}}\left[\frac{1}{5}\left(\frac{u}{R}\right)^2+\frac{2}{15}\right]\right]_x^{\varepsilon_rR+(1-\varepsilon_r)x}$$

$$+YR\left[-\frac{1}{4}\left(\frac{u}{R}\right)^3\sqrt{1-\left(\frac{u}{R}\right)^2}+\frac{1}{8}\left(\frac{u}{R}\right)\sqrt{1-\left(\frac{u}{R}\right)^2}+\frac{1}{8}\cos^{-1}\left(\frac{u}{R}\right)\right]_x^{\varepsilon_rR+(1-\varepsilon_r)x}$$

$$-\frac{Z}{3}\left[\left[1-\left(\frac{u}{R}\right)^2\right]\sqrt{1-\left(\frac{u}{R}\right)^2}\right]_x^{\varepsilon_r R+(1-\varepsilon_r)x} \qquad \text{(Eqn E-24)}$$

Case 2 : For $x<-R$ and $x+\varepsilon_r(R-x)>-R$, i.e. the concrete section contains only part of the parabolic stress profile as shown in Figure E-8

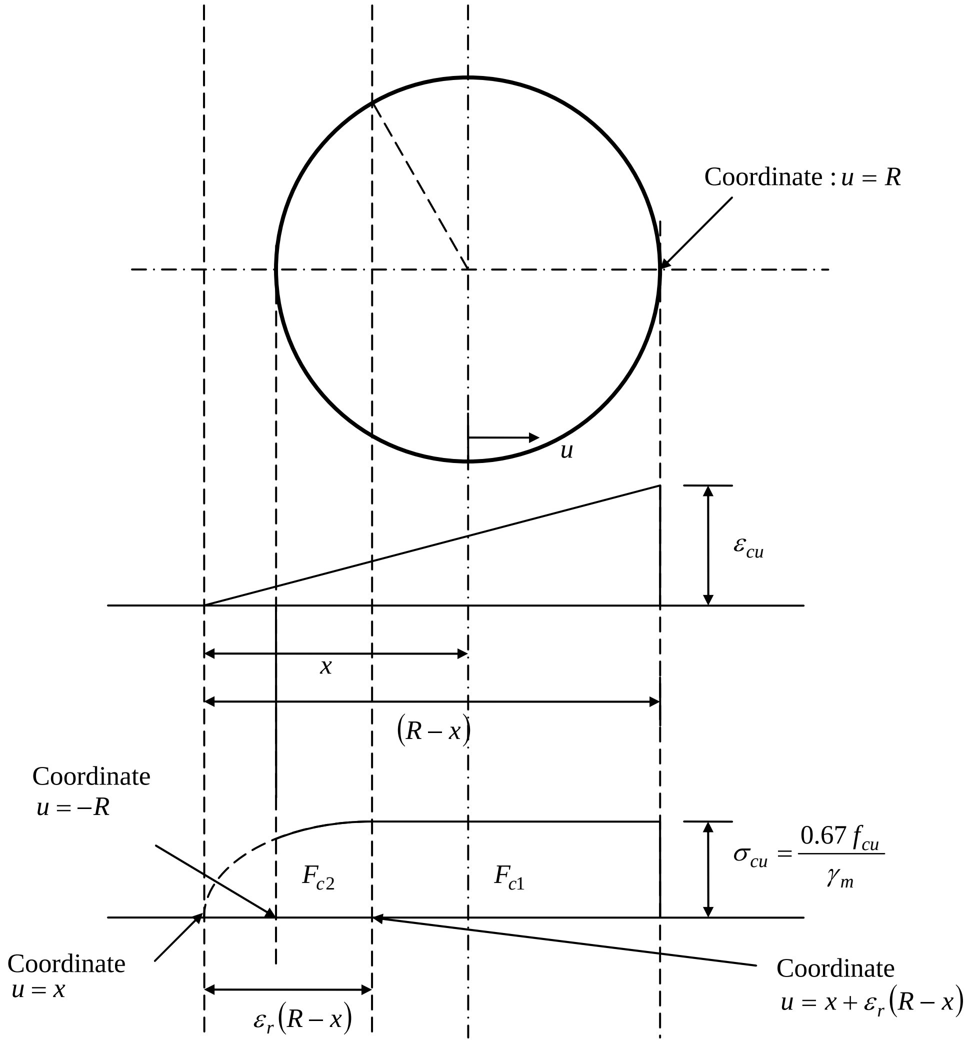


Figure E-8 – Concrete Stress Relationship in Circular Column (Case 2)

F_{c1} and M_{c1} will be identical to that shown in (Eqn E-20) and (Eqn E-21) while for F_{c2} and M_{c2} will be based on (Eqn E-23) and (Eqn E-24) but with the lower limits of integration shall be taken to $-R$, i.e.

$$\frac{F_{c2}}{R^2} = XR^2\left[-\frac{1}{4}\left(\frac{u}{R}\right)^3\sqrt{1-\left(\frac{u}{R}\right)^2}+\frac{1}{8}\left(\frac{u}{R}\right)\sqrt{1-\left(\frac{u}{R}\right)^2}+\frac{1}{8}\cos^{-1}\left(\frac{u}{R}\right)\right]_{-R}^{\varepsilon_r R+(1-\varepsilon_r)x}$$

$$-\frac{YR}{3}\left[\left[1-\left(\frac{u}{R}\right)^2\right]\sqrt{1-\left(\frac{u}{R}\right)^2}\right]_x^{\varepsilon_r R+(1-\varepsilon_r)x}+\frac{Z}{2}\left[\frac{u}{R}\sqrt{1-\left(\frac{u}{R}\right)^2}-\cos^{-1}\left(\frac{u}{R}\right)\right]_{-R}^{\varepsilon_r R+(1-\varepsilon_r)x} \qquad \text{(Eqn E-25)}$$

$$\frac{M_{c2}}{R^3} = -XR^2\left[\left[1-\left(\frac{u}{R}\right)^2\right]^{\frac{3}{2}}\left[\frac{1}{5}\left(\frac{u}{R}\right)^2+\frac{2}{15}\right]\right]_{-R}^{\varepsilon_r R+(1-\varepsilon_r)x}$$

$$+YR\left[-\frac{1}{4}\left(\frac{u}{R}\right)^3\sqrt{1-\left(\frac{u}{R}\right)^2}+\frac{1}{8}\left(\frac{u}{R}\right)\sqrt{1-\left(\frac{u}{R}\right)^2}+\frac{1}{8}\cos^{-1}\left(\frac{u}{R}\right)\right]_{-R}^{\varepsilon_r R+(1-\varepsilon_r)x}$$

$$-\frac{Z}{3}\left[\left[1-\left(\frac{u}{R}\right)^2\right]\sqrt{1-\left(\frac{u}{R}\right)^2}\right]_{-R}^{\varepsilon_r R+(1-\varepsilon_r)x} \qquad \text{(Eqn E-26)}$$

Case 3, For $x+\varepsilon_r(R-x)<-R$, i.e. the entire circular section is under constant stress,

$$\frac{F_c}{R^2} = \frac{0.67 f_{cu}\pi}{\gamma_m}, \qquad \text{(Eqn E-27)}$$

$$\frac{M_c}{R^3} = 0 \qquad \text{(Eqn E-28)}$$

E.4 Computing Force and Moments of the Reinforcement Bars

The stresses of the rebars depend on the strains as illustrated in Figure E-9. For pre-determined neutral axis depth value, the strain ε_s at the reinforcement bars positions can be worked out and subsequently the stresses. The stresses of the reinforcement bars will generally be $E_s\varepsilon_s$ but is capped at $0.87 f_y$ for both tension and compression, depending on the signs of the strains. As $f_y = 500$ N/mm^2 and $E_s = 200000$ N/mm^2, the limiting strains at $0.87 f_y$ is $0.87 f_y / E_s = \pm 0.002175$.

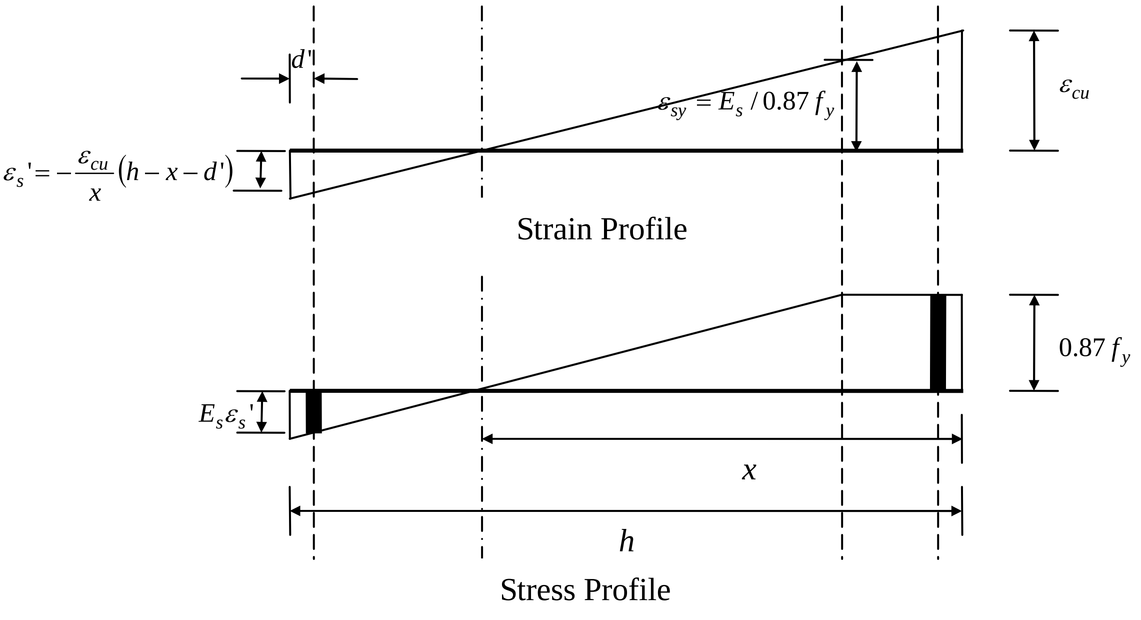


Figure E-9 – Stress Strain relationship for Rebars

So the total force by steel rebars is

$$F_s = \sum E_s \varepsilon_{si} A_{si} \qquad \text{(Eqn E-29)}$$

where ε_{si} and A_{si} are respectively the strain and cross sectional area of the i^{th} bar.

Similarly the total moment by the steel rebars is

$$M_s = \sum E_s \varepsilon_{si} A_{si} x_{si} \qquad \text{(Eqn E-30)}$$

where x_{si} is the distance of the i^{th} bar from the centre line of the column section.

For rectangular column we may further express in the following form

$$\frac{F}{bh} = \frac{F_c}{bh} + \frac{F_s}{bh} \quad \text{and} \quad \frac{M}{bh^2} = \frac{M_c}{bh^2} + \frac{M_s}{bh^2}$$

Similar expression can be obtained for circular sections as $\frac{F}{h^2} = \frac{F_c}{h^2} + \frac{F_s}{h^2}$ and $\frac{M}{h^3} = \frac{M_c}{h^3} + \frac{M_s}{h^3}$

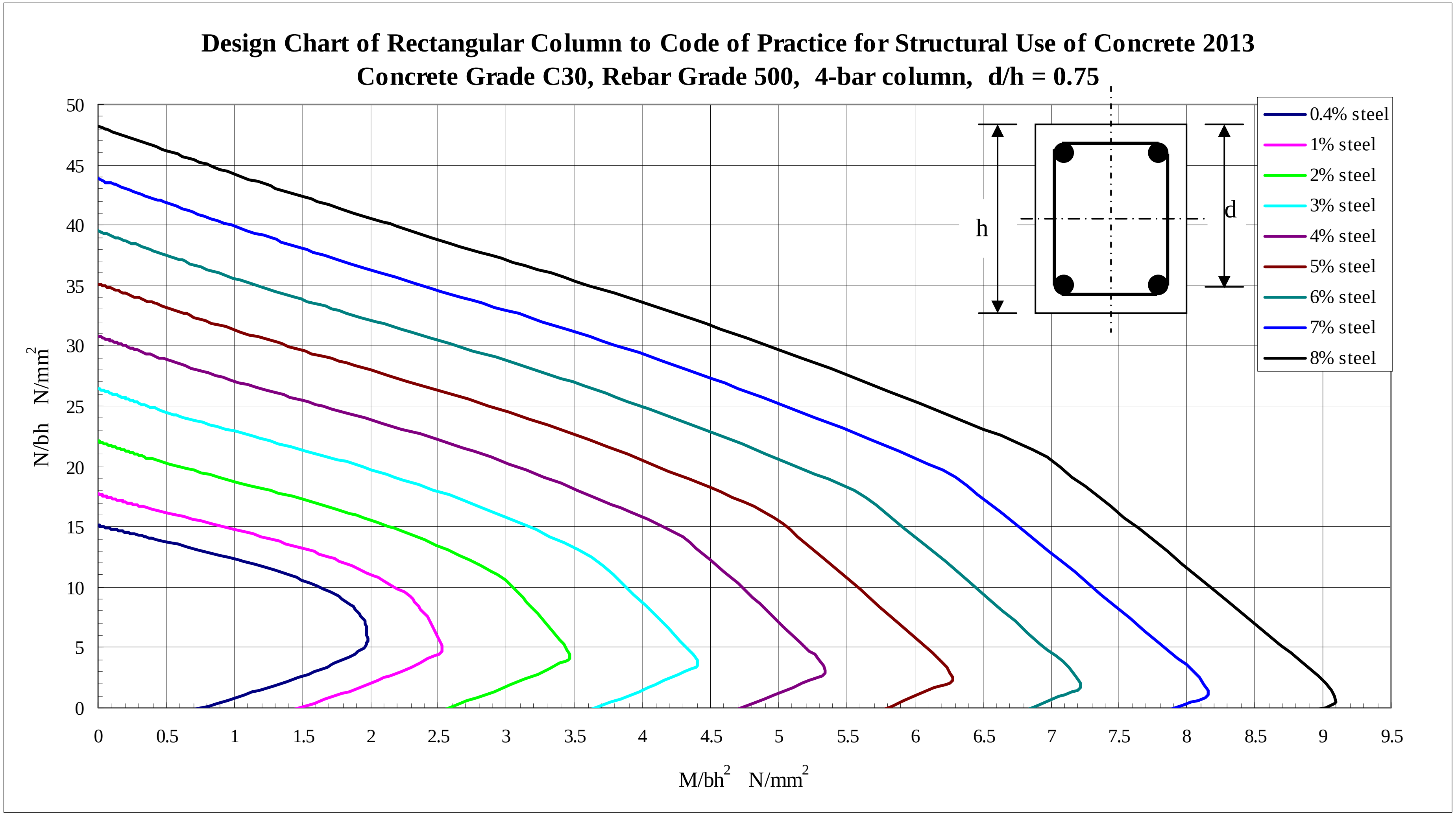
where $h = 2R$ is the diameter of the circular section.

E.5 Analysis of Column Section with Pre-determined Sizes and Reinforcement Bars

With pre-determined column plan sizes and rebars (pre-determined quantities and locations) the neutral axis depth for both rectangular and circular column can be solved so that the applied axial loads can be balanced at the appropriate case. For a rectangular column section, the process may start with the assumption of the neutral axis depth x at certain range, say the entirely within the section, then use (Eqn E-10) in Case 1 of E2 to express F_c in term of x and similarly express the F_s in term of x by summing the forces of the individual steel bars in accordance with the strain with respect to their locations. x can then be solved by $F = F_c + F_s$. If x does not exist, try other range such as Case 2 and using (Eqn E-14). The solution of x should more effectively by carried out by a systematic trial and error process which has to be used for circular columns which are more than polynominals. With the neutral axis depth obtained, the total moment of resistance can also be calculated and the section with the reinforcement bars can be considered adequate if the calculated moment of resistance is greater than the applied moment.

E.6 Plotting the *F/bh* verse *M/bh*^{2} Diagrams

From the above equations, it has been shown that the *F/bh* and *M/bh*^{2} where *F* and *M* are the total force and moments of resistance on the section are dependent on (i) the concrete grade; (ii) the reinforcement bars percentages ρ_s; (iii) the reinforcement bar locations (for 4 bar column, the "cover ratio" which is the concrete cover (d') to the centre of the steel divided by the length of the column (*h*)". Keeping (i), (ii) and (iii) of the above as constant, a *F/bh* and *M/bh*^{2} curve can be plotted for varying values of neutral axis depth ratios (*x/h*). An interaction diagram for a concrete grade and constant d'/h ratio can be plotted for ρ_s ranging from 0.4% to 8%. The diagram can be used for structural design by which, with the known values of column size (*b* and *h*), force and moment on the column section (*N* and *M*) and d'/h ratio, the required ρ_s can be estimated. For the plotting, the limits are from neutral axis = 0 up to (i) where the column section is under the constant part of the concrete stress block only; and (ii) the strain of the reinforcement bars are stressed up to $0.87 f_y$ in compression where the axial load capacity is maximum with moment of resistance = 0. Beyond the greater of these two limits, the column section will be having the maximum axial load capacity but zero moment of resistance. For the achievement of the former, $(1-\varepsilon_r)x \ge h \Rightarrow \frac{x}{h} \ge \frac{1}{1-\varepsilon_r}$ by Figure E-6 for rectangular section and $\frac{x}{R} \ge \frac{2}{1-\varepsilon_r}$ for circular section. For achievement of the latter, $\frac{\varepsilon_{sy}}{d_n - h + d'} \le \frac{\varepsilon_{cu}}{d_n} \Rightarrow \frac{d_n}{h} \ge \frac{\varepsilon_{cu}(1-d'/h)}{\varepsilon_{cu}-\varepsilon_{sy}}$ for rectangular section (where ε_{sy} is the yield strain of steel taken as $\frac{0.87f_y}{E_s}$) and $\frac{\varepsilon_{sy}}{d_n - 2R + d'} \le \frac{\varepsilon_{cu}}{d_n} \Rightarrow \frac{d_n}{R} \ge \frac{\varepsilon_{cu}(2-d'/h)}{\varepsilon_{cu}-\varepsilon_{sy}}$ for circular section.


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C30, Rebar Grade 500, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
2
2.5
3
3.5
4
4.5
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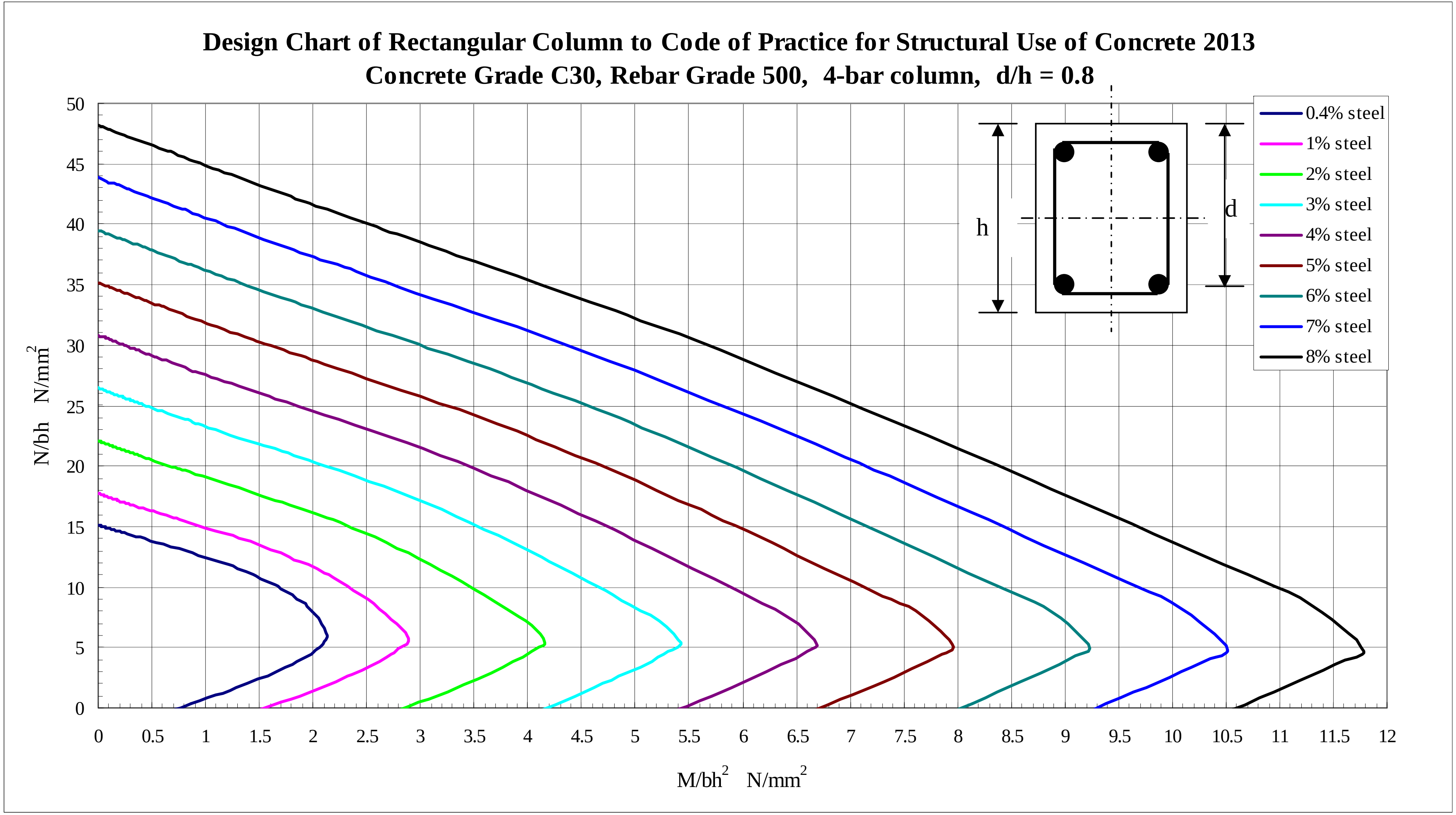
8
8.5
9
9.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C30, Rebar Grade 500, 4-bar column, d/h = 0.8
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
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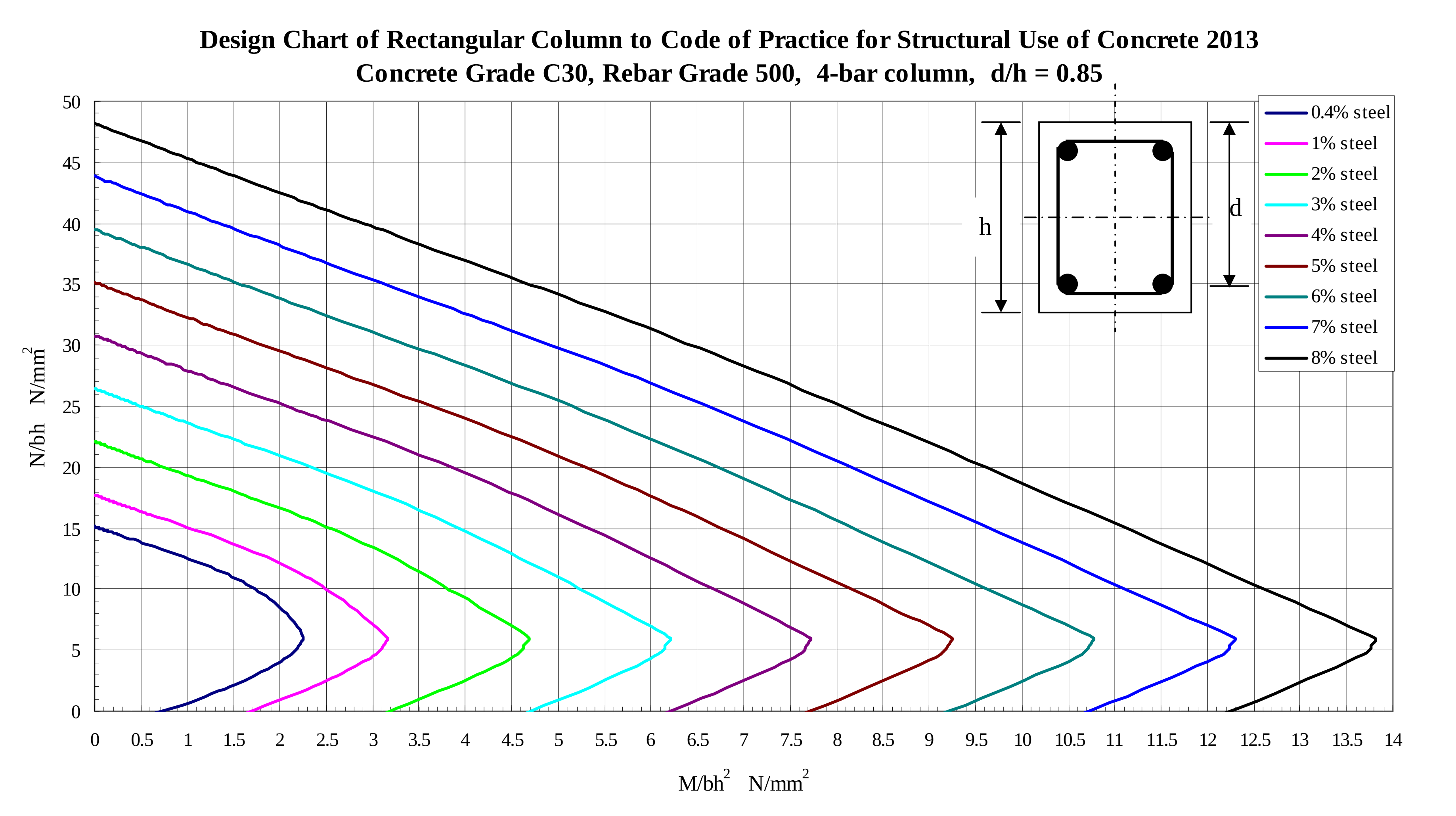
10.5
11
11.5
12



Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C30, Rebar Grade 500, 4-bar column, d/h = 0.85
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
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4
4.5
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5.5
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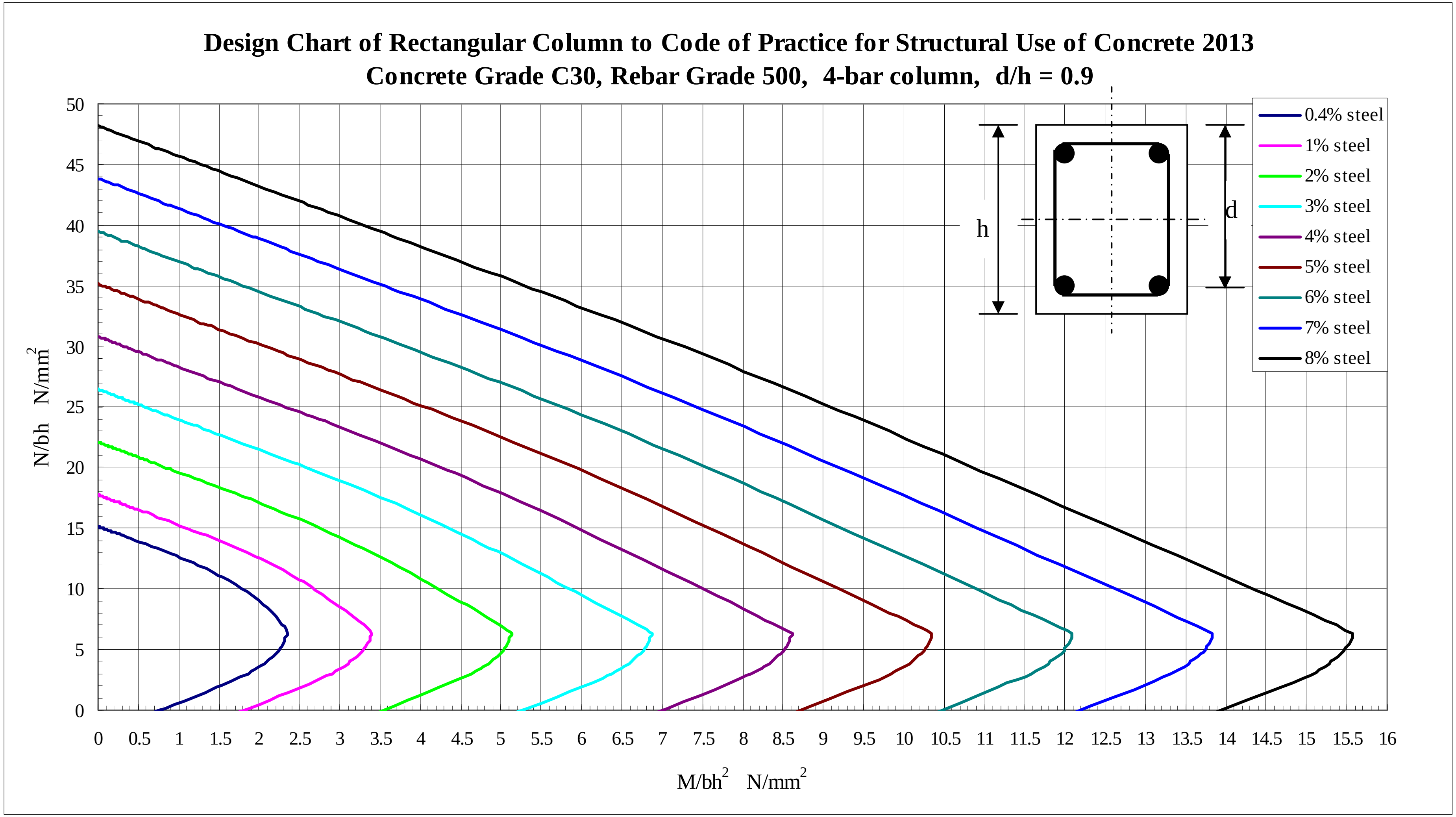
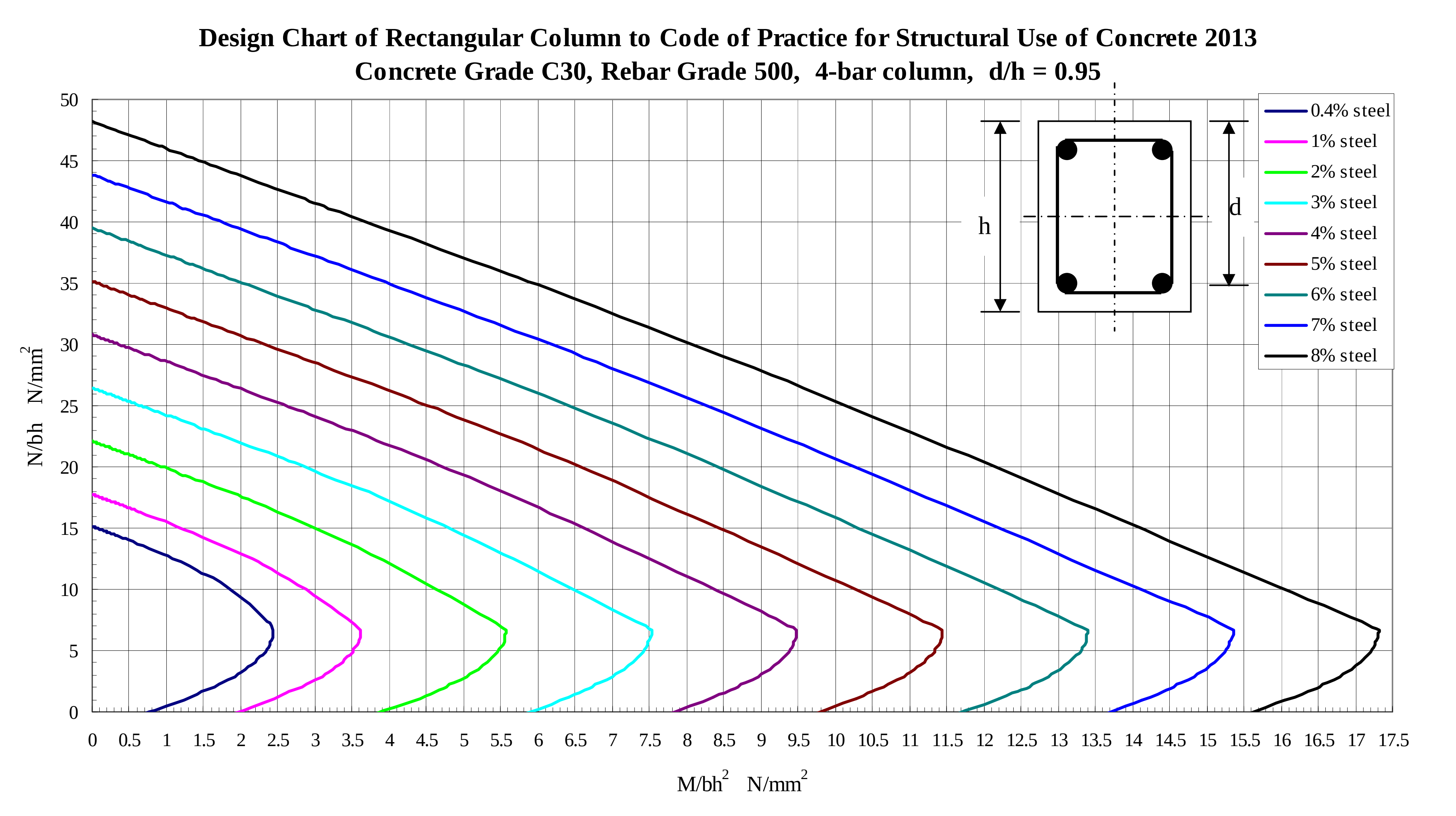
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C30, Rebar Grade 500, 4-bar column, d/h = 0.9
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C30, Rebar Grade 500, 4-bar column, d/h = 0.95
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
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35
40
45
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0.5
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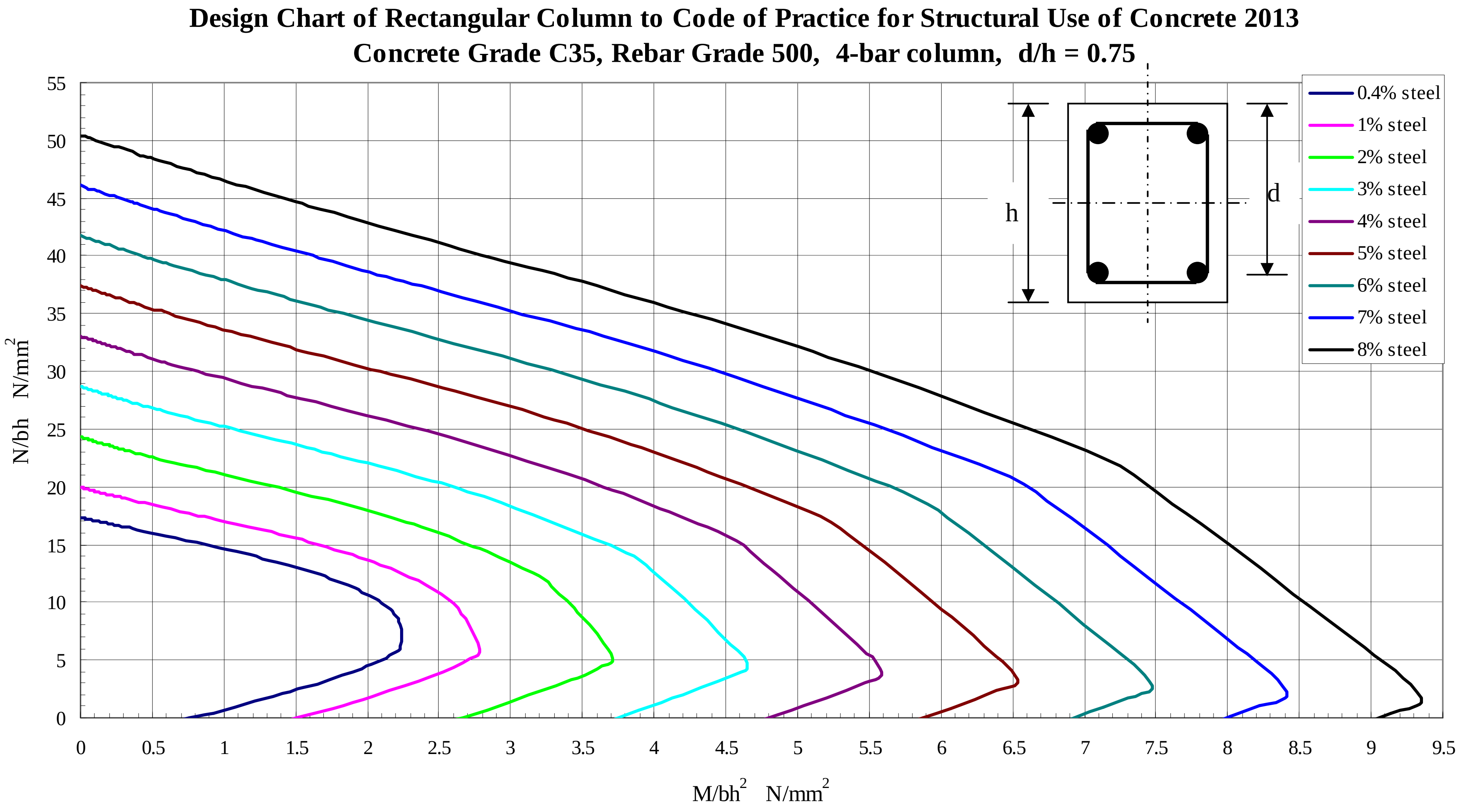
17.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C35, Rebar Grade 500, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
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0.5
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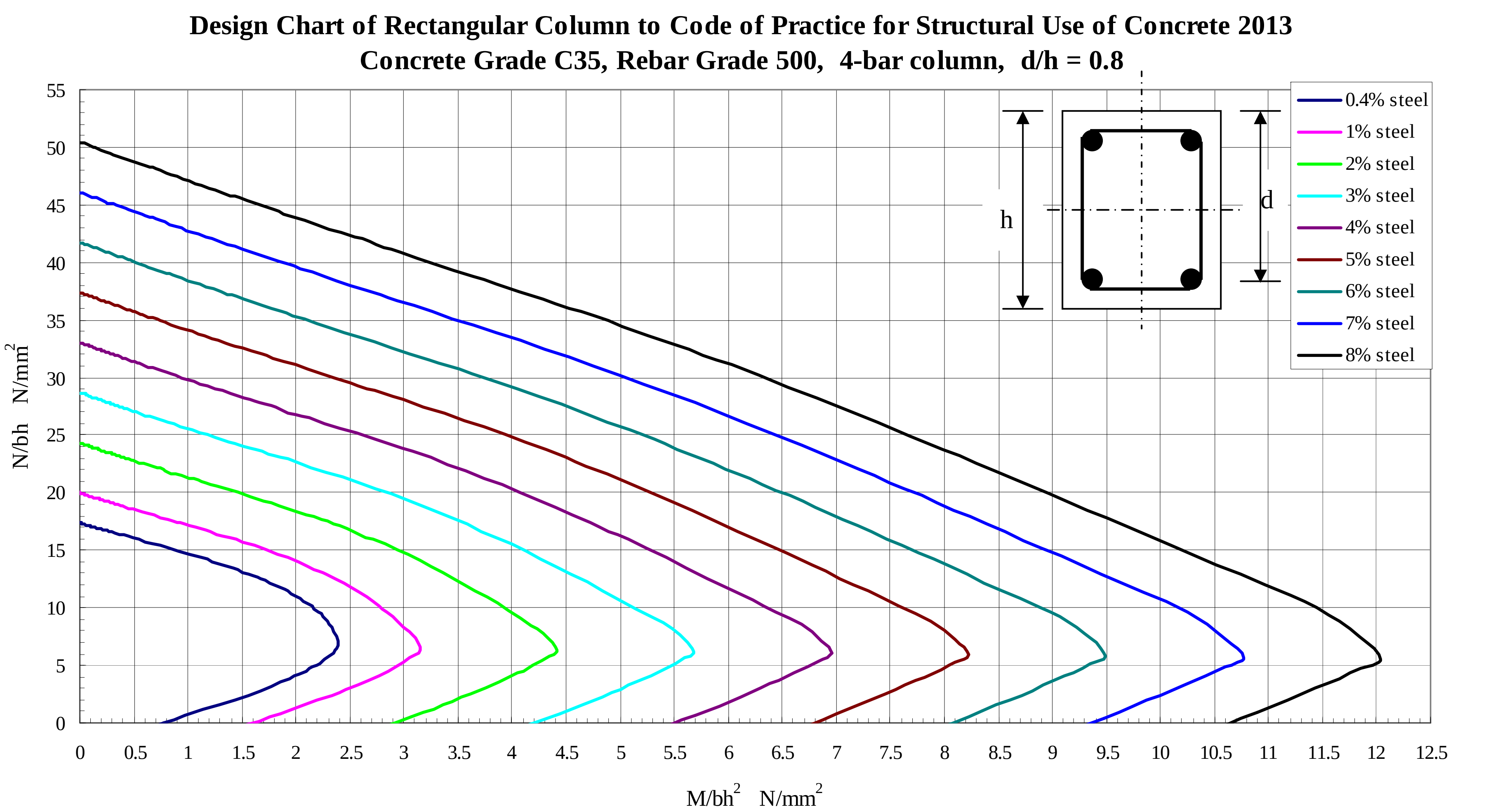
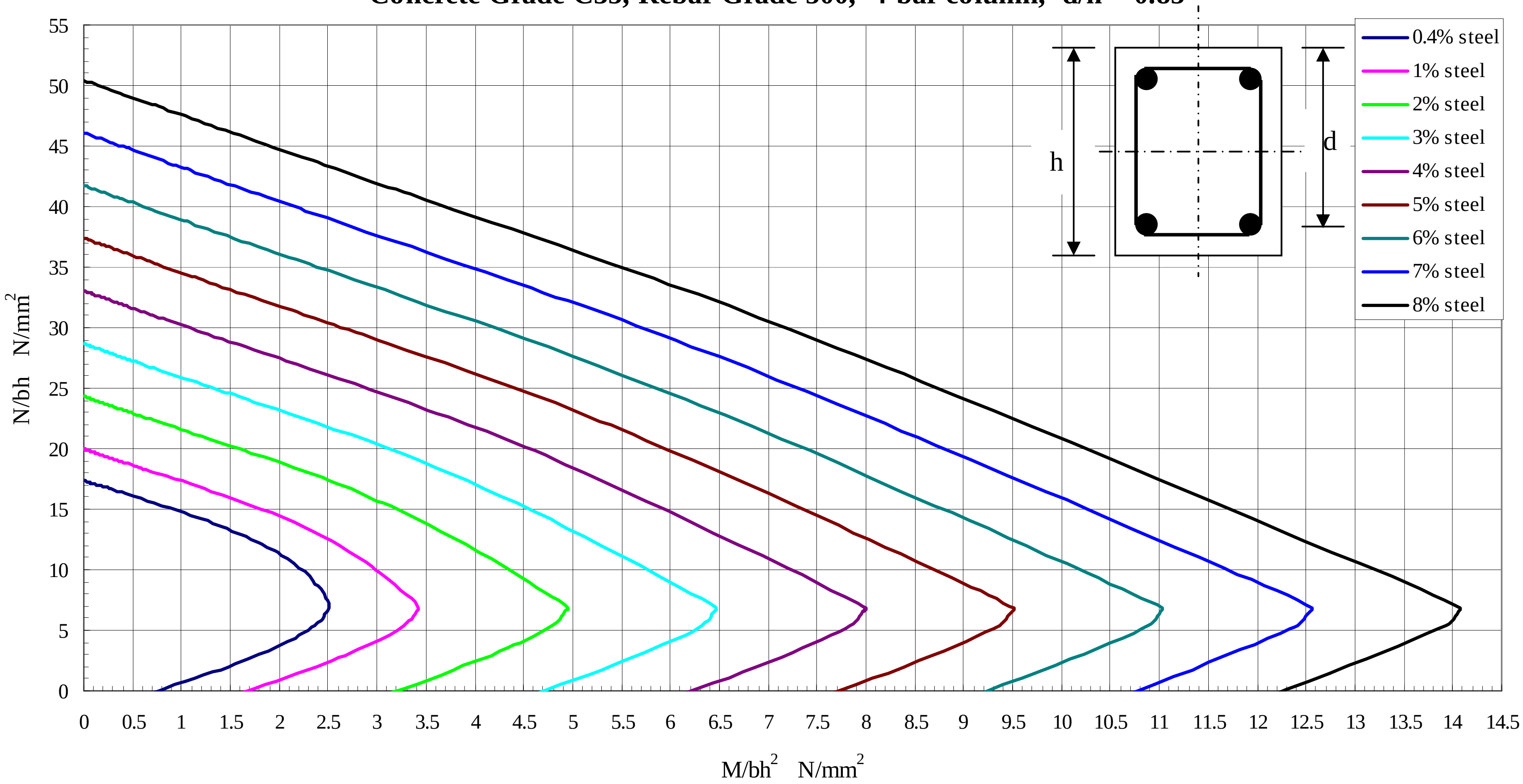
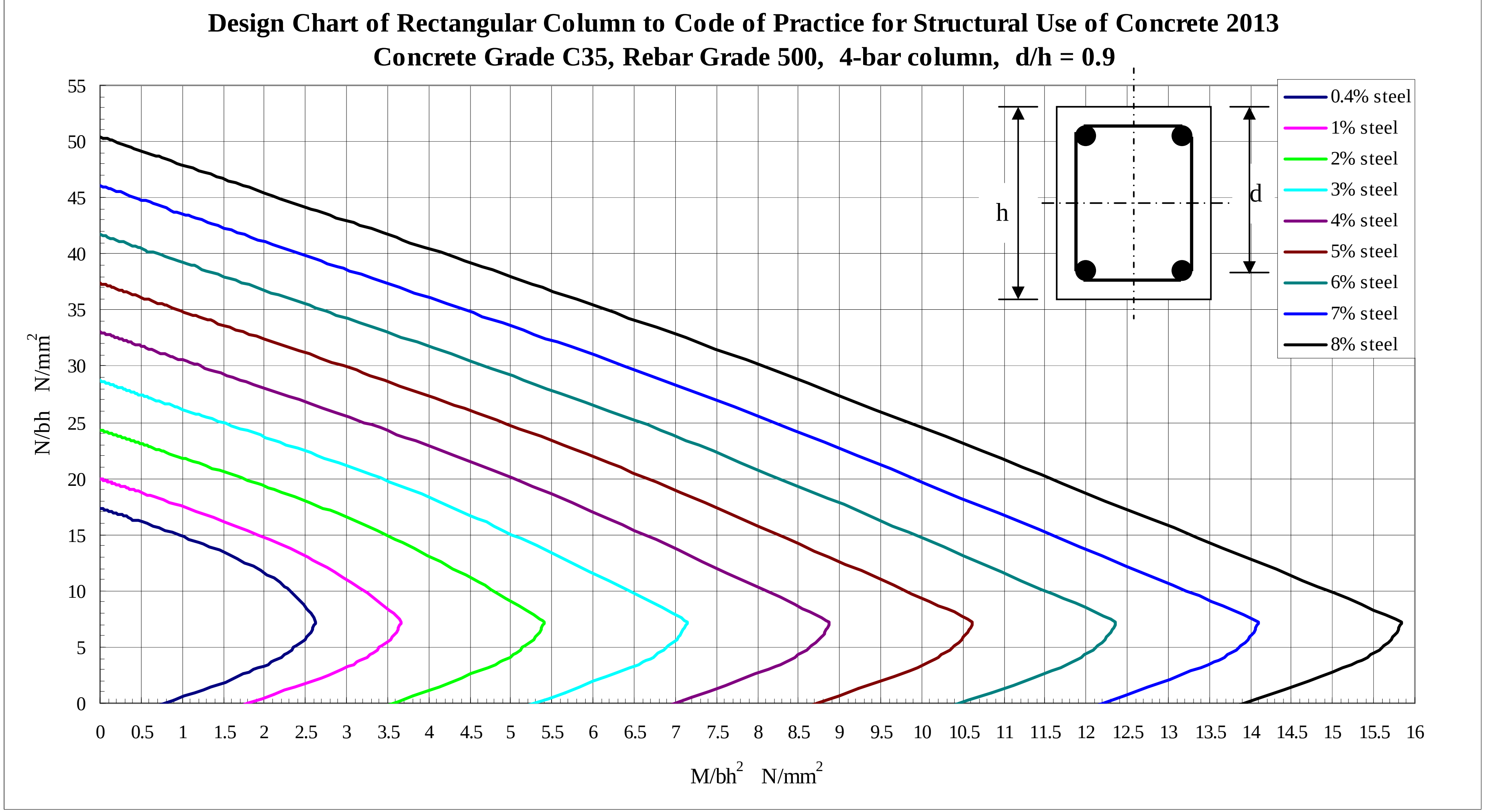
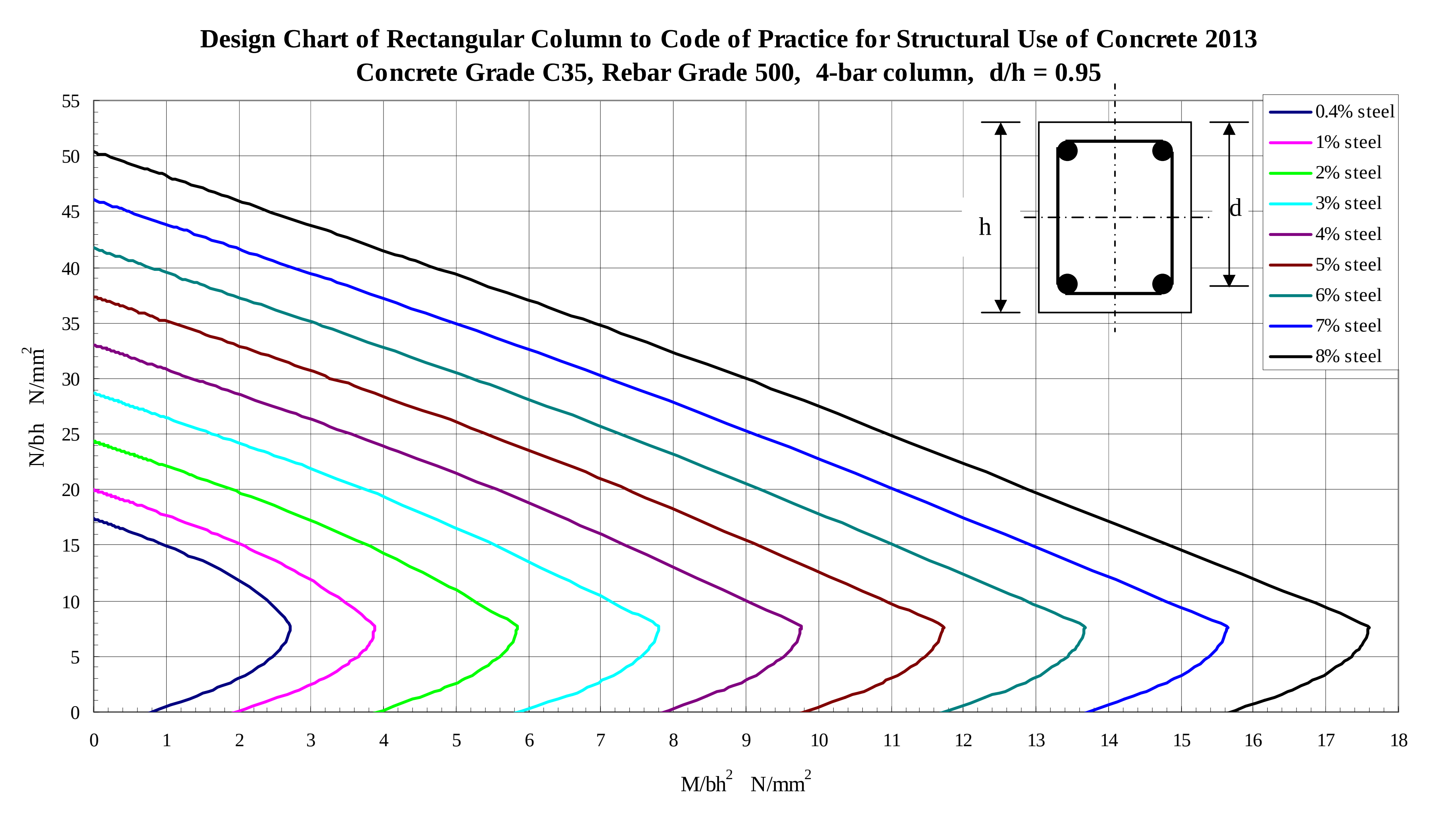

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C35, Rebar Grade 500, 4-bar column, d/h = 0.8
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d
N/bh N/mm2
M/bh2 N/mm2

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013

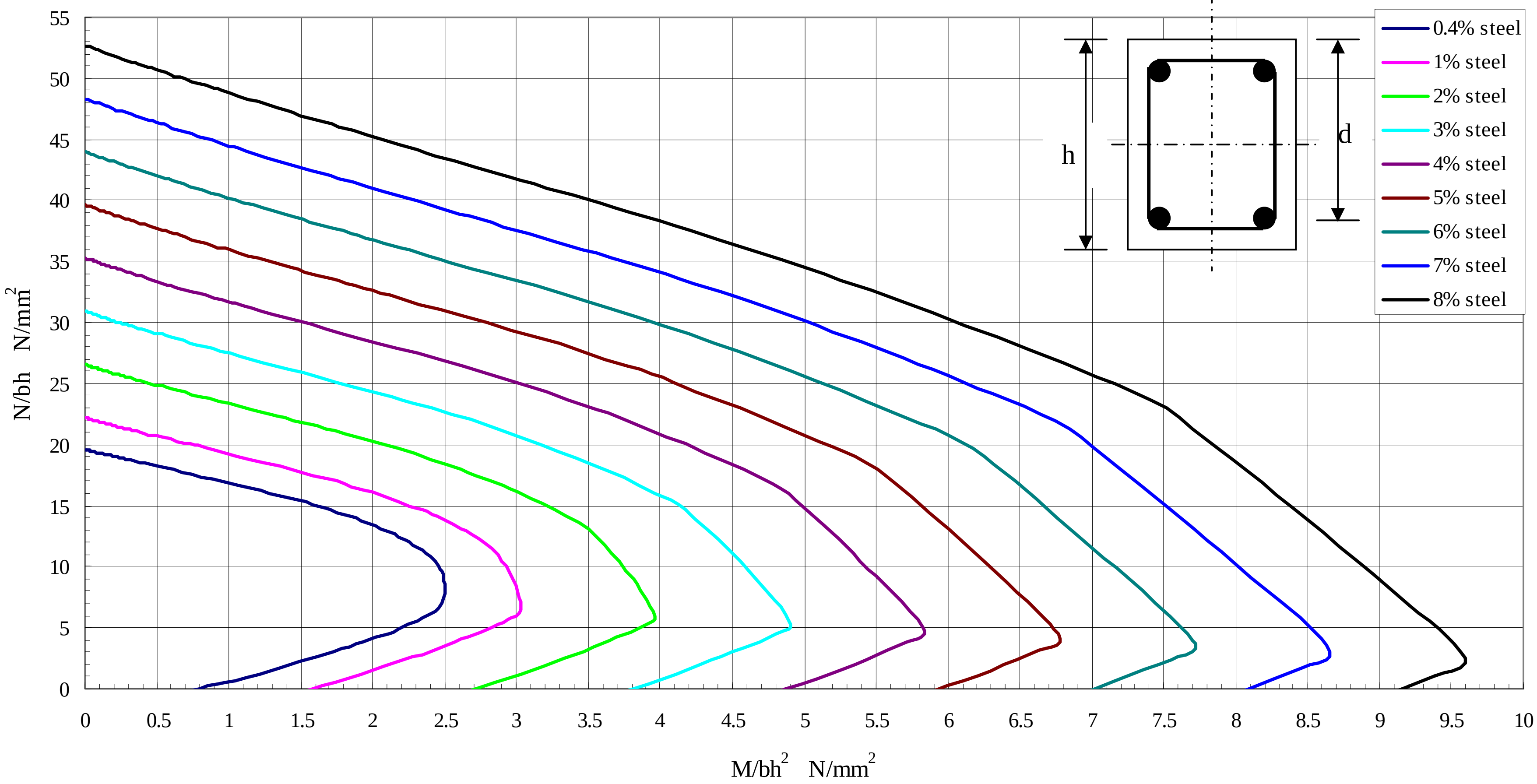
Concrete Grade C35, Rebar Grade 500, 4-bar column, d/h = 0.85

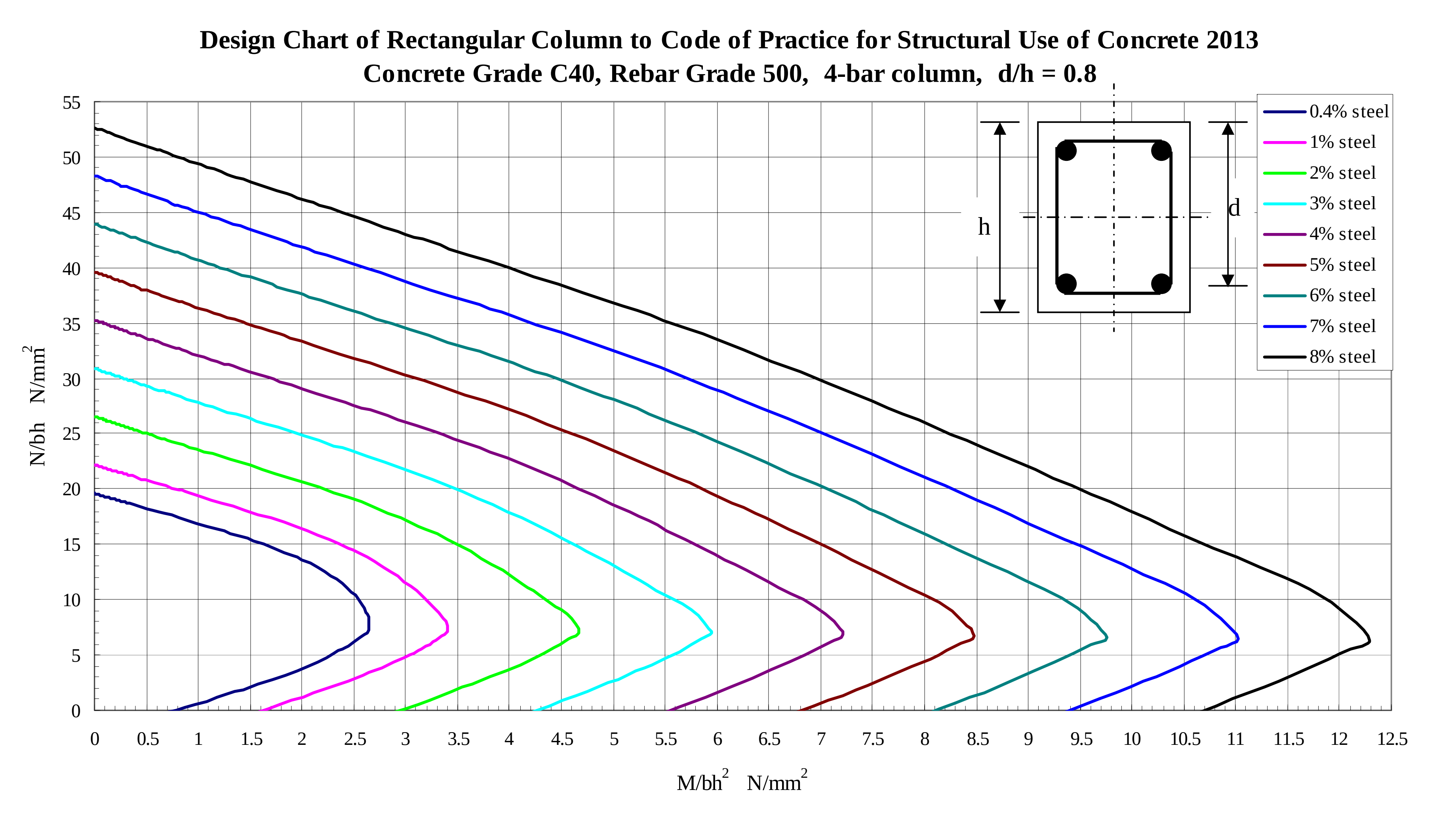



Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C35, Rebar Grade 500, 4-bar column, d/h = 0.9
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C35, Rebar Grade 500, 4-bar column, d/h = 0.95
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
0 5 10 15 20 25 30 35 40 45 50 55

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C40, Rebar Grade 500, 4-bar column, d/h = 0.75




Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C40, Rebar Grade 500, 4-bar column, d/h = 0.8
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
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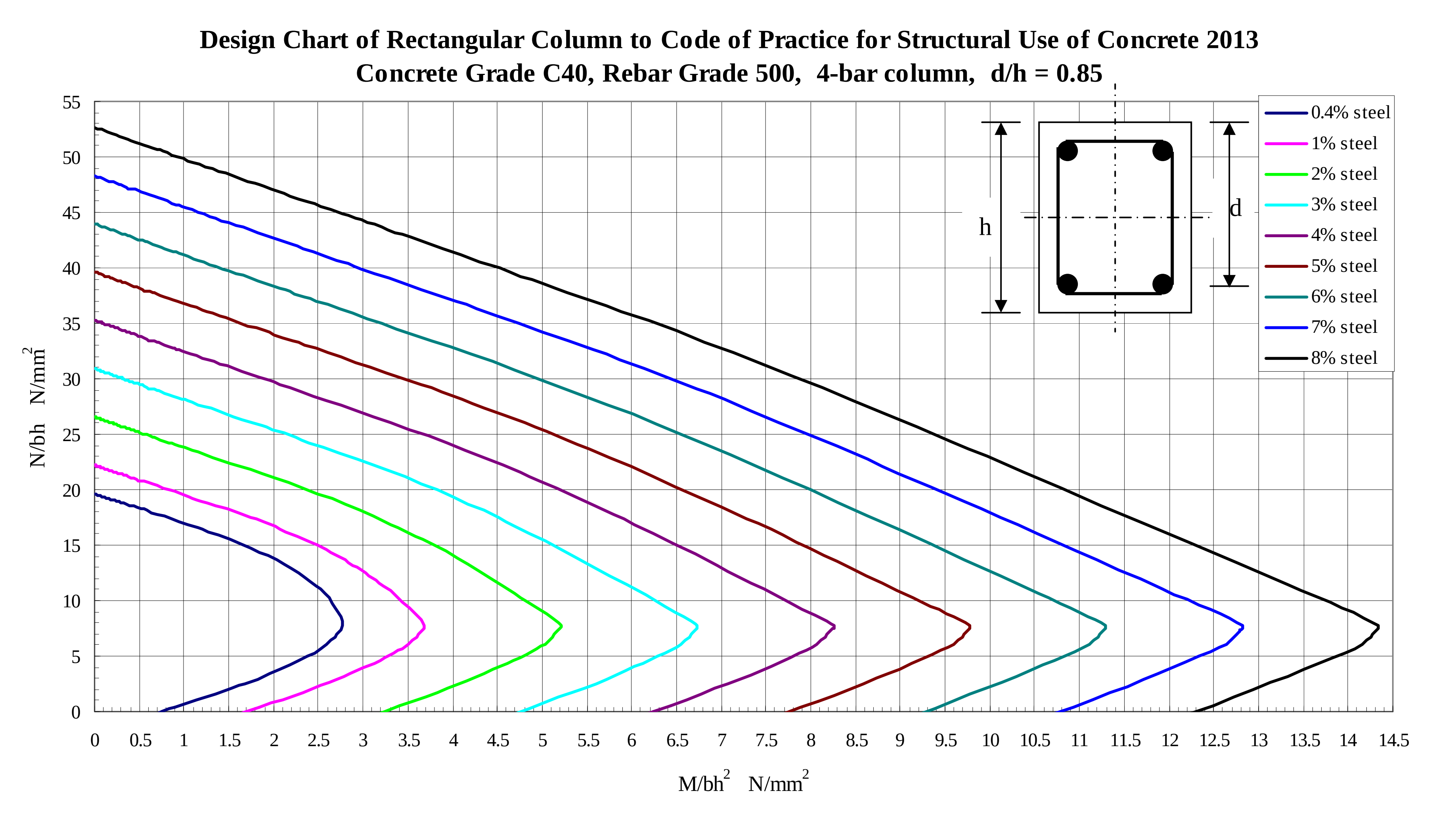
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C40, Rebar Grade 500, 4-bar column, d/h = 0.85
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d
0
5
10
15
20
25
30
35
40
45
50
55
0.5
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1.5
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2.5
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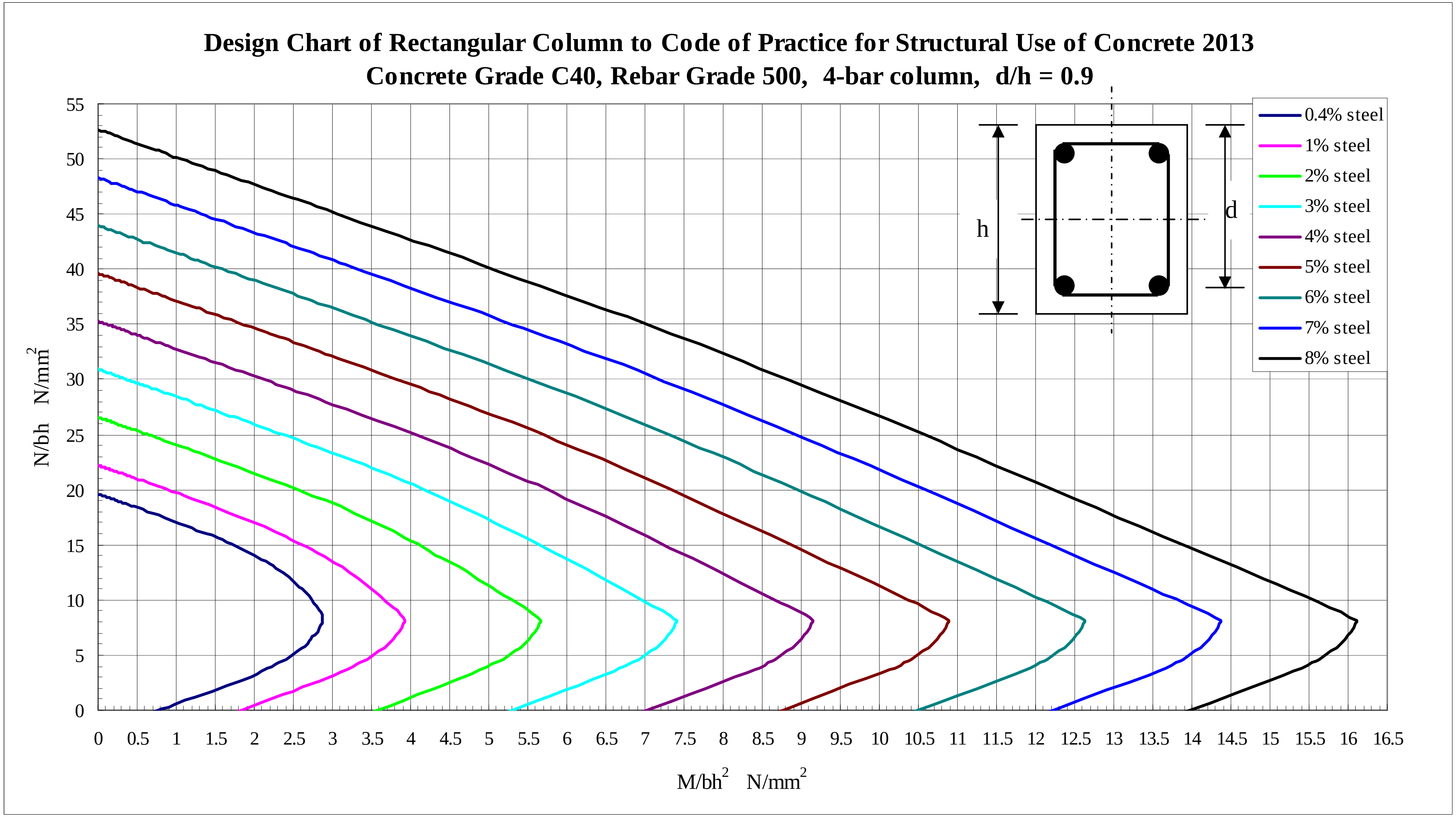
14.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C40, Rebar Grade 500, 4-bar column, d/h = 0.9
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
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0.5
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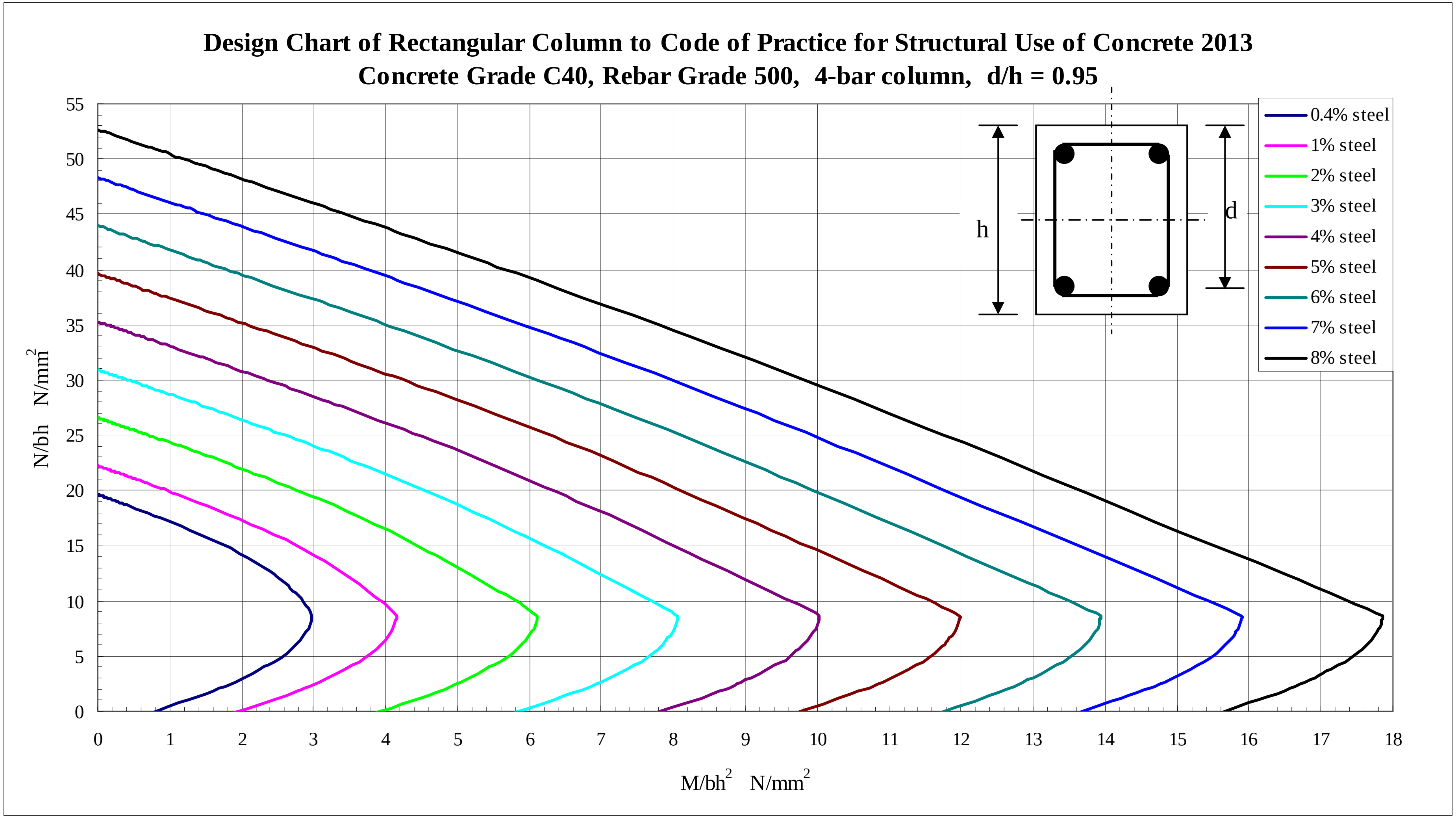
15.5
16
16.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C40, Rebar Grade 500, 4-bar column, d/h = 0.95
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
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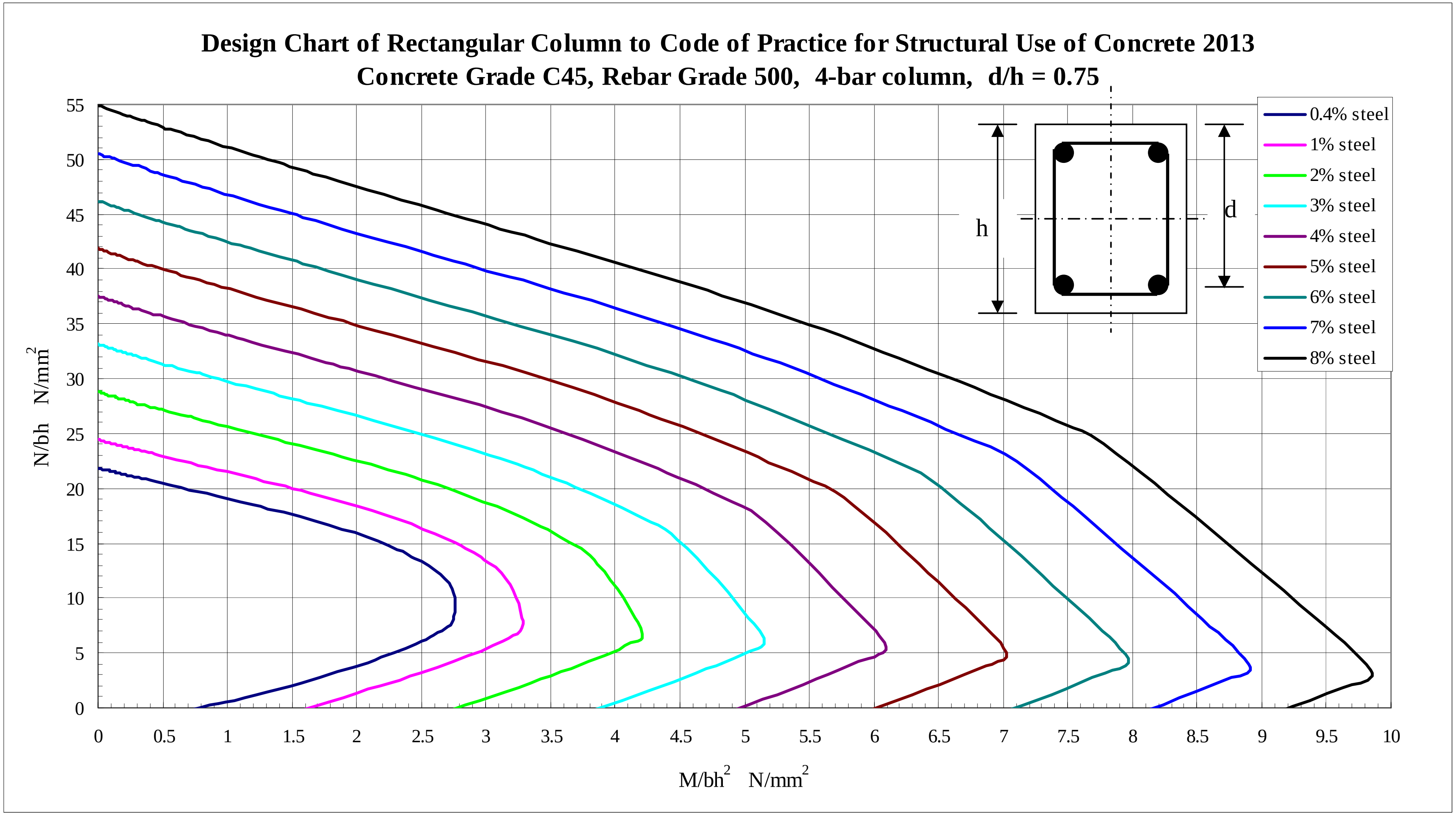
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C45, Rebar Grade 500, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d
0
5
10
15
20
25
30
35
40
45
50
55
0.5
1
1.5
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2.5
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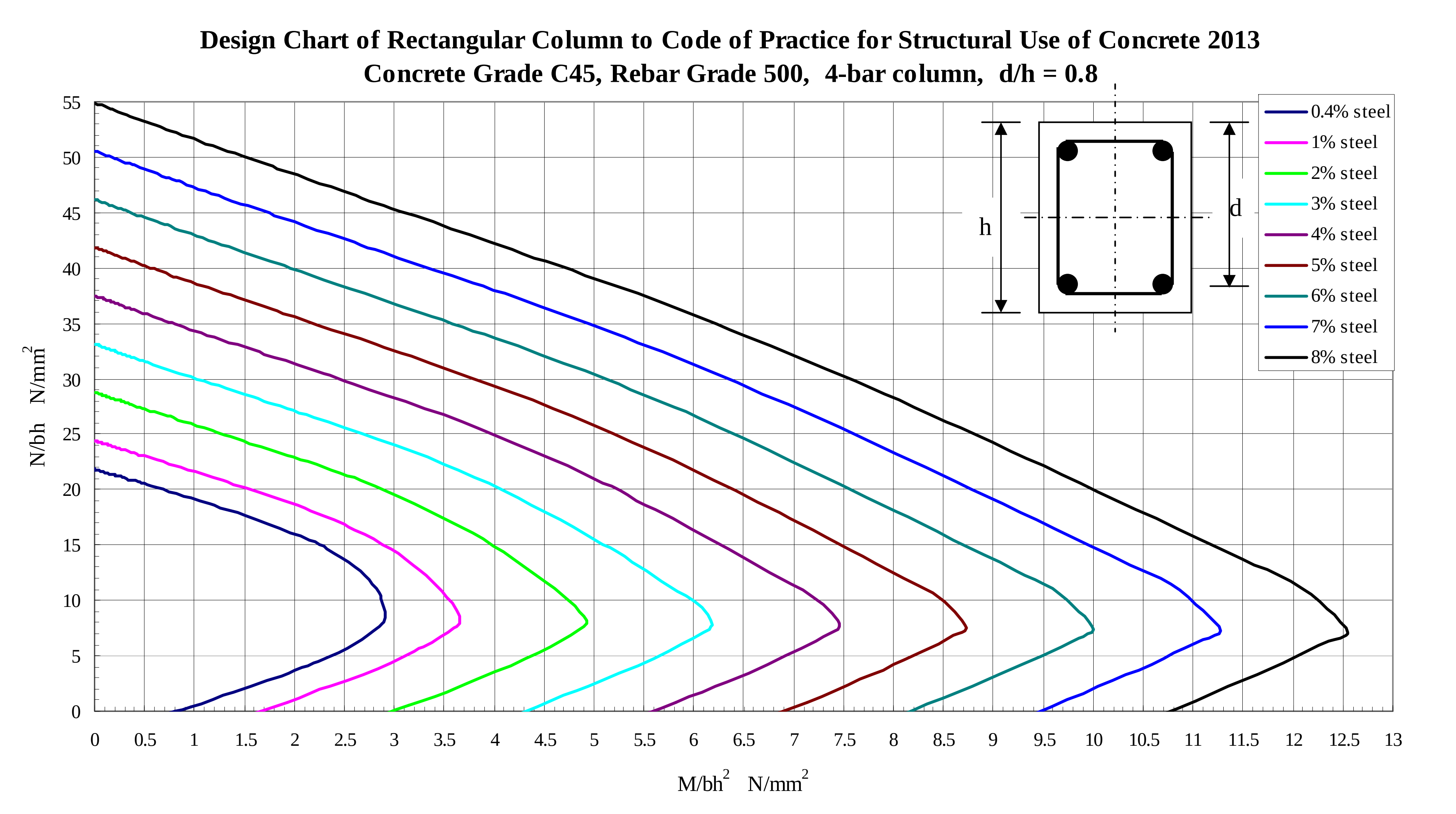
9.5
10


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C45, Rebar Grade 500, 4-bar column, d/h = 0.8
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d
0
5
10
15
20
25
30
35
40
45
50
55
0.5
1
1.5
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12.5
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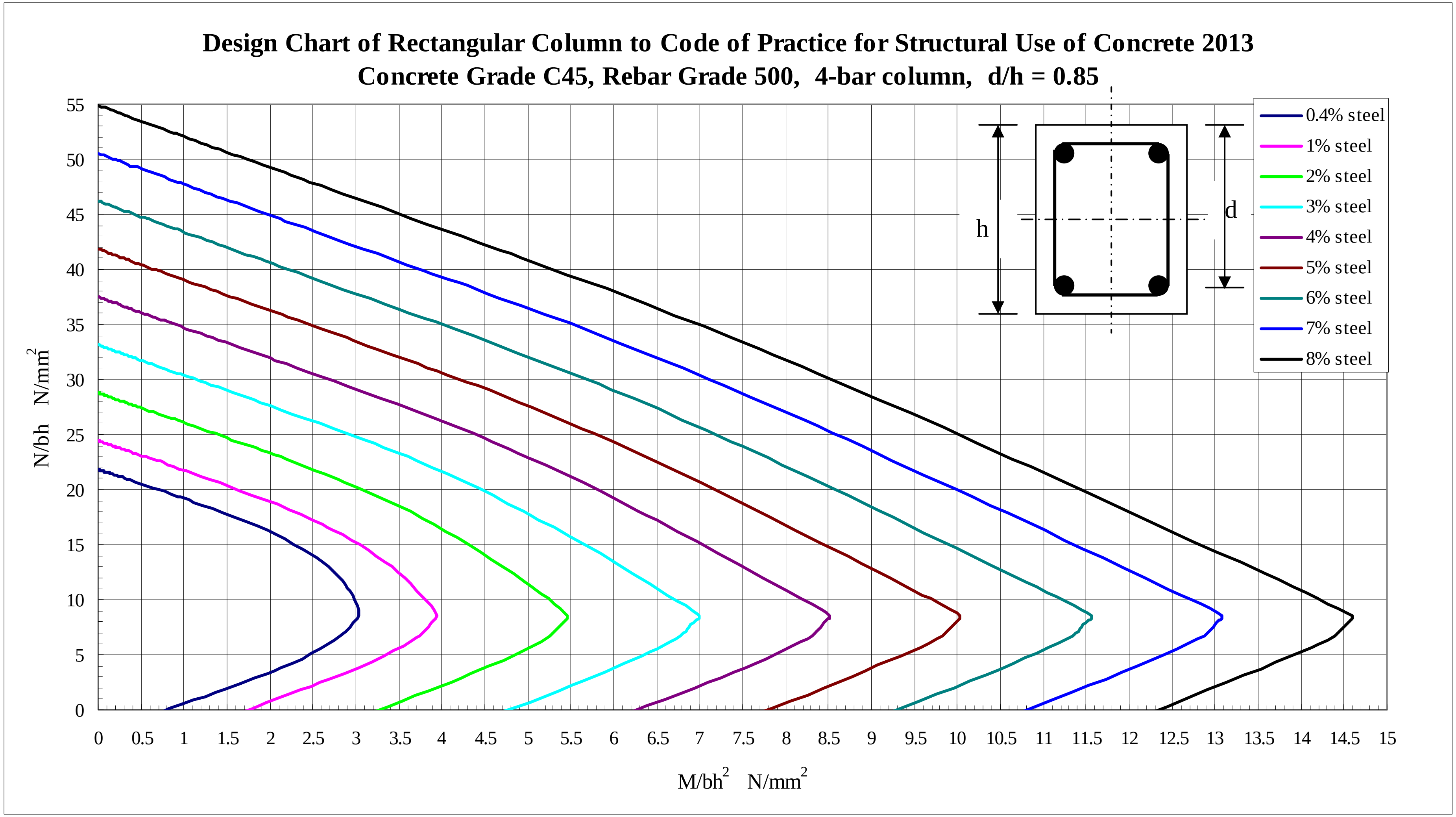
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C45, Rebar Grade 500, 4-bar column, d/h = 0.85
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
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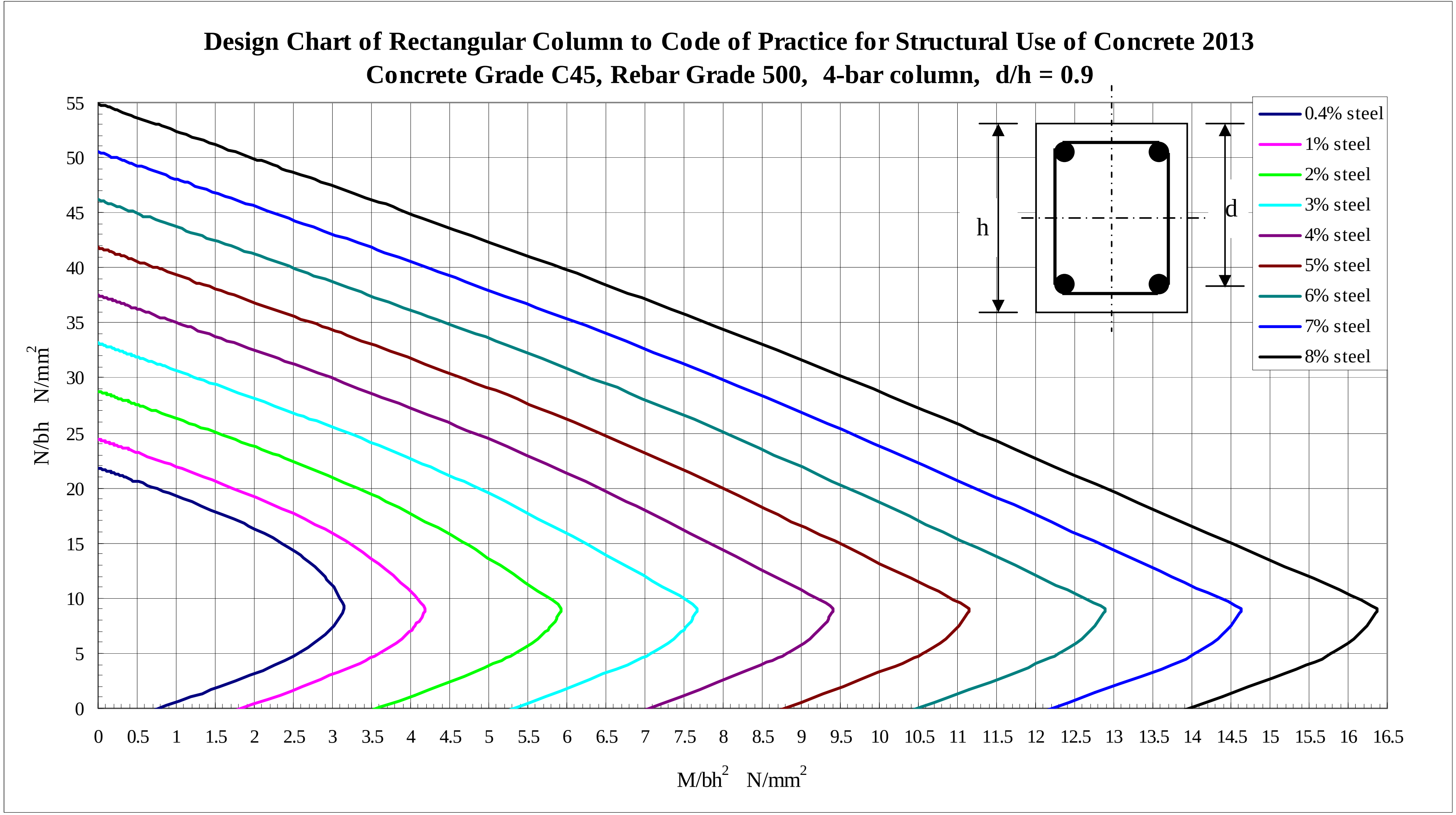
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C45, Rebar Grade 500, 4-bar column, d/h = 0.9
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
0
0.5
1
1.5
2
2.5
3
3.5
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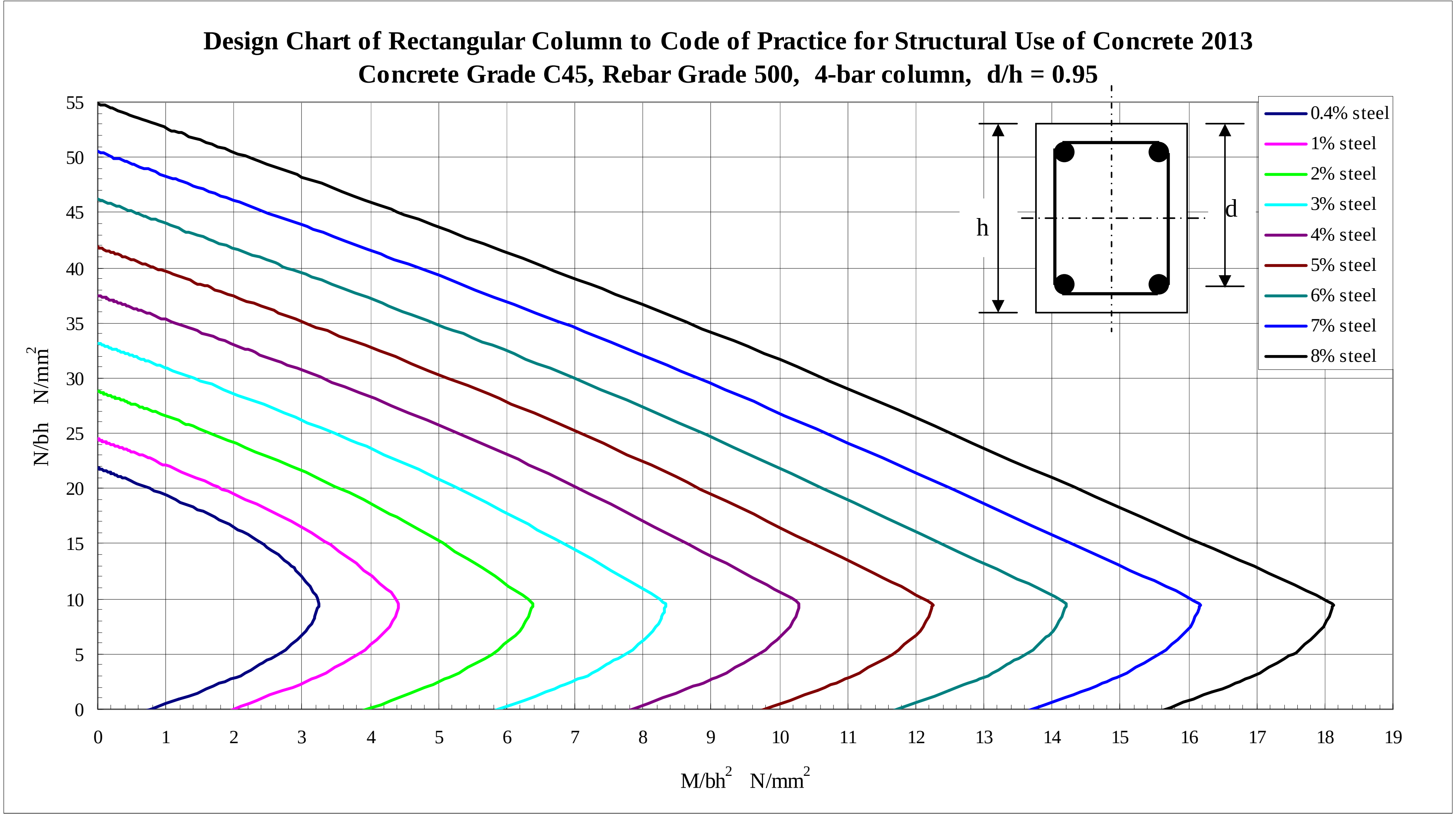
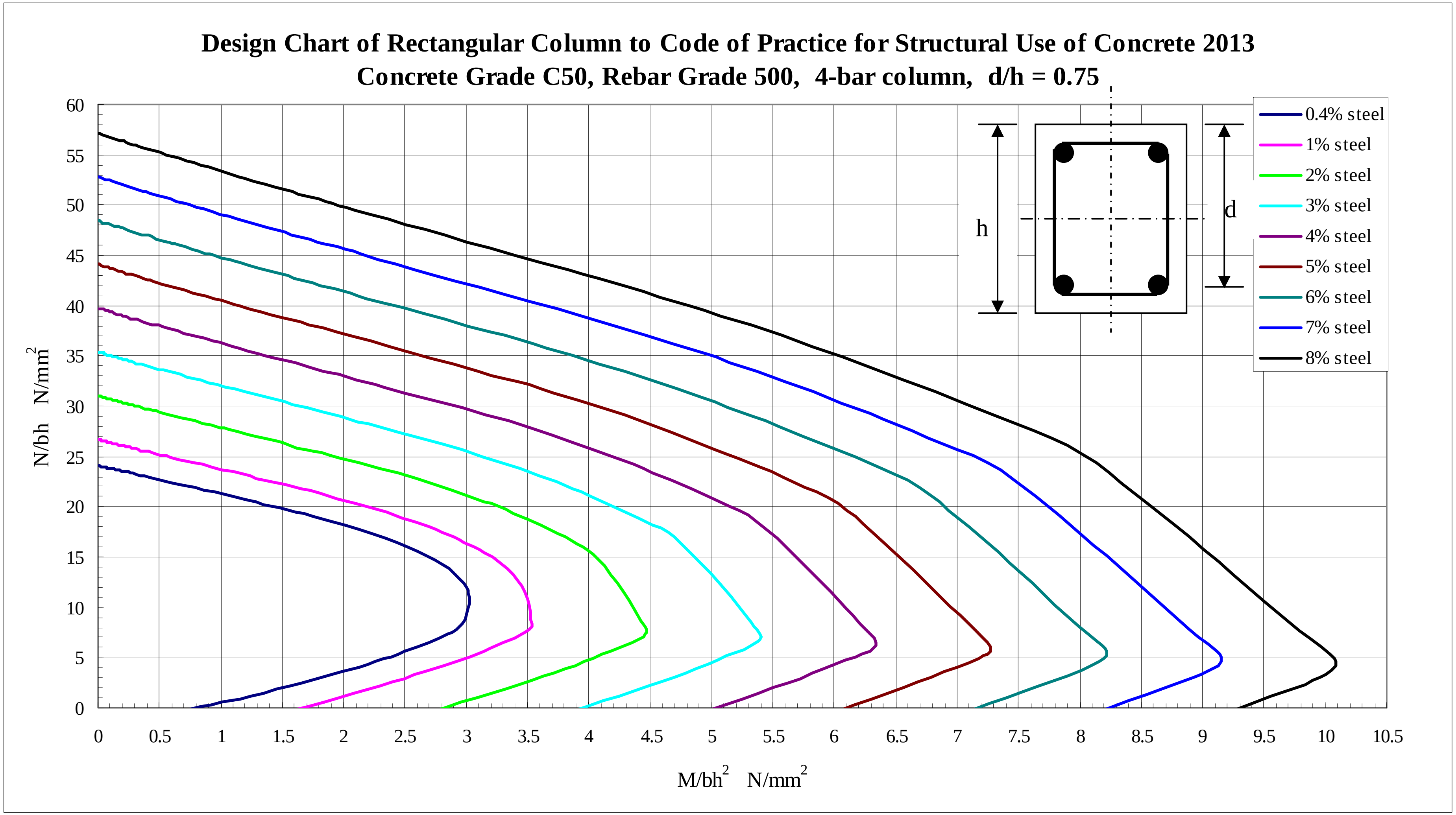
14
14.5
15
15.5
16
16.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C45, Rebar Grade 500, 4-bar column, d/h = 0.95
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C50, Rebar Grade 500, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
60
0
0.5
1
1.5
2
2.5
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8.5
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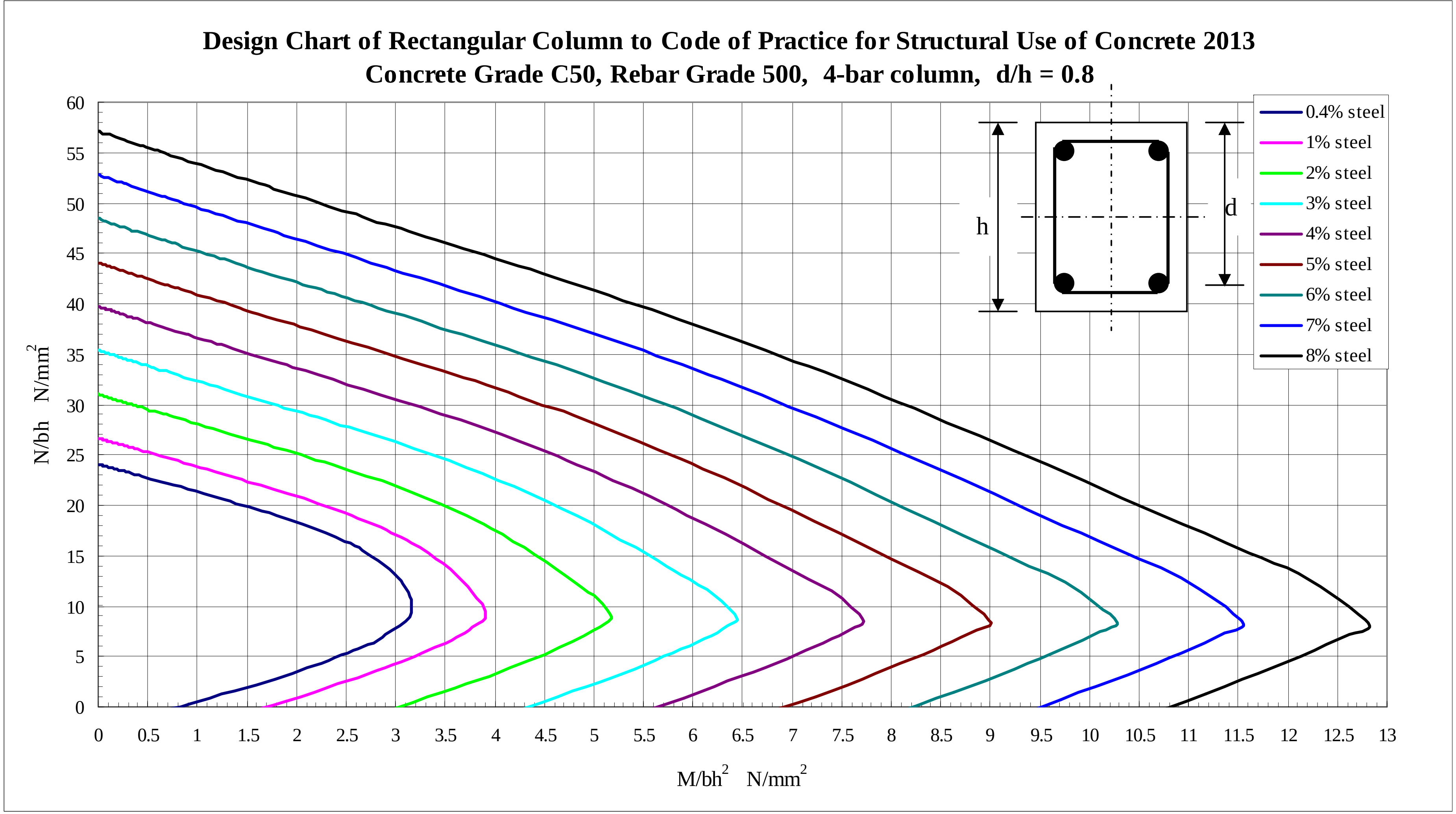
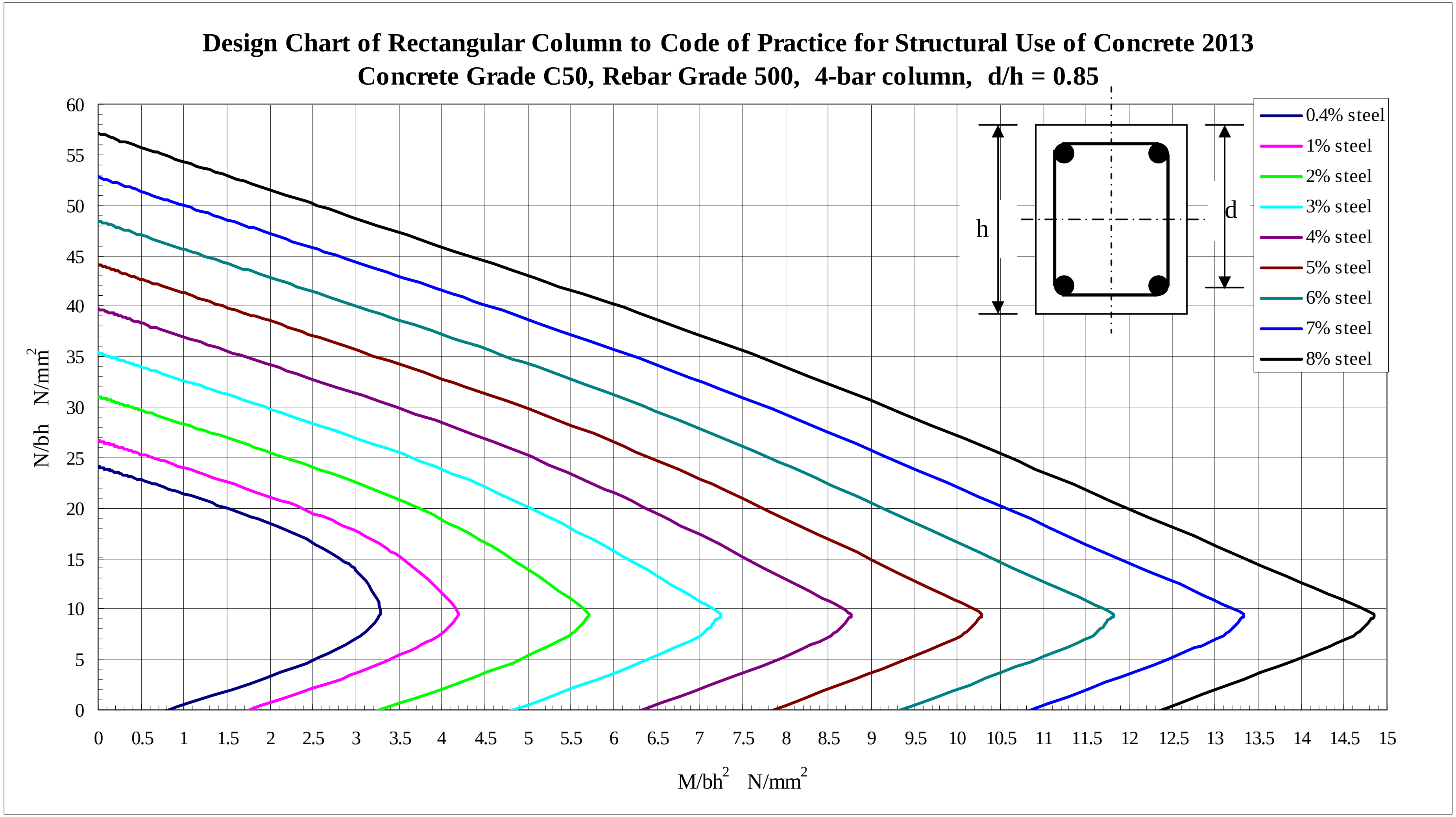
9.5
10
10.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C50, Rebar Grade 500, 4-bar column, d/h = 0.8
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C50, Rebar Grade 500, 4-bar column, d/h = 0.85
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d
N/bh² N/mm²
M/bh² N/mm²
0
5
10
15
20
25
30
35
40
45
50
55
60
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5.5
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6.5
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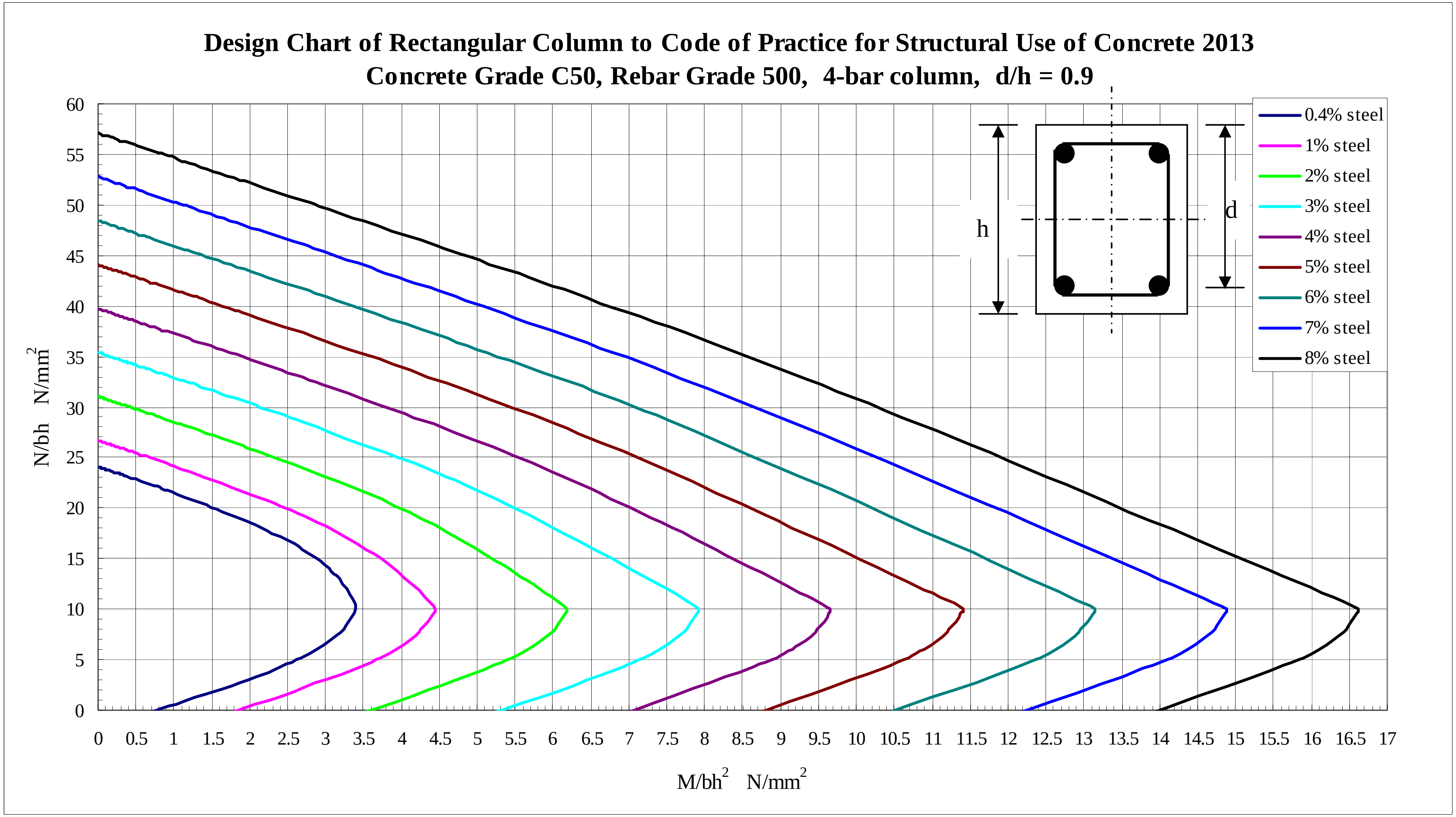
14.5
15


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C50, Rebar Grade 500, 4-bar column, d/h = 0.9
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
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55
60
0.5
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16.5
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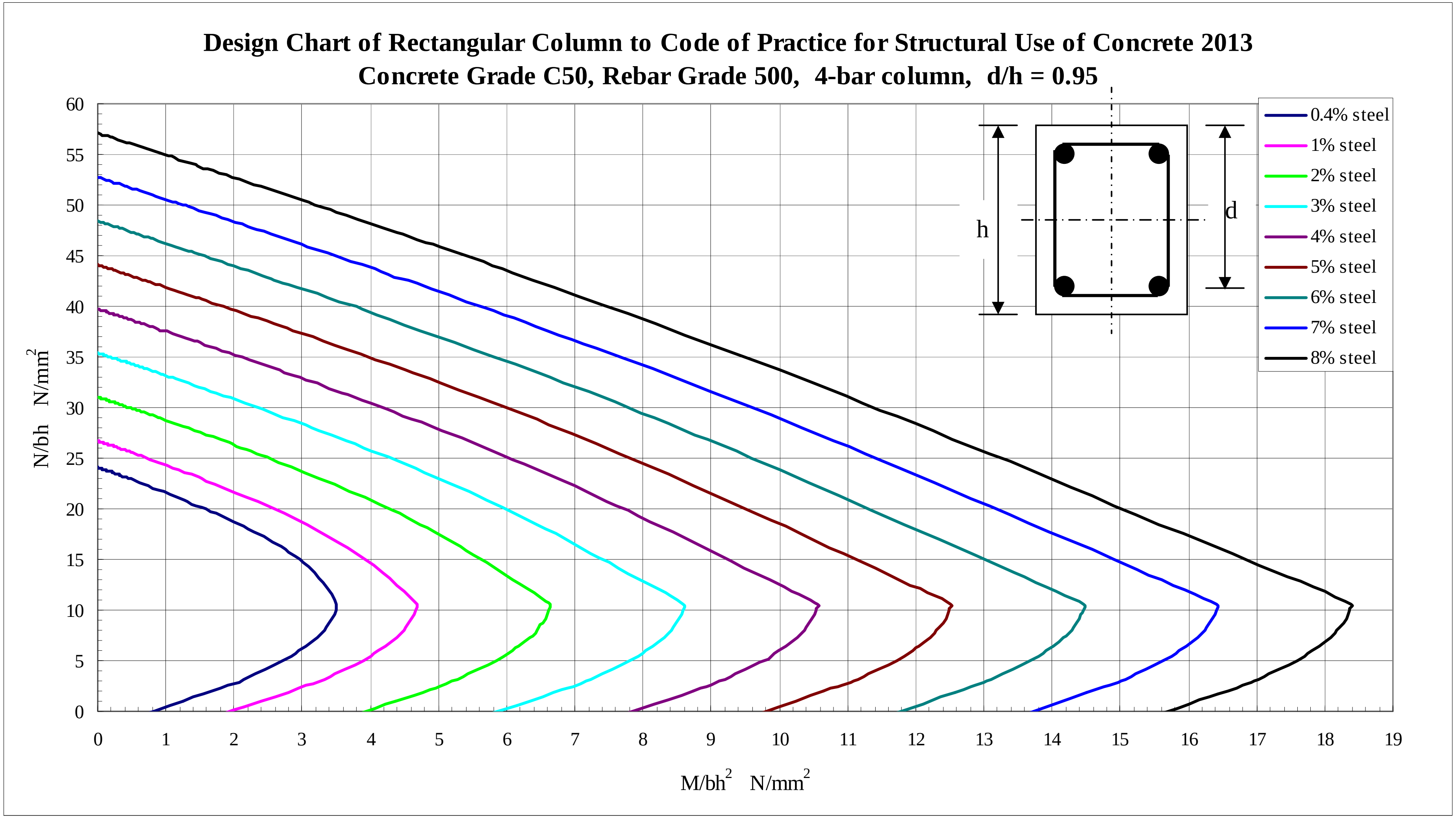
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C50, Rebar Grade 500, 4-bar column, d/h = 0.95
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
1
2
3
4
5
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7
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9
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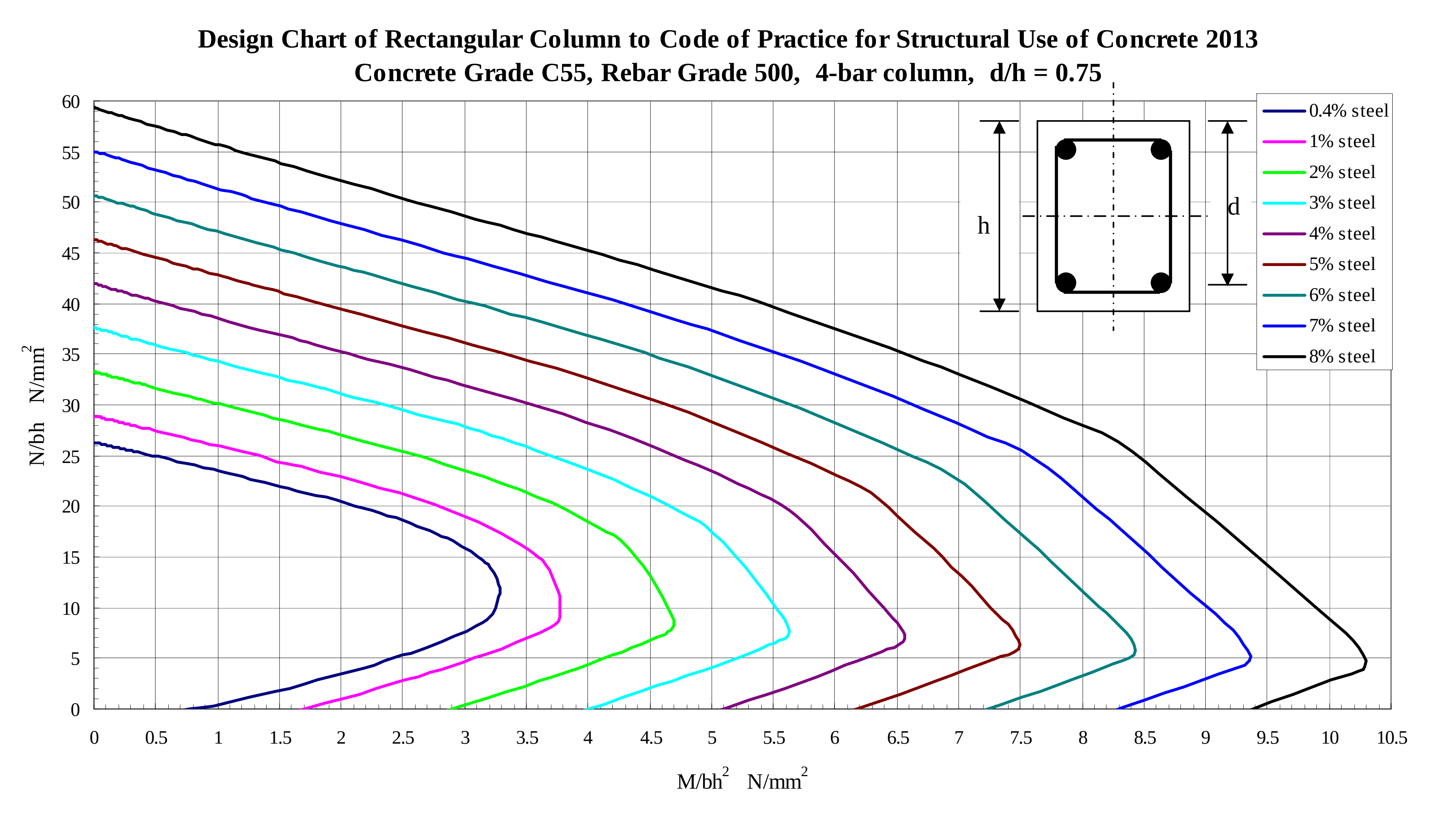
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C55, Rebar Grade 500, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
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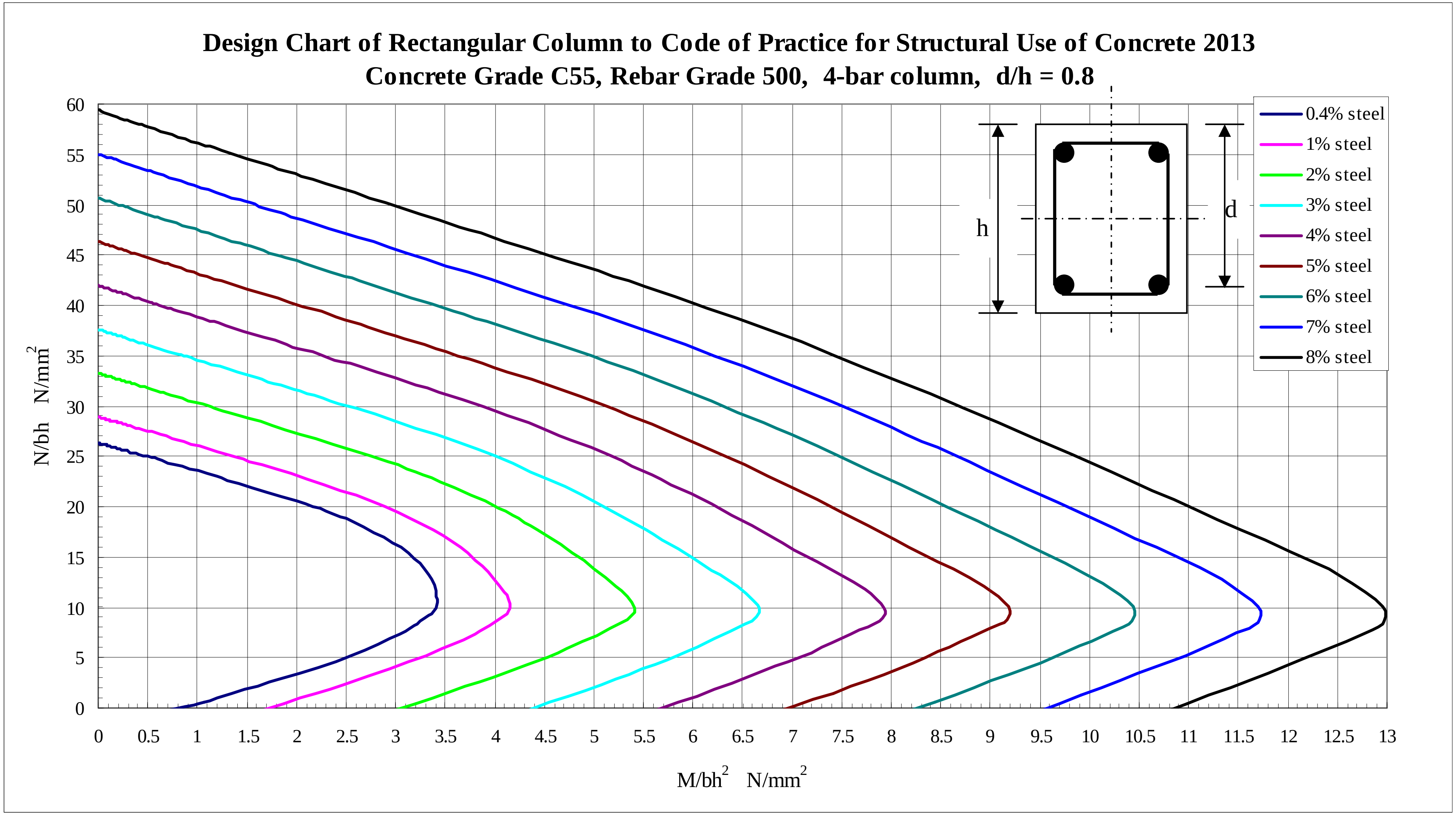
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C55, Rebar Grade 500, 4-bar column, d/h = 0.8
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d
0
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35
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50
55
60
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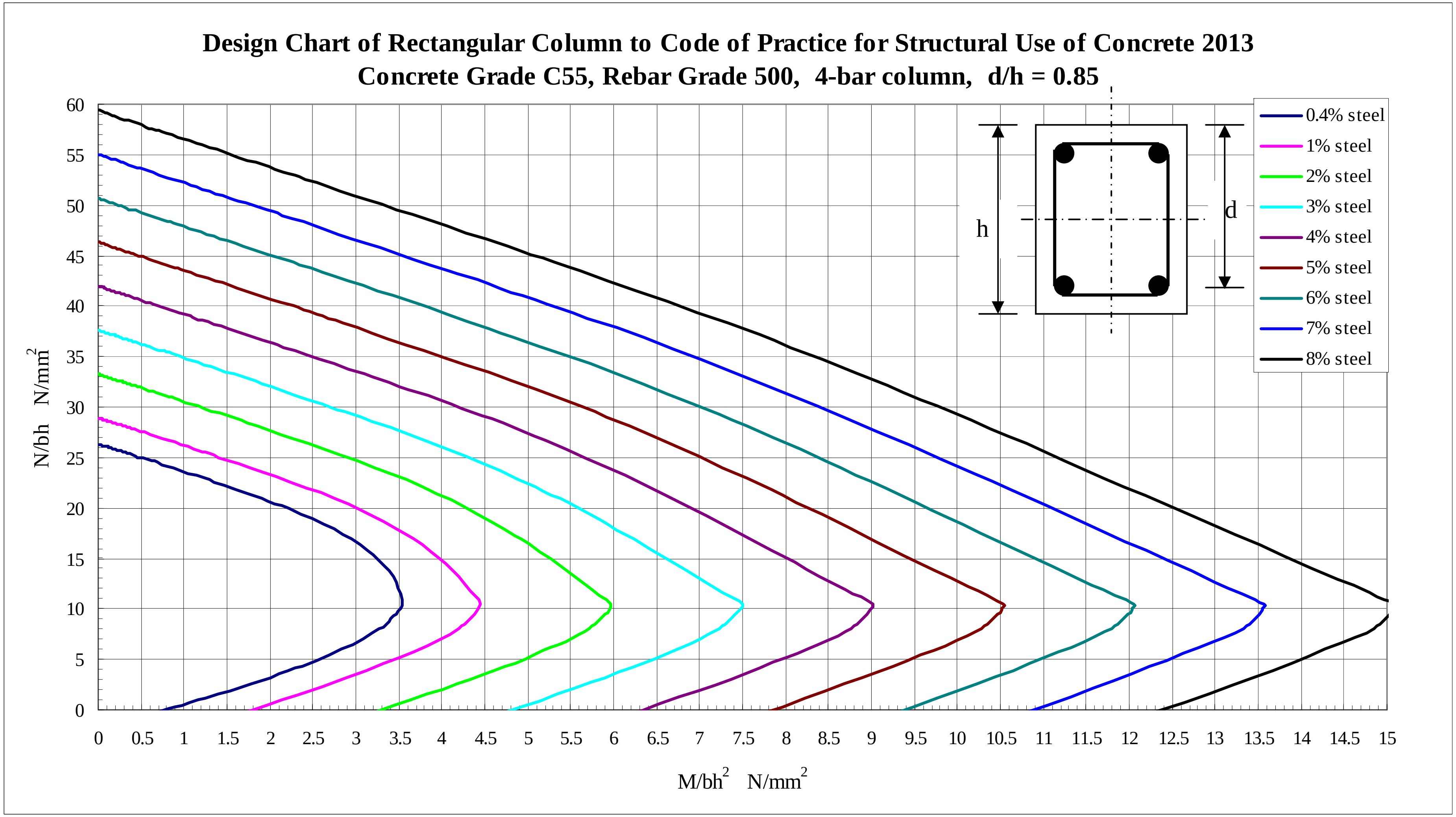
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C55, Rebar Grade 500, 4-bar column, d/h = 0.85
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d
0
5
10
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35
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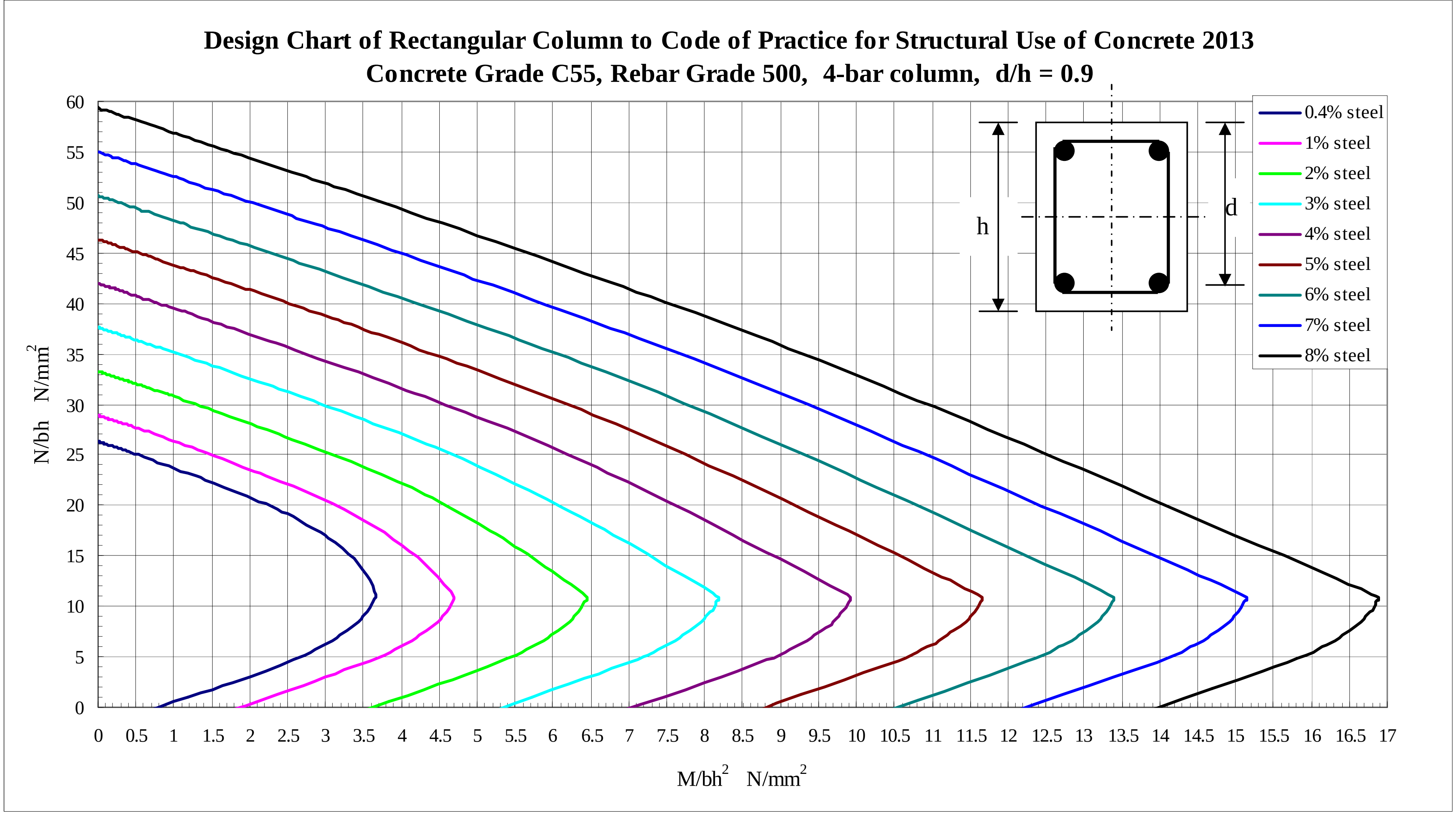
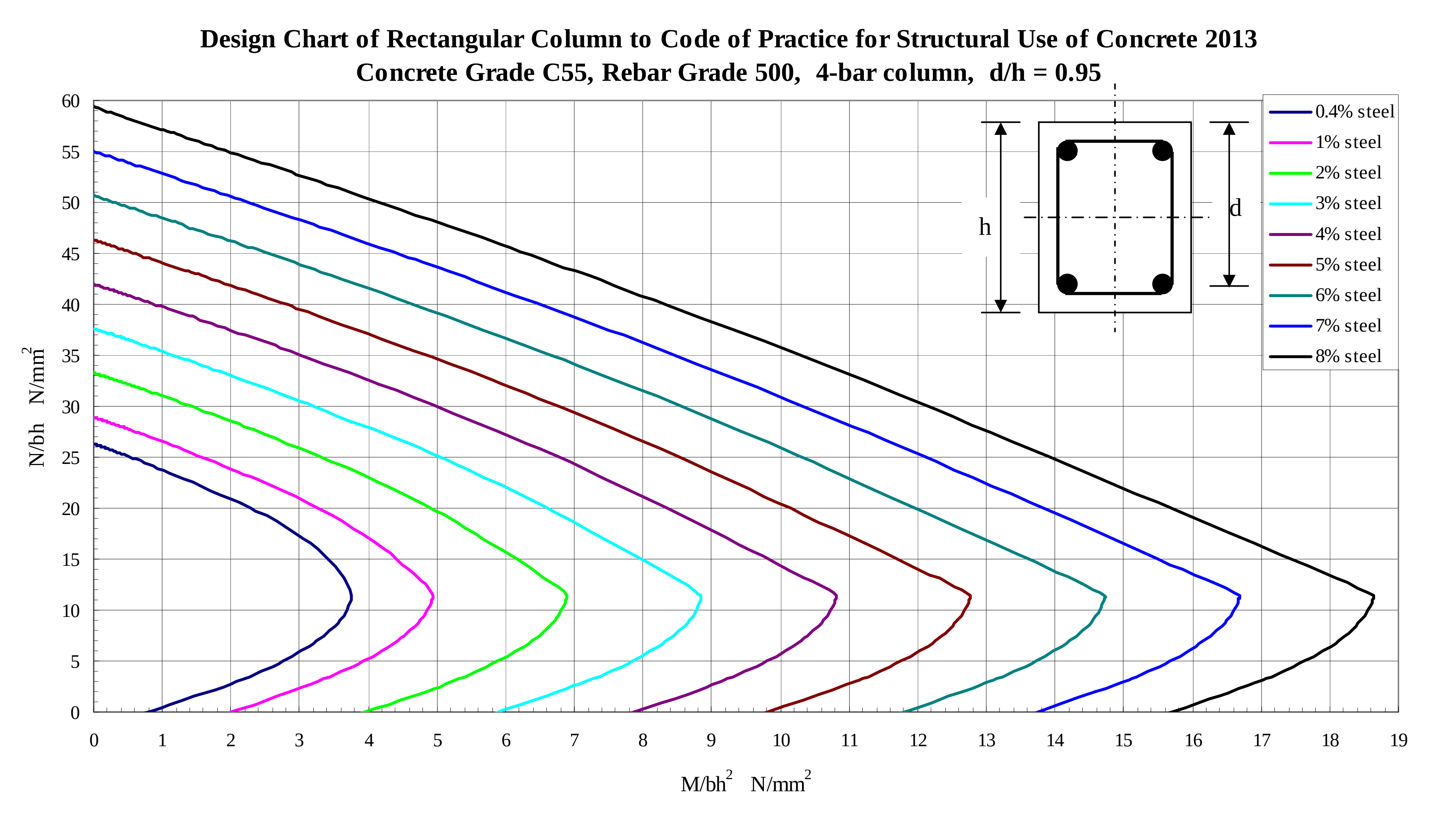
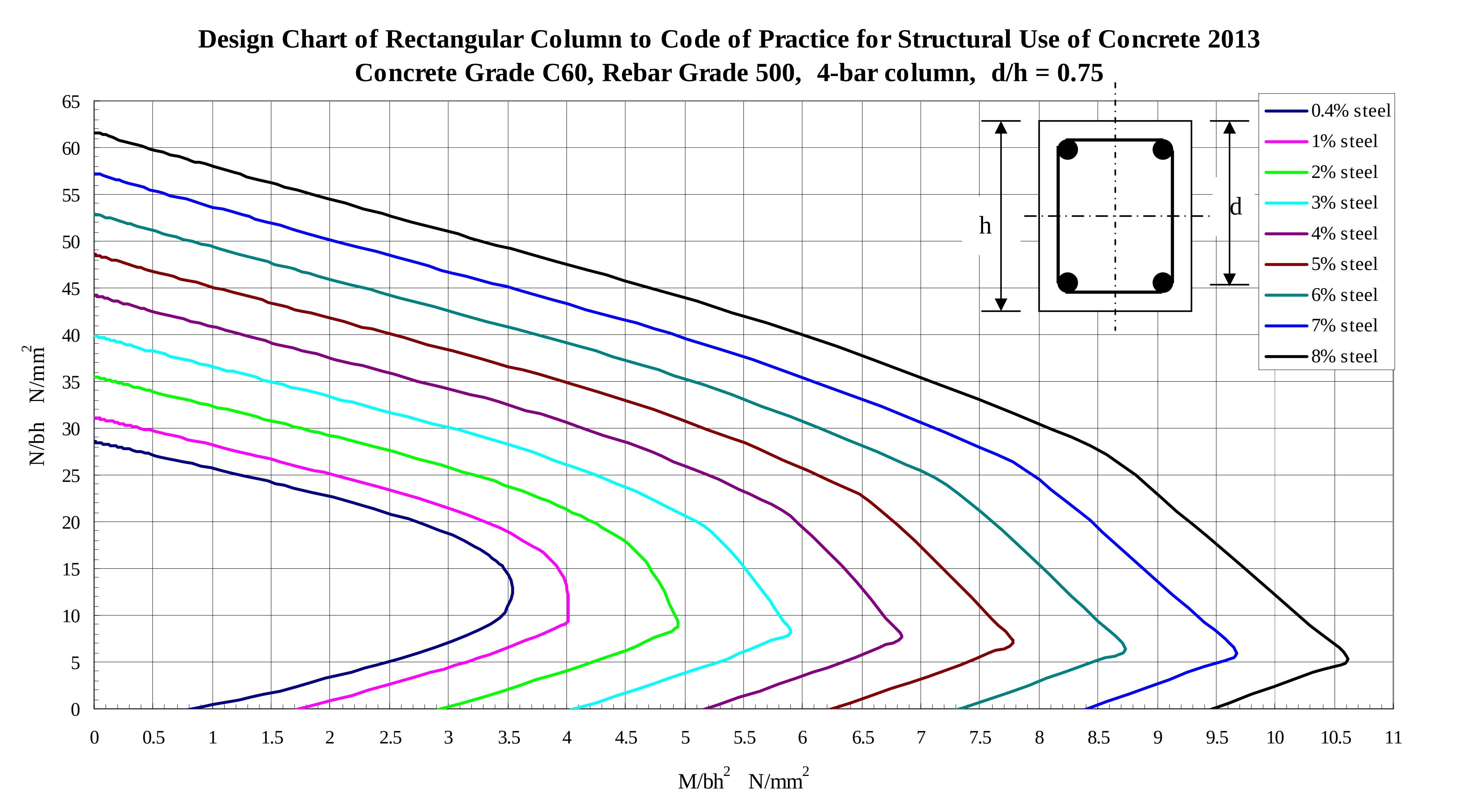
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C55, Rebar Grade 500, 4-bar column, d/h = 0.9
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C55, Rebar Grade 500, 4-bar column, d/h = 0.95
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
60
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C60, Rebar Grade 500, 4-bar column, d/h = 0.75
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
h
d
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
60
65
0
0.5
1
1.5
2
2.5
3
3.5
4
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5
5.5
6
6.5
7
7.5
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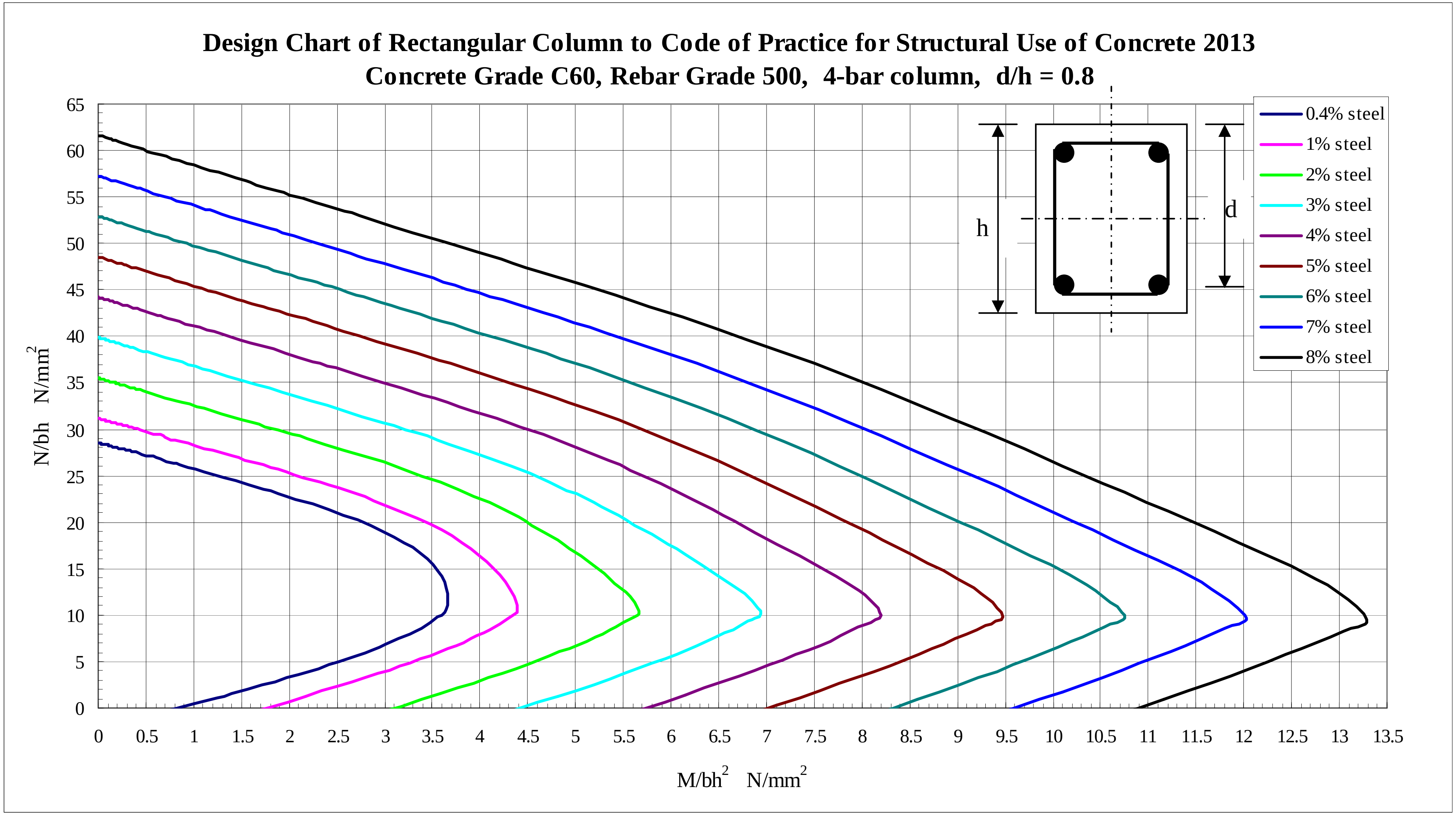
9.5
10
10.5
11


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C60, Rebar Grade 500, 4-bar column, d/h = 0.8
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
60
65
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
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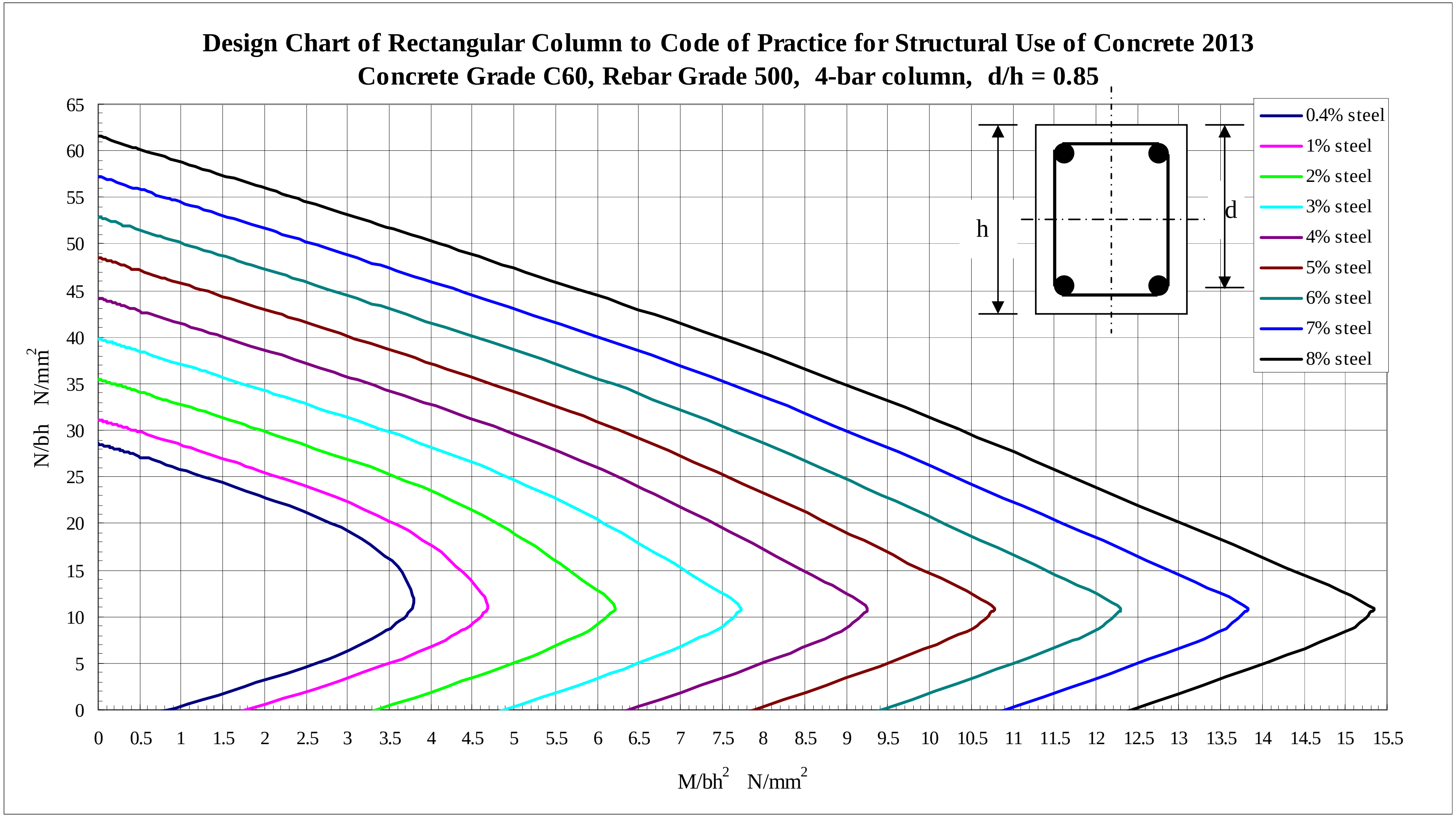
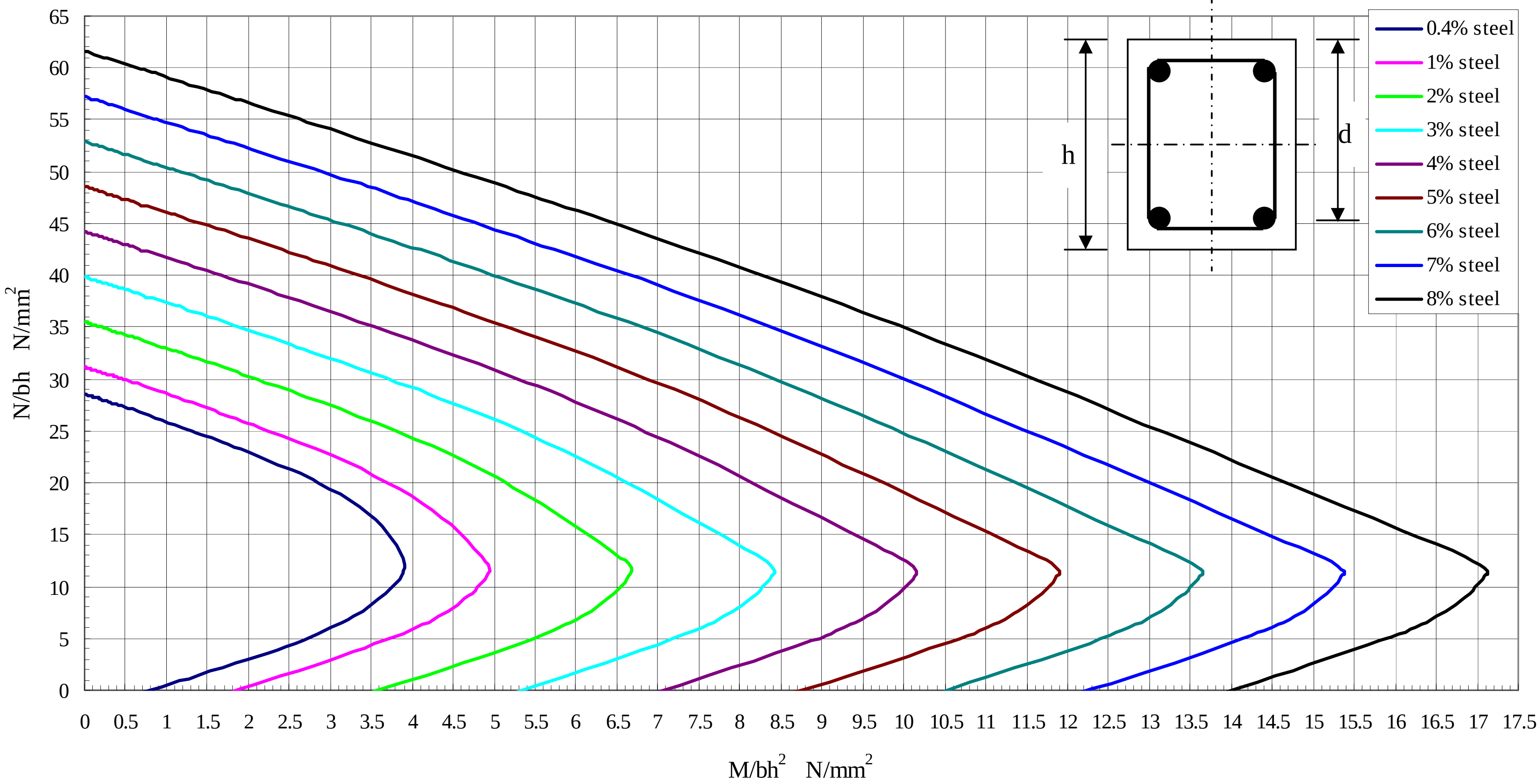
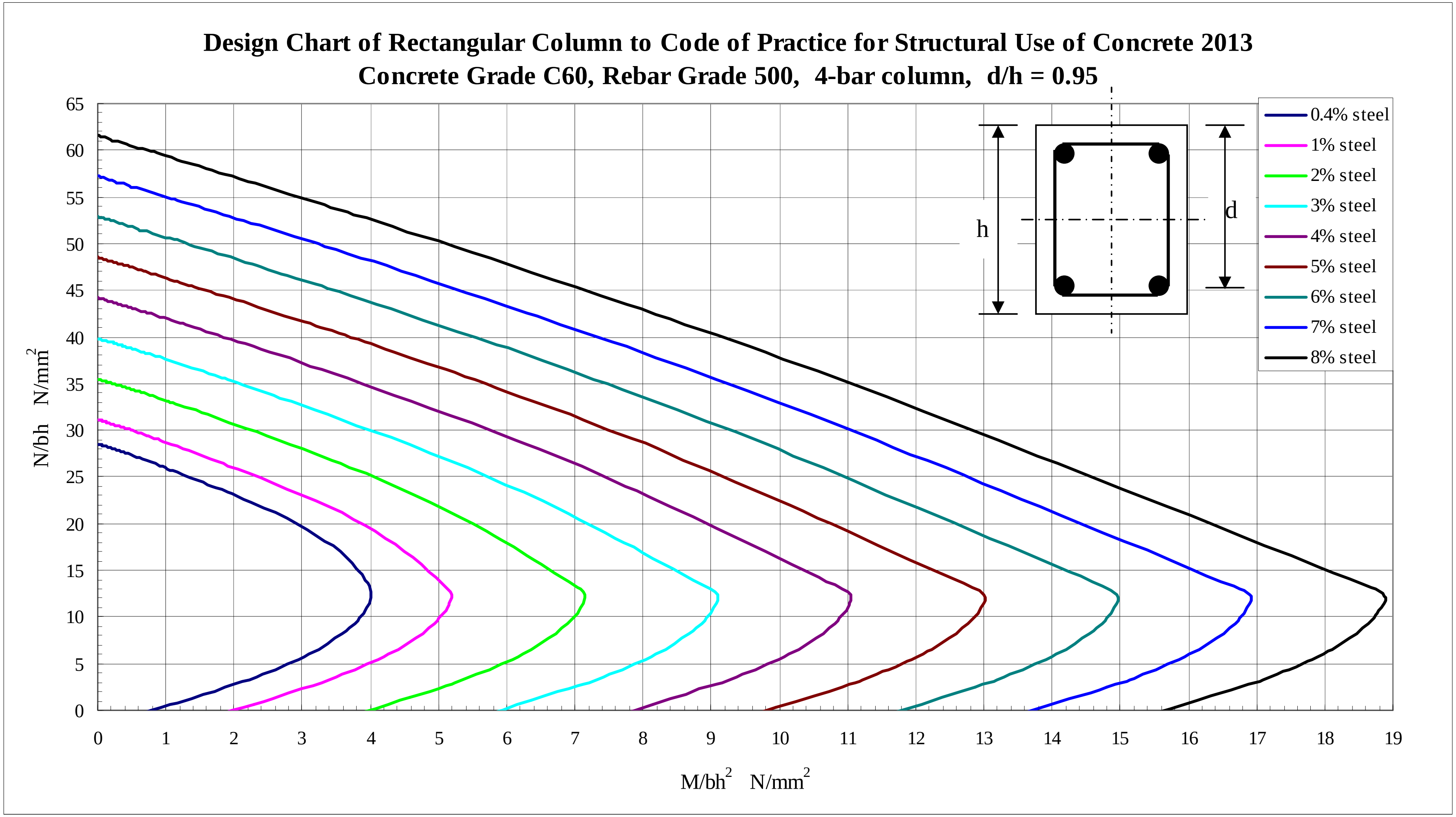
10
10.5
11
11.5
12
12.5
13
13.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C60, Rebar Grade 500, 4-bar column, d/h = 0.85
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
60
65
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15.5

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013

Concrete Grade C60, Rebar Grade 500, 4-bar column, d/h = 0.9




Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C60, Rebar Grade 500, 4-bar column, d/h = 0.95
N/bh N/mm2
M/bh2 N/mm2
h
d
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
60
65
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19

Rectangular Column Designed to CoPConc2013 - Equivalent 4-Bar Column with Predetermined Rebars

f_{cu} = 45 MPa f_y = 500 MPa E_c = 22161 MPa E_s = 200000 MPa

b = 1500 mm h = 2000 mm Cover to main rebar = 50 mm

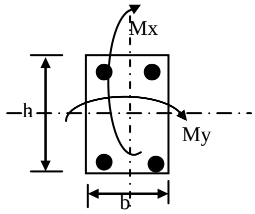
Corner Rebar: 4 T 40

Rebar along each b face (exclude corner bar) : 12 T 40 Bar spacing = 104.62 mm

Rebar along each h face (exclude corner bar) : 15 T 40 Bar spacing = 116.25 mm

b' = 1430.00 mm h' = 1930.00 mm Steel Percentage = 2.4295 %

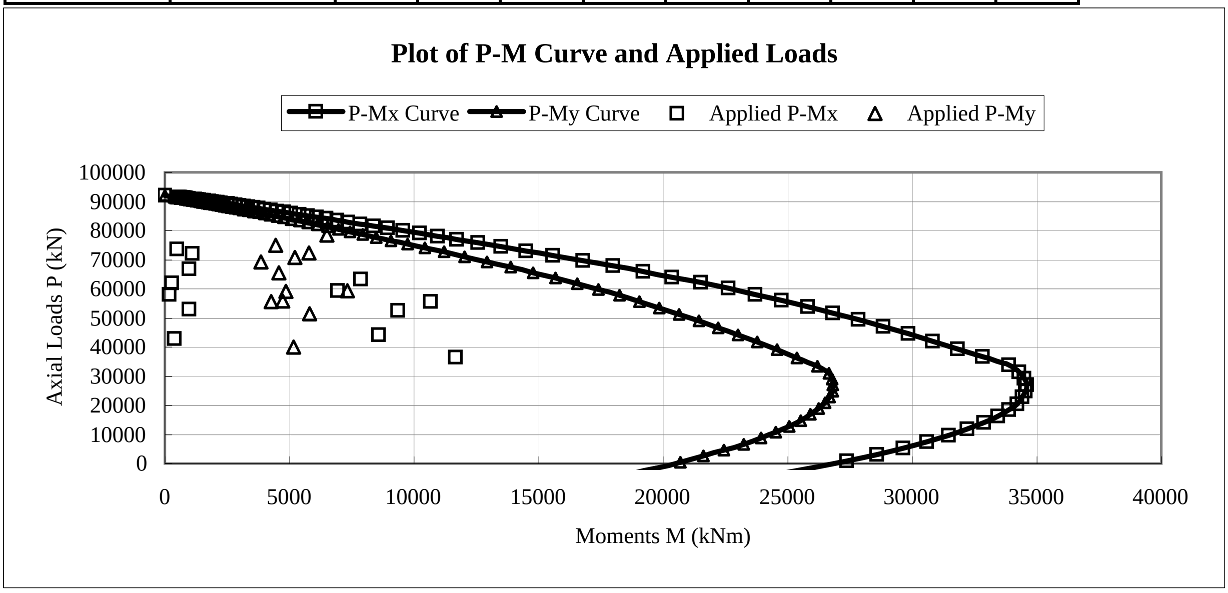
Axial Load Capacity = 92005 kN



Basic Load Case

Load Case No.	1	2	3	4	5	6
Load Case	D.L.	L.L.	Wx	Wy	W45	W135
Axial Load P (kN)	47872	4101	-3628.1	-2611.1	-5692.3	8209.2
Moment M_x (kNm)	-291.3	-37.11	470.81	-3700	-1750.3	4892.9
Moment M_y (kNm)	-31.33	16.09	5.17	2700	2764	-3520.2

		P (kN)	M_x (kNm)	M_y (kNm)		M_{design} (kNm)	d_{nx} or d_{ny} (mm)		$M_{resistance}$ (kNm)	Results
Load Comb 1	1.4D+1.6L	73582	-467.2	-18.118	Mx' =	476.14	2120.3	Mux =	14075	OK
Load Comb 2	1.2(D+L+Wx)	58013	170.88	-12.084	Mx' =	178.99	1698.3	Mux =	23735	OK
Load Comb 3	1.2(D+L-Wx)	66721	-959.06	-24.492	Mx' =	973.16	1920.3	Mux =	18721	OK
Load Comb 4	1.2(D+L+Wy)	59234	-4834.1	3221.7	Mx' =	6953.2	1728.1	Mux =	23096	OK
Load Comb 5	1.2(D+L-Wy)	65500	4045.9	-3258.3	My' =	4566.2	1423	Muy =	14839	OK
Load Comb 6	1.2(D+L+W45)	55536	-2494.5	3298.5	My' =	4254.9	1235.9	Muy =	19091	OK
Load Comb 7	1.2(D+L-W45)	69198	1706.3	-3335.1	My' =	3847	1497.5	Muy =	13004	OK
Load Comb 8	1.2(D+L+W135)	72218	5477.4	-4242.6	My' =	5776.9	1564.4	Muy =	11429	OK
Load Comb 9	1.2(D+L-W135)	52516	-6265.5	4206	Mx' =	9349	1569.9	Mux =	26415	OK
Load Comb 10	1.4(D+Wx)	61941	251.31	-36.624	Mx' =	274.31	1795.9	Mux =	21598	OK
Load Comb 11	1.4(D-Wx)	72099	-1067	-51.1	Mx' =	1093.1	2071.4	Mux =	15107	OK
Load Comb 12	1.4(D+Wy)	63365	-5587.8	3736.1	Mx' =	7875.6	1832.3	Mux =	20773	OK
Load Comb 13	1.4(D-Wy)	70676	4772.2	-3823.9	My' =	5209.1	1528.8	Muy =	12236	OK
Load Comb 14	1.4(D+W45)	59051	-2858.3	3825.7	My' =	4861	1299.2	Muy =	17704	OK
Load Comb 15	1.4(D-W45)	74989	2042.6	-3913.4	My' =	4448.3	1638.1	Muy =	9948.5	OK
Load Comb 16	1.4(D+W135)	78513	6442.2	-4972.2	My' =	6509.7	1755.4	Muy =	8038.3	OK
Load Comb 17	1.4(D-W135)	55527	-7257.9	4884.4	Mx' =	10670	1639.2	Mux =	24983	OK
Load Comb 18	1.0D+1.4Wx	42792	367.83	-24.092	Mx' =	388.31	1364.3	Mux =	30540	OK
Load Comb 19	1.0D-1.4Wx	52951	-950.43	-38.568	Mx' =	978.51	1579.7	Mux =	26213	OK
Load Comb 20	1.0D+1.4Wy	44216	-5471.3	3748.7	Mx' =	8592.8	1392.5	Mux =	29982	OK
Load Comb 21	1.0D-1.4Wy	51527	4888.7	-3811.3	My' =	5810.7	1166.8	Muy =	20561	OK
Load Comb 22	1.0D+1.4W45	39902	-2741.8	3838.3	My' =	5169.5	986.33	Muy =	24329	OK
Load Comb 23	1.0D-1.4W45	55841	2159.2	-3900.9	My' =	4724.8	1241.2	Muy =	18975	OK
Load Comb 24	1.0D+1.4W135	59364	6558.7	-4959.6	My' =	7322.8	1305.1	Muy =	17574	OK
Load Comb 25	1.0D-1.4W135	36379	-7141.3	4897	Mx' =	11679	1244.5	Mux =	32951	OK



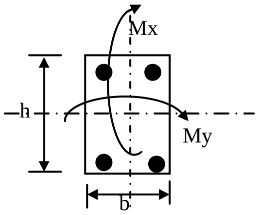
Rectangular Column Designed to CoPConc2013 - Minimum Rebar Percentage of Equivalent 4-Bar Column

f_{cu} = 45 MPa f_y = 500 MPa E_c = 22161.2 MPa E_s = 200000 MPa

b = 1500 mm h = 2000 mm Reinforcement Cover Ratios c_x = 0.12 c_y = 0.15

b' = 1275.00 mm h' = 1760.00 mm Min Steel Percentage = 0.08 % Steel Percentage = 2.303 %

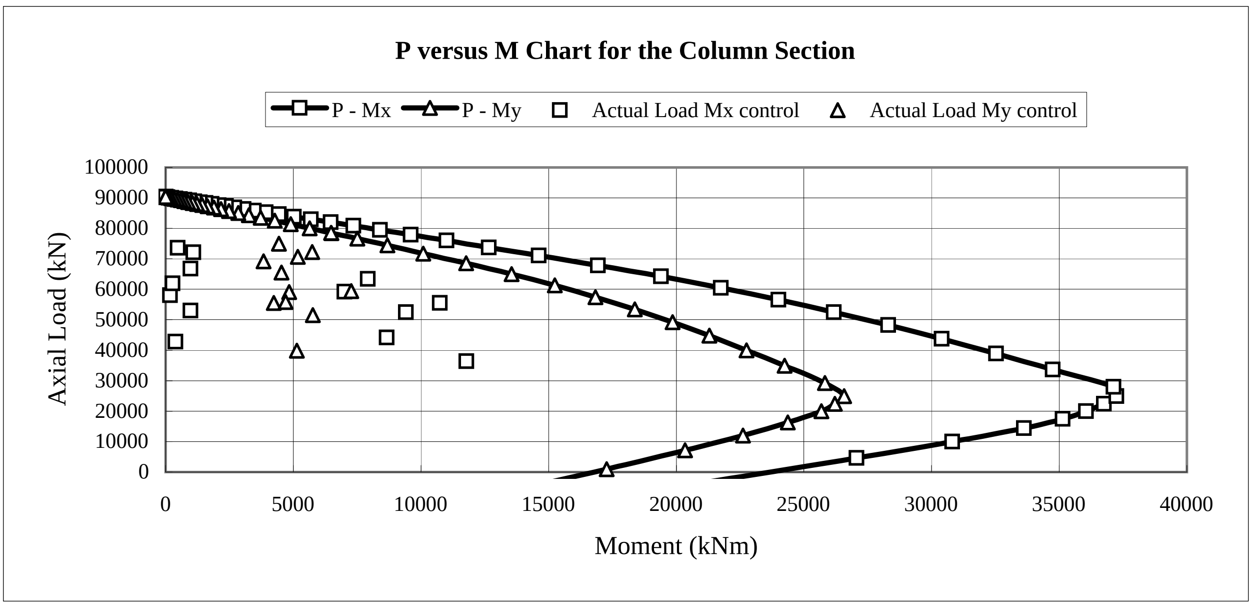
Axial Load Capacity = 90351.2 kN



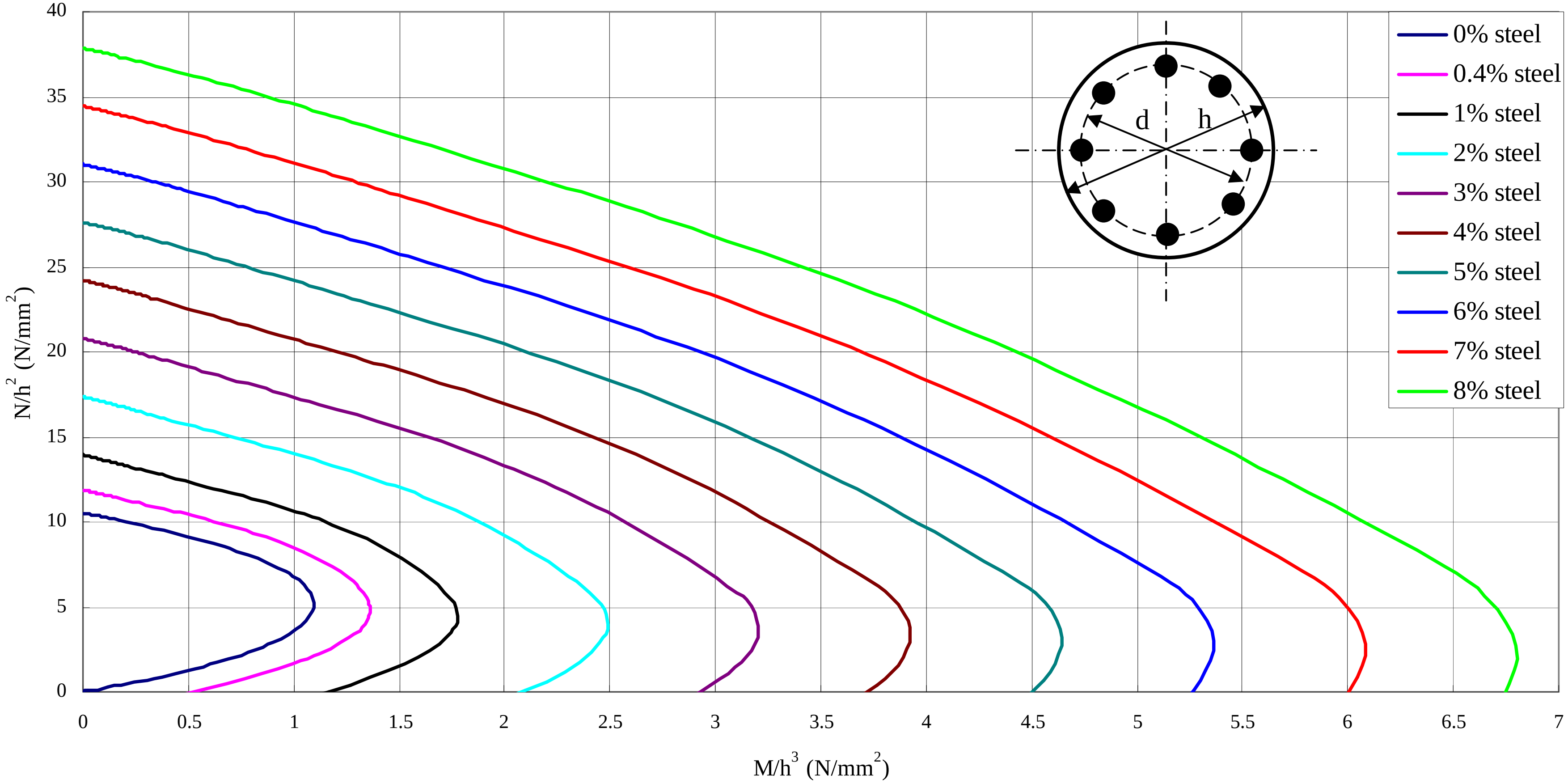
Basic Load Case

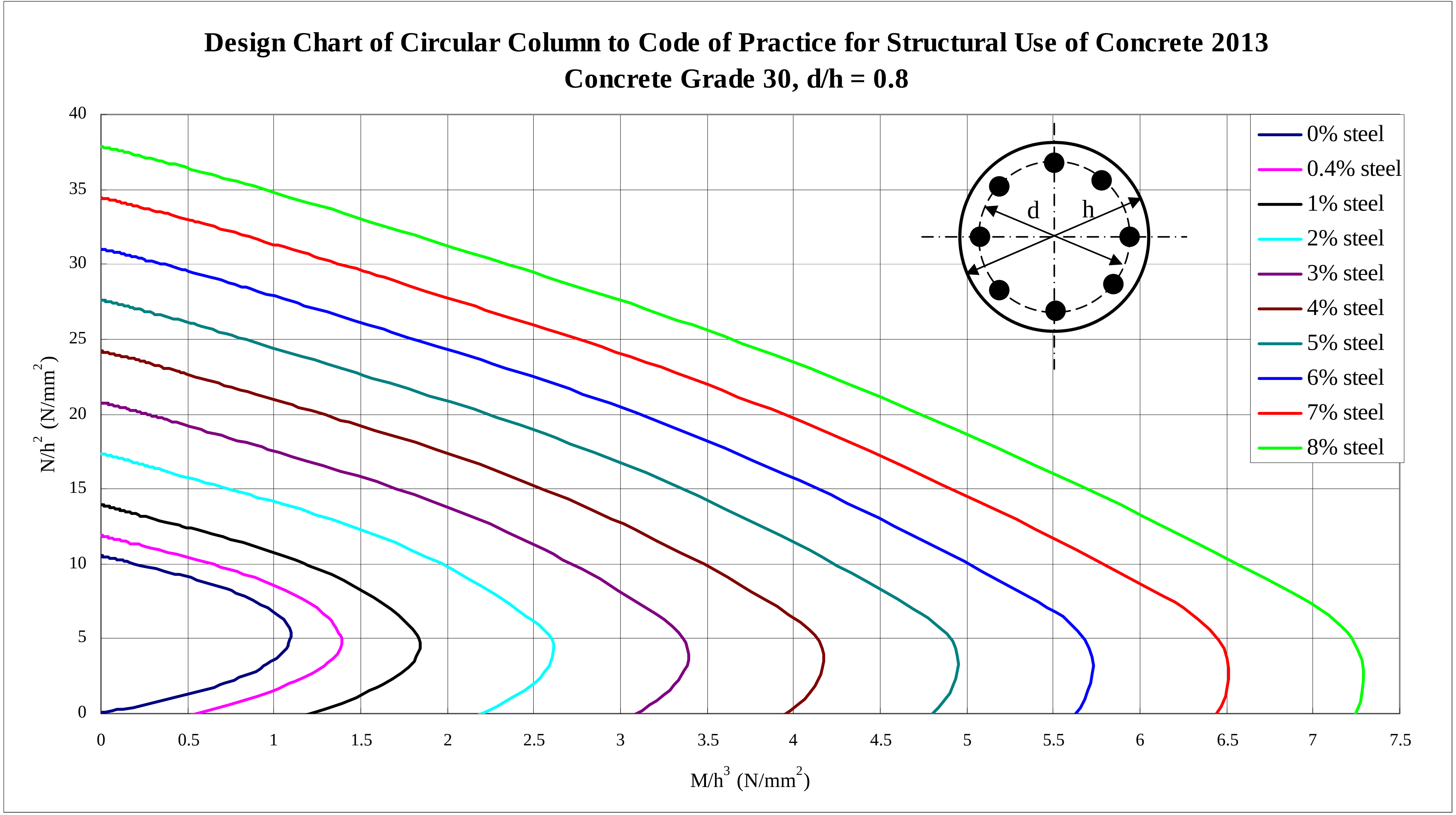
Load Case No.	1	2	3	4	5	6
Load Case	D.L.	L.L.	Wx	Wy	W45	W135
Axial Load P (kN)	47871.5	4101.01	-3628.1	-2611.1	-5692.3	8209.16
Moment M_x (kNm)	-291.3	-37.11	470.81	-3700	-1750.3	4892.88
Moment M_y (kNm)	-31.33	16.09	5.17	2700	2763.99	-3520.2

		N (kN)	M_x (kN-m)	M_y (kN-m)			N/bh (N/mm^2)	M/bh^2 (N/mm^2)	d_n/h or d_n/b	Min. p (%)	Steel area (mm^2)
Load Comb 1	1.4D+1.6L	73581.8	-467.2	-18.118	Mx' =	476.348	24.5273	0.07939	2.02511	1.066	31973
Load Comb 2	1.2(D+L+Wx)	58013.4	170.88	-12.084	Mx' =	179.175	19.3378	0.02986	1.55718	0.080	2400
Load Comb 3	1.2(D+L-Wx)	66720.7	-959.06	-24.492	Mx' =	973.478	22.2402	0.16225	1.63905	0.587	17599
Load Comb 4	1.2(D+L+Wy)	59233.7	-4834.1	3221.71	Mx' =	7001.47	19.7446	1.16691	1.10007	0.621	18630
Load Comb 5	1.2(D+L-Wy)	65500.4	4045.91	-3258.3	My' =	4537.06	21.8335	1.00823	1.17532	1.020	30610
Load Comb 6	1.2(D+L+W45)	55536.3	-2494.5	3298.5	My' =	4233.64	18.5121	0.94081	1.09457	0.207	6202
Load Comb 7	1.2(D+L-W45)	69197.8	1706.3	-3335.1	My' =	3835.58	23.0659	0.85235	1.24874	1.208	36244
Load Comb 8	1.2(D+L+W135)	72218	5477.36	-4242.6	My' =	5742.69	24.0727	1.27615	1.18131	1.712	51350
Load Comb 9	1.2(D+L-W135)	52516.1	-6265.5	4205.98	Mx' =	9419.26	17.5054	1.56988	0.99458	0.375	11263
Load Comb 10	1.4(D+Wx)	61940.9	251.314	-36.624	Mx' =	274.837	20.647	0.04581	1.72181	0.151	4545
Load Comb 11	1.4(D-Wx)	72099.4	-1067	-51.1	Mx' =	1093.7	24.0331	0.18228	1.74939	1.013	30387
Load Comb 12	1.4(D+Wy)	63364.5	-5587.8	3736.14	Mx' =	7927.68	21.1215	1.32128	1.11623	1.029	30860
Load Comb 13	1.4(D-Wy)	70675.7	4772.18	-3823.9	My' =	5178.26	23.5586	1.15072	1.19265	1.512	45362
Load Comb 14	1.4(D+W45)	59050.9	-2858.3	3825.72	My' =	4837.94	19.6836	1.0751	1.10405	0.565	16943
Load Comb 15	1.4(D-W45)	74989.4	2042.64	-3913.4	My' =	4436.43	24.9965	0.98587	1.26966	1.742	52249
Load Comb 16	1.4(D+W135)	78513	6442.21	-4972.2	My' =	6475.42	26.171	1.43898	1.20113	2.303	69083
Load Comb 17	1.4(D-W135)	55527.3	-7257.9	4884.45	Mx' =	10747.4	18.5091	1.79124	0.99813	0.739	22162
Load Comb 18	1.0D+1.4Wx	42792.2	367.834	-24.092	Mx' =	388.773	14.2641	0.0648	1.10904	0.080	2400
Load Comb 19	1.0D-1.4Wx	52950.8	-950.43	-38.568	Mx' =	979.147	17.6503	0.16319	1.27138	0.080	2400
Load Comb 20	1.0D+1.4Wy	44215.9	-5471.3	3748.67	Mx' =	8663.89	14.7386	1.44398	0.92765	0.080	2400
Load Comb 21	1.0D-1.4Wy	51527.1	4888.7	-3811.3	My' =	5766.19	17.1757	1.28138	1.01328	0.122	3659
Load Comb 22	1.0D+1.4W45	39902.3	-2741.8	3838.26	My' =	5139.85	13.3008	1.14219	0.92163	0.080	2400
Load Comb 23	1.0D-1.4W45	55840.8	2159.16	-3900.9	My' =	4706.46	18.6136	1.04588	1.08022	0.299	8958
Load Comb 24	1.0D+1.4W135	59364.4	6558.73	-4959.6	My' =	7270.18	19.7881	1.61559	1.03419	0.942	28249
Load Comb 25	1.0D-1.4W135	36378.7	-7141.3	4896.98	Mx' =	11782.8	12.1262	1.9638	0.75732	0.080	2400
									Max =	2.303	69083

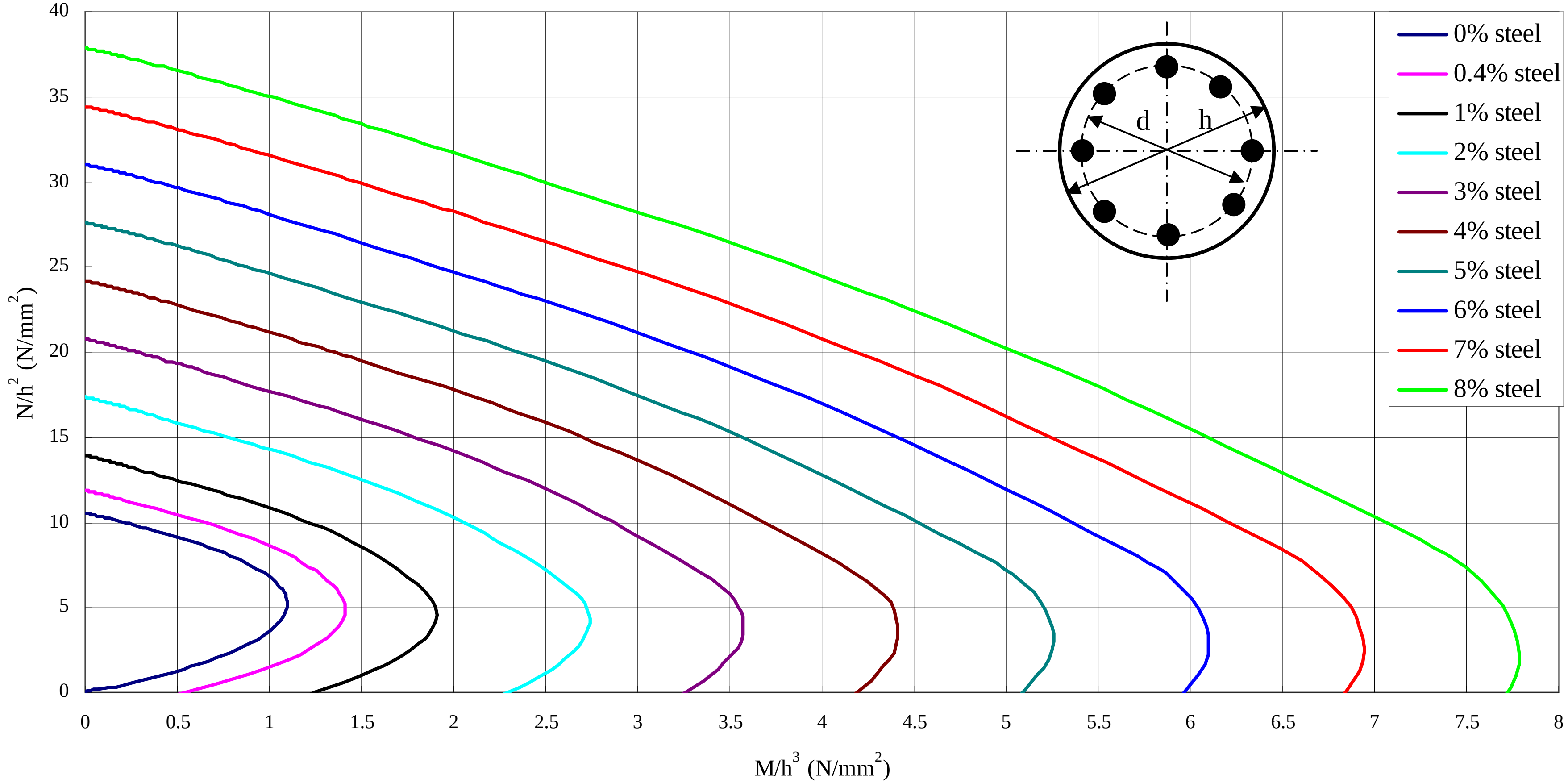


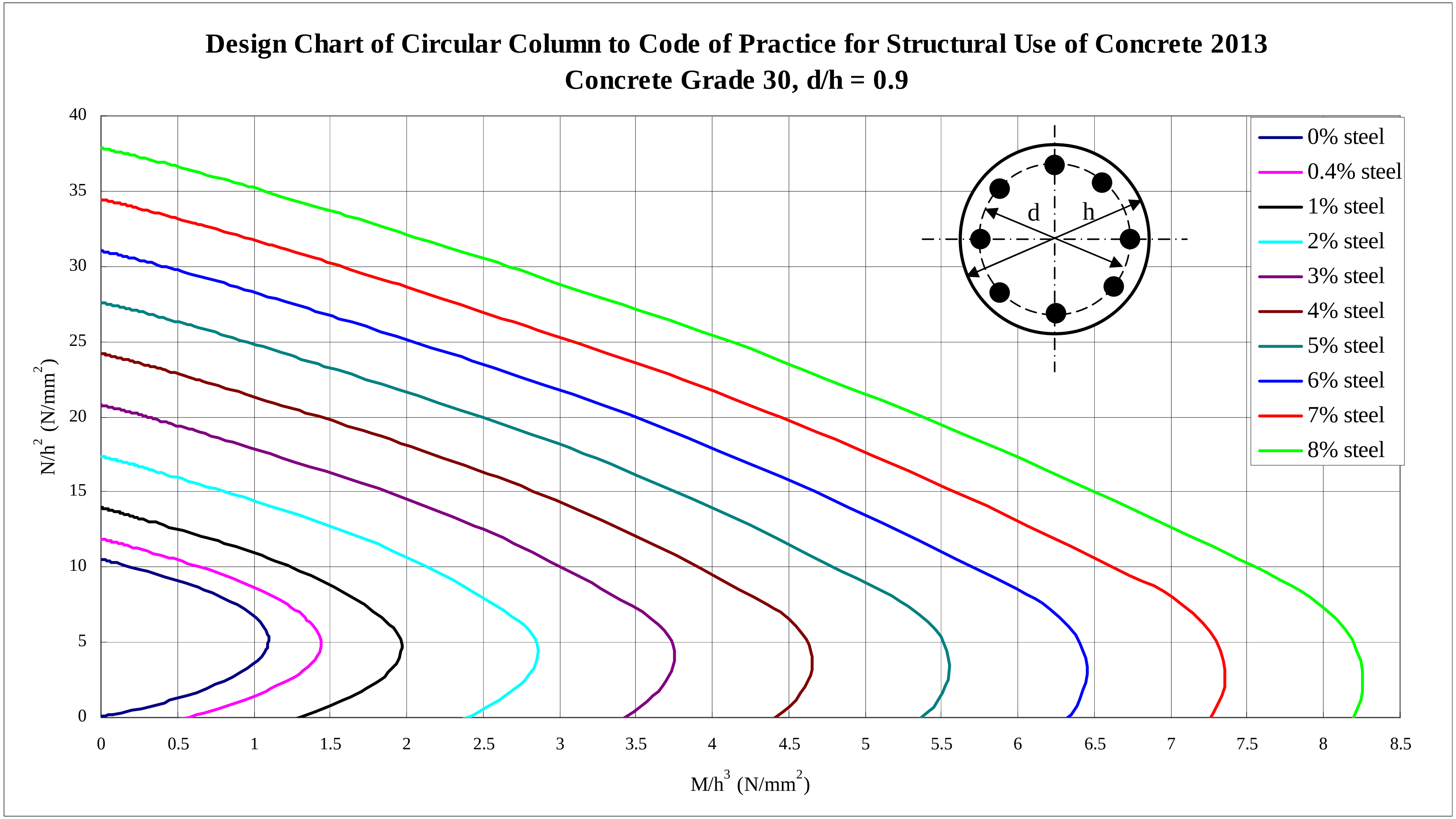
Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 30, d/h = 0.75

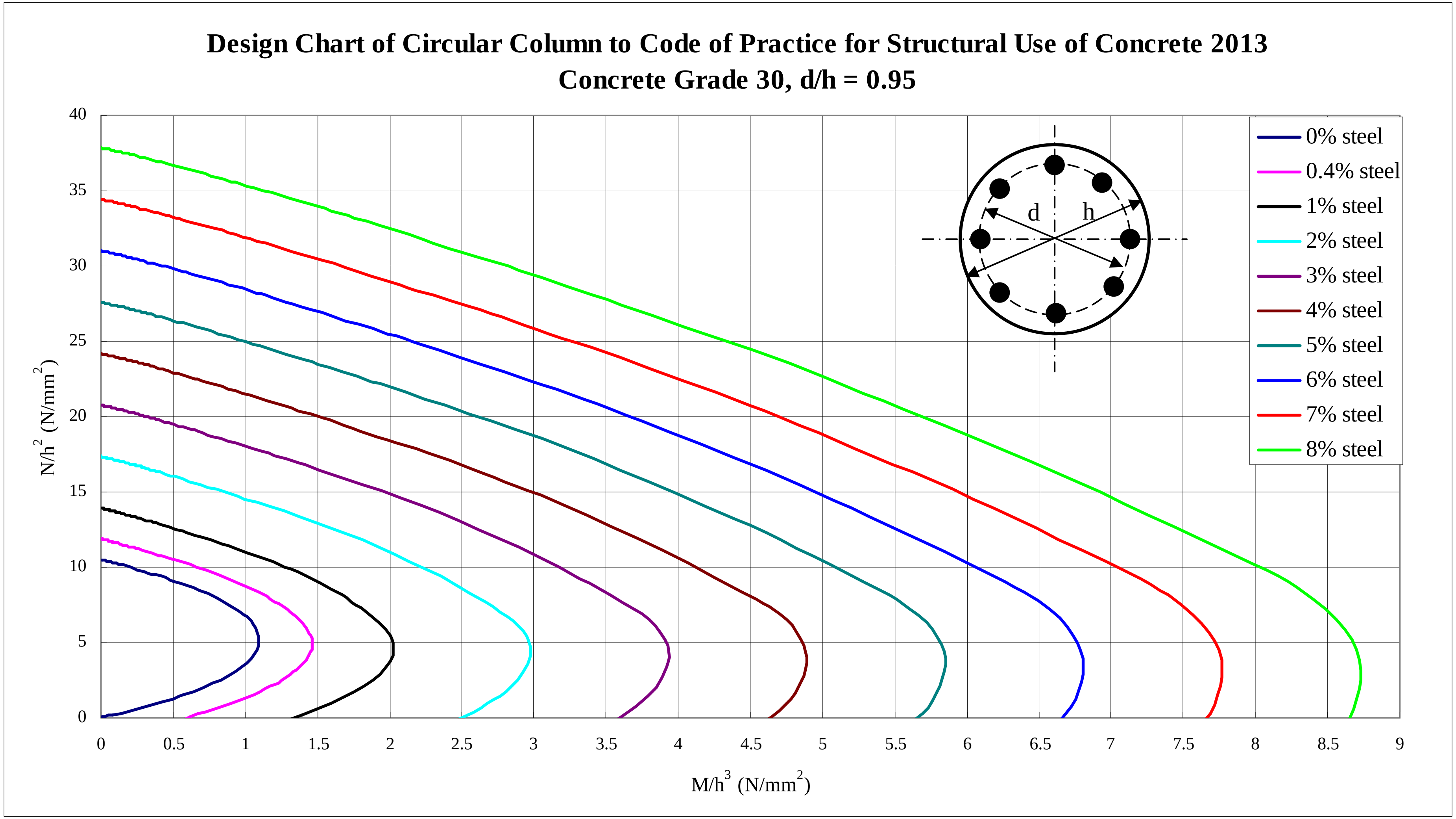


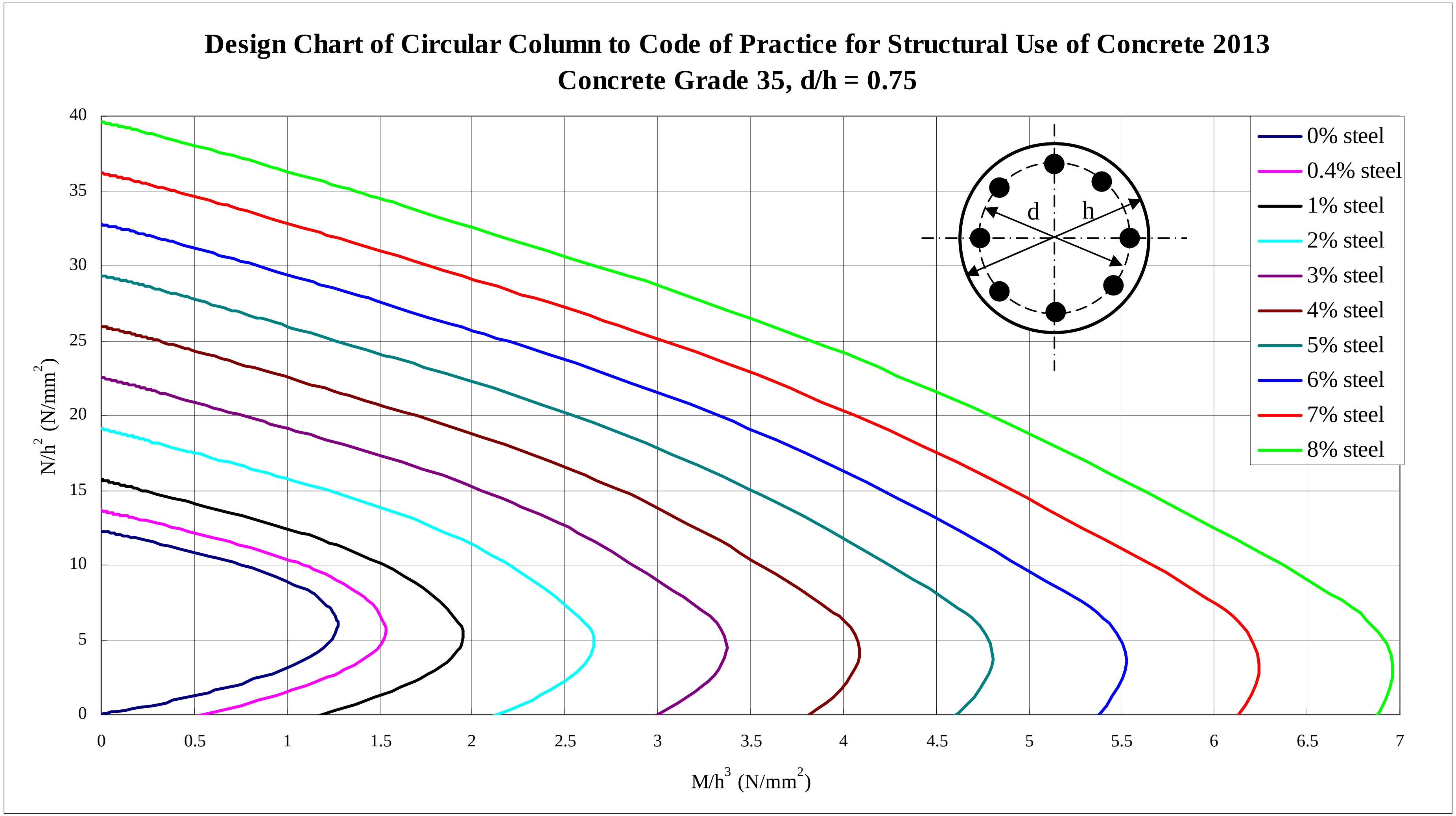

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 30, d/h = 0.8
N/h2 (N/mm2)
M/h3 (N/mm2)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h

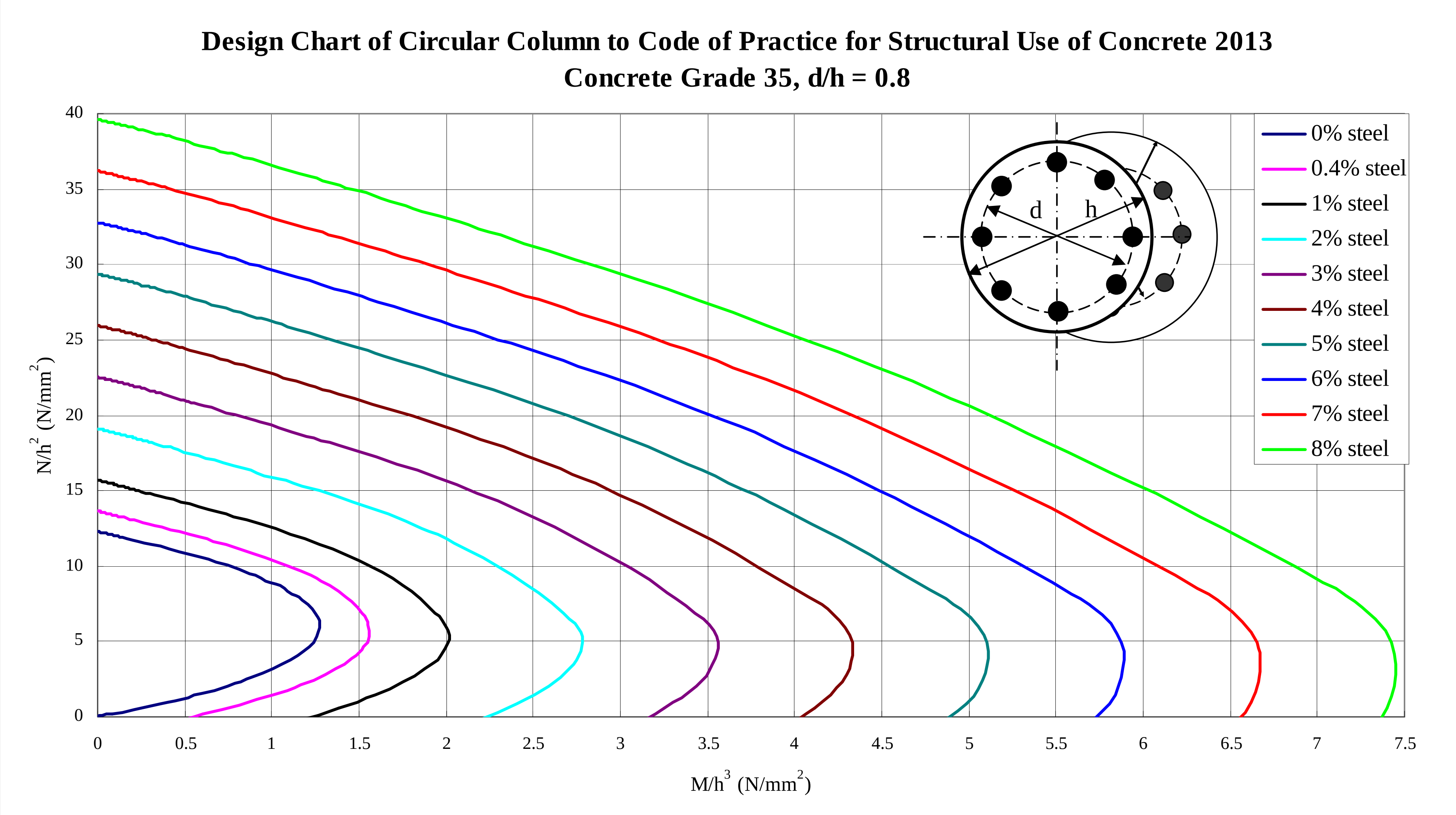
Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 30, d/h = 0.85

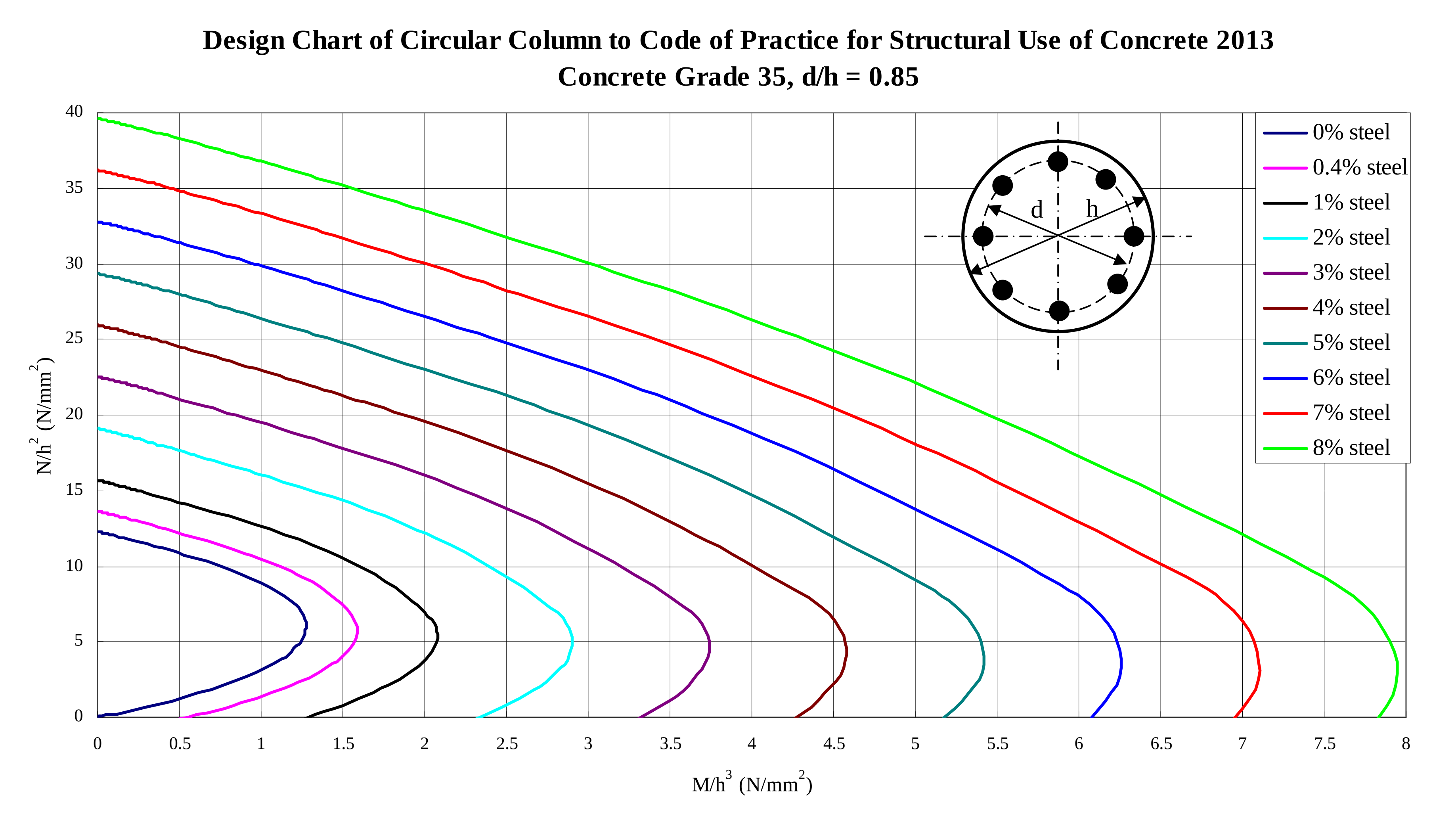



Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 30, d/h = 0.9
N/h2 (N/mm2)
M/h3 (N/mm2)
0
5
10
15
20
25
30
35
40
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
d
h
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel

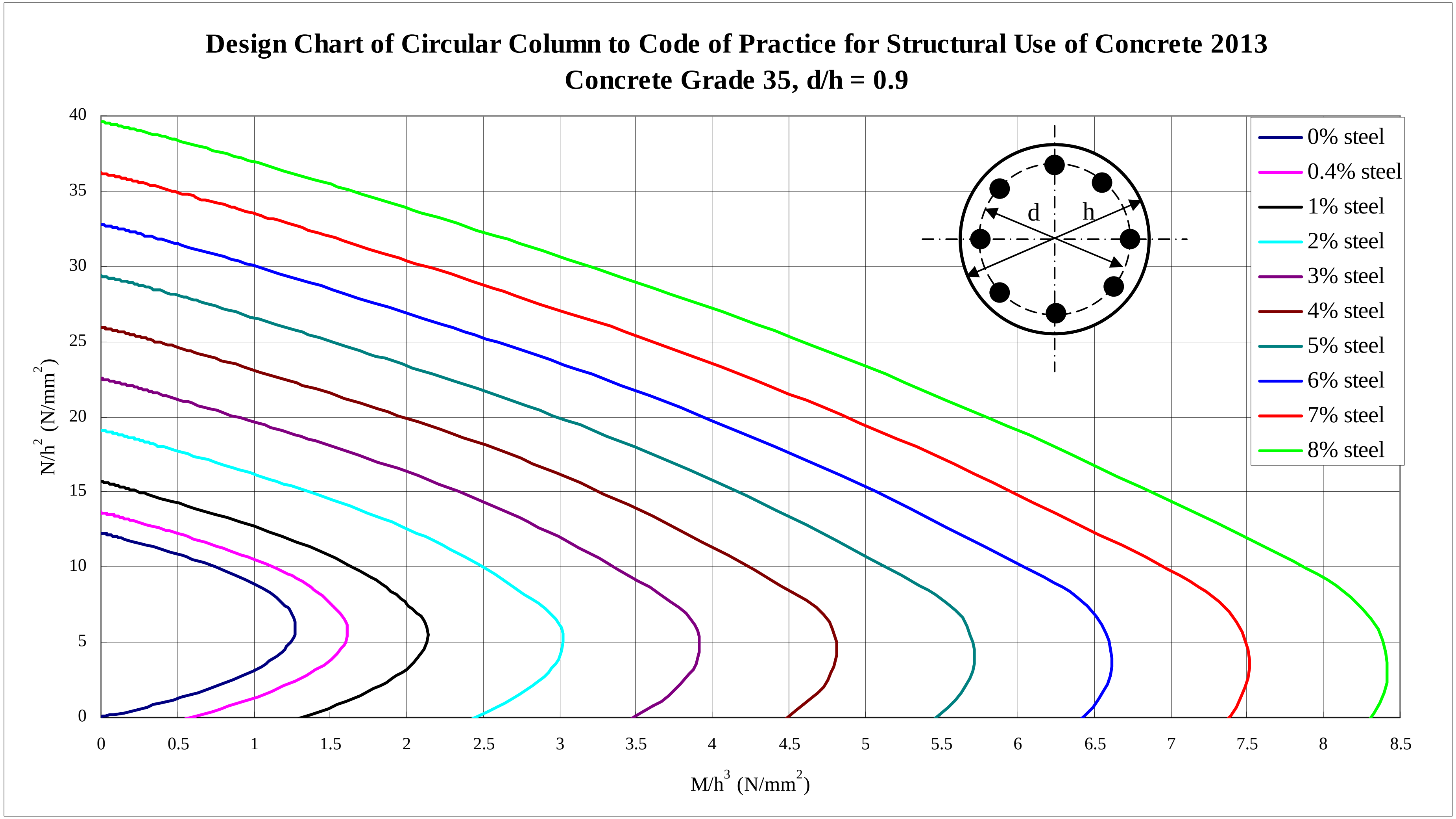

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 30, d/h = 0.95
N/h2 (N/mm2)
M/h3 (N/mm2)
0
5
10
15
20
25
30
35
40
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
d
h
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel


Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 35, d/h = 0.75
N/h² (N/mm²)
M/h³ (N/mm²)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
0
5
10
15
20
25
30
35
40
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7

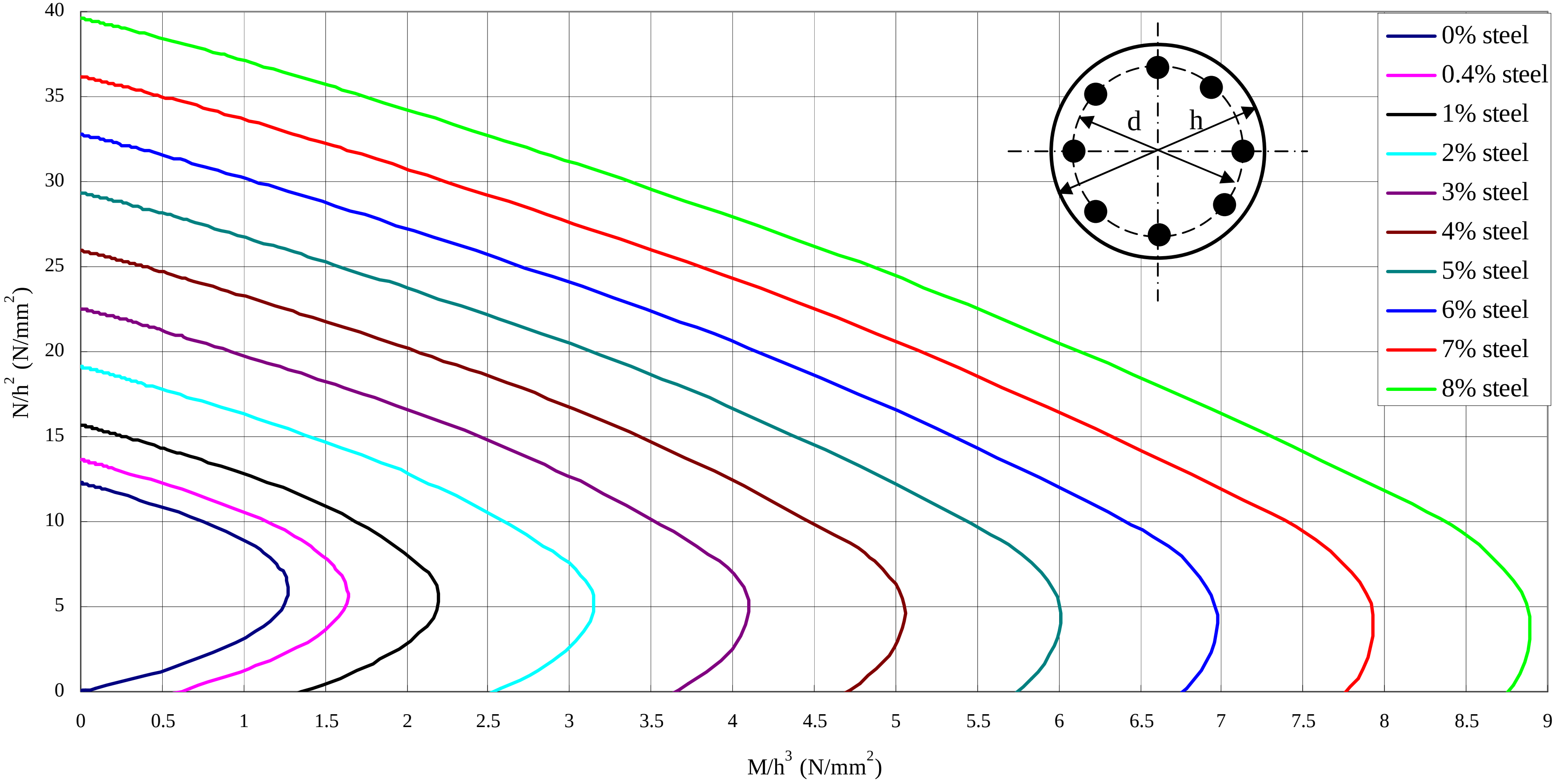

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 35, d/h = 0.8
N/h² (N/mm²)
M/h³ (N/mm²)
0
5
10
15
20
25
30
35
40
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
d
h
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel



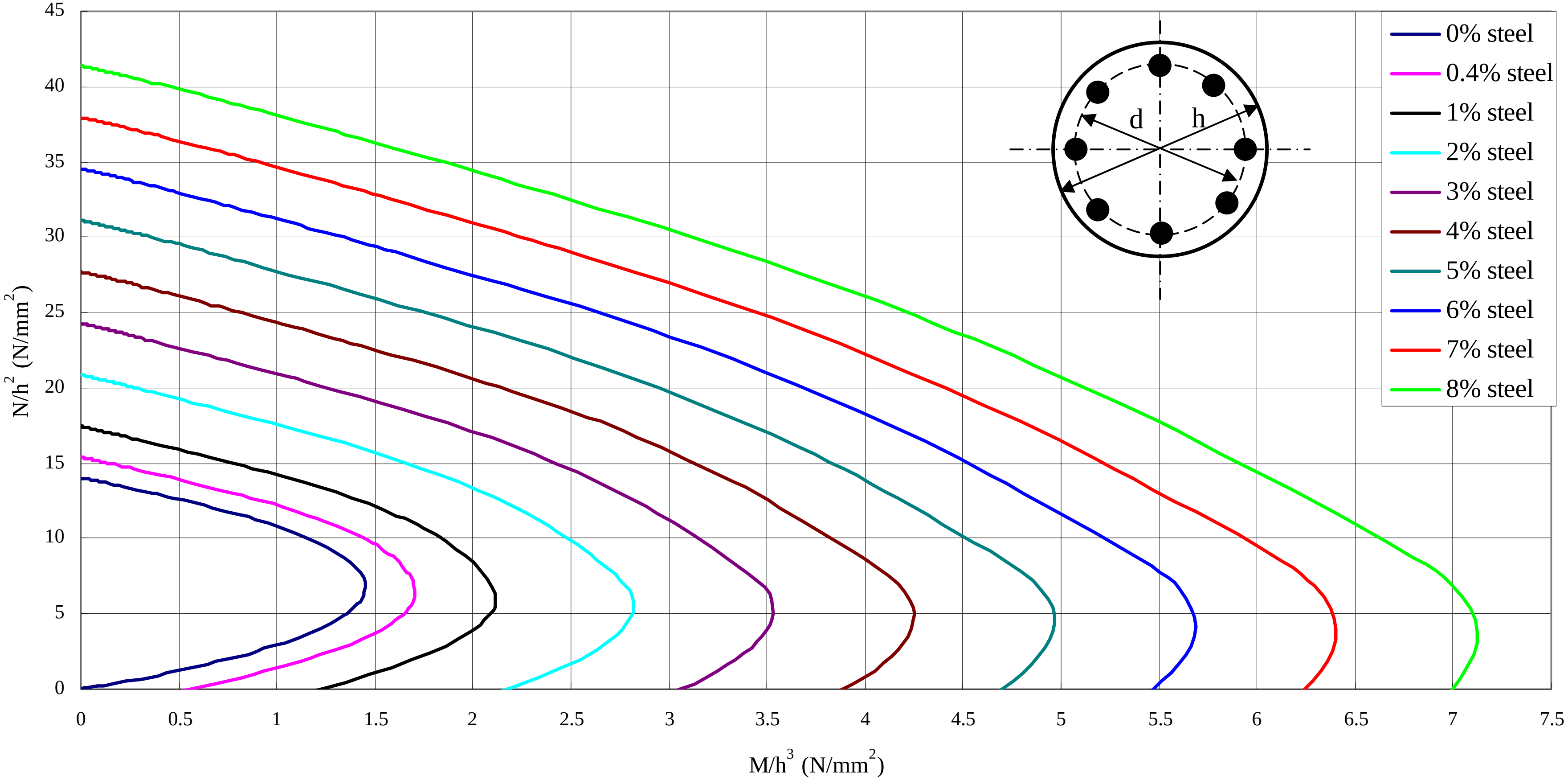
Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 35, d/h = 0.85
N/h² (N/mm²)
M/h³ (N/mm²)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h

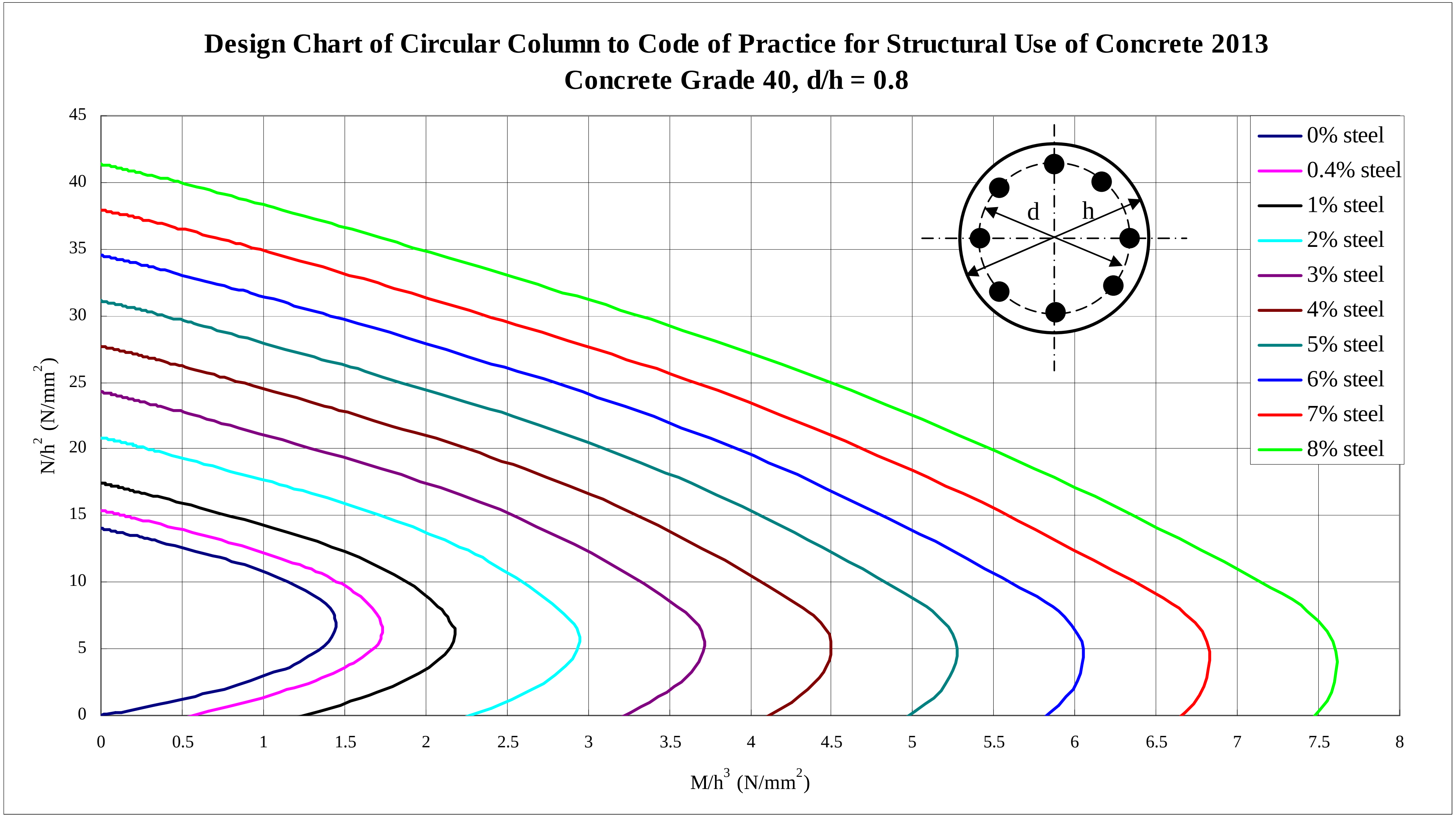

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 35, d/h = 0.9
N/h2 (N/mm2)
M/h3 (N/mm2)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
0
5
10
15
20
25
30
35
40
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5

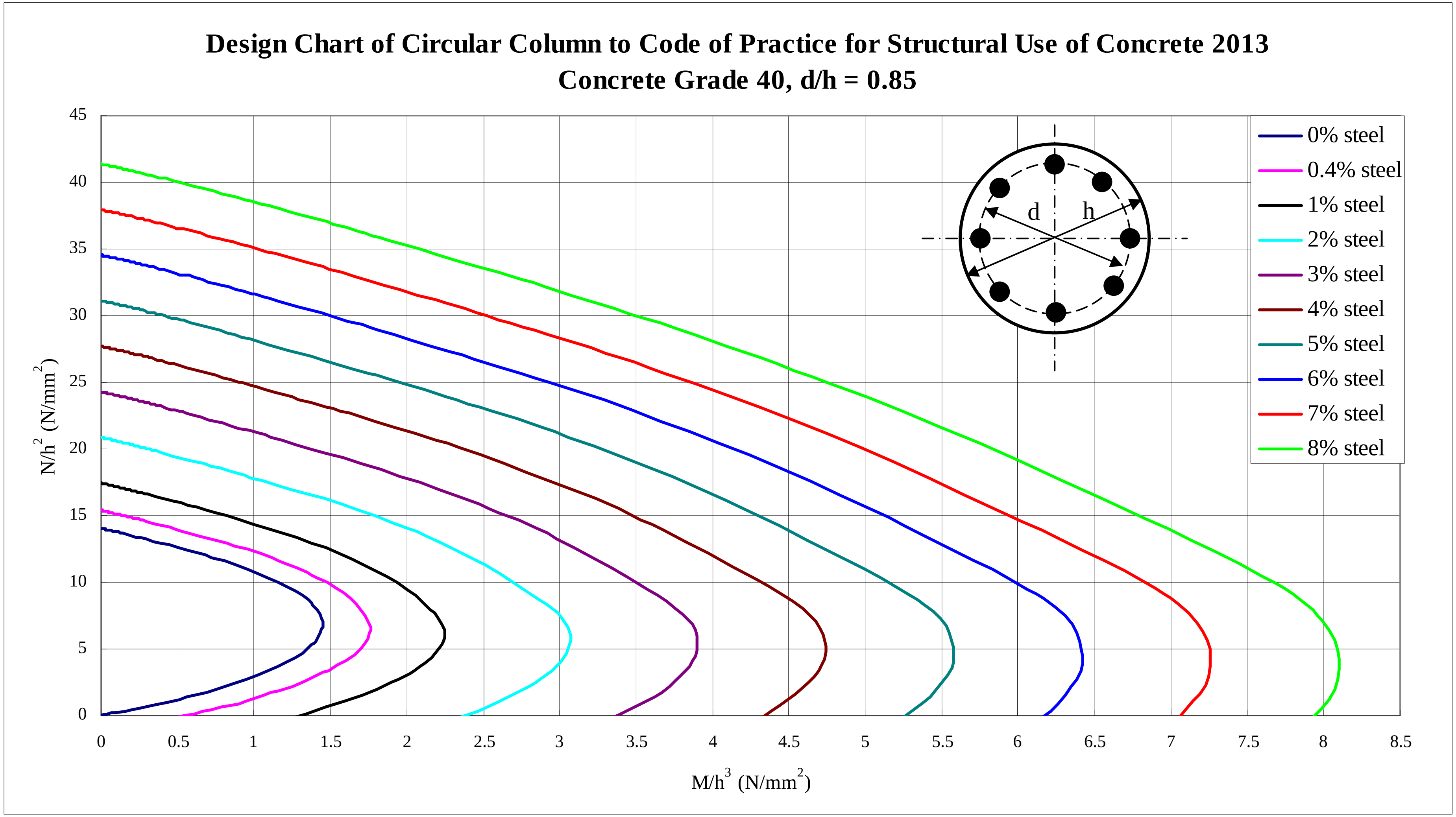
Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 35, d/h = 0.95



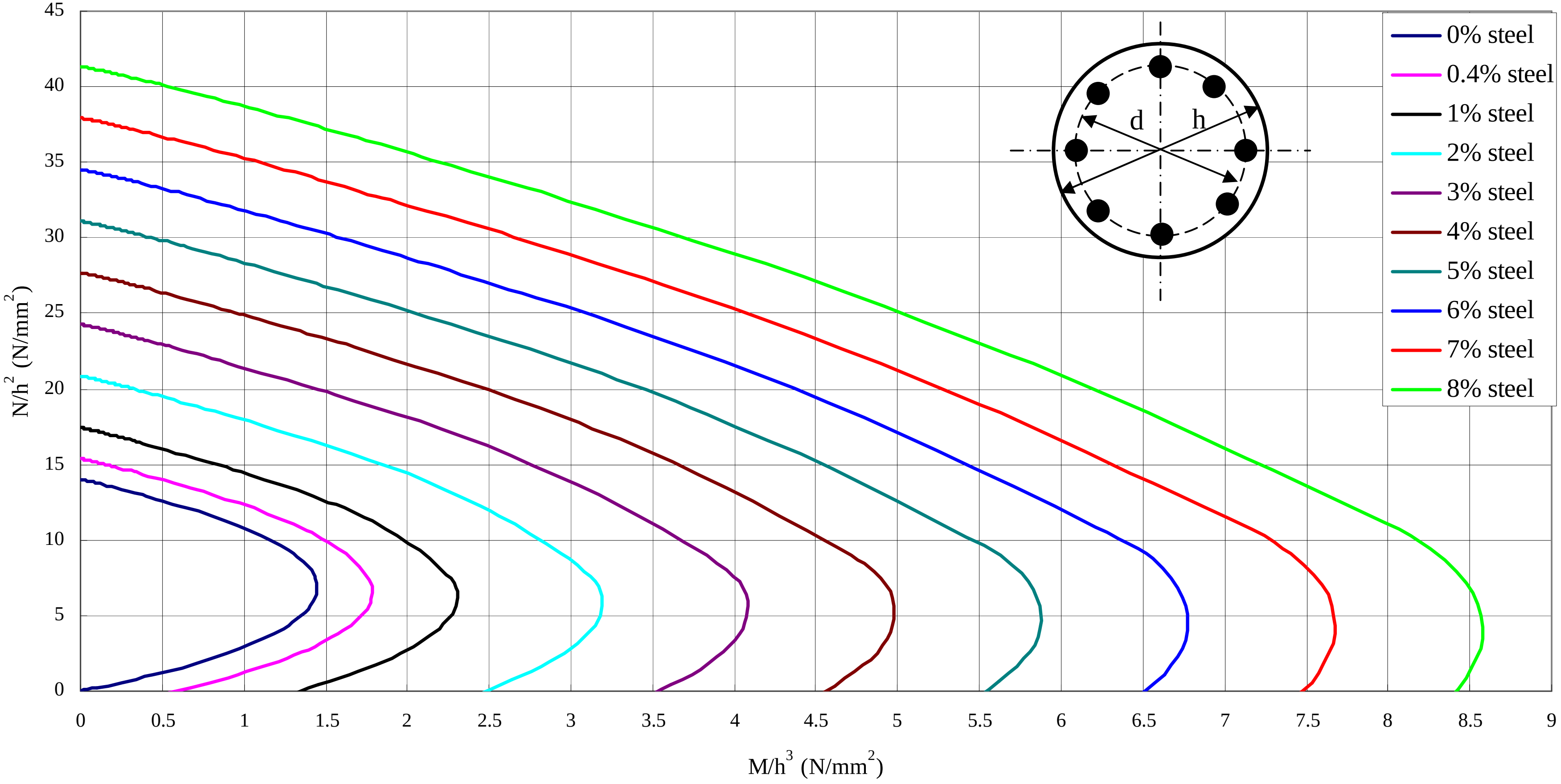
Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 40, d/h = 0.75

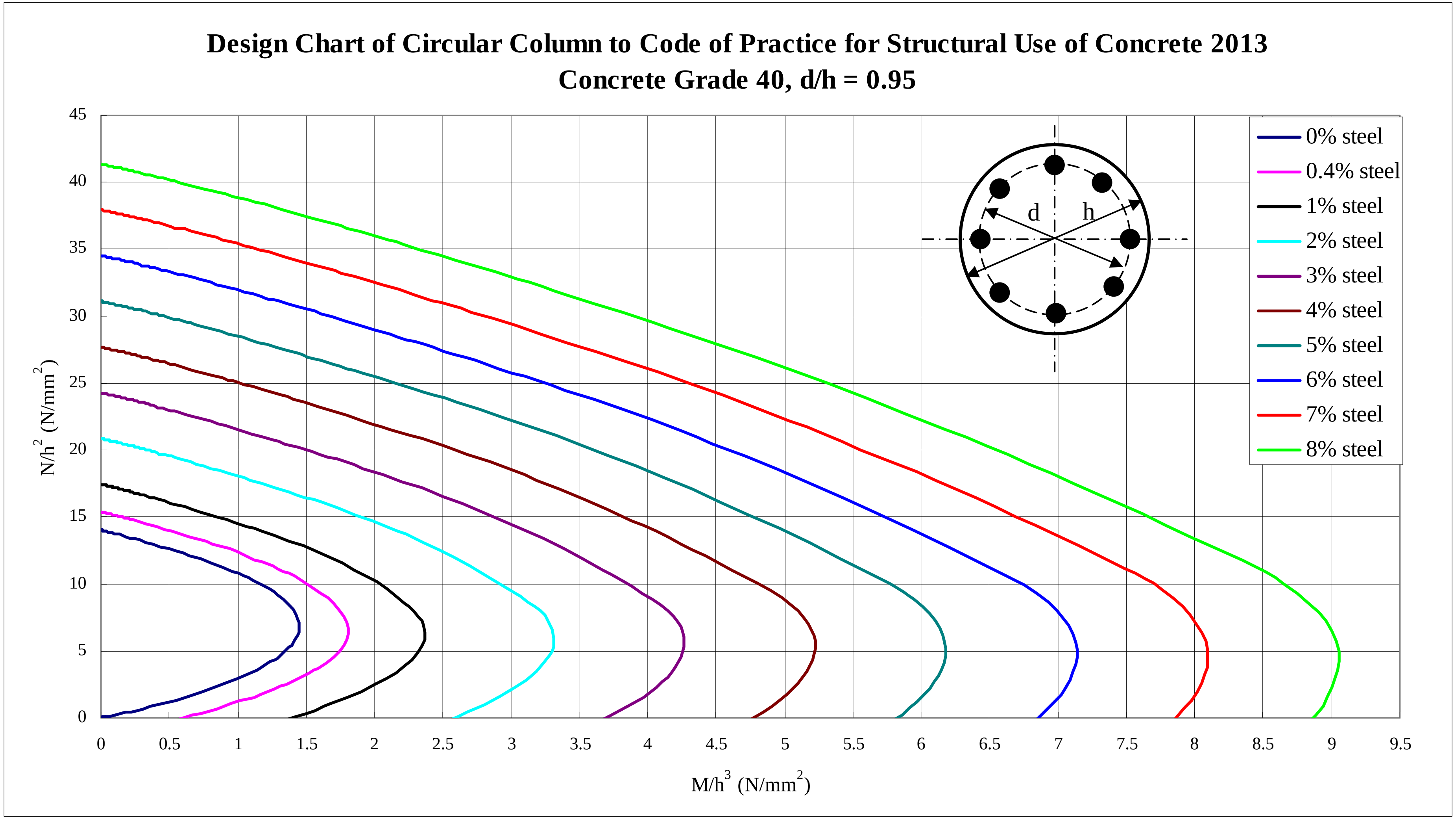


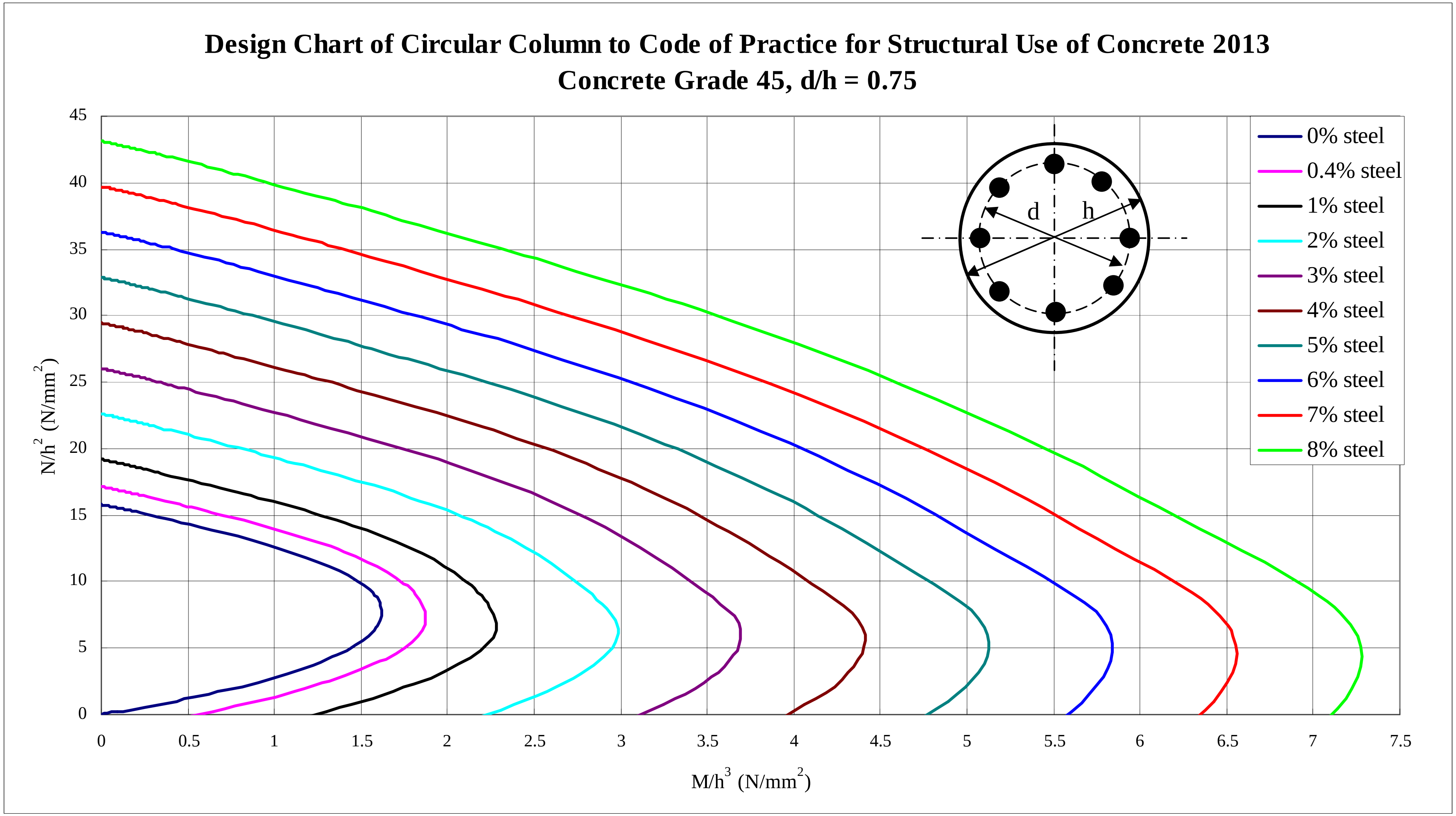

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 40, d/h = 0.8
N/h2 (N/mm2)
M/h3 (N/mm2)
0
5
10
15
20
25
30
35
40
45
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
d
h
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel


Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 40, d/h = 0.85
N/h² (N/mm²)
M/h³ (N/mm²)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
0
5
10
15
20
25
30
35
40
45
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5

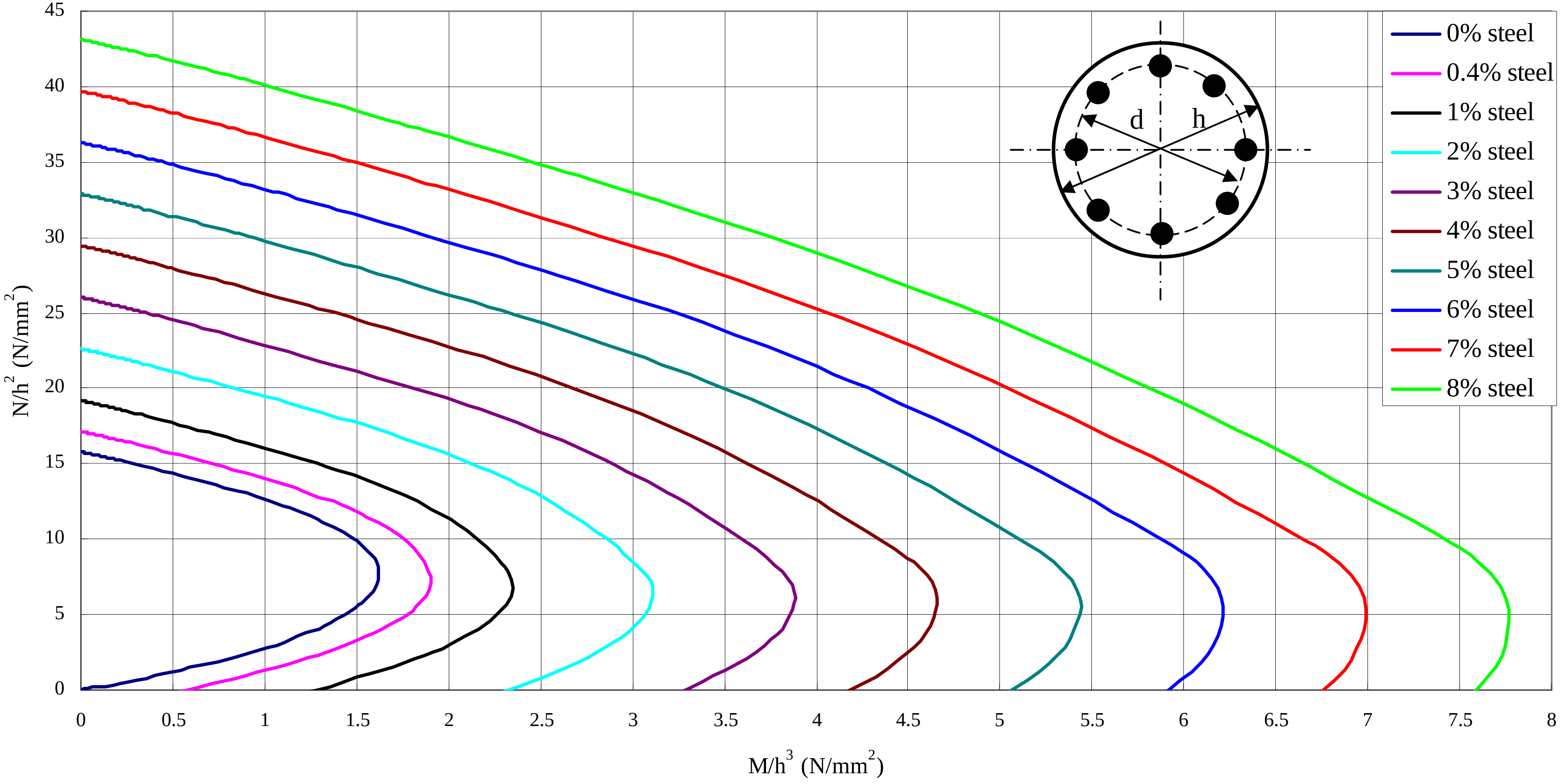
Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 40, d/h = 0.9

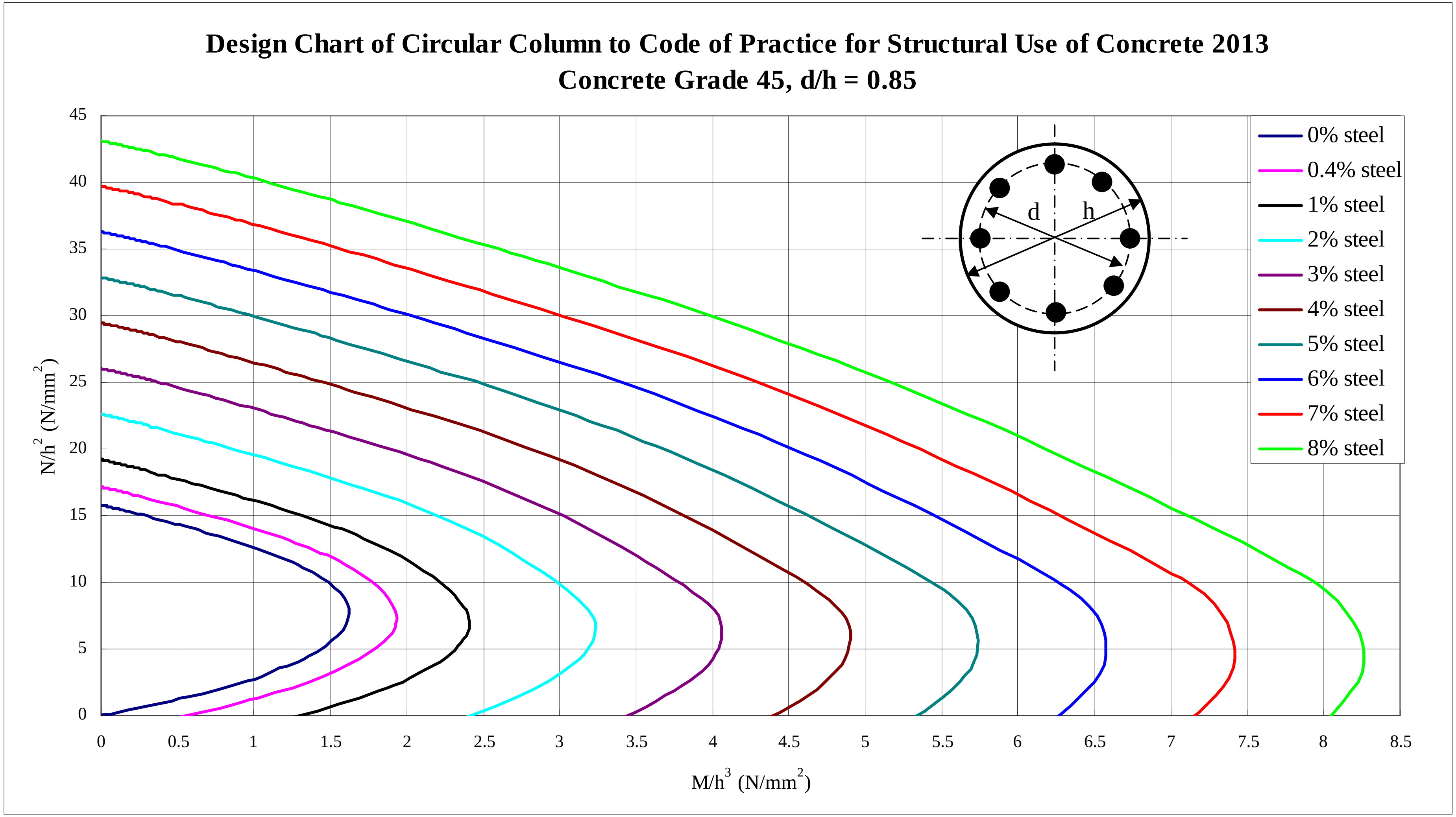



Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 40, d/h = 0.95
N/h2 (N/mm2)
M/h3 (N/mm2)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h

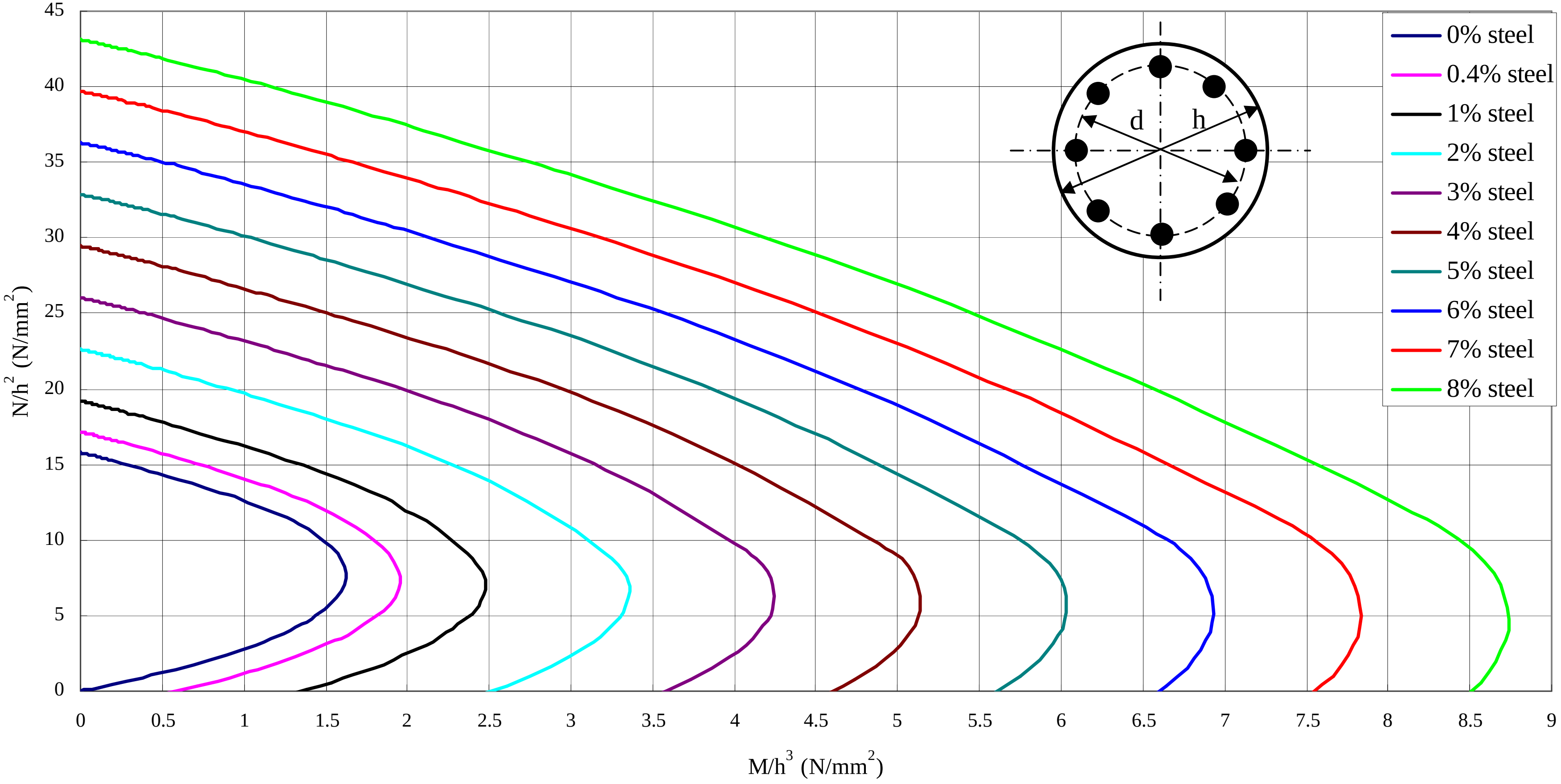

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 45, d/h = 0.75
N/h2 (N/mm2)
M/h3 (N/mm2)
0
5
10
15
20
25
30
35
40
45
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
d
h
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel

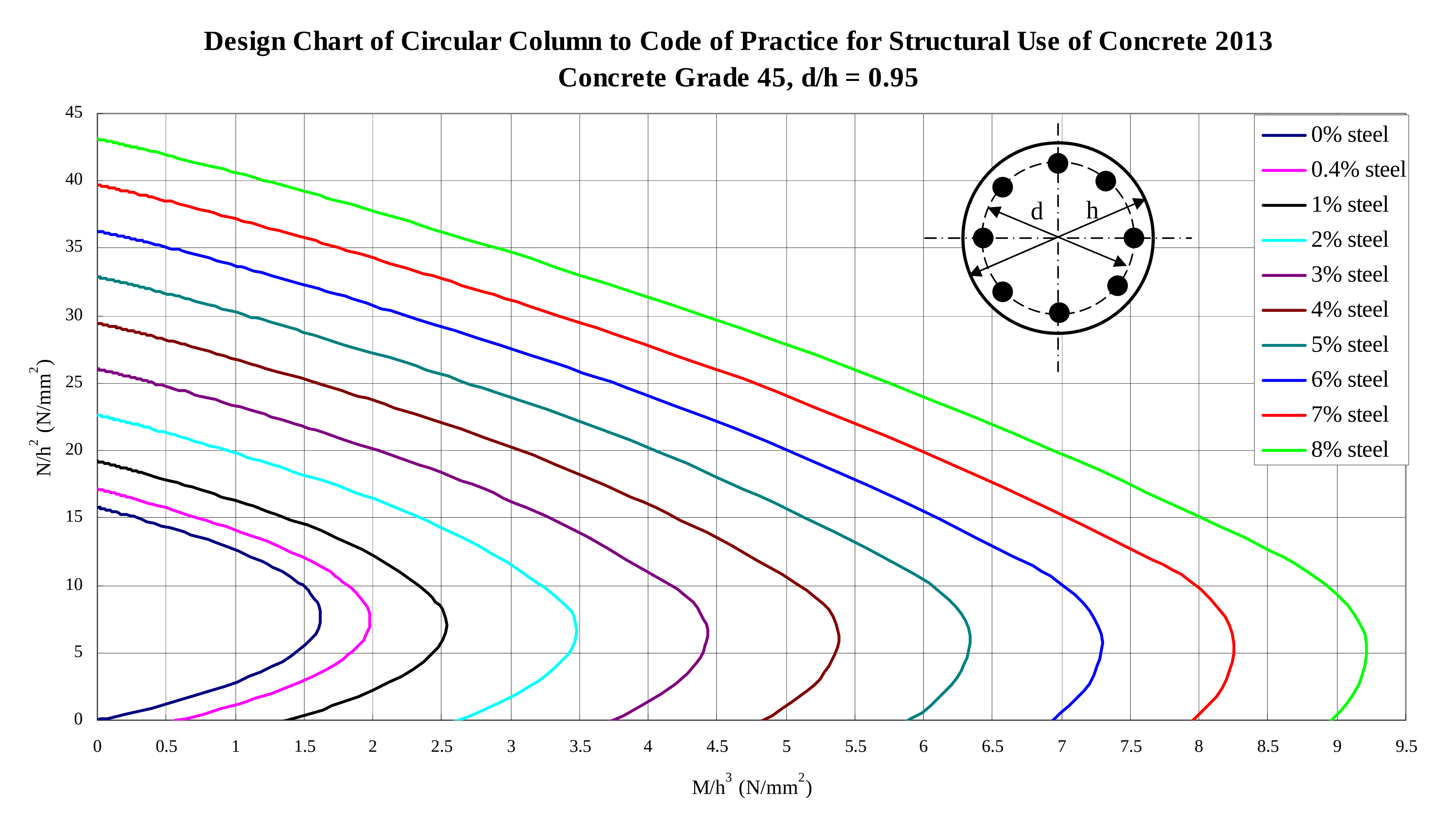
Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 45, d/h = 0.8

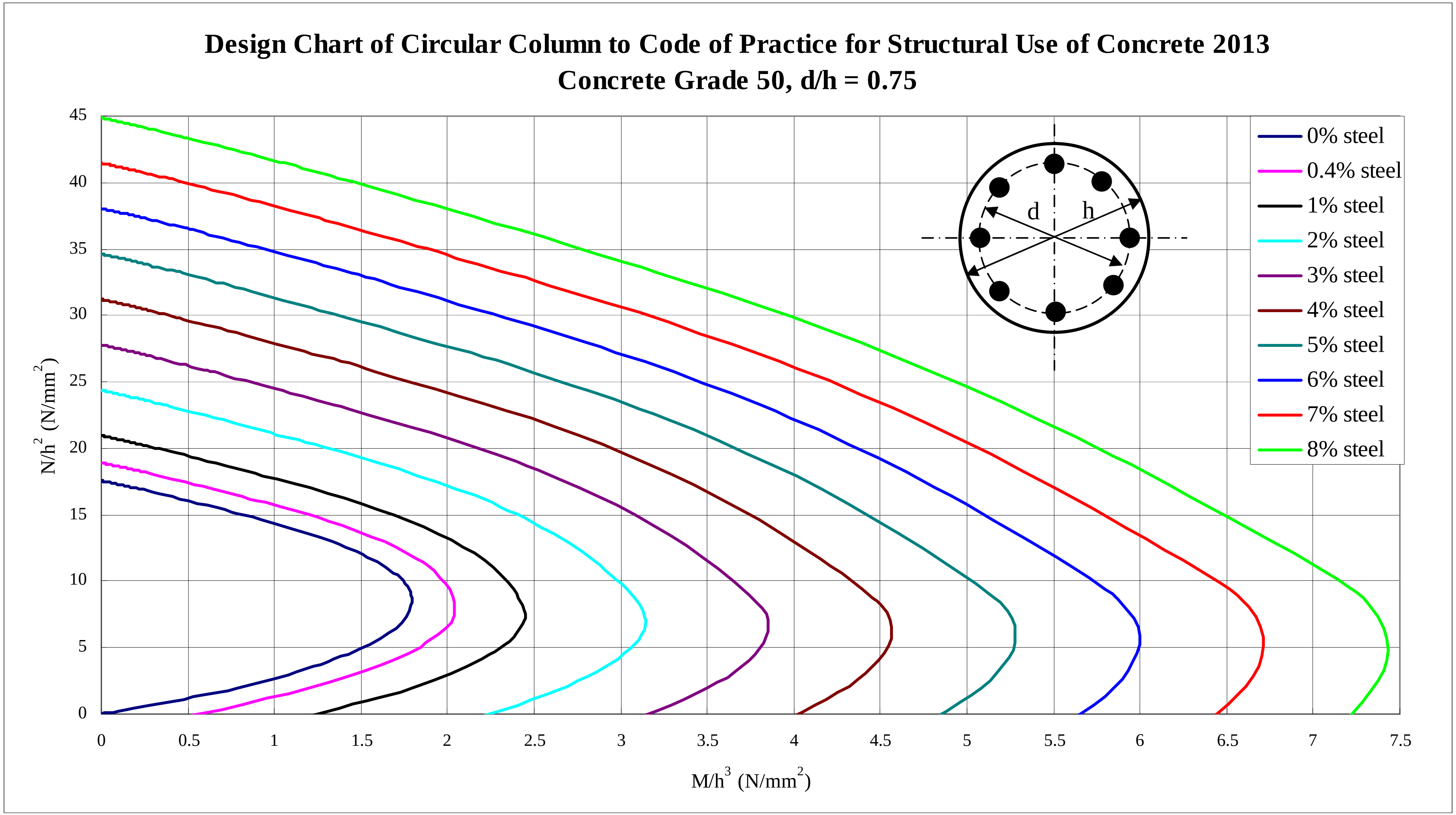



Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 45, d/h = 0.85
N/h² (N/mm²)
M/h³ (N/mm²)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h

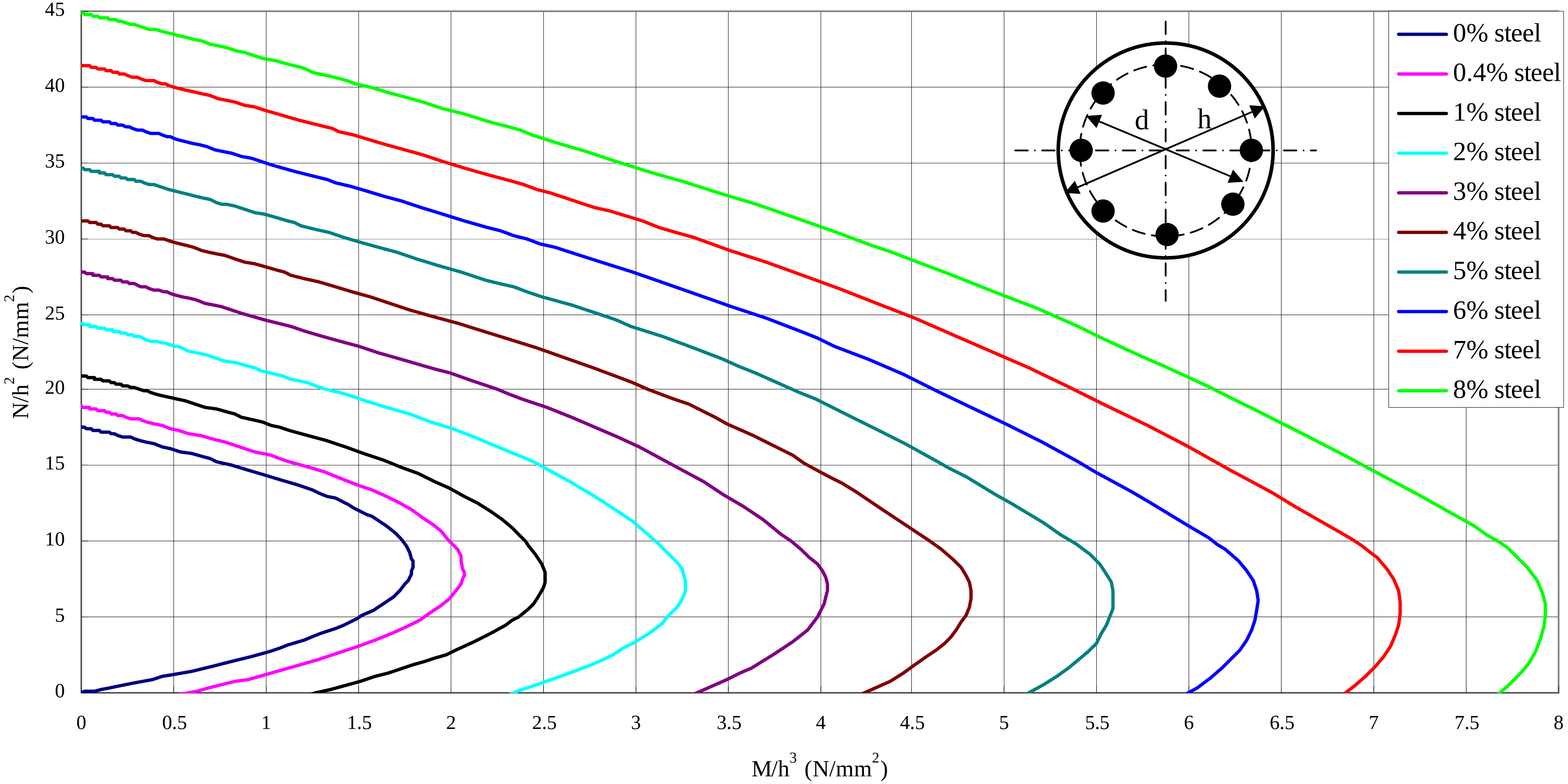
Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 45, d/h = 0.9

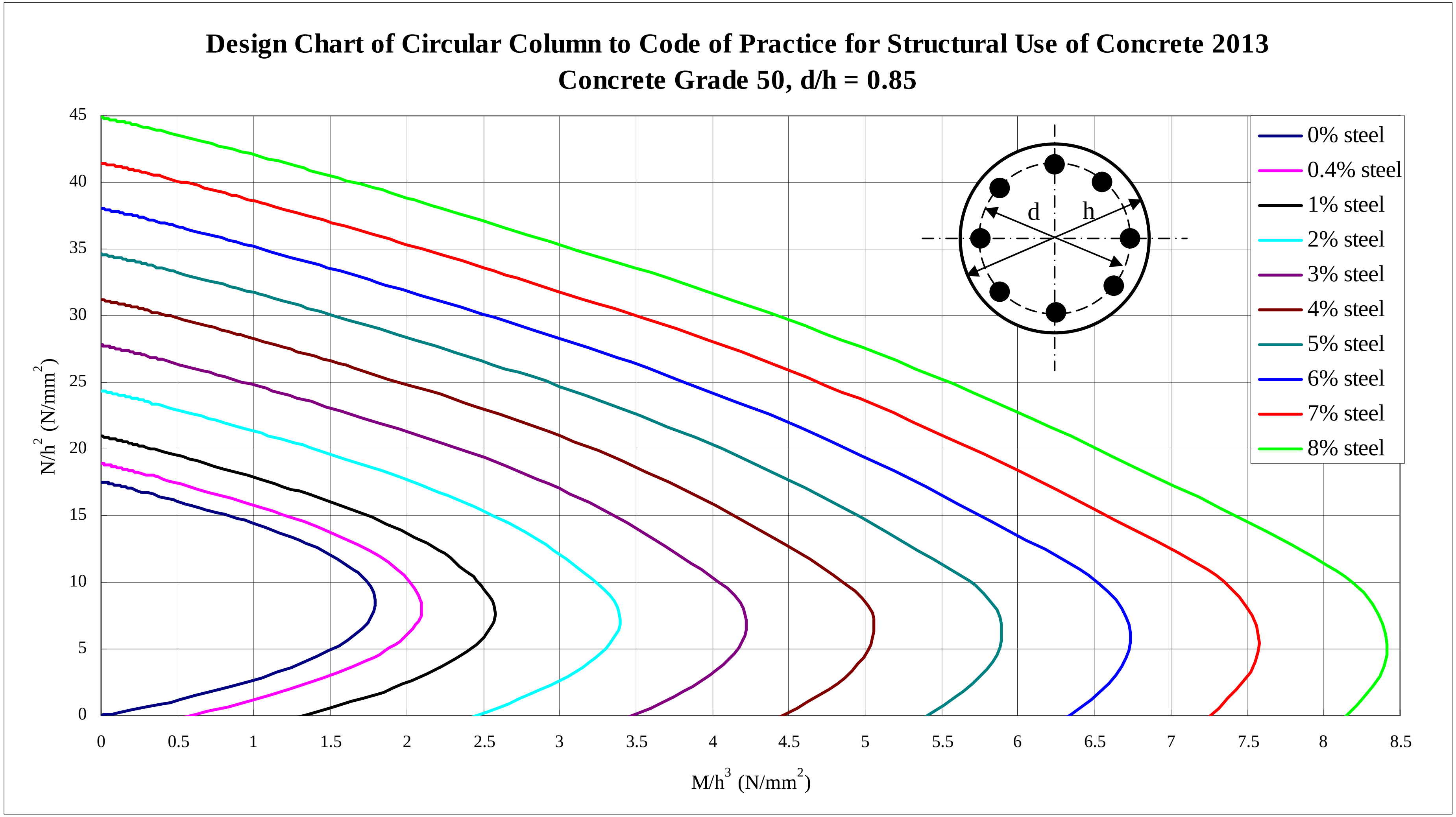


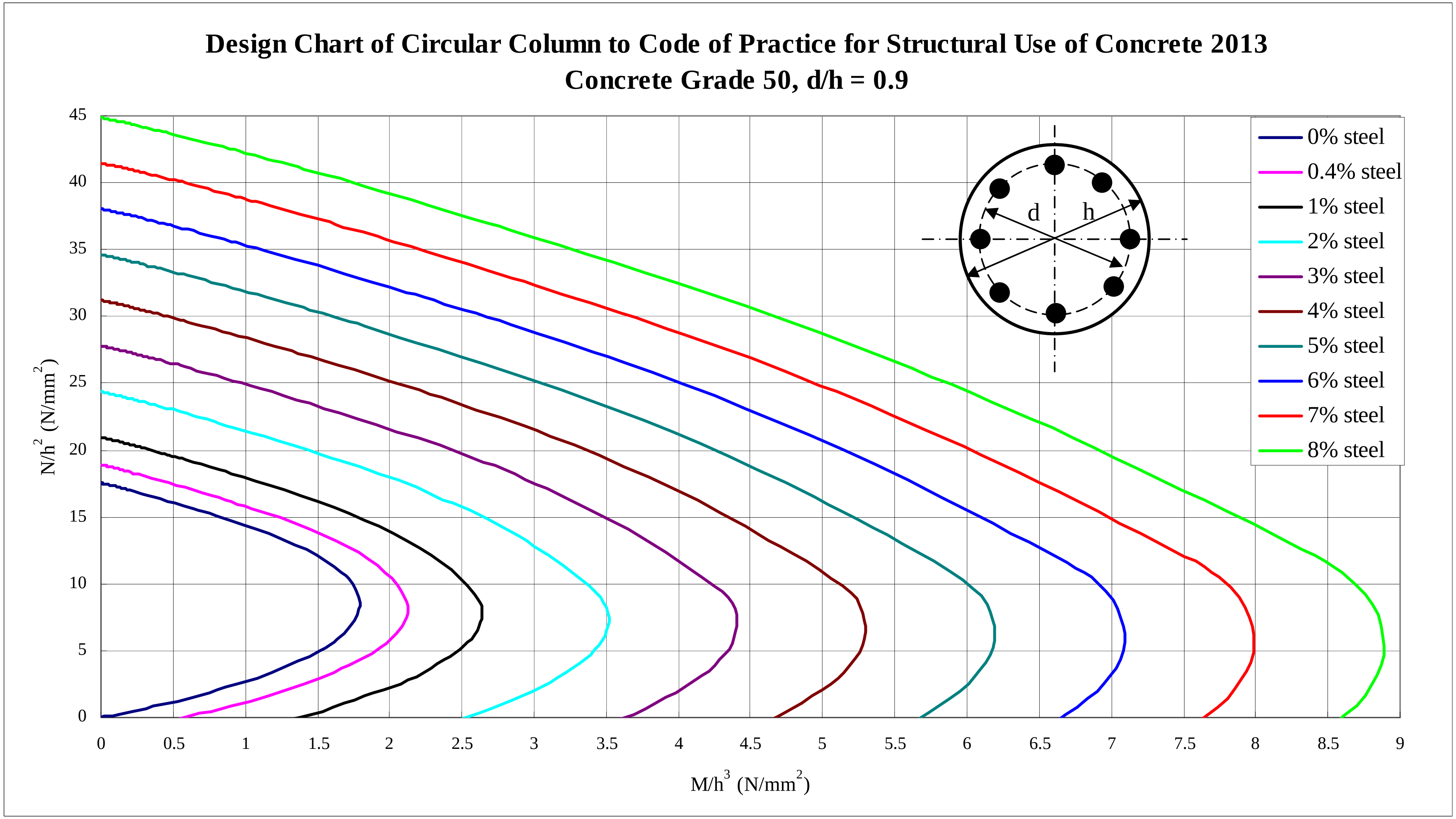

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 45, d/h = 0.95
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
N/h2 (N/mm2)
M/h3 (N/mm2)
0
5
10
15
20
25
30
35
40
45
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5


Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 50, d/h = 0.75
N/h2 (N/mm2)
M/h3 (N/mm2)
0
5
10
15
20
25
30
35
40
45
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
d
h
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel

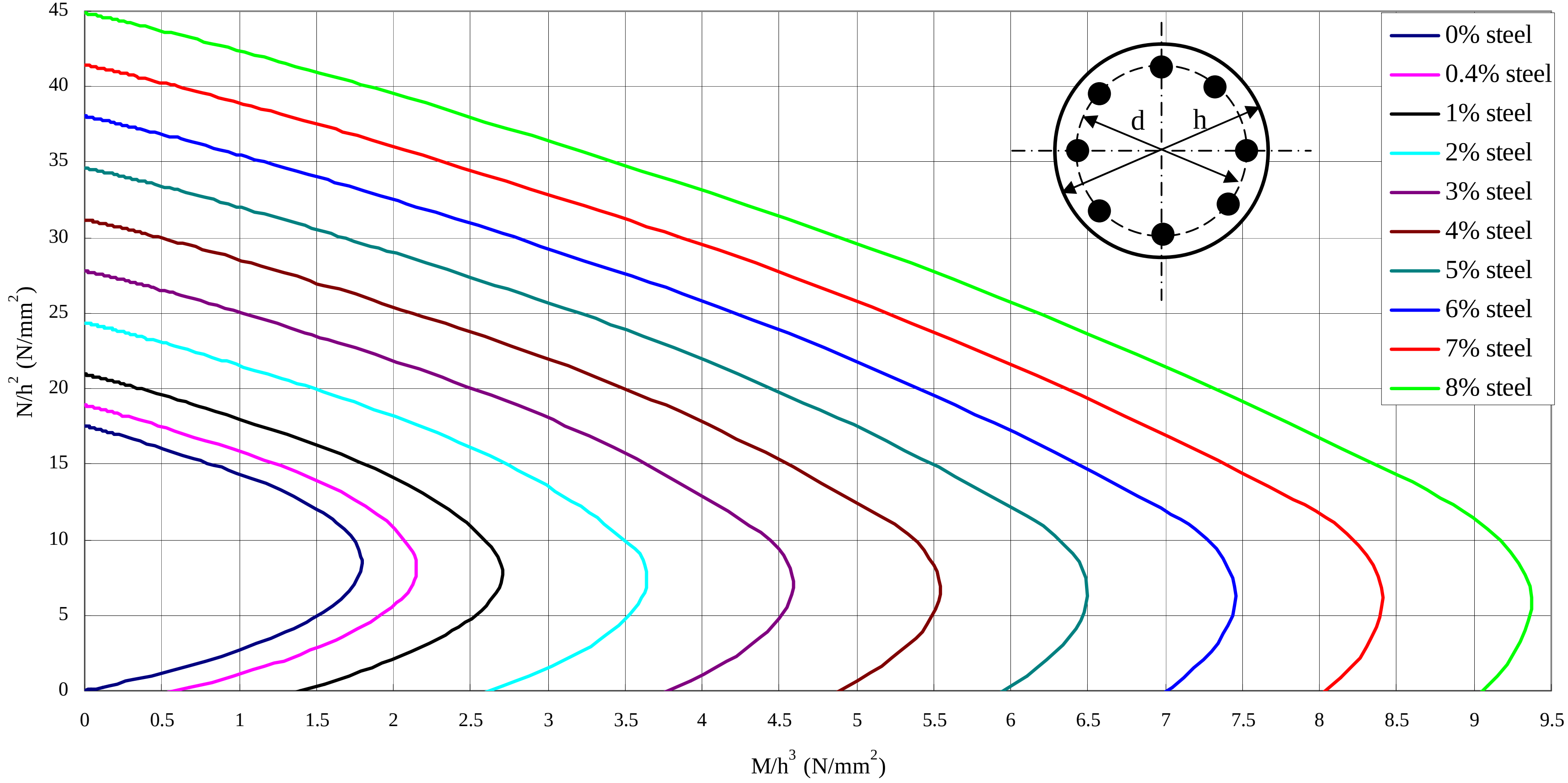
Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 50, d/h = 0.8

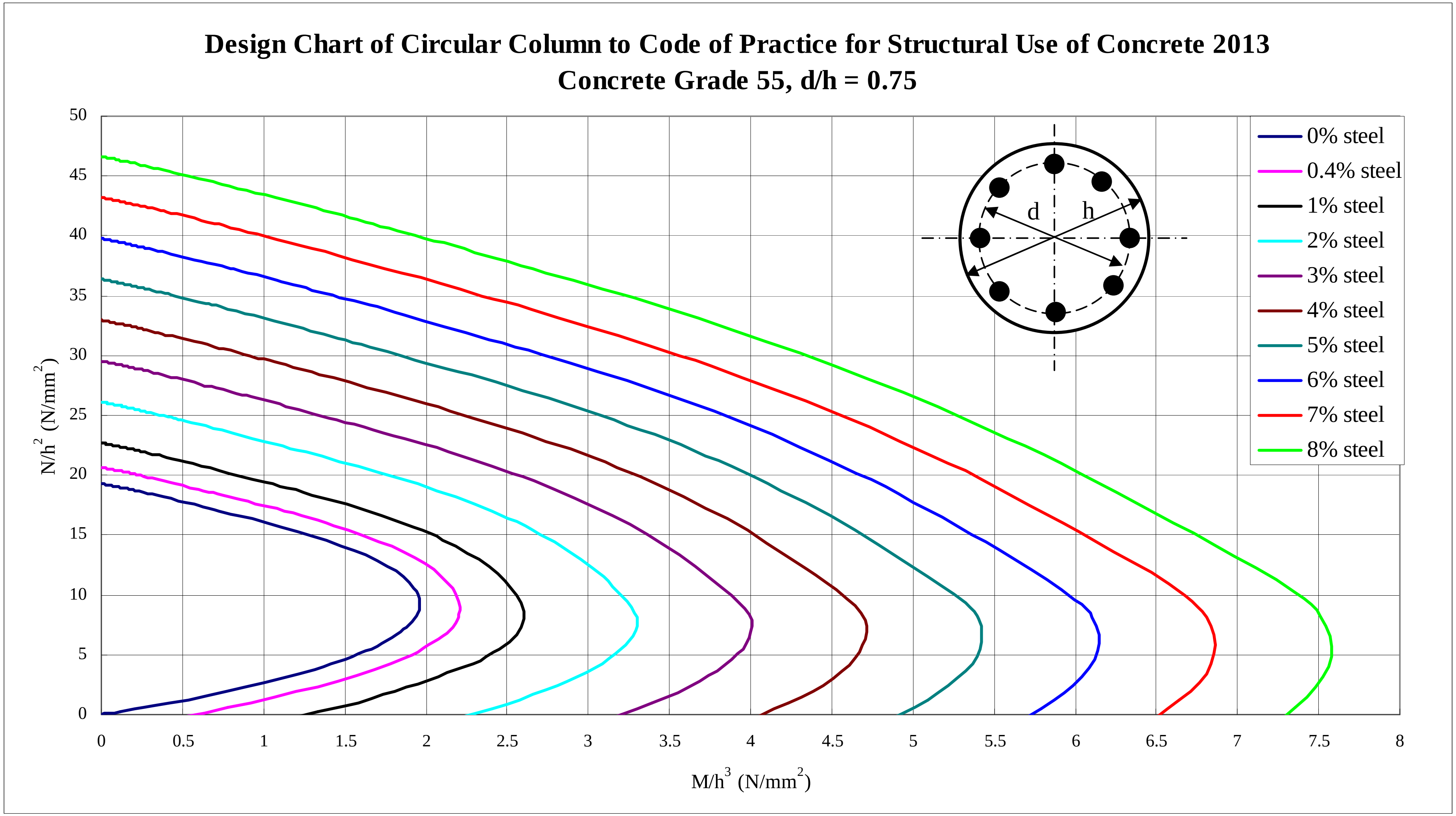


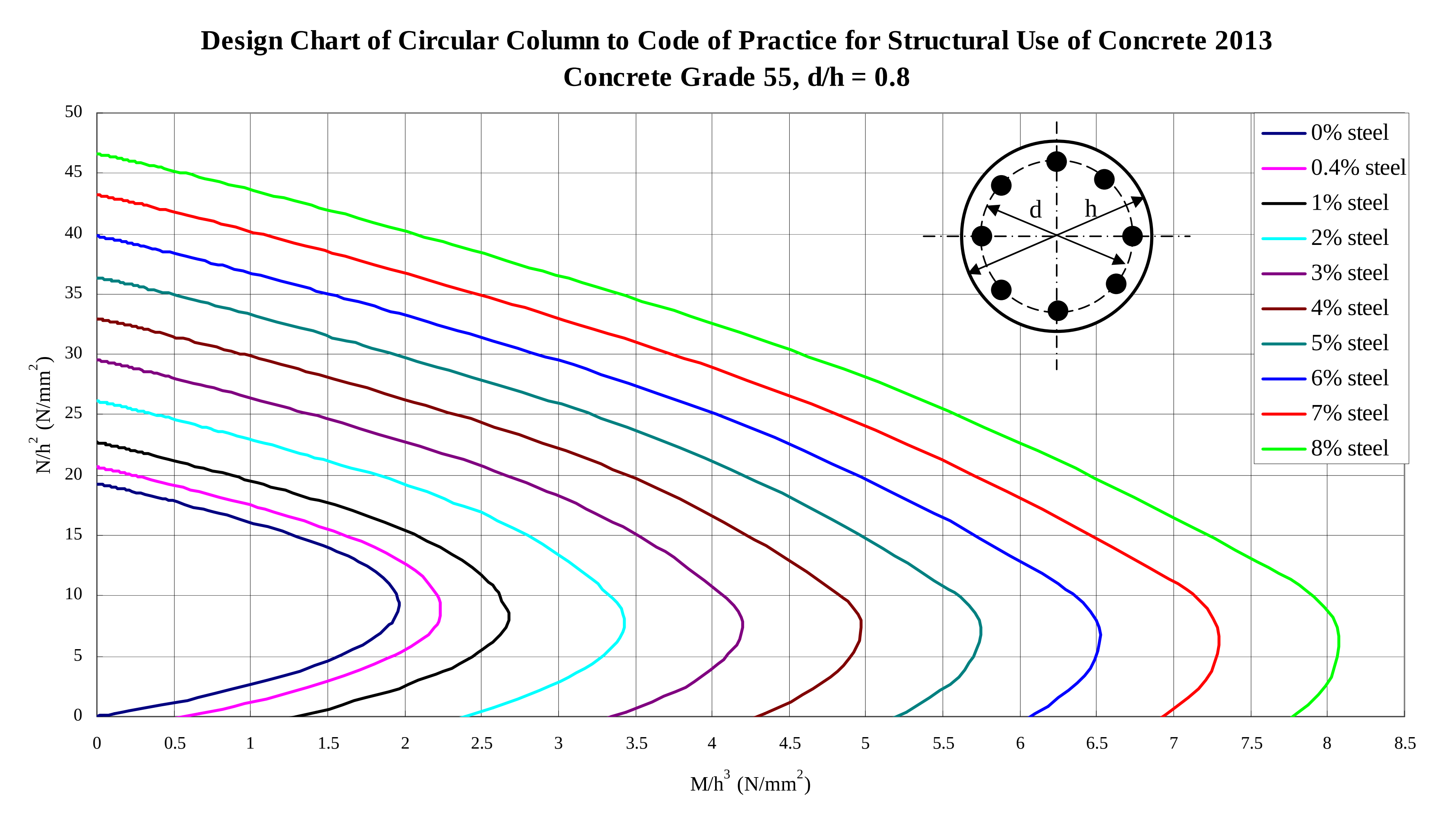

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 50, d/h = 0.85
N/h² (N/mm²)
M/h³ (N/mm²)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
0
5
10
15
20
25
30
35
40
45
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5

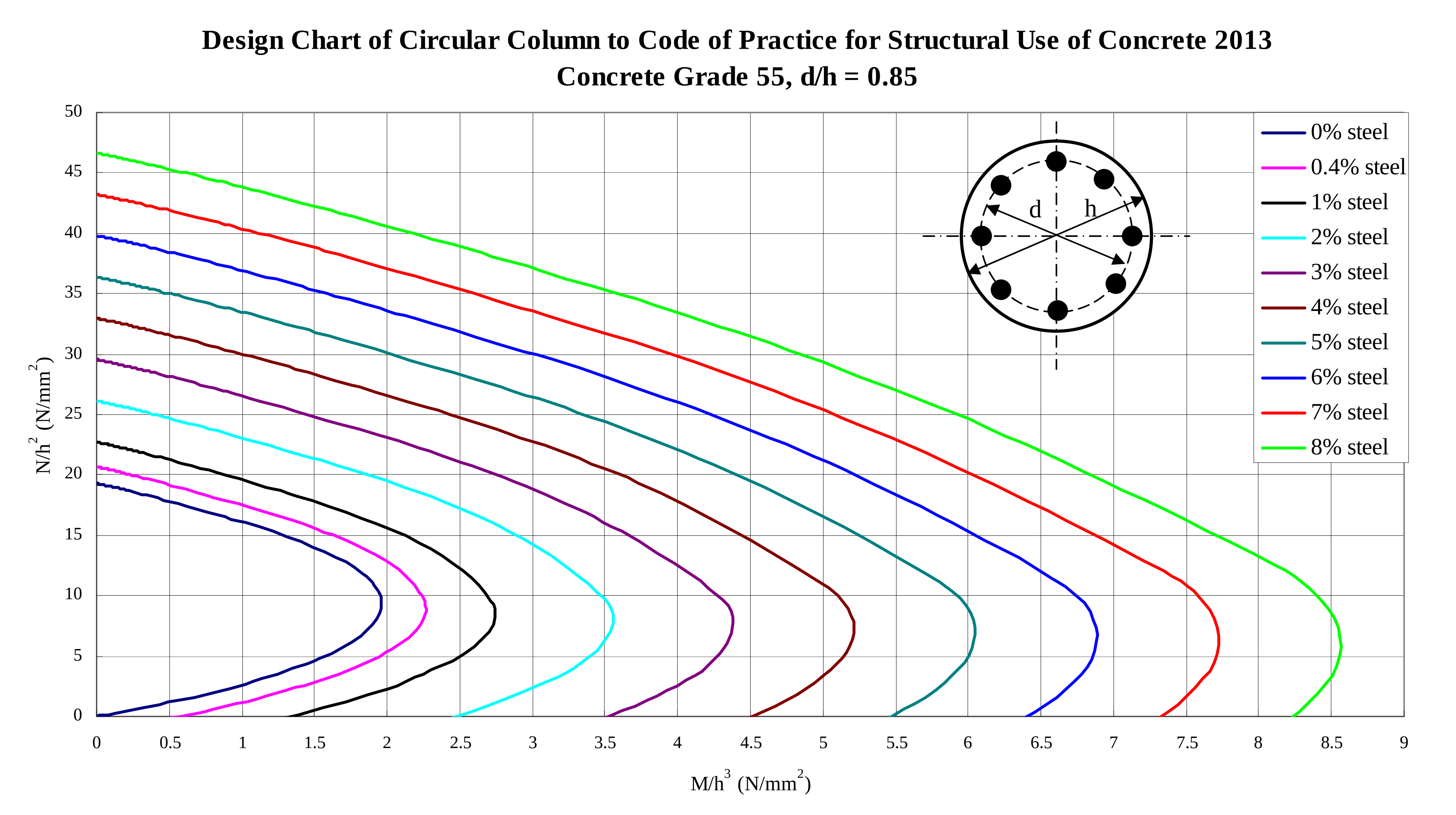

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 50, d/h = 0.9
45
40
35
30
25
20
15
10
5
0
N/h2 (N/mm2)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
M/h3 (N/mm2)
d
h
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 50, d/h = 0.95




Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 55, d/h = 0.75
N/h² (N/mm²)
M/h³ (N/mm²)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h


Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 55, d/h = 0.8
N/h² (N/mm²)
M/h³ (N/mm²)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5


Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 55, d/h = 0.85
N/h2 (N/mm2)
M/h3 (N/mm2)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
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6.5
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8.5
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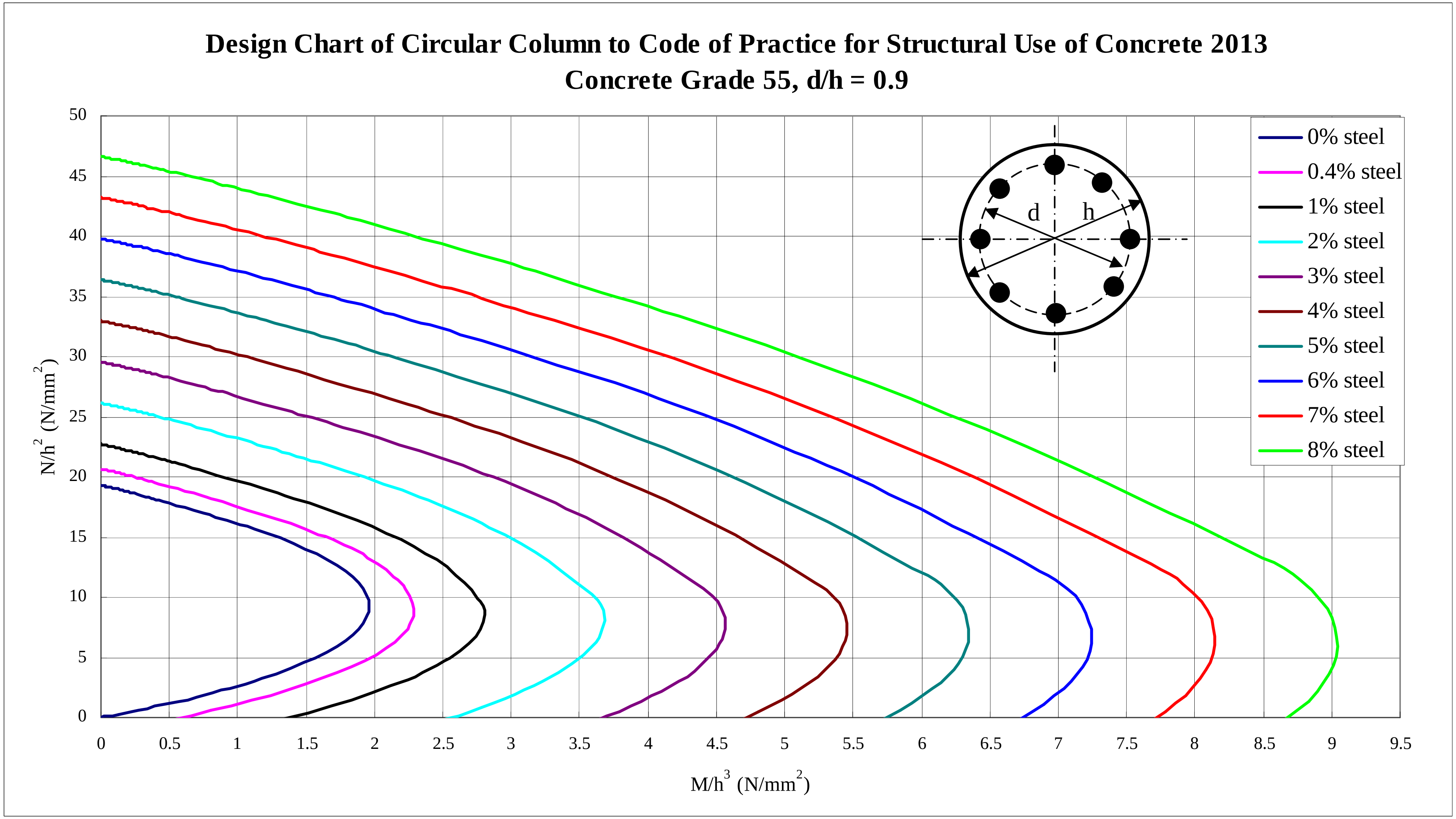
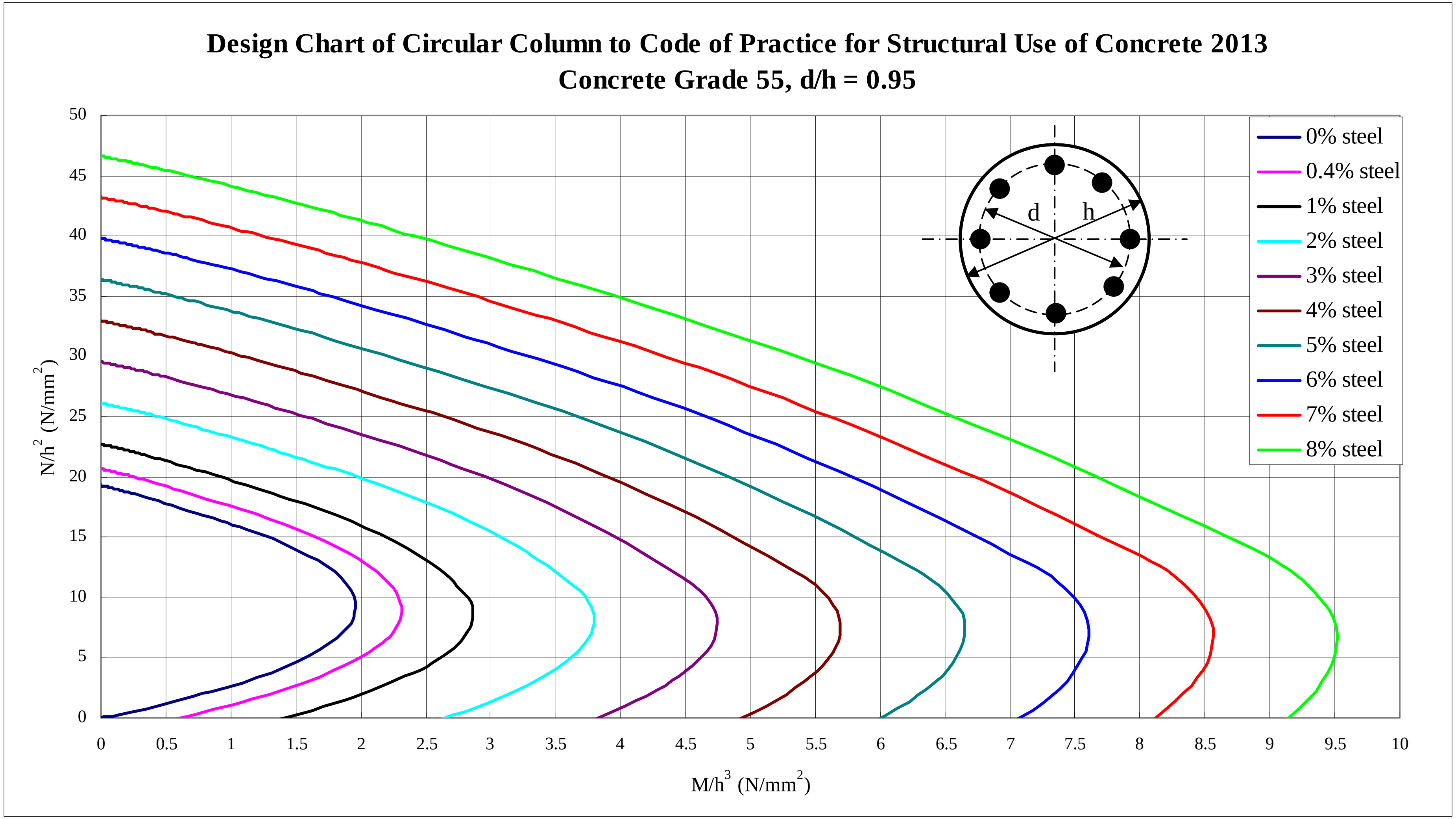

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 55, d/h = 0.9
N/h² (N/mm²)
M/h³ (N/mm²)
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
d
h
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel


Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 55, d/h = 0.95
N/h² (N/mm²)
M/h³ (N/mm²)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
2
2.5
3
3.5
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5.5
6
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7.5
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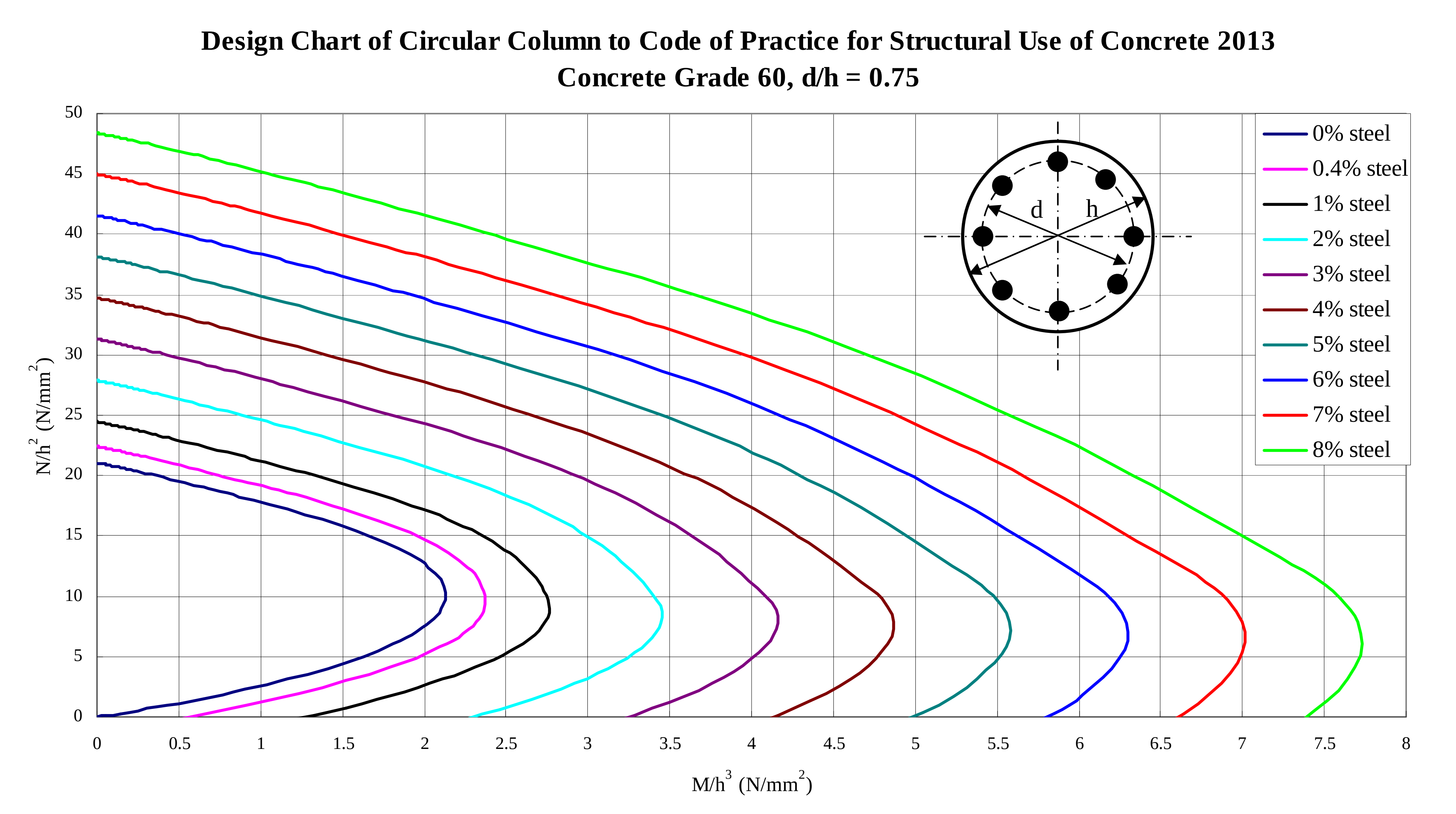
8.5
9
9.5


Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 60, d/h = 0.75
N/h² (N/mm²)
M/h³ (N/mm²)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
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5.5
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6.5
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7.5
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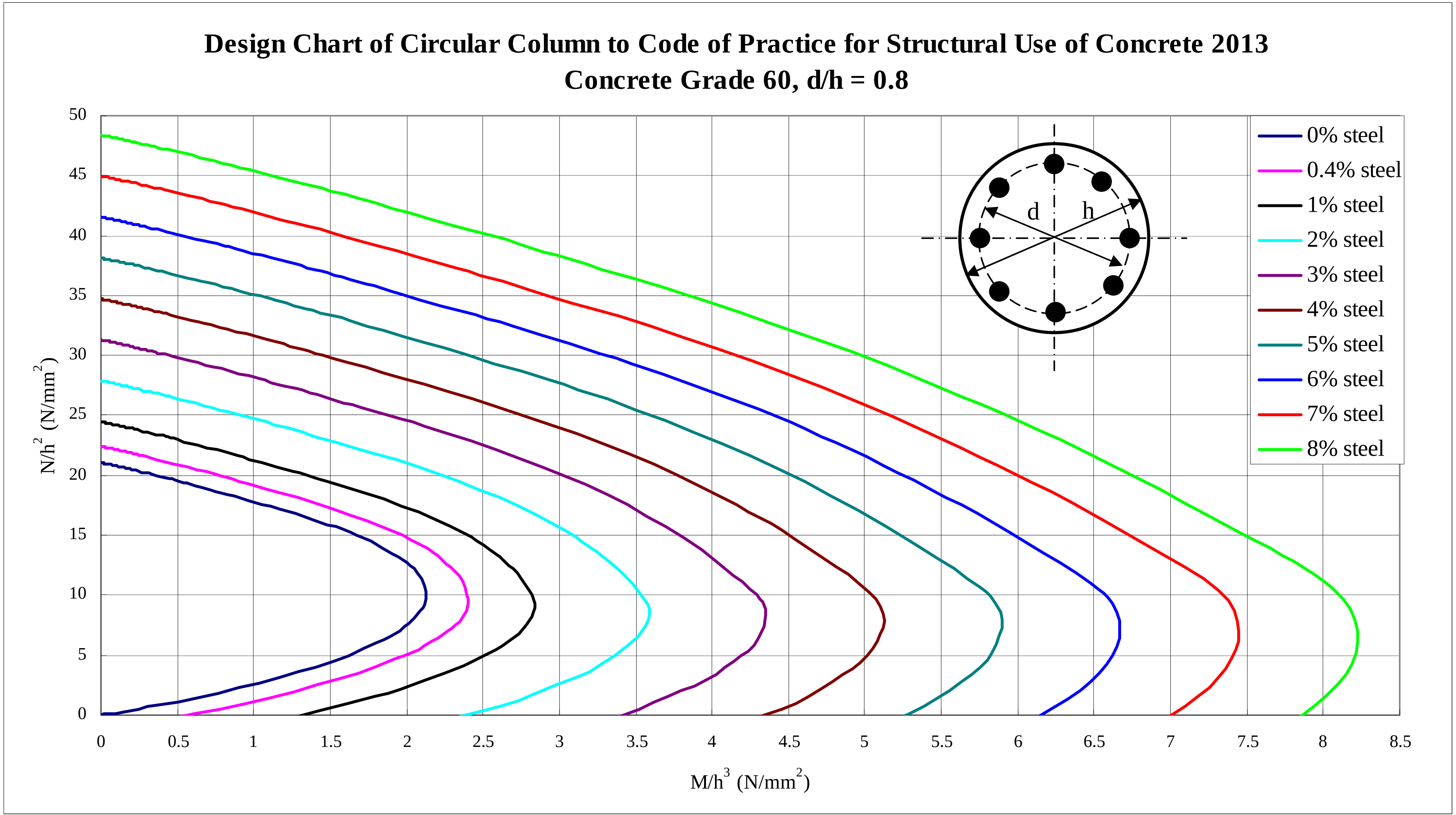
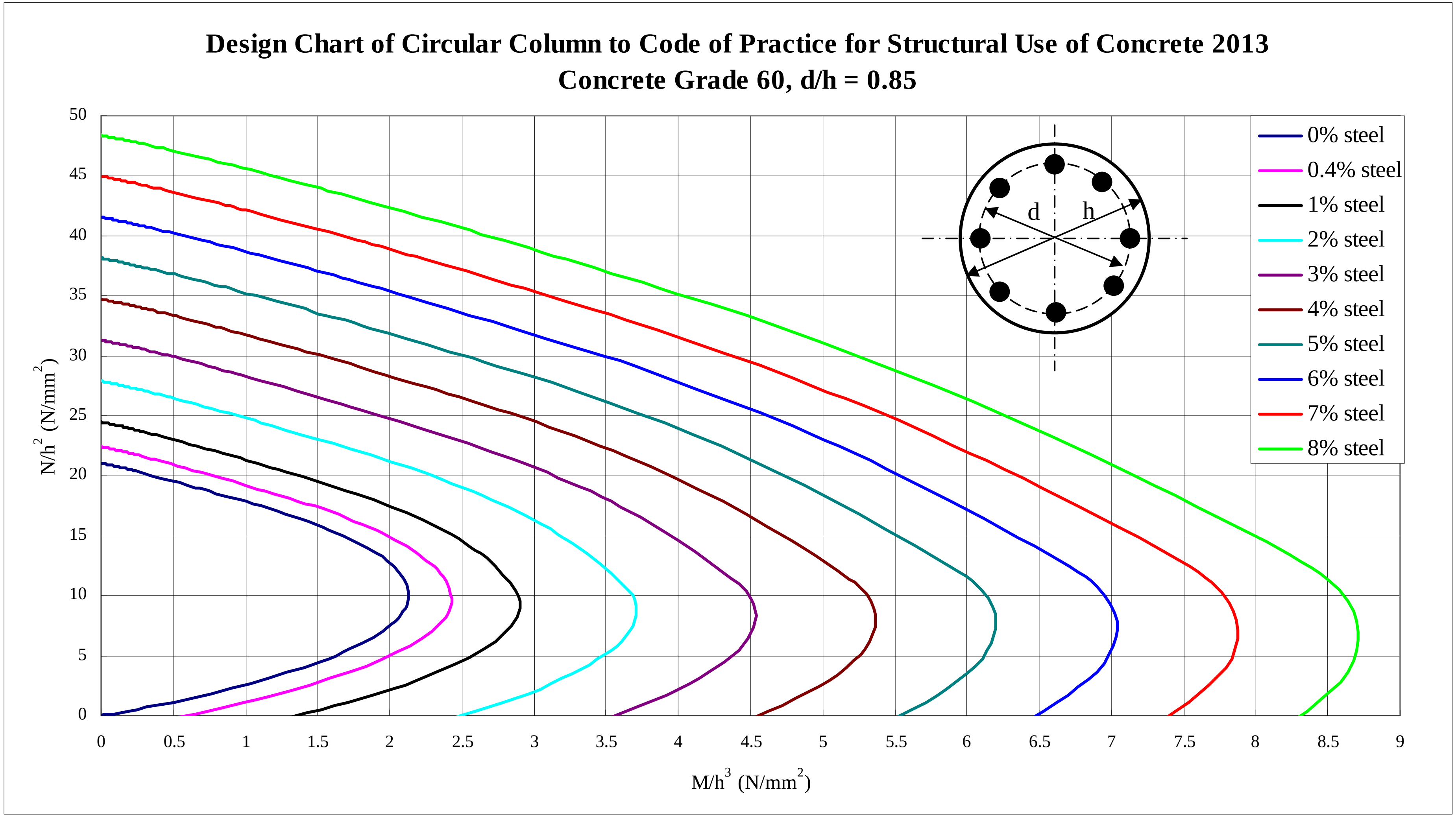

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 60, d/h = 0.8
N/h2 (N/mm2)
M/h3 (N/mm2)
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
d
h
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel


Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 60, d/h = 0.85
N/h2 (N/mm2)
M/h3 (N/mm2)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
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4.5
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7.5
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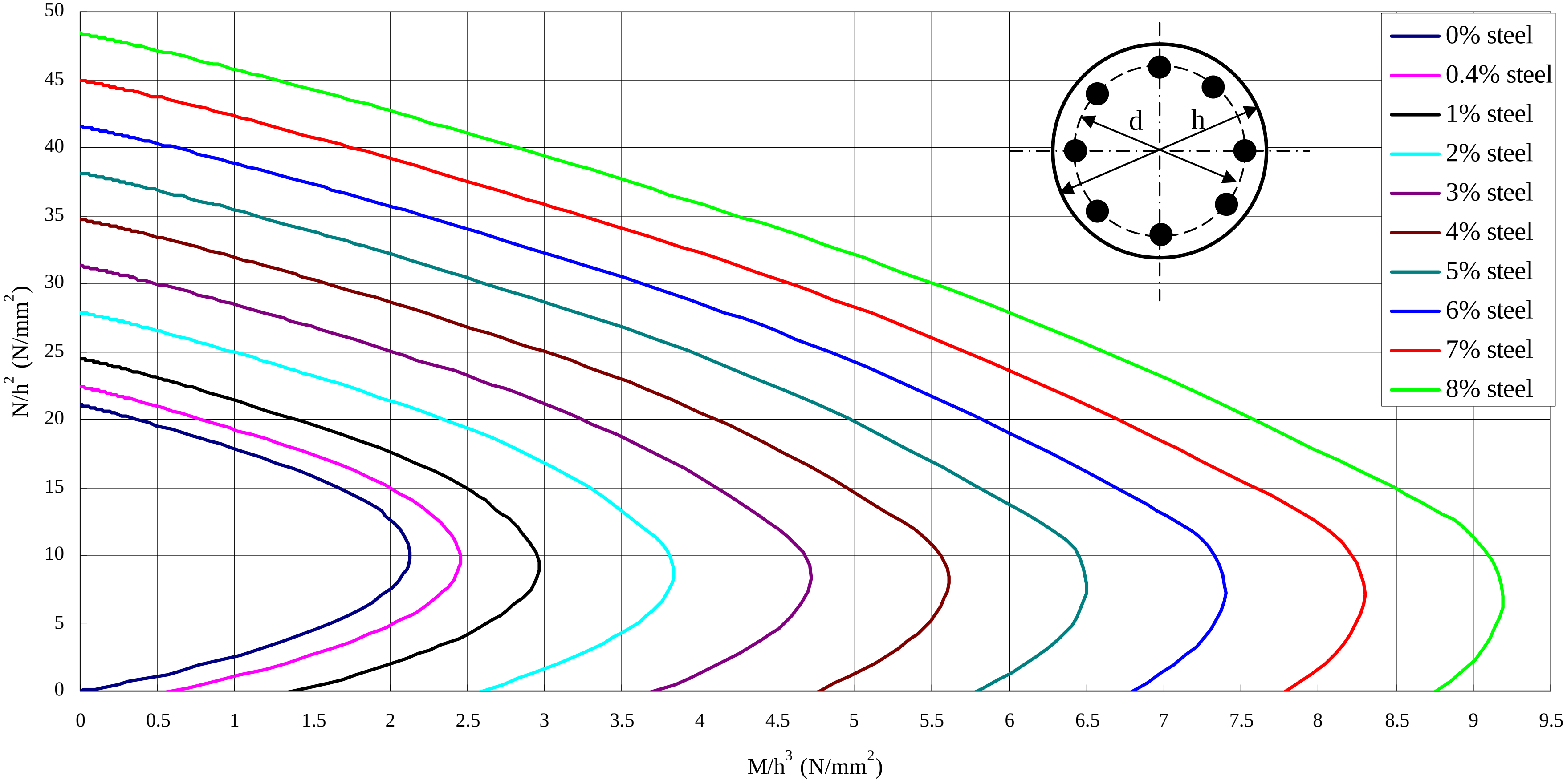
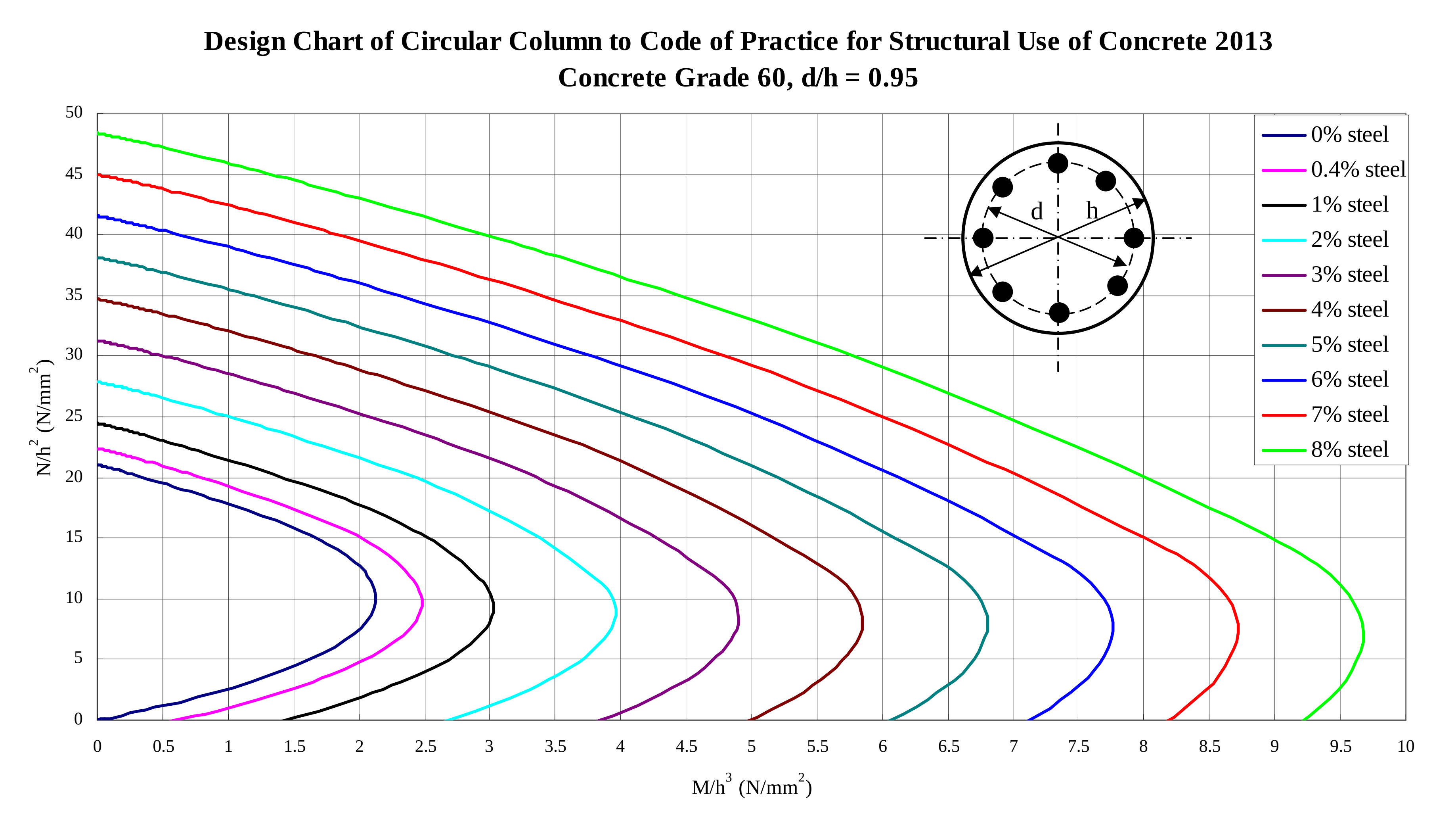
8.5
9

Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 60, d/h = 0.9




Design Chart of Circular Column to Code of Practice for Structural Use of Concrete 2013
Concrete Grade 60, d/h = 0.95
N/h2 (N/mm2)
M/h3 (N/mm2)
0% steel
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
d
h
0
5
10
15
20
25
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35
40
45
50
0.5
1
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2
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6.5
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7.5
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8.5
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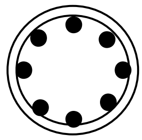
9.5
10

Circular Column (Bored Pile) Designed to CoPConc2013 - with Predetermined Rebars

Project :

Column Mark **Floor**

f_{cu} =	45	N/mm^2	E_c =	22161.2 N/mm^2	Col. Dia. =	900	mm	cover=	50	mm
f_y =	500	N/mm^2	E_s =	200000 N/mm^2	Steel Provided :	20	T	32		
Steel Percentage :	2.53	%		Bar c/c spacing 120.637 mm		r/R = 0.85333				



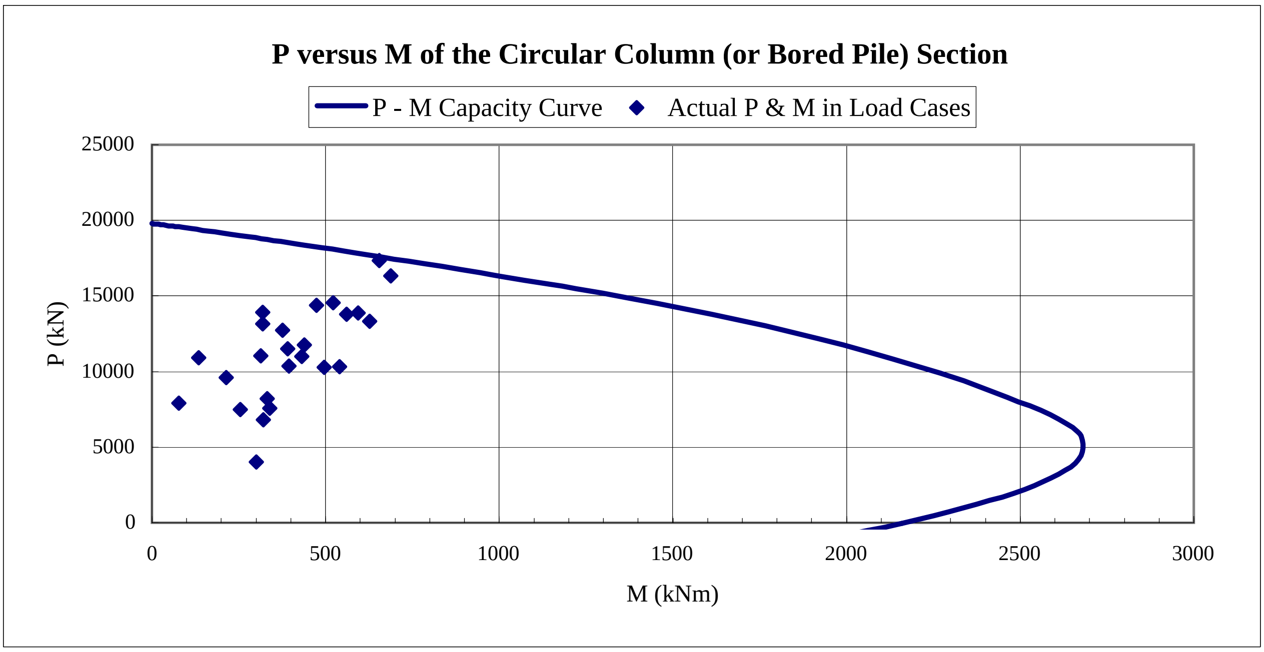
Basic Load Case

Load Case No.	1	2	3	4	5	6
Load Case	D.L.	L.L.	Wx	Wy	W45	W135
Axial Load P (kN)	8900	1200	-1500	-1000	500	3500
Moment M_x (kNm)	100	200	150	-200	10	250
Moment M_y (kNm)	-250	150	120	50	50	-60

Max. Axial Load Capacity

Plain Concrete	12787.1	kN
2.53 % steel	19784	kN

		N (kN)	M_x (kNm)	M_y (kNm)	$M_{resultant}$ (kNm)	N/D^2 (N/mm^2)	M/D^3 (N/mm^2)	x/R	$M_{resistance}$ (kNm)	Pass / Fail
Load Comb 1	1.4D+1.6L	14380	460	-110	472.969	17.7531	0.64879	-0.8107	1484.83	Pass
Load Comb 2	1.2(D+L+Wx)	10320	540	24	540.533	12.7407	0.74147	-0.4064	2211	Pass
Load Comb 3	1.2(D+L-Wx)	13920	180	-264	319.525	17.1852	0.43831	-0.758	1584.65	Pass
Load Comb 4	1.2(D+L+Wy)	10920	120	-60	134.164	13.4815	0.18404	-0.4591	2123.02	Pass
Load Comb 5	1.2(D+L-Wy)	13320	600	-180	626.418	16.4444	0.85928	-0.6926	1707.38	Pass
Load Comb 6	1.2(D+L+W45)	12720	372	-60	376.808	15.7037	0.51688	-0.6304	1822.05	Pass
Load Comb 7	1.2(D+L-W45)	11520	348	-180	391.796	14.2222	0.53744	-0.5139	2029.27	Pass
Load Comb 8	1.2(D+L+W135)	16320	660	-192	687.36	20.1481	0.94288	-1.0788	1002.58	Pass
Load Comb 9	1.2(D+L-W135)	7920	60	-48	76.8375	9.77778	0.1054	-0.2169	2505.75	Pass
Load Comb 10	1.4(D+Wx)	10360	350	-182	394.492	12.7901	0.54114	-0.4098	2205.31	Pass
Load Comb 11	1.4(D-Wx)	14560	-70	-518	522.708	17.9753	0.71702	-0.832	1444.38	Pass
Load Comb 12	1.4(D+Wy)	11060	-140	-280	313.05	13.6543	0.42942	-0.4717	2101.68	Pass
Load Comb 13	1.4(D-Wy)	13860	420	-420	593.97	17.1111	0.81477	-0.7513	1597.3	Pass
Load Comb 14	1.4(D+W45)	13160	154	-280	319.556	16.2469	0.43835	-0.6757	1738.73	Pass
Load Comb 15	1.4(D-W45)	11760	126	-420	438.493	14.5185	0.6015	-0.5364	1990.02	Pass
Load Comb 16	1.4(D+W135)	17360	490	-434	654.566	21.4321	0.8979	-1.3014	720.692	Pass
Load Comb 17	1.4(D-W135)	7560	-210	-266	338.904	9.33333	0.46489	-0.1912	2543.78	Pass
Load Comb 18	1.0D+1.4Wx	6800	310	-82	320.662	8.39506	0.43987	-0.1385	2620.24	Pass
Load Comb 19	1.0D-1.4Wx	11000	-110	-418	432.231	13.5802	0.59291	-0.4663	2110.87	Pass
Load Comb 20	1.0D+1.4Wy	7500	-180	-180	254.558	9.25926	0.34919	-0.187	2550.03	Pass
Load Comb 21	1.0D-1.4Wy	10300	380	-320	496.79	12.716	0.68147	-0.4047	2213.84	Pass
Load Comb 22	1.0D+1.4W45	9600	114	-180	213.063	11.8519	0.29227	-0.3457	2309.85	Pass
Load Comb 23	1.0D-1.4W45	8200	86	-320	331.355	10.1235	0.45453	-0.2373	2475.49	Pass
Load Comb 24	1.0D+1.4W135	13800	450	-334	560.407	17.037	0.76873	-0.7447	1609.87	Pass
Load Comb 25	1.0D-1.4W135	4000	-250	-166	300.093	4.93827	0.41165	0.08032	2662.72	Pass
									Overall	**Pass**

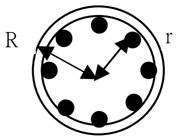


Circular Column (Bored Pile) Designed to CoPConc2013 - Minimum Rebar Percentage

Project :

Column Mark **Floor**

f_{cu} =	45	N/mm^2	E_c = 22161.2 N/mm^2	Col. Dia. =	900	mm	r/R =	0.85	
f_y =	500	N/mm^2	E_s = 200000 N/mm^2	No. of Bar =	20	Bar c/c spacing	120.166	mm	
Min % =	0.8	%	Steel Percentage : 2.475 %	Equivalent Bar dia. =	31.6596	mm			



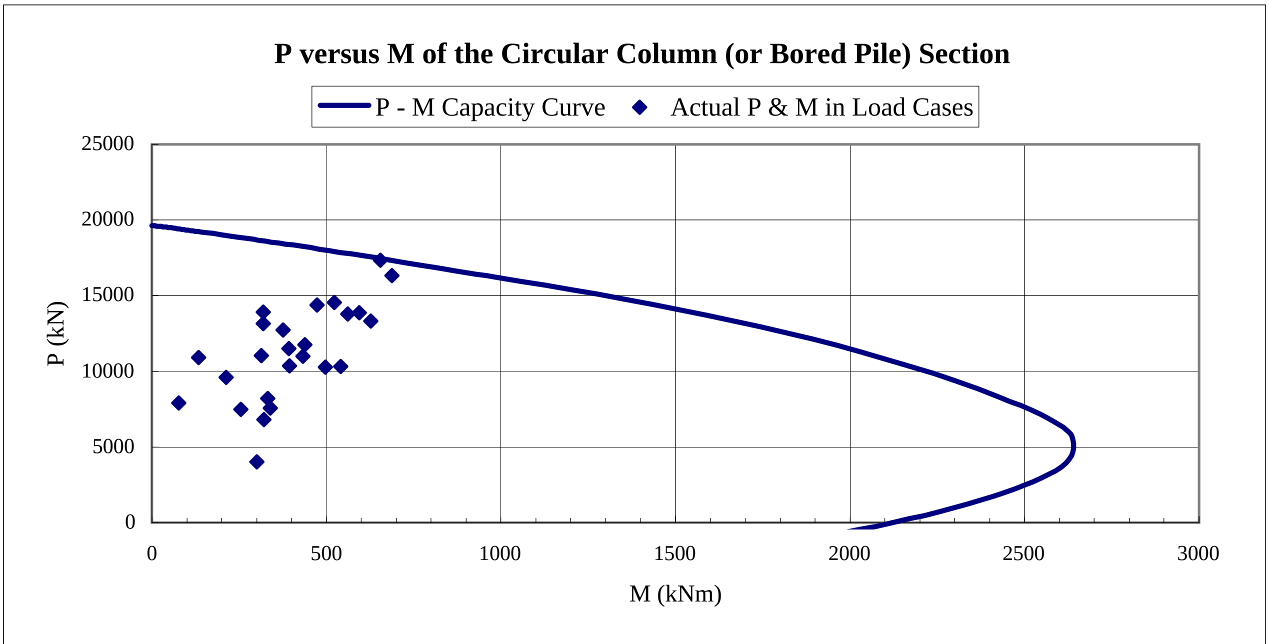
Basic Load Case

Load Case No.	1	2	3	4	5	6
Load Case	D.L.	L.L.	Wx	Wy	W45	W135
Axial Load P (kN)	8900	1200	-1500	-1000	500	3500
Moment M_x (kNm)	100	200	150	-200	10	250
Moment M_y (kNm)	-250	150	120	50	50	-60

Max. Axial Load Capacity

Plain Concrete	12787.1	kN
2.475 % steel	19636	kN

		N (kN)	M_x (kNm)	M_y (kNm)	$M_{resultant}$ (kNm)	N/D^2 (N/mm^2)	M/D^3 (N/mm^2)	x/R	Min p (%)	Rebar (mm^2)
Load Comb 1	1.4D+1.6L	14380	460	-110	472.969	17.7531	0.64879	-1.2777	1.147	7298
Load Comb 2	1.2(D+L+Wx)	10320	540	24	540.533	12.7407	0.74147	-0.9186	0.800	5089
Load Comb 3	1.2(D+L-Wx)	13920	180	-264	319.525	17.1852	0.43831	-1.3936	0.800	5089
Load Comb 4	1.2(D+L+Wy)	10920	120	-60	134.164	13.4815	0.18404	-1.3016	0.800	5089
Load Comb 5	1.2(D+L-Wy)	13320	600	-180	626.418	16.4444	0.85928	-1.071	0.998	6351
Load Comb 6	1.2(D+L+W45)	12720	372	-60	376.808	15.7037	0.51688	-1.2131	0.800	5089
Load Comb 7	1.2(D+L-W45)	11520	348	-180	391.796	14.2222	0.53744	-1.0925	0.800	5089
Load Comb 8	1.2(D+L+W135)	16320	660	-192	687.36	20.1481	0.94288	-1.2456	2.131	13559
Load Comb 9	1.2(D+L-W135)	7920	60	-48	76.8375	9.77778	0.1054	-0.9921	0.800	5089
Load Comb 10	1.4(D+Wx)	10360	350	-182	394.492	12.7901	0.54114	-0.9923	0.800	5089
Load Comb 11	1.4(D-Wx)	14560	-70	-518	522.708	17.9753	0.71702	-1.2464	1.281	8147
Load Comb 12	1.4(D+Wy)	11060	-140	-280	313.05	13.6543	0.42942	-1.1105	0.800	5089
Load Comb 13	1.4(D-Wy)	13860	420	-420	593.97	17.1111	0.81477	-1.1321	1.141	7259
Load Comb 14	1.4(D+W45)	13160	154	-280	319.556	16.2469	0.43835	-1.3108	0.800	5089
Load Comb 15	1.4(D-W45)	11760	126	-420	438.493	14.5185	0.6015	-1.0865	0.800	5089
Load Comb 16	1.4(D+W135)	17360	490	-434	654.566	21.4321	0.8979	-1.3308	2.475	15745
Load Comb 17	1.4(D-W135)	7560	-210	-266	338.904	9.33333	0.46489	-0.765	0.800	5089
Load Comb 18	1.0D+1.4Wx	6800	310	-82	320.662	8.39506	0.43987	-0.6608	0.800	5089
Load Comb 19	1.0D-1.4Wx	11000	-110	-418	432.231	13.5802	0.59291	-1.0242	0.800	5089
Load Comb 20	1.0D+1.4Wy	7500	-180	-180	254.558	9.25926	0.34919	-0.8142	0.800	5089
Load Comb 21	1.0D-1.4Wy	10300	380	-320	496.79	12.716	0.68147	-0.9367	0.800	5089
Load Comb 22	1.0D+1.4W45	9600	114	-180	213.063	11.8519	0.29227	-1.0686	0.800	5089
Load Comb 23	1.0D-1.4W45	8200	86	-320	331.355	10.1235	0.45453	-0.8473	0.800	5089
Load Comb 24	1.0D+1.4W135	13800	450	-334	560.407	17.037	0.76873	-1.1533	1.069	6802
Load Comb 25	1.0D-1.4W135	4000	-250	-166	300.093	4.93827	0.41165	0.2685	0.800	5089
								Max	2.475	15745



Derivation of Formulae for Structural Design of Wall

The design formulae of wall are similar to those derived in Column with the typical stress strain relationship shown in Figure F-1.

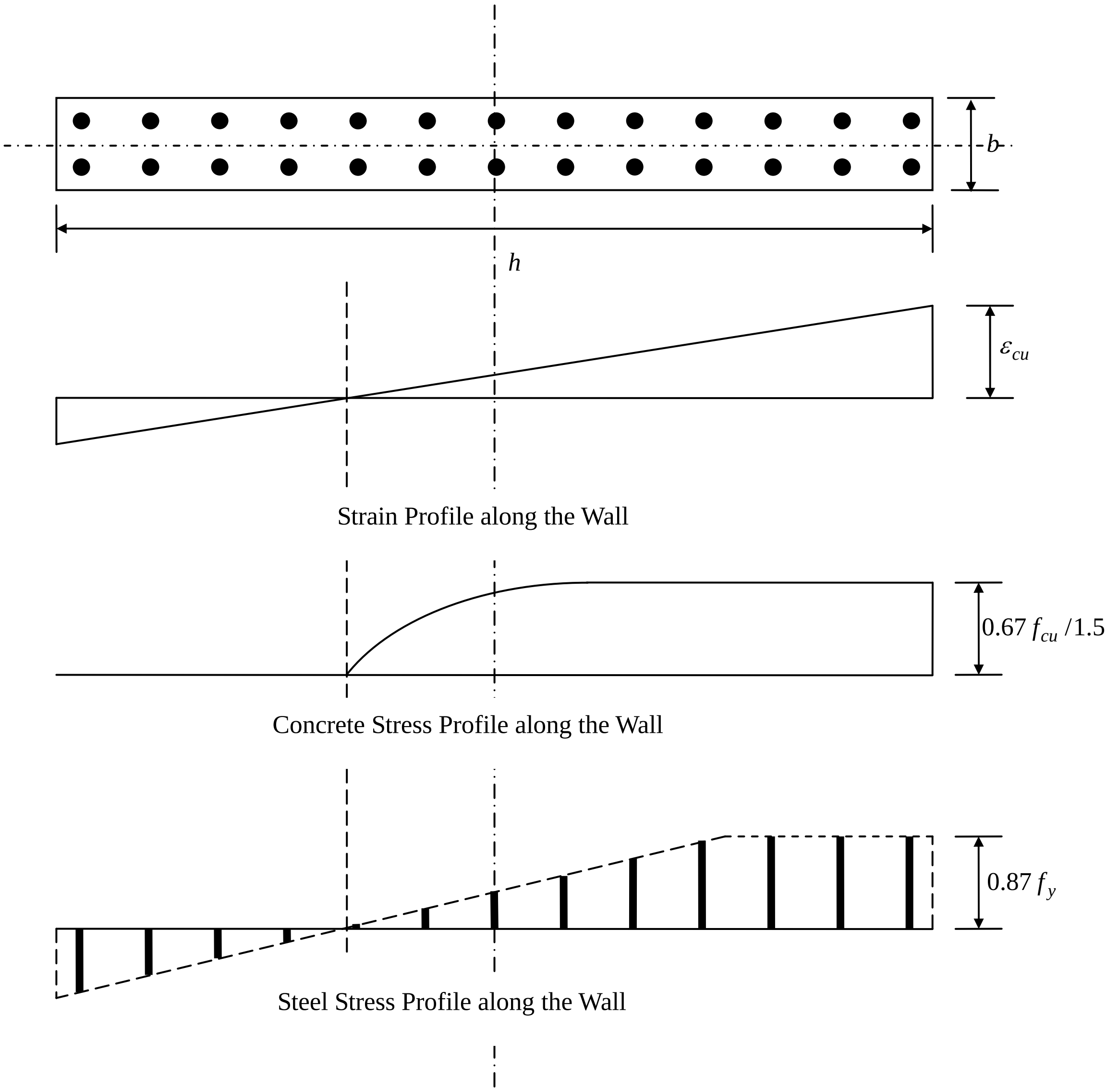
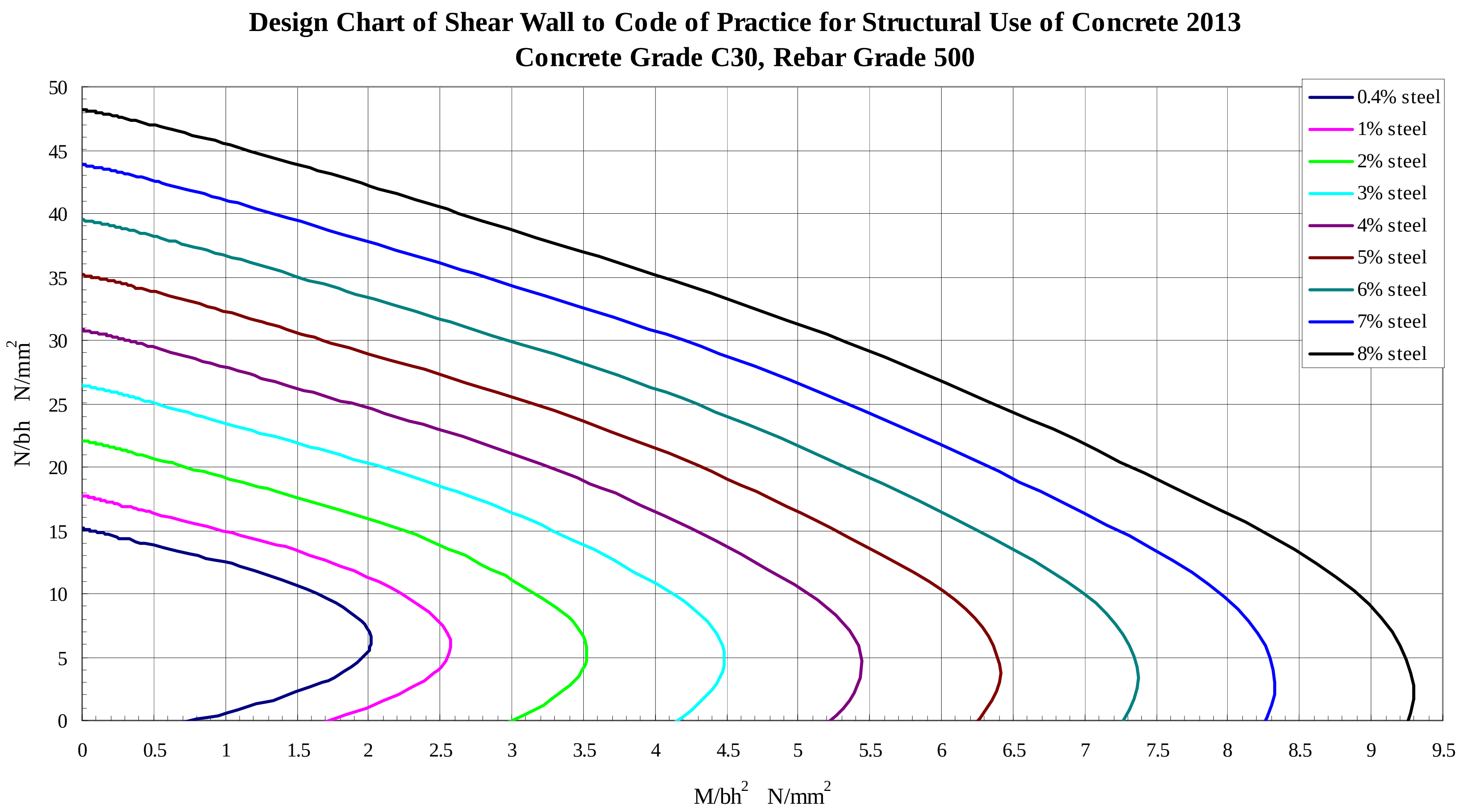


Figure F-1 – Typical Stress / Strain Profile of Wall

The computation of stresses of concrete and concrete are identical to that of column in Appendix E. The design charts produced are therefore based on a number of bars uniformly distributed along the length of the wall. For convenience and to attain reasonable number, distribution of 30 nos. of bars at each side of the wall is chosen and the design charts are produced. Unlike column, the ratio of the "effective length" to the actual length of the column is no longer applicable. Thus the charts depend on concrete grades only.


Design Chart of Shear Wall to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C30, Rebar Grade 500
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
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10
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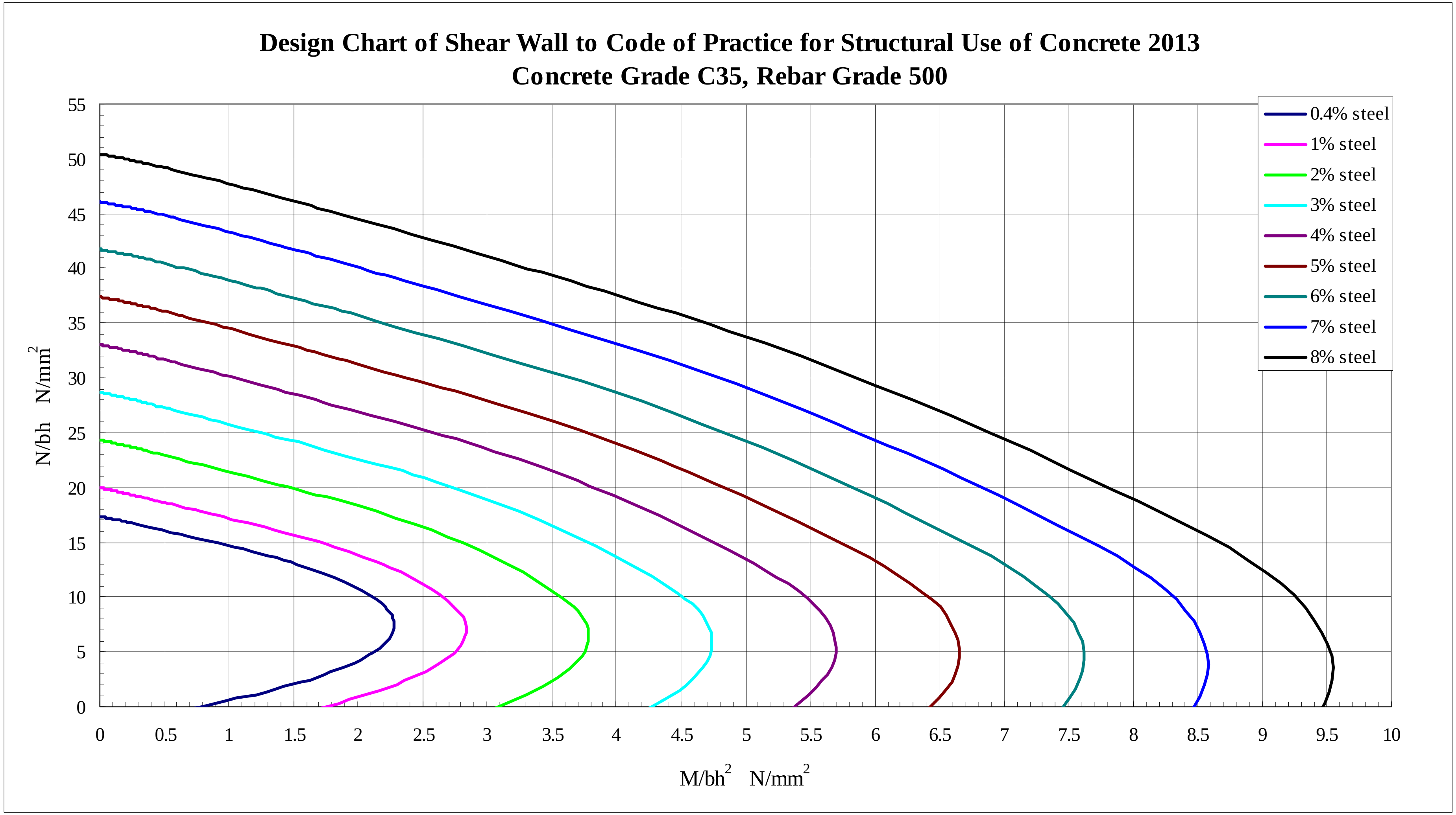
30
35
40
45
50


Design Chart of Shear Wall to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C35, Rebar Grade 500
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
0.5
1
1.5
2
2.5
3
3.5
4
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7
7.5
8
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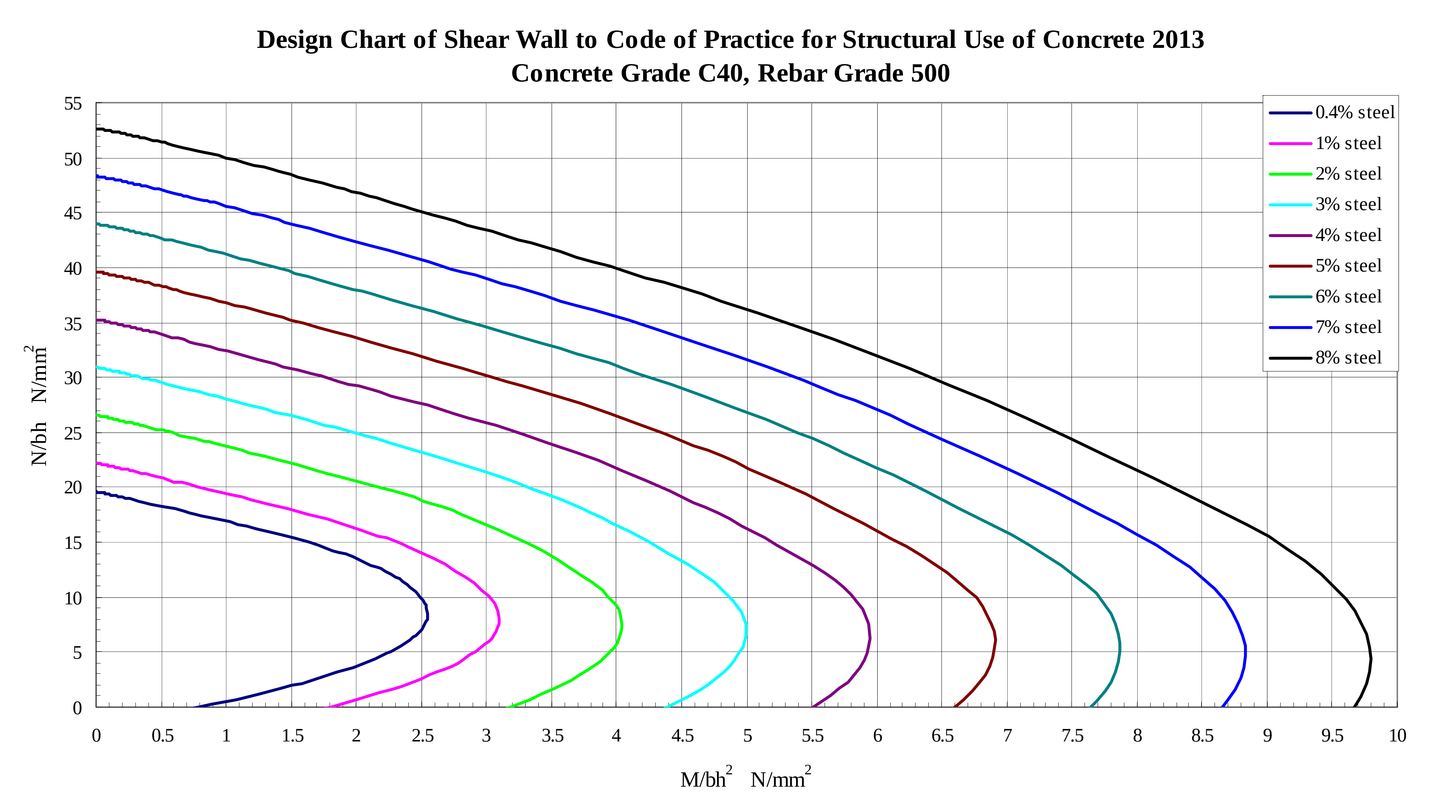
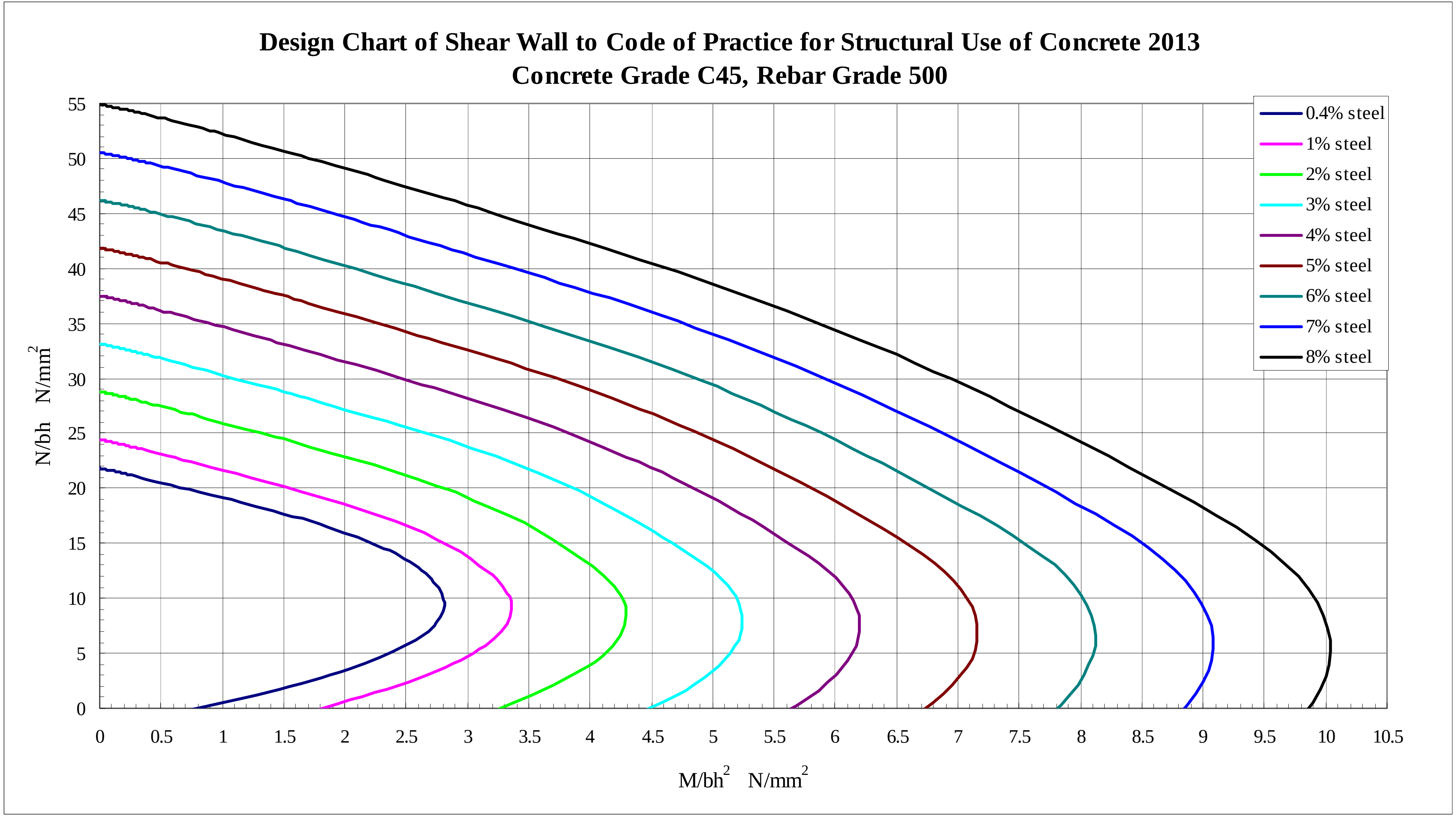
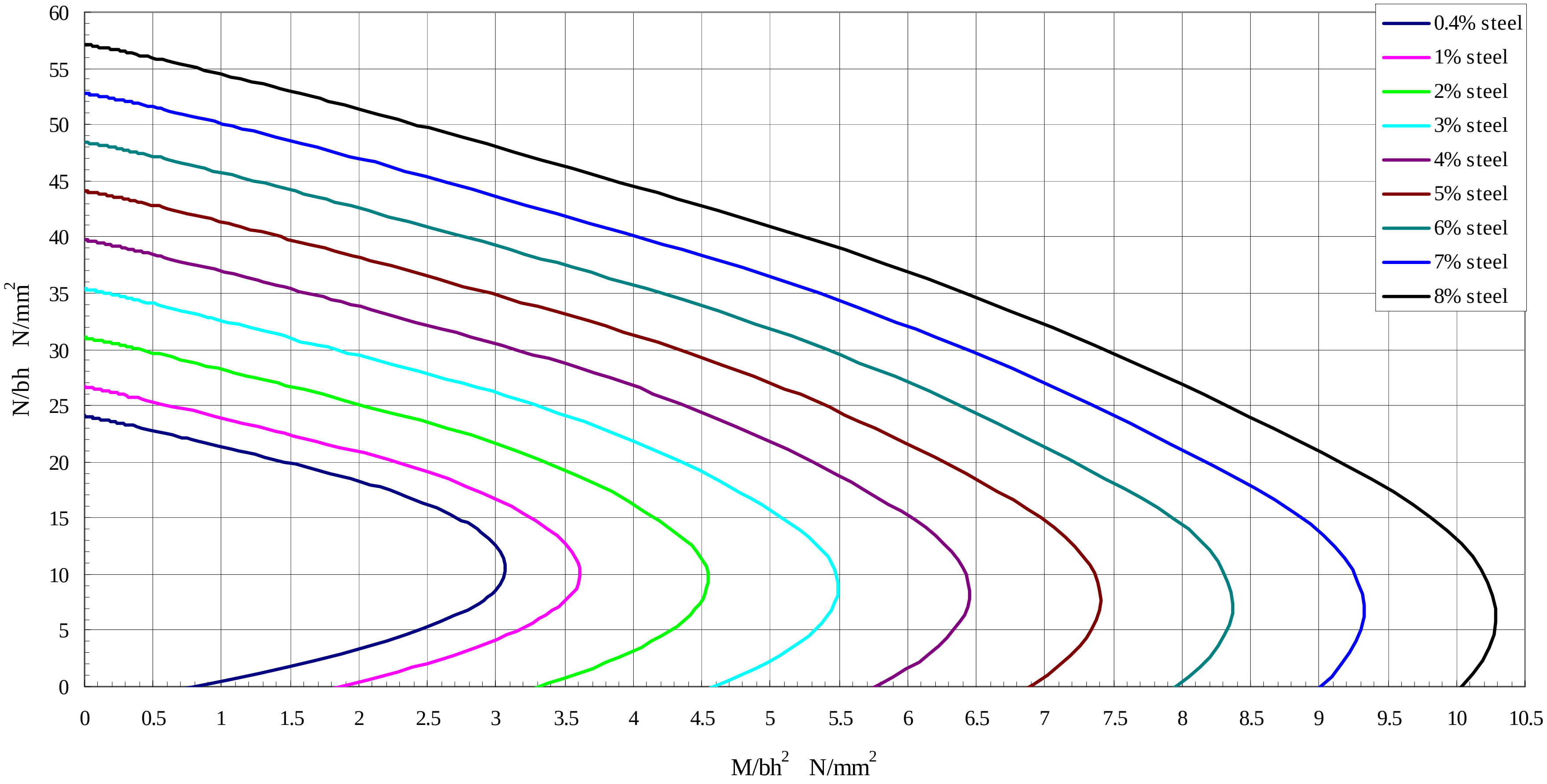
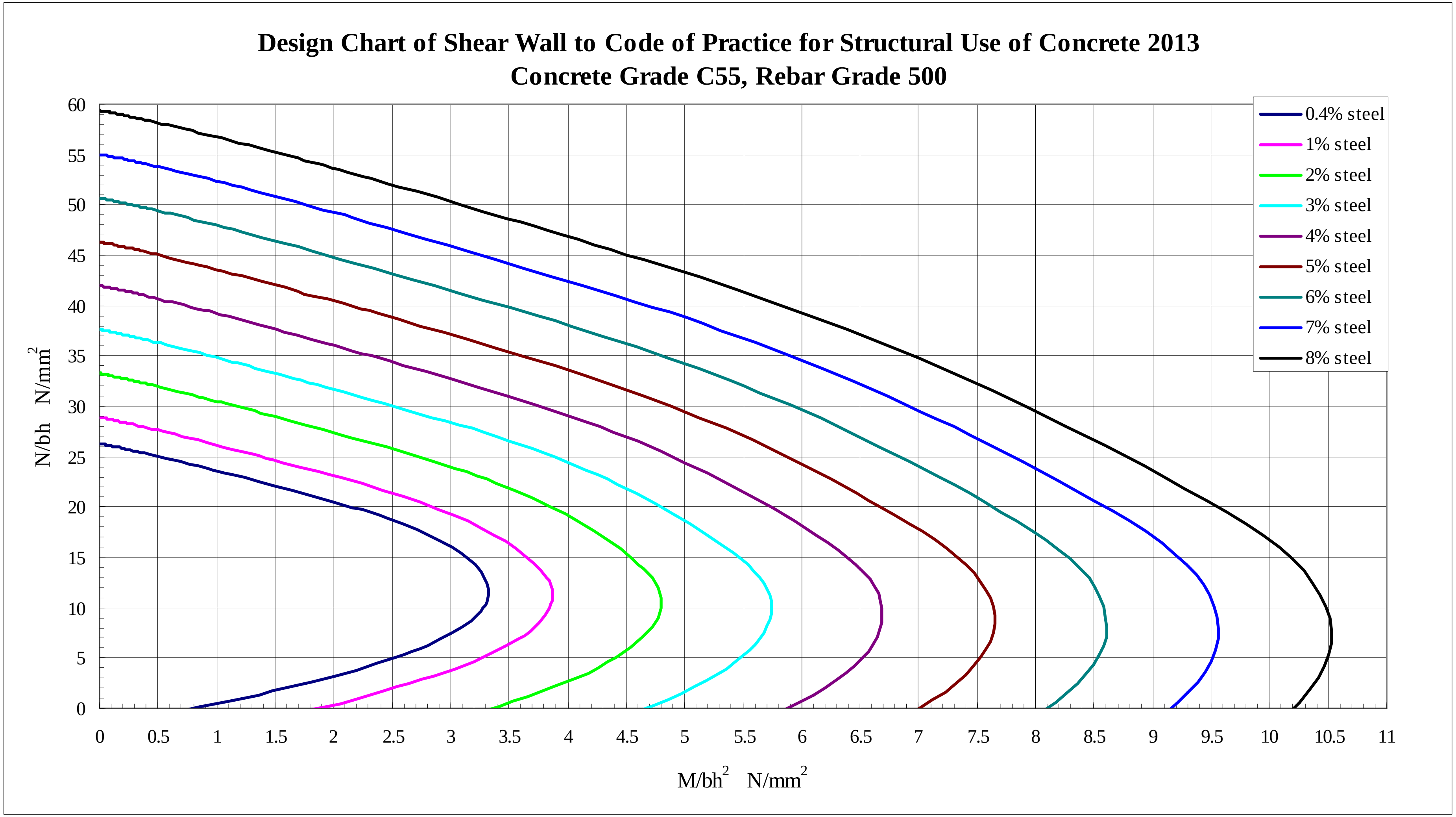
9
9.5
10

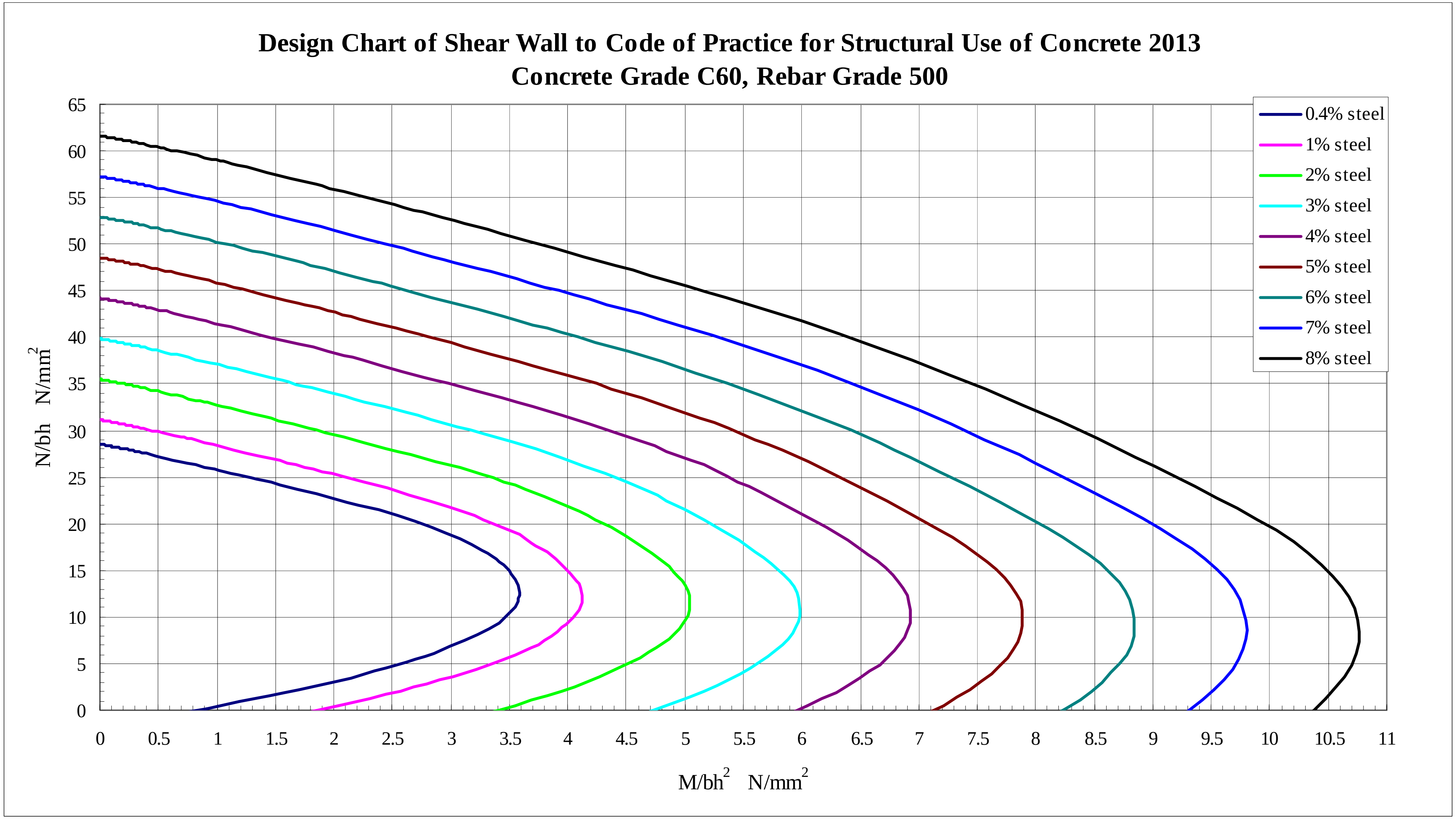

Design Chart of Shear Wall to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C40, Rebar Grade 500
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10


Design Chart of Shear Wall to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C45, Rebar Grade 500
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5

Design Chart of Shear Wall to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C50, Rebar Grade 500



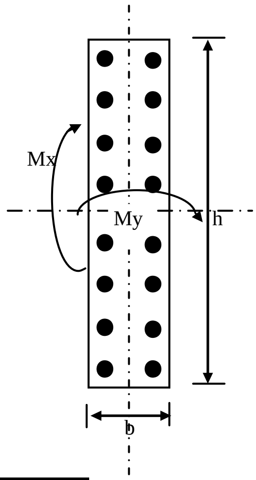

Design Chart of Shear Wall to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C55, Rebar Grade 500
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
60
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11


Design Chart of Shear Wall to Code of Practice for Structural Use of Concrete 2013
Concrete Grade C60, Rebar Grade 500
N/bh N/mm2
M/bh2 N/mm2
0 5 10 15 20 25 30 35 40 45 50 55 60 65
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10 10.5 11
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel

Wall Designed to CoPConc2013 - Predetermined Rebars

f_{cu} = 45 MPa f_y = 500 MPa E_c = 22161 MPa E_s = 200000 MPa

b = 300 mm h = 3000 mm Cover to main rebar = 45 mm



Rebar along each h face (include corner bar) : 21 T 32 Bar spacing = 143.9 mm

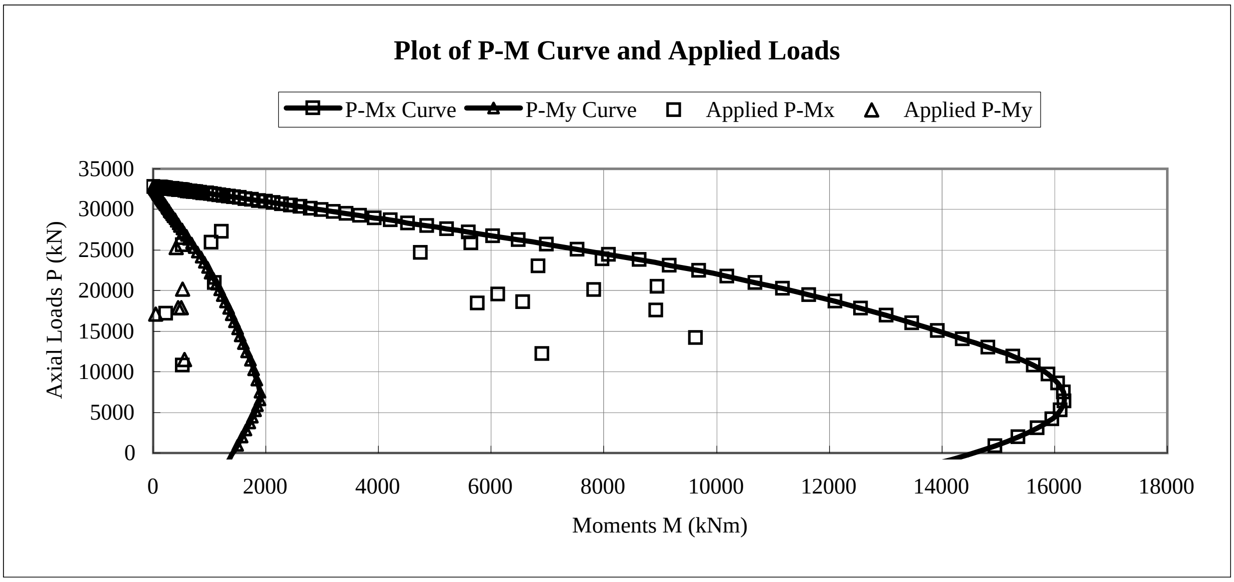
b' = 239.00 mm h' = 2250.00 mm Steel Percentage = 3.7532 %

Axial Load Capacity = 32784 kN

Basic Load Case

Load Case No.	1	2	3	4	5	6
Load Case	D.L.	L.L.	Wx	Wy	W45	W135
Axial Load P (kN)	15872	2101	-3628.1	-2611.1	-3092.3	1209.2
Moment M_x (kNm)	-291.3	-37.11	470.81	-3700	-1750.3	4892.9
Moment M_y (kNm)	-31.33	16.09	5.17	190	280	-342

		N (kN)	M_x (kNm)	M_y (kNm)			d_{nx} or d_{ny} (mm)		$M_{resistance}$ (kNm)	Pass / Fail
Load Comb 1	1.4D+1.6L	25582	-467.2	-18.118	Mx' =	518.37	3063.2	Mux =	7065.5	Pass
Load Comb 2	1.2(D+L+Wx)	17213	170.88	-12.084	Mx' =	228.04	2198.4	Mux =	12876	Pass
Load Comb 3	1.2(D+L-Wx)	25921	-959.06	-24.492	Mx' =	1028.2	3108	Mux =	6758.1	Pass
Load Comb 4	1.2(D+L+Wy)	18434	-4834.1	209.71	Mx' =	5760.7	2305.2	Mux =	12227	Pass
Load Comb 5	1.2(D+L-Wy)	24700	4045.9	-246.29	Mx' =	4741.5	2955.7	Mux =	7860.1	Pass
Load Comb 6	1.2(D+L+W45)	17856	-2494.5	317.71	My' =	446.23	225.2	Muy =	1334.2	Pass
Load Comb 7	1.2(D+L-W45)	25278	1706.3	-354.29	My' =	408.66	307.22	Muy =	732.06	Pass
Load Comb 8	1.2(D+L+W135)	23018	5477.4	-428.69	Mx' =	6841.4	2763.6	Mux =	9246.5	Pass
Load Comb 9	1.2(D+L-W135)	20116	-6265.5	392.11	Mx' =	7829.4	2462.2	Mux =	11251	Pass
Load Comb 10	1.4(D+Wx)	17141	251.31	-36.624	My' =	50.09	218.54	Muy =	1380.7	Pass
Load Comb 11	1.4(D-Wx)	27299	-1067	-51.1	Mx' =	1211.3	3321.1	Mux =	5490.9	Pass
Load Comb 12	1.4(D+Wy)	18565	-5587.8	222.14	Mx' =	6561.9	2317	Mux =	12155	Pass
Load Comb 13	1.4(D-Wy)	25876	4772.2	-309.86	Mx' =	5647.3	3101.9	Mux =	6799.1	Pass
Load Comb 14	1.4(D+W45)	17891	-2858.3	348.14	My' =	495.11	225.53	Muy =	1331.9	Pass
Load Comb 15	1.4(D-W45)	26549	2042.6	-435.86	My' =	500.95	326.24	Muy =	606.67	Pass
Load Comb 16	1.4(D+W135)	23913	6442.2	-522.66	Mx' =	7974.8	2864	Mux =	8534.9	Pass
Load Comb 17	1.4(D-W135)	20527	-7257.9	434.94	Mx' =	8943.9	2502.9	Mux =	10990	Pass
Load Comb 18	1.0D+1.4Wx	10792	367.83	-24.092	Mx' =	524.38	1725.3	Mux =	15628	Pass
Load Comb 19	1.0D-1.4Wx	20951	-950.43	-38.568	Mx' =	1095.4	2545.5	Mux =	10713	Pass
Load Comb 20	1.0D+1.4Wy	12216	-5471.3	234.67	Mx' =	6903	1822.1	Mux =	15146	Pass
Load Comb 21	1.0D-1.4Wy	19527	4888.7	-297.33	Mx' =	6119.3	2405.8	Mux =	11607	Pass
Load Comb 22	1.0D+1.4W45	11542	-2741.8	360.67	My' =	555.22	173.53	Muy =	1714.4	Pass
Load Comb 23	1.0D-1.4W45	20201	2159.2	-423.33	My' =	519.96	248.56	Muy =	1170.7	Pass
Load Comb 24	1.0D+1.4W135	17564	6558.7	-510.13	Mx' =	8926.1	2228.5	Mux =	12694	Pass
Load Comb 25	1.0D-1.4W135	14179	-7141.3	447.47	Mx' =	9626.3	1958.7	Mux =	14315	Pass



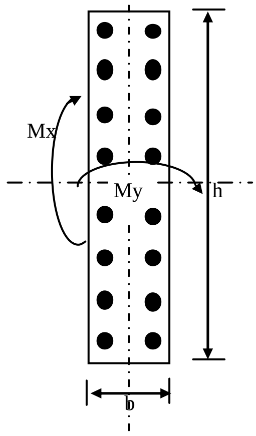
Wall Designed to CoPConc2013 - Minimum Rebar Percentage

f_{cu} =	45	MPa	f_y =	500	MPa	E_c =	22161	MPa	E_s =	200000 MPa
b =	300	mm	h =	3000	mm					

Cover to main rebar / b = 0.2033 b' = 239.00 mm h' = 2250.00 mm

No. of bars on one side = 21 Equivalent Bar size = 31.028 mm

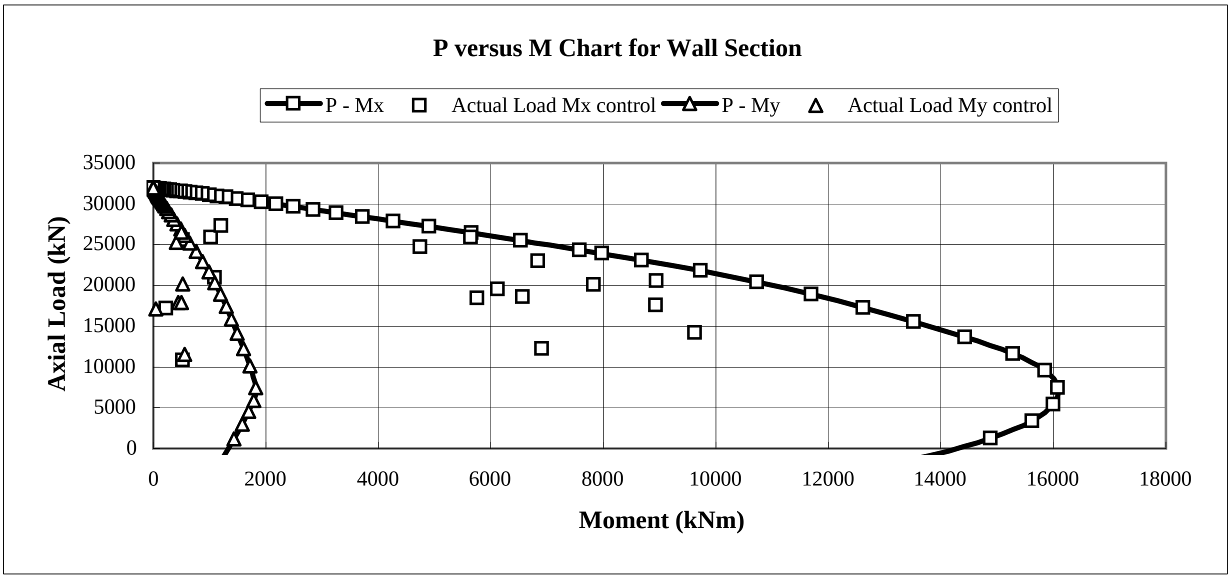
Min. Steel Percentage = 0.40 %



Basic Load Case

Load Case No.	1	2	3	4	5	6
Load Case	D.L.	L.L.	Wx	Wy	W45	W135
Axial Load P (kN)	15872	2101	-3628.1	-2611.1	-3092.3	1209.2
Moment M_x (kNm)	-291.3	-37.11	470.81	-3700	-1750.3	4892.9
Moment M_y (kNm)	-31.33	16.09	5.17	190	280	-342

		N (kN)	M_x (kNm)	M_y (kNm)			N/bd (N/mm^2)	M/bh^2 (N/mm^2)	d_n/h or d_n/b	Min. p (%)	Steel area (mm^2)
Load Comb 1	1.4D+1.6L	25582	-467.2	-18.118	Mx' =	518.37	28.424	0.192	1.795	2.021	18189
Load Comb 2	1.2(D+L+Wx)	17213	170.88	-12.084	Mx' =	228.04	19.126	0.0845	1.4775	0.400	3600
Load Comb 3	1.2(D+L-Wx)	25921	-959.06	-24.492	Mx' =	1028.2	28.801	0.3808	1.5848	2.224	20016
Load Comb 4	1.2(D+L+Wy)	18434	-4834.1	209.71	Mx' =	5760.7	20.482	2.1336	0.9708	1.514	13626
Load Comb 5	1.2(D+L-Wy)	24700	4045.9	-246.29	Mx' =	4741.5	27.445	1.7561	1.117	2.851	25655
Load Comb 6	1.2(D+L+W45)	17856	-2494.5	317.71	My' =	446.23	19.84	1.6527	1.0126	1.030	9270
Load Comb 7	1.2(D+L-W45)	25278	1706.3	-354.29	My' =	408.66	28.086	1.5136	1.1674	2.897	26073
Load Comb 8	1.2(D+L+W135)	23018	5477.4	-428.69	Mx' =	6841.4	25.576	2.5338	1.0047	2.985	26868
Load Comb 9	1.2(D+L-W135)	20116	-6265.5	392.11	Mx' =	7829.4	22.351	2.8998	0.9286	2.537	22833
Load Comb 10	1.4(D+Wx)	17141	251.31	-36.624	My' =	50.09	19.045	0.1855	1.3704	0.400	3600
Load Comb 11	1.4(D-Wx)	27299	-1067	-51.1	Mx' =	1211.3	30.333	0.4486	1.5761	2.617	23557
Load Comb 12	1.4(D+Wy)	18565	-5587.8	222.14	Mx' =	6561.9	20.627	2.4303	0.9439	1.776	15986
Load Comb 13	1.4(D-Wy)	25876	4772.2	-309.86	Mx' =	5647.3	28.751	2.0916	1.0919	3.391	30519
Load Comb 14	1.4(D+W45)	17891	-2858.3	348.14	My' =	495.11	19.879	1.8337	0.9939	1.168	10513
Load Comb 15	1.4(D-W45)	26549	2042.6	-435.86	My' =	500.95	29.499	1.8554	1.1365	3.473	31259
Load Comb 16	1.4(D+W135)	23913	6442.2	-522.66	Mx' =	7974.8	26.57	2.9536	0.9821	3.529	31758
Load Comb 17	1.4(D-W135)	20527	-7257.9	434.94	Mx' =	8943.9	22.808	3.3126	0.9014	2.982	26837
Load Comb 18	1.0D+1.4Wx	10792	367.83	-24.092	Mx' =	524.38	11.991	0.1942	1.0876	0.400	3600
Load Comb 19	1.0D-1.4Wx	20951	-950.43	-38.568	Mx' =	1095.4	23.279	0.4057	1.3877	0.973	8761
Load Comb 20	1.0D+1.4Wy	12216	-5471.3	234.67	Mx' =	6903	13.573	2.5567	0.762	0.456	4107
Load Comb 21	1.0D-1.4Wy	19527	4888.7	-297.33	Mx' =	6119.3	21.697	2.2664	0.9765	1.893	17039
Load Comb 22	1.0D+1.4W45	11542	-2741.8	360.67	My' =	555.22	12.825	2.0564	0.7942	0.400	3600
Load Comb 23	1.0D-1.4W45	20201	2159.2	-423.33	My' =	519.96	22.445	1.9258	1.0234	1.847	16623
Load Comb 24	1.0D+1.4W135	17564	6558.7	-510.13	Mx' =	8926.1	19.516	3.306	0.845	2.288	20592
Load Comb 25	1.0D-1.4W135	14179	-7141.3	447.47	Mx' =	9626.3	15.754	3.5653	0.7417	1.839	16547
									Max =	3.529	31758



Design of Shear Wall against In-plane Shear

Two approaches are formulated in this Appendix. The first one is an analytical approach based on Law & Su and the other is based on modification of the codified approach of ACI 318-11.

G.1 Analytical Approach

The analytical is based on the paper by Law & Su titled "An Analytical Approach for Design of Reinforced Concrete Shear Walls against Lateral In-plane Shear and Comparison with Codified Methods". The formulation of the approach is based on Clark (1976) and Subedi (1975) which is a "holistic" one to determine the horizontal and vertical reinforcement ratios (ρ_x and ρ_y) so that the concrete section is adequate to resist the vertical stress σ_y and shear stress τ_{xy} at individual points. Denoting concrete compression and tensile strengths as f_{cc} and f_{ct}, the determination of ρ_x and ρ_y is as shown in Table G-1 which is extracted from the paper by Law & Su.

Case No.	ρ_x	ρ_y	Constraints
1	0	0	$\left(\frac{\tau_{xy}}{f_{cc}}\right)^2 \le \left(\frac{f_{ct}}{f_{cc}}\right)^2 + \frac{f_{ct}}{f_{cc}}\left(\frac{\sigma_y}{f_{cc}}\right)$; $\left(\frac{\tau_{xy}}{f_{cc}}\right)^2 \le 1 - \left(\frac{\sigma_y}{f_{cc}}\right)$
2	0	$\frac{1}{f_{sc}}\left[\sigma_y - f_c + \frac{\tau_{xy}^2}{f_{cc}}\right]$	$\sqrt{\frac{f_{ct}}{f_{cc}}} \ge \frac{\tau_{xy}}{f_c} \ge 0$; $\frac{\sigma_y}{f_{cc}} \ge 1 - \left(\frac{\tau_{xy}}{f_{cc}}\right)^2$
3	$\frac{1}{2f_{st}}\left[(f_{cc} - f_{ct}) - \alpha\right]$	$\frac{1}{f_{sc}}\left[\sigma_y - \frac{1}{2}\left[(f_{cc} - f_{ct}) + \alpha\right]\right]$	$\left(\frac{\tau_{xy}}{f_{cc}}\right)^2 \ge -\left(\frac{\sigma_y}{f_{cc}}\right)^2 + \left(\frac{\sigma_y}{f_{cc}}\right)\left(1 - \frac{f_{ct}}{f_{cc}}\right) + \frac{f_{ct}}{f_{cc}}$; $\sqrt{\frac{f_{ct}}{f_{cc}}} \le \frac{\tau_{xy}}{f_{cc}} \le \frac{1}{2}\left(1 + \frac{f_{ct}}{f_{cc}}\right)$
4	$\frac{1}{f_{st}\sigma_y}\left[\frac{\tau_{xy}^2}{\sigma_y + f_{ct}} - \sigma_y f_{ct}\right]$	0	$\left(\frac{\tau_{xy}}{f_{cc}}\right)^2 \le -\left(\frac{\sigma_y}{f_{cc}}\right)^2 + \frac{\sigma_y}{f_{cc}}\left(1 - \frac{f_{ct}}{f_{cc}}\right) + \frac{f_{ct}}{f_{cc}}$; $\left(\frac{\tau_{xy}}{f_{cc}}\right)^2 \ge \left(\frac{f_{ct}}{f_{cc}}\right)^2 + \frac{f_{ct}}{f_{cc}}\left(\frac{\sigma_y}{f_{cc}}\right)$; $\frac{\tau_{xy}}{f_{cc}} \le \frac{\sigma_y}{f_{cc}} + \frac{f_{ct}}{f_{cc}}$
5	$\frac{\lvert\tau_{xy}\rvert - f_{ct}}{f_{st}}$	$\frac{1}{f_{st}}\left(-\sigma_y + \lvert\tau_{xy}\rvert - f_{ct}\right)$	$\frac{\tau_{xy}}{f_{cc}} \ge \frac{1}{2}\left(1 + \frac{f_{ct}}{f_{cc}}\right)$; $\frac{\tau_{xy}}{f_{cc}} \ge \frac{\sigma_y}{f_{cc}} + \frac{f_{ct}}{f_{cc}}$;
6	0	$\frac{1}{f_{st}}\left(-\sigma_y + \frac{\tau_{xy}^2}{f_{ct}} - f_{ct}\right)$	$\left(\frac{\tau_{xy}}{f_{cc}}\right)^2 \ge \left(\frac{f_{ct}}{f_{cc}}\right)^2 + \frac{f_{ct}}{f_{cc}}\left(\frac{\sigma_y}{f_{cc}}\right)$; $\frac{\tau_{xy}}{f_{cc}} \le \frac{f_{ct}}{f_{cc}}$

where $\alpha = \sqrt{(f_{cc} + f_{ct})^2 - 4\tau_{xy}^2}$

Table G-1 – Summary of Reinforcement Ratios and Constraints for Various Cases

Graphically, the various cases can be represented on charts such as Figures G-2 and G-3 under ratios of $f_{ct}/f_{cc} = 0.1$ and 0.05 respectively. In the two figures, Area I is applicable to Case 1, Area II to Case 2 and so on. Though there are altogether 6 cases to be considered, they can be easily be programmed on spreadsheets for quick solutions, as have been done by the authors.

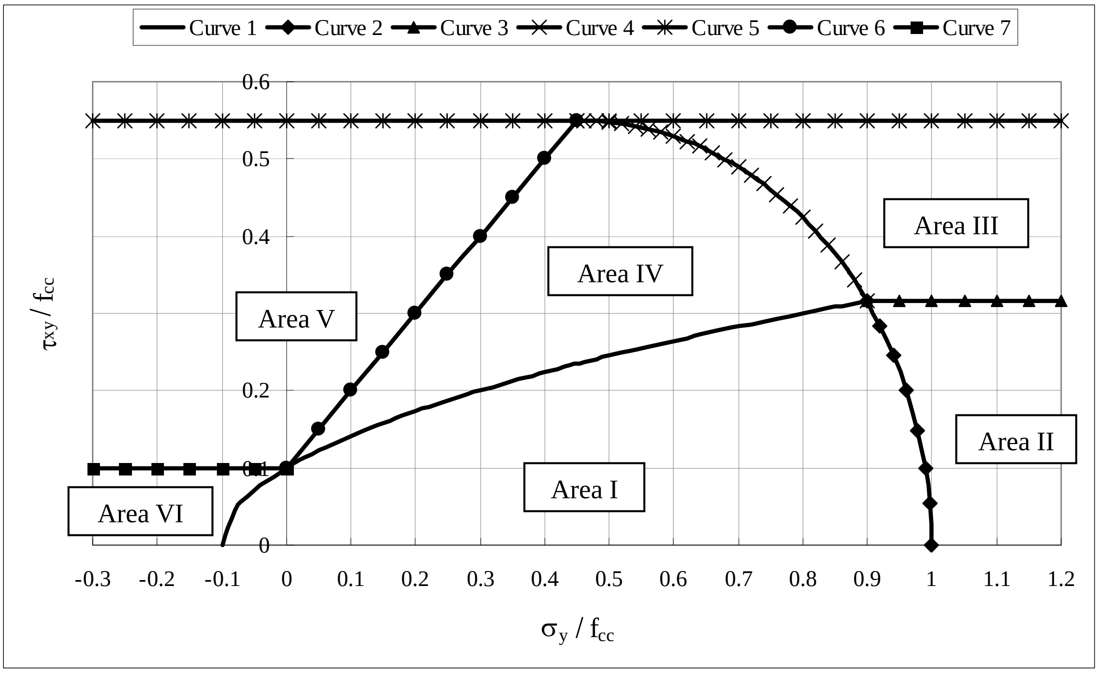


Figure G-2 – Boundary Chart for the Analytical Approach $f_{ct} = 0.1 f_{cc}$

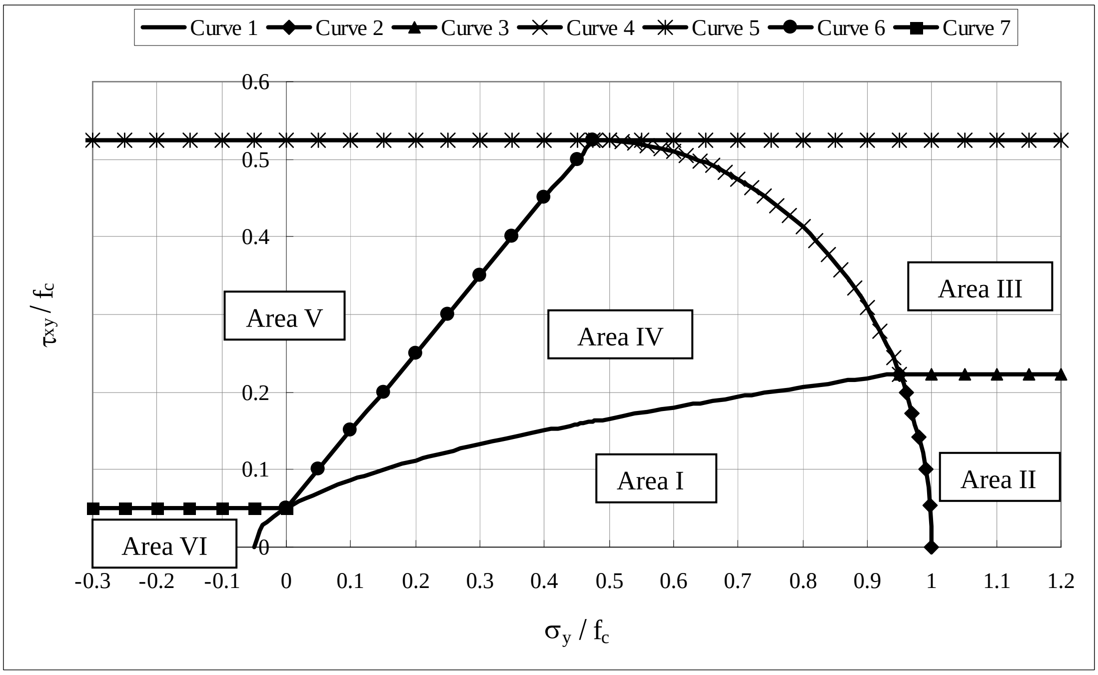


Figure G-3 – Boundary Chart for the Analytical Approach $f_{ct} = 0.05 f_{cc}$

The boundary curves in the charts are also listed as follows :

Curve 1 : $\left(\frac{\tau_{xy}}{f_{cc}}\right)^2 = \left(\frac{f_{ct}}{f_{cc}}\right)^2 + \frac{f_{ct}}{f_{cc}}\left(\frac{\sigma_y}{f_{cc}}\right);$

Curve 2 : $\left(\frac{\tau_{xy}}{f_{cc}}\right)^2 = 1 - \left(\frac{\sigma_y}{f_{cc}}\right)$

Curve 3 : $\frac{\tau_{xy}}{f_{cc}} = \sqrt{\frac{f_{ct}}{f_{cc}}}$

Curve 4 : $\left(\frac{\tau_{xy}}{f_{cc}}\right)^2 = -\left(\frac{\sigma_y}{f_{cc}}\right)^2 + \left(\frac{\sigma_y}{f_{cc}}\right)\left(1 - \frac{f_{ct}}{f_{cc}}\right) + \frac{f_{ct}}{f_{cc}}$

Curve 5 : $\frac{\tau_{xy}}{f_{cc}} = \frac{1}{2}\left(1 + \frac{f_{ct}}{f_{cc}}\right)$

Curve 6 : $\frac{\tau_{xy}}{f_{cc}} = \frac{\sigma_y}{f_{cc}} + \frac{f_{ct}}{f_{cc}}$

Curve 7 : $\frac{\tau_{xy}}{f_{cc}} = \frac{f_{ct}}{f_{cc}}$

As demonstration of the use of the analytical method formulated in the foregoing, 8 numerical Works Examples analyzed by the analytical approach covering various areas in Figures 8 and 9 are worked out by spreadsheets and tabulated in Table H-2 which is also extracted from Law & Su's papr. In the examples, $f_{cc} = 0.45 f_{cu}$, $f_{cu} = 45$ MPa, $f_{ct} = 0.1 f_{cc}$, $f_y = 500$ MPa and $f_{sc} = f_{st} = 0.87 f_y = 435$ MPa.

	Worked Examples							
	G-1	G-2	G-3	G-4	G-5	G-6	G-7	G-8
σ_y (MPa)	-1	7	25	25	10	2	-5	-10
τ_{xy} (MPa)	1	1	5	9	10	10	10	1
Area (Figure G-2 & G-3)	I	I	II	III	IV	V	V	VI
$\sigma_{x(excess)}$ (MPa)	0	0	0	2.552	6.291	7.975	7.975	0
$\sigma_{y(excess)}$ (MPa)	0	0	5.985	9.327	0	5.975	12.975	8.469
ρ_x (%)	0	0	0	0. 59	1.45	1.83	1.83	0
ρ_y (%)	0	0	1.38	2.14	0	1.37	2.98	1.95

Table G-2 – Demonstration by Worked Problems of the Analytical Approach

G.2 Modification of the Codified Approach of ACI 318-11

The next approach formulated in this appendix is a modified version of the codified method in ACI318-11. By ACI 318-11: the shear capacity of a wall contributed by concrete alone should be the lesser of its equations (11-27) and (11-28) reproduced as follows :

$$V_c = 0.27\lambda\sqrt{f_c'}hd + \frac{N_u d}{4\ell_w};\qquad \text{(Eqn G-1)}$$

$$V_c = \left[0.05\lambda\sqrt{f_c'} + \frac{\ell_w\left(0.1\lambda\sqrt{f_c'} + 0.2N_u/(\ell_w h)\right)}{M_u/V_u - \ell_w/2}\right]hd \qquad \text{(Eqn G-2)}$$

where $\lambda = 1.0$ for normal concrete;

f_c' is the cylinder strength of concrete in MPa which is taken as $0.8f_{cu}$;

- N_u is the factored axial force on the wall;
- M_u is the factored moment on the wall;
- V_u is the factored shear on the wall
- ℓ_w is the length of the wall;
- h is the thickness of the wall;
- d is the distance between the extreme compressive fibre to the centre of the steel in tension and can be taken as $0.8\ell_w$ in the absence of compatibility analysis.

By (Eqn G-1), the design tensile strength of concrete is $0.27\times1\times0.8\sqrt{0.8}\times\sqrt{f_{cu}} = 0.19\sqrt{f_{cu}}$ which is 1.04MPa, 1.12MPa, 1.2MPa, 1.27MPa fore grades C30, C35, C40, C45 respectively. They are all less than $0.1f_{cc}$ which is considered reasonable.

The excess shear force $V_u - V_c$ will be resisted by horizontal bars alone by the following equation :

$$\frac{A_{sh}}{s} = \frac{V_u - V_c}{0.87f_y d} \qquad \text{(Eqn G-3)}$$

where A_{sh} is the cross sectional area of the shear reinforcements in form of horizontal bars and s is the spacing of the shear reinforcements.

Take an example of a wall of length $\ell_w = 3$ m, thickness $h = 300$ mm of grade C45 under factored horizontal shear $V_u = 4000$ kN; axial load $N_u = 9000$ kN and moment $M_u = 15000$ kNm, $V_c = 2966.4$ kN by (Eqn G-1) and $V_c = 2462.4$ kN by (Eqn G-2). So V_c is taken as 2462.4kN. The excess shear to be taken up by horizontal reinforcement is $V_u - V_c = 4000 - 2462.4 = 1537.6$ kN. If $f_y = 500$ MPa, $\frac{A_{sh}}{s} = \frac{V_u - V_c}{0.87f_y d} = 1.473$. Assuming T12 bars on both faces, $A_{sh} = 226\,\text{mm}^2$, $s = 153$ mm. So provide horizontal links as T12@150(B.F.)

Estimation of Support Stiffnesses of Vertical Supports to Transfer Structures and other Plate Bending Structures

The discussion in this Appendix is to arrive at more accurate estimate for the support stiffness to a 2-D mathematical model for plate bending structures which we often use for analysis and design of transfer structures and other plate bending structures such as pile caps. For support stiffness, we are referring to the force or moment required to produce unit vertical movement or unit rotation at a support of the transfer structure which are denoted by K_Z , $K_{\theta X}$, $K_{\theta Y}$ for settlement stiffness along the Z direction, and rotational stiffnesses about X and Y directions. These stiffnesses are independent parameters which can interact only through the plate structure. Most softwares allow the user either to input numerical values or structural sizes and heights of the support (which are usually walls or columns) by which the softwares can calculate numerical values for the support stiffnesses as follows :

H.1 Vertical Support Stiffness

For the settlement stiffness K_Z , the value is simply AE/L where A is the cross sectional of the support which is either a column or a wall, E is the Young's Modulus of the construction material and L is the free length of the column / wall. The AE/L simply measures the 'elastic shortening' of the column / wall.

Strictly speaking, the expression AE/L is only correct if the column / wall is one storey high and restrained completely from settlement at the bottom. However, if the column / wall is more than one storey high, the settlement stiffness estimation can be very complicated. It will not even be a constant value. The settlement of the support is, in fact, 'interacting' with that of others through the structural frame linking them together by transferring the axial loads in the column / wall to others through shears in the linking beams. Nevertheless, if the linking beams (usually floor beams) in the structural frame are comparatively 'flexible', the transfer of loads from one column / wall through the linking beams to the rest of the frame will generally be negligible. By ignoring such transfer, the settlement stiffness of a column / wall can be obtained by 'compounding' the settlement stiffness of the individual settlement stiffness at each floor as

$$K_Z = \frac{1}{\dfrac{L_1}{A_1E_1} + \dfrac{L_2}{A_2E_2} + \dfrac{L_3}{A_3E_3} + ...\dfrac{L_n}{A_nE_n}} = \frac{1}{\sum \dfrac{L_i}{A_iE_i}} \qquad \text{(Eqn H-1)}$$

H.2 Rotational Stiffness

For the rotational stiffness, most of the existing softwares calculate the numerical values either by $\frac{4EI}{L}$ or $\frac{3EI}{L}$, depending on whether the far end of the supporting column / wall is assumed fixed or pinned (where I is the second moment of area of the column / wall section). However, apart from the assumed fixity at the far end, the formulae $\frac{4EI}{L}$ or $\frac{3EI}{L}$ are also based on the assumption that both ends of the column / wall are restrained from lateral movement (sidesway). It is obvious that the assumption will not be valid if the out-of-plane load or the structural layout is unsymmetrical where the plate will have lateral movement even under gravity load. The errors may be significant if the structure is to simulate a transfer plate under wind load which is in the form of an out-of-plane moment tending to overturn the structure. Nevertheless, the errors can be minimized by finding the force that will be required to restrain the slab structure from sidesway and applying a force of the same magnitude but of opposite direction to nullify this force. This magnitude of this restraining force or nullifying force is the sum of the total shears produced in the supporting walls / columns due to the moments induced on the walls / columns from the plate analysis. However, the analysis to find the effects on the plate by the "nullifying force" has to be done on a plane frame or a space frame structure as the 2-D plate bending model cannot cater for lateral in-plane loads. This approach is adopted by some local engineers and the procedure for analysis is illustrated in Figure H-1.

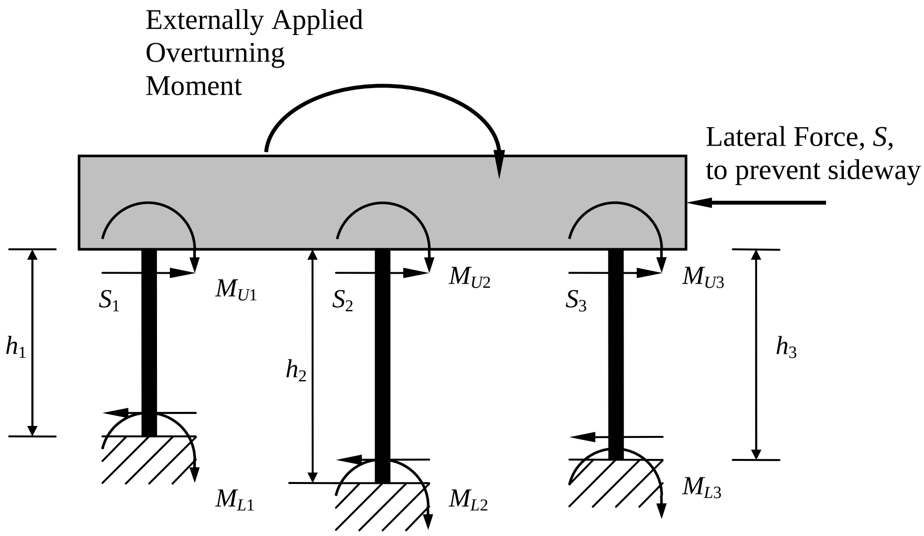


Figure H-1 – Diagrammatic Illustration of the Restraining Shear

In addition, the followings should be noted :

Note : 1. If the wall / column is prismatic and the lower end is restrained from rotation, the moment at the lower end will be $M_{Li} = 0.5M_{Ui}$ (carry-over from the top); if the lower end is assumed pinned, the moment at it will be zero;

2. The shear on the wall / column will be $S_i = \frac{M_{Ui} + M_{Li}}{h_i}$ where M_{Ui} is obtained from plate bending analysis and the total restraining shear is $S = \sum S_i$

To "nullify" the restraining shear which does not really exist, a "nullifying shear" should be applied as shown in Figure H-2 and the internal forces created in the structures (including the plate structure and the supporting column) should be added back to the original model for design. The mathematical model for this action is either by a 3-D mathematical model (normally too rough for normal design) and 2-D plane frame ones.

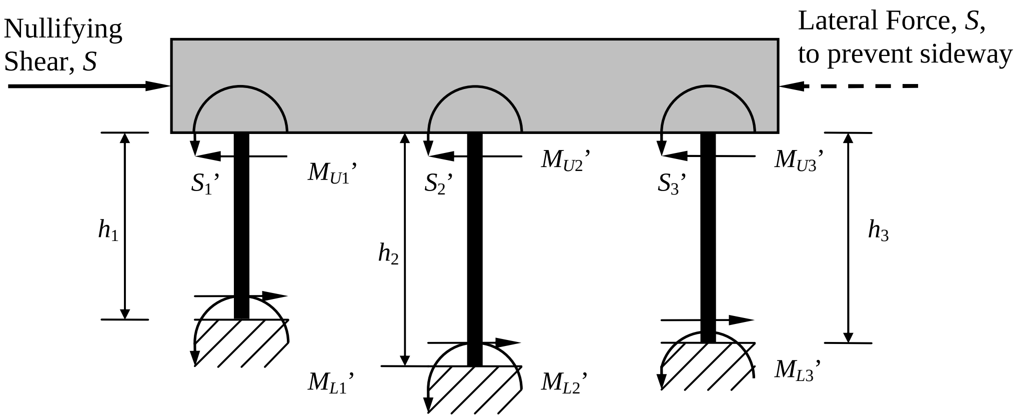


Figure H-2 – Diagrammatic Illustration of the Application of the Nullifying Shear

Nevertheless, if the out-of-plane effects are to be examined by the original 2-D plate bending mathematical model, the moments created at the upper end of the column in the 3-D or 2-D plane frame models should be added to the original 2-D plate bending model with the support rotational stiffnesses removed for analysis. The reason of removing the support rotational stiffnesses is to avoid redistribution of moments among the supports and the plate structures. The internal forces in the plate thus created can be added to the original plate bending structure for design.

The analysis can be done in one single attempt if a 3-D mathematical model (fine enough for the design of the transfer plate) is used in analysis which automatically cater for the sidesway effects.

A fuller discussion can be found in Cheng & Law (2005).

Derivation of Formulae for Rigid Cap Analysis

I.1 Underlying Principles of the Rigid Cap Analysis

The “Rigid Cap Analysis” method utilizes the assumption of “Rigid Cap” in the solution of pile loads beneath a single cap against out-of-plane loads, i.e. the cap is a perfectly rigid body which does not deform upon the application of loads. The cap itself may settle, translate or rotate, but as a rigid body. The deflections of a connecting pile will therefore be well defined by the movement of the cap and the location of the pile beneath the cap, taking into consideration of the connection conditions of the piles.

I.2 Derivation of the General Formulae

Consider a Pile i situated in a coordinate system with point O as the origin on the pile cap as shown in Figure I-1 with settlement stiffness K_{iZ}. By settlement stiffness, we are referring to the force required to produce unit settlement at the pile head. For a pile with cross sectional area A and Young’s modulus E and length L, K_{iZ} may be set to AE/L if it is end-bearing on very stiff stratum and the soil restraint along the pile shaft is ignored. Nevertheless, more sophisticated formulation can be adopted to take soil restraint along pile shaft and settlement at pile tip into account. In fact, the analytical approach so described in this Appendix is called the “Winkler spring” approach without consideration of the so called “pile interaction” through the soil medium. Though considered deficit in the underlying principles, the approach without this consideration of “pile interaction” is still widely accepted. Discussion of the approach with pile interactions is beyond the scope of this Manual.

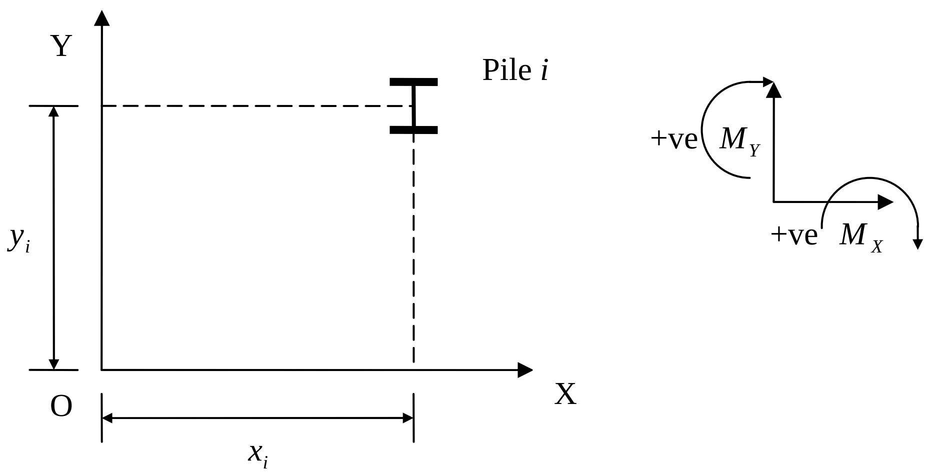


Figure I-1 – Derivation of Pile Loads under Rigid Cap

As the settlement of all piles beneath the Cap will lie in the same plane after the application of the out-of-plane load, the settlement of Pile i denoted by Δ_{iZ} can be defined by $b_0 + b_1 x_i + b_2 y_i$ which is the equation for a plane in “co-ordinate geometry” where b_0, b_1 and b_2 are constants.

The upward reaction by Pile i is

$$P_i = K_{iZ}(b_0 + b_1 x_i + b_2 y_i) \quad \text{(Eqn I-1)}$$

Summing all pile loads and balancing the externally applied load P on the whole cap, we can list

$$P = \sum P_i = \sum K_{iZ}(b_0 + b_1 x_i + b_2 y_i)$$

$$\Rightarrow P = b_0 \sum K_{iZ} + b_1 \sum K_{iZ} x_i + b_2 \sum K_{iZ} y_i \quad \text{(Eqn I-2)}$$

Balancing the externally applied Moment M_X and M_Y along X and Y respectively, we can list

$$M_X = \sum P_i x_i = \sum K_{iZ}(b_0 + b_1 x_i + b_2 y_i) x_i$$

$$\Rightarrow M_X = b_0 \sum K_{iZ} x_i + b_1 \sum K_{iZ} x_i^2 + b_2 \sum K_{iZ} x_i y_i \quad \text{(Eqn I-3)}$$

$$M_Y = \sum P_y y_i = \sum K_{iZ}(b_0 + b_1 x_i + b_2 y_i) y_i$$

$$\Rightarrow M_Y = b_0 \sum K_{iZ} y_i + b_1 \sum K_{iZ} x_i y_i + b_2 \sum K_{iZ} y_i^2 \quad \text{(Eqn I-4)}$$

In Matrix form, we can write

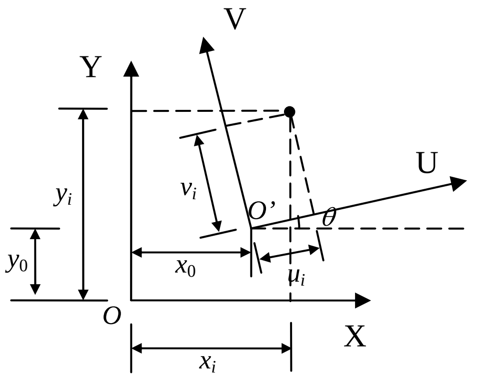
$$\begin{bmatrix} \sum K_{iZ} & \sum K_{iZ} x_i & \sum K_{iZ} y_i \\ \sum K_{iZ} x_i & \sum K_{iZ} x_i^2 & \sum K_{iZ} x_i y_i \\ \sum K_{iZ} y_i & \sum K_{iZ} x_i y_i & \sum K_{iZ} y_i^2 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} P \\ M_X \\ M_Y \end{bmatrix} \quad \text{(Eqn I-5)}$$

(Eqn I-5) can then be used to solve the three unknowns b_0, b_1 and b_2 which can then used to back-calculate the individual pile loads by (Eqn I-1).

I.3 Alternative Approach by Locating the Pile Group Centre

Conventionally, it is considered not convenient to solve matrix or simultaneous equations. So instead there is a practice by which the centre O is re-located to O' and the axes X-X and Y-Y are rotated as shown in Figure I-2 such that the new coordinate axes becomes U-U and V-V. The coordinates in the former system as (x_i, y_i) now become (u_i, v_i). The purpose of this relocation and axes rotation is to make $\sum K_{iZ} u_i = \sum K_{iZ} v_i = \sum K_{iZ} u_i v_i = 0$ so that the three unknowns b_0, b_1 and b_2 in (Eqn I-5) which become b_0', b_1' and b_2' can then be readily solved. Physically, the relocated centre O' is the "Pile Group Centre". It represent the point at which the resultant load acting on the pile group will create equal settlements on all piles. The rotated axes U-U and V-V are the "principal axes" of the pile group at which piles with equal distance from U-U will have equal settlements when moment is acting along V-V alone and similarly for piles with equal distance from V-V will have equal settlements when moment is acting along U-U alone. For the conversion of the coordinate system as shown in Figure I-2, the "translation" of the centre O to O' by x_0

and y_0 and the rotation θ needs be determined as shown in the derivations that follow.



$$u_i = (x_i - x_0)\cos\theta + (y_i - y_0)\sin\theta$$
$$v_i = -(x_i - x_0)\sin\theta + (y_i - y_0)\cos\theta$$

Figure I-2 – Transformation of Coordinate Axes

By $\sum K_{iZ} u_i = 0$

$$\sum K_{iZ}\left[(x_i - x_0)\cos\theta + (y_i - y_0)\sin\theta\right] = 0 \qquad \text{(Eqn I-6)}$$

By $\sum K_{iZ} v_i = 0$

$$\sum K_{iZ}\left[-(x_i - x_0)\sin\theta + (y_i - y_0)\cos\theta\right] = 0 \qquad \text{(Eqn I-7)}$$

(Eqn I-6)$\times\cos\theta$ + (Eqn I-7)$\times\sin\theta$

$$\Rightarrow \sum K_{iZ}(x_i - x_0) = 0$$

$$\Rightarrow x_0 = \frac{\sum K_{iZ} x_i}{\sum K_{iZ}} \qquad \text{(Eqn I-8)}$$

Similarly by (Eqn I-6)$\times\sin\theta$ + (Eqn I-7)$\times\cos\theta$

$$\Rightarrow y_0 = \frac{\sum K_{iZ} y_i}{\sum K_{iZ}} \qquad \text{(Eqn I-9)}$$

By $\sum K_{iZ} u_i v_i = 0$

$$\sum K_{iZ}\left[(x_i - x_0)\cos\theta + (y_i - y_0)\sin\theta\right]\left[-(x_i - x_0)\sin\theta + (y_i - y_0)\cos\theta\right] = 0$$

$$\Rightarrow \sum K_{iZ}\left[\left[-(x_i - x_0)^2 + (y_i - y_0)^2\right]\sin\theta\cos\theta + (x_i - x_0)(y_i - y_0)\left(\cos^2\theta - \sin^2\theta\right)\right] = 0$$

$$\Rightarrow \sum K_{iZ}\left[\left[-(x_i - x_0)^2 + (y_i - y_0)^2\right]\frac{1}{2}\sin 2\theta + (x_i - x_0)(y_i - y_0)\cos 2\theta\right] = 0$$

$$\Rightarrow \tan 2\theta = \frac{2\sum K_{iZ}(x_i - x_0)(y_i - y_0)}{\sum K_{iZ}(x_i - x_0)^2 - \sum K_{iZ}(y_i - y_0)^2}$$

$$\Rightarrow \theta = \frac{1}{2}\tan^{-1}\frac{2\sum K_{iZ}(x_i - x_0)(y_i - y_0)}{\sum K_{iZ}(x_i - x_0)^2 - \sum K_{iZ}(y_i - y_0)^2} \qquad \text{(Eqn I-10)}$$

With determination of x_0, y_0 and θ by (Eqn I-8) to (Eqn I-10), the new coordinate system U-V can be defined and the coordinates of all the piles can be converted by the equations shown in Figure I-2. So (Eqn I-5) can be re-written as

$$\begin{bmatrix} \sum K_{iZ} & 0 & 0 \\ 0 & \sum K_{iZ} u_i^2 & 0 \\ 0 & 0 & \sum K_{iZ} v_i^2 \end{bmatrix} \begin{bmatrix} b_0' \\ b_1' \\ b_2' \end{bmatrix} = \begin{bmatrix} P \\ M_U \\ M_V \end{bmatrix} \qquad \text{(Eqn I-11)}$$

where b_0', b_1' and b_2' now refer to the new system U-V and M_U, M_V also be the moments about U-V as converted from M_X, M_Y.

By (Eqn I-11)

$$b_0' = \frac{P}{\sum K_{iZ}}; \qquad b_1' = \frac{M_U}{\sum K_{iZ} u_i^2}; \qquad b_2' = \frac{M_V}{\sum K_{iZ} v_i^2} \qquad \text{(Eqn I-12)}$$

Back-substituting into (Eqn I-1)

$$P_i = \frac{P}{\sum K_{iZ}} K_{iZ} + \frac{M_U u_i}{\sum K_{iZ} u_i^2} K_{iZ} + \frac{M_V v_i}{\sum K_{iZ} v_i^2} K_{iZ} \qquad \text{(Eqn I-13)}$$

If all piles (N nos.) are identical, i.e. K_{iZ} is a constant, (Eqn I-13) can be reduced to

$$P_i = \frac{P}{N} + \frac{M_U u_i}{\sum u_i^2} + \frac{M_V v_i}{\sum v_i^2} \qquad \text{(Eqn I-14)}$$

Transverse Reinforcements in Lap Zones

The requirements of adding transverse reinforcements in lap zones of bars are stipulated in 8.7.4 of the Code which are identical to EC2 of the same clause no. By the clause, transverse reinforcements located between the bars and the surface of the concrete are required to be provided at the lap zones of main reinforcements of diameter not less than 20mm. The provision serves to reduce cracking or "splitting" of the concrete cover and therefore increases bond strength of the main bars as splitting of the concrete cover is a cause of bond failure. In fact, the provision also improves ductility as failure of splices without transverse reinforcement is sudden and complete while those with transverse reinforcements tend to exhibit a less brittle failure and also possess residual strength beyond the maximum load (Beeby and Narayanan 1995). The followings may be helpful in understanding the background of the requirement :

(i) It is well established that a deformed bar bonded in concrete under tension or compression will create "bond force" which "radiates" out into the surrounding concrete from the bonding surface. This force is originated from the component of the bearing force on the concrete by the ribs of the deformed bar in the radiating direction perpendicular to the bar. The bond force will in turn generate tensile stress in the circumferential direction which may split the concrete as illustrated in Figures J-1 and Figure J-2.

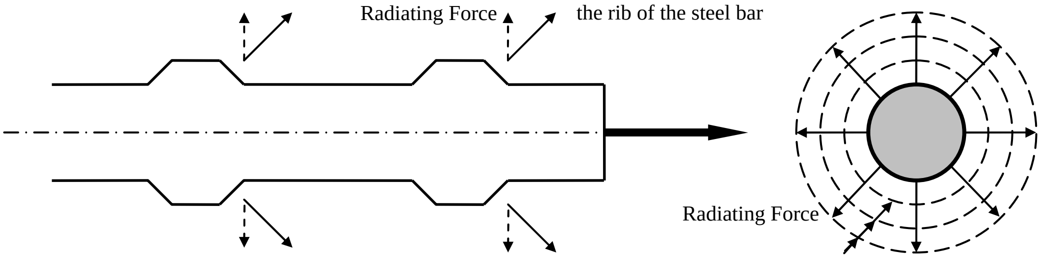
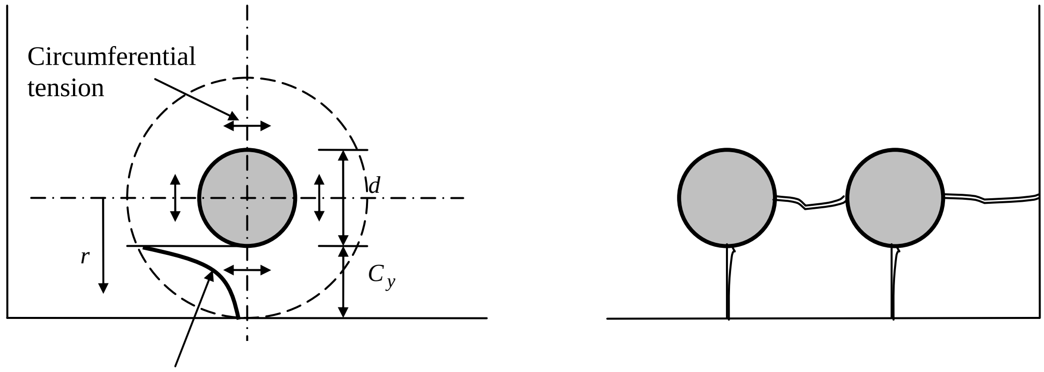


Figure J-1 – Illustration of Creation of Circumferential Tension in Concrete by an Embedded Bar under Tension or Compression

Tepfers (1979) and Kemp (1986) had put forward equations for calculation of the tensile stress by regarding the radial bond stress as hydraulic pressure on the inner surface of a thick-walled concrete cylinder surrounding the steel bar in accordance with the work of Timoshenko (1956). The equation for the elastic stage is shown as follows:

$$\sigma_t = \frac{(d/2)^2 \tau \tan\alpha}{(C_y + d/2)^2 - (d/2)^2}\left[1 + \frac{(C_y + d/2)^2}{r^2}\right] \qquad \text{(Eqn J-1)}$$

where the symbols are defined in Figure J-2 except α which relates the shear of the bar to the bond force and is taken as 45^o when cover cracking begins. By (Eqn J-1), thinner concrete cover and higher shear stress will incur higher tensile stress that may crack the concrete. In addition, if the tensile stress induced on the concrete exceeds the tensile strength of concrete, tension cracks will take place in the weaker locations such as the concrete cover and the space between 2 bars also illustrated in Figure J-2.



$$\sigma_t = \frac{(d/2)^2 \tau \tan\alpha}{(C_y + d/2)^2 - (d/2)^2}\left[1 + \frac{(C_y + d/2)^2}{r^2}\right]$$

where σ_t is the concrete tensile stress in the circumferential direction and τ is the shear stress by the reinforcement bar on concrete

Tension cracks induced in concrete due to shear by the reinforcement bar on concrete

Figure J-2 – Tensile Stress induced on Concrete due to Bonds by Rebar

Tepfers (1979) had further considered the “partly cracked elastic” stage by which the maximum tensile stress of concrete had been exceeded and concrete cracks within an inner core surrounding the reinforcement bar while the outer core remains in the elastic stage. He considers that the bond force from the reinforcement bar is transferred through the “concrete teeth’ in the cracked inner core to the outer elastic core and put forward a theoretical expression relating the maximum bond stress f_{cbc} that can be sustained by the bar to the tensile strength of concrete as

$$f_{cbc} = \frac{C_y / d + 0.5}{1.664} f_{ct} \qquad \text{(Eqn J-2)}$$

where f_{ct} is the tensile strength of concrete and the radius of the inner core with cracked concrete is

$$e = 0.486(C_y + d/2) \qquad \text{(Eqn J-3)}$$

The expression was later modified by Esfahani M.R. and Rangan B.V. (1998) to account for the plastic deformation of the concrete by regression analysis on experimental data as

$$u_c = 4.9\frac{C/d_b + 0.5}{C/d_b + 3.6} f_{ct} \qquad \text{(Eqn J-4)}$$

In (Eqn J-4) the symbols f_{cbc} has been replaced by u_c as the bond strength of concrete; and C_y and d have been replaced by C and d_b respectively.

(ii) As concrete is weak in tension, transverse reinforcements can be placed to "intercept" the tension cracks and thus act as some provisions against "bond failure" due to concrete splitting (while another failure mode by bond is the pulling out of the bar). So alternatively saying, bond strength of concrete may be increased. In locations of laps where the splitting forces from the bars are additive, cracking will take place more easily. Examples of provisions of the transverse reinforcements in critical locations of laps to intercept the cracking (or potential cracking) are shown in Figure J-3.

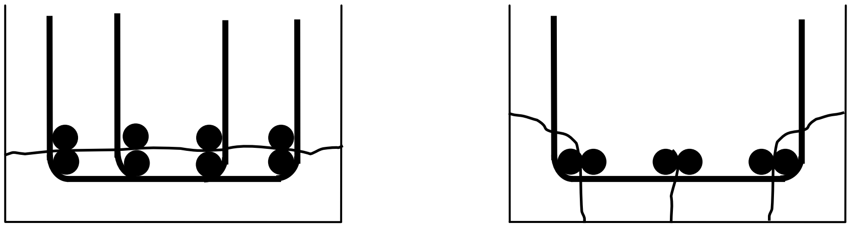


Figure J-3 – Cracks (Potential Cracks) created in Lapped Bars Intercepted by Transverse Reinforcements

(iii) Cl. 8.7.4 of the Code requires transverse reinforcements to be placed at the end thirds of the lap because higher splitting stresses are found at the ends due to the higher bond stresses in the locations. By the work of Tepfers R. (1980) and Giuriani E. (1982) and others, the shear stress (the bond stress) is not uniform along the lap. In Tepfers's "modulus of displacement theory" in which a constant named the K-modulus (the proportionality relating the shear stress of the bar to the "slip" between the bar and concrete) was introduced and by considering equilibrium and compatibility and with the assumption of no through cracks within the lap zone, Tepfers R. (1980) has derived the formulae of shear stress distribution of the two equal bars in a lap under constant bending moment as shown in Figure J-4 as :

$$\tau_1 = -\frac{\sigma_{s0}A_s\kappa_2}{2u}\frac{\cosh\kappa_2 x}{\sinh(\kappa_2 l/2)} + \frac{\sigma_{s0}A_s\kappa_1}{2u(1+2n\rho)}\frac{\sinh\kappa_2 x}{\cosh(\kappa_2 l/2)} \quad \text{(Eqn J-5)}$$

$$\tau_2 = \frac{\sigma_{s0}A_s\kappa_2}{2u}\frac{\cosh\kappa_2 x}{\sinh(\kappa_2 l/2)} + \frac{\sigma_{s0}A_s\kappa_1}{2u(1+2n\rho)}\frac{\sinh\kappa_1 x}{\cosh(\kappa_1 l/2)} \quad \text{(Eqn J-6)}$$

where τ_1, τ_2 are the shear stress of the two bars in the lap at location x;
x is zero at middle of the lap; $l/2$ at the right end $-l/2$ at the left end and l is the length of the lap
σ_{s0} is the axial stress of each of the bar at its end
A_s is the cross sectional area of each of the reinforcement bar
u is the perimeter of each of the reinforcement bar
$\kappa_2 = \frac{uK}{A_s E_s}$ with K as the K-modulus measured by Tepfers (1980) as the secant modulus ranging from 50×10^3 MN/m^3 to 100×10^3 MN/m^3 for 0.2mm slip depending on concrete grade; and E_s is the Young's modulus of steel.
n is ratio of Young's Modulus of steel to concrete

ρ is the steel ratio as illustrated in Figure J-4

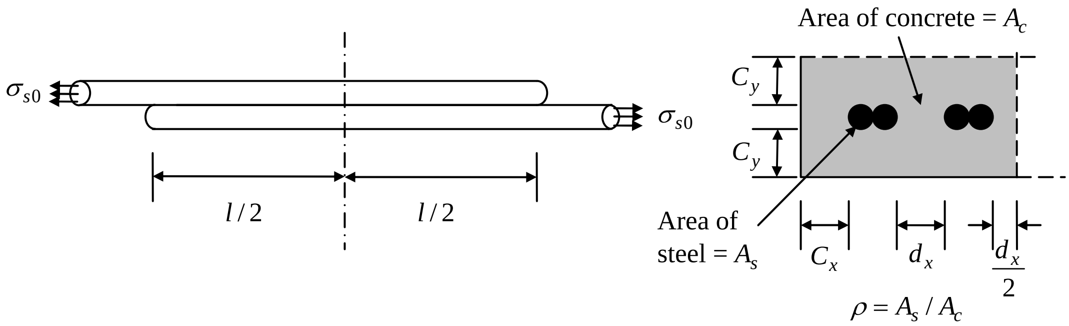


Figure J-4 – Illustration of the Modulus of Displacement Theory

A typical plot for the shear stress of a T40 bar lapping with another T40 bar for a lap length of 2 m by Tepfer's approach is presented in Figure J-5. The bars are stressed to 0.87 times of steel yield stress at the ends of the lap, i.e. $\sigma_{s0} = 435\,\text{N/mm}^2$ and embedded in grade C35 concrete (long term Young's modulus taken as 11.85kN/mm^2) and K is taken as $50 \times 10^3\,\text{MN/m}^3$. It can be seen that the shear stresses are high at the ends of the laps and drop rapidly to low values in the middle for the elastic stage. If the bond stresses of concrete at the ends are capped (i) to the ultimate value imposed by EC2 as 4.65MPa and (ii) after enhancement by 38% to 6.42MPa due to the addition of transverse reinforcement as required by Cl. 8.7.4 of the Code, the stress profiles are re-analyzed as the "plastic stage" and added in Figure J-5. The enhancement of 38% is arrived at by the approach of Esfahani M.R. and Kianoush M.R. (2005) which will be described later.

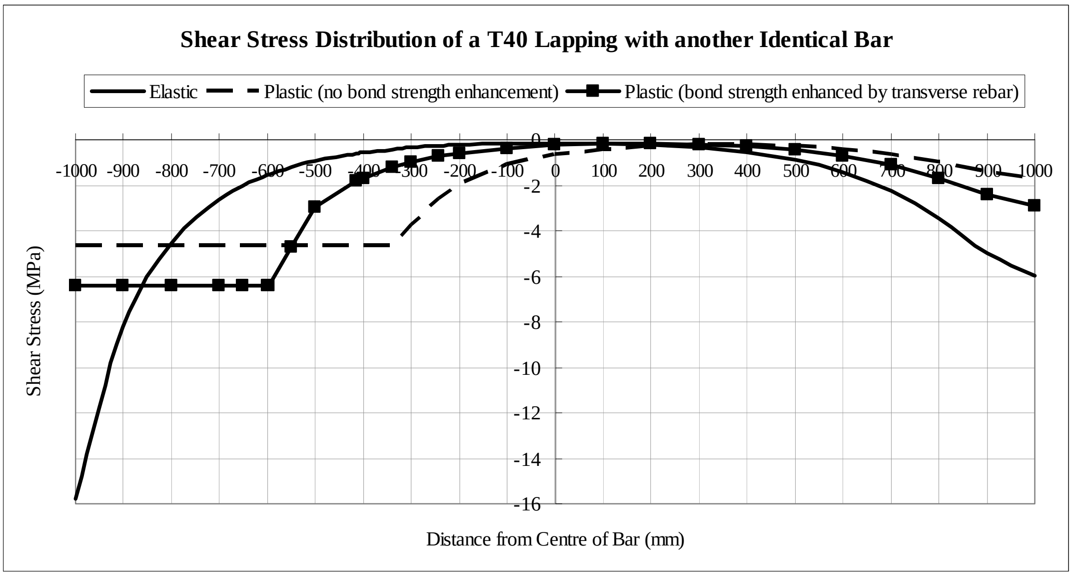


Figure J-5 – Typical Plots of Variation of Shear Stress along Lapped Bars

Bars of other sizes exhibit similar behaviour. So placing transverse reinforcements at the outer zones of the lap is an effective means against concrete splitting due to high bond stresses and thus enhances bond strength.

(iv) The Code has provided simple rules in its Cl. 8.7.4 for provision of transverse reinforcements at laps to reduce cracking of concrete. By the rules, no transverse reinforcement is required when the diameter of the lapped bar is 16mm or less, or the lapped bars in any section are less than 20%. Otherwise transverse reinforcements of total areas not less than the cross sectional area of the spliced bar should be provided in the space between the spliced bar and the concrete surface. It is because for the smaller bars, the minimum reinforcement is considered adequate to cope with the tensile stresses generated at the laps in accordance with Beeby and Narayanan (1995). In addition, the transverse reinforcements are to be positioned at the outer thirds of the lap. The reason for such positioning has been discussed in (iii) above. The Code also requires extra transverse bars outside the lap for compression lap as some end-bearing by the main bars on the concrete may create bursting force outside the lap zone as explained by Kwan (2006).

(v) It may be worthwhile to check the structural adequacy of the Code requirement of transverse bars by the work of Tepfers (1980). It can easily be deduced from Figure J-6 that the tensile force required to maintain equilibrium by the cover is $F_t = d\tau/2$ where τ is the shear per circumferential length of the bar over the entire lap length. If the longitudinal bar is stressed to $0.87 f_y$ so that $\pi\left(d^2/4\right)0.87 f_y = F_t = d\tau/2$ and the minimum cross sectional area of the transverse reinforcement according to the Code is provided which is assumed to take up all the tension force in the cover, it can easily be proven that the factor of safety defined as the load carrying capacity of the transverse bar (also stressed to $0.87 f_y$) divided by F_t will be $\pi = 3.14$. So the provision by the Code is very adequate for the ultimate limit state.

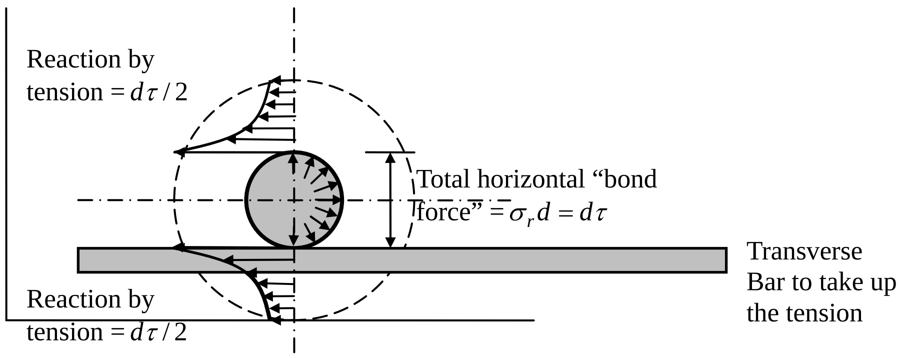


Figure J-6 – Tensile Stress induced on Concrete due to Bonds by Reinforcement

(vi) It should be re-emphasized that transverse reinforcements can only increase bond strength by strengthening concrete against splitting whereas bond failure can still take place by "pulling out". So the provision of transverse reinforcement can only increase the bond carrying capacity to certain extent. In addition, Esfahani M.R. and Kianoush M.R. (2005) pointed out that the stress in the transverse reinforcement depends on the state of cracking of concrete cover. For non-cracked cover, the stress of transverse reinforcement is negligible; thus the presence of transverse reinforcement may not increase the bond strength significantly. After concrete cracks, the force transfer will be larger especially at high C_d / d ratio.

Esfahani M.R. and Kianoush M.R. (2005) have by experiments established empirical formulae to account for the contribution of the transverse reinforcements to the bond strength. They concluded that the bond strength can be increased by a factor of $1+0.015 f_R \dfrac{A_t A_b}{CS}$ if transverse reinforcement of area A_t in mm^2 with spacing S in mm is provided. In the factor, A_b is the area of the single spliced bar in mm^2 and C (in mm) is the minimum of (1) the side cover to the spliced bar; (2) bottom (or top) cover to the spliced bar; and (3) half of the clear spacing with the adjacent bar plus half of diameter of the spliced bar in mm; f_R is the factor to account for the "rib effects" of the reinforcing bar which is 1 for bar with relative rib area ratios $R < 0.11$ and 1.6 for $R \geq 0.11$. So for the T40 bars with transverse reinforcements of total area equal to that of the T40 added in the lap according to the Code, the bond strength can be increased by 38%.

Similarly other codes including EC2, ACI 318-08 and NZS3101-2006 have included the effects of the transverse reinforcements together with concrete cover thickness in the determination of the bond and lap lengths. However, the Code has adopted a simple approach by BS8110 in its Cl. 8.4.4 by relating bond strength to concrete grade and bar type (plain or deformed bar) only. A comparison of the required tension lap lengths for the 4 codes for T12 to T40 bars in concrete grade C35 are plotted in Figure J-7. The concrete covers assumed are 25mm for T12 to T20; 40mm for T25 and 50mm for T32 and T40.

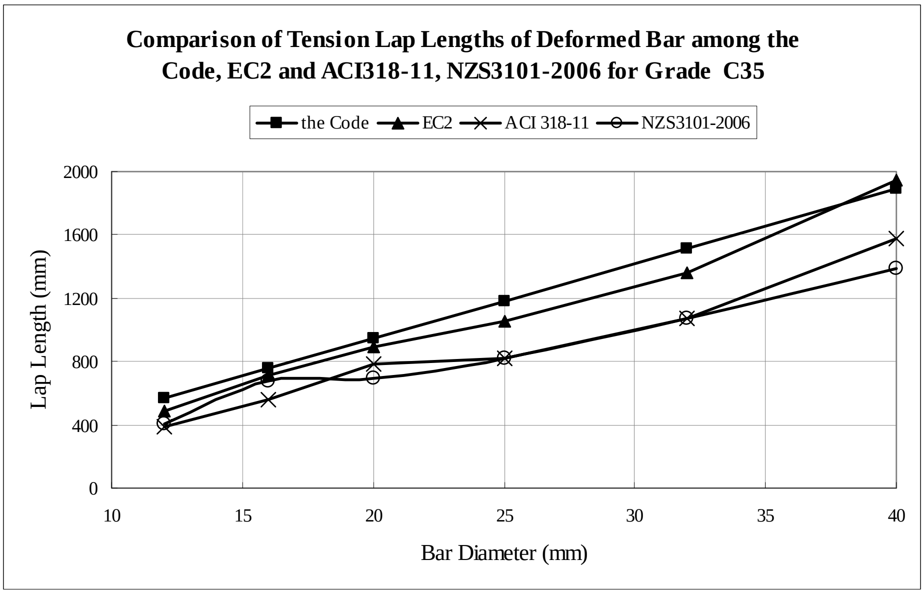


Figure J-7 – Comparison of Lap Lengths among the Code, EC2 and ACI 318-11 in Grade C35 Concrete

In Figure J-7, transverse reinforcements required by the Code are provided where bond strength enhancements applicable to EC2, ACI 318-11 and NZS3101-2006 are adopted. It can be seen that the lap lengths required by the Code are generally the longest whilst that of NZS3101 and ACI318 are the shortest as they allow the enhancement which can be up to 2.25 and 2.5 times the basic bond strength respectively by the aggregate factors due to cover and transverse reinforcements. In addition, NZS3101 allows lap length to be identical to anchorage lengths while other codes require additional factors in the order of 1.3 to 1.4 generally. The sudden increase of lap length for T40 bar by EC2 is due to the reduction of ultimate bond strength for bar size in excess of T32 by EC2 (from 2.9MPa to 2.67MPa).

So it is advisable to take cover and transverse reinforcements into account in the determination of lap and anchorage lengths in the Code which is more realistic. In addition, shorter lap and anchorage lengths can be achieved. As the provision of transverse reinforcement in the Code follows that of EC2, reference should therefore be made to EC2 Cl. 8.4.4 for the determination of bond strength enhancement due to cover and transverse reinforcements and subsequently the reduction of lap and anchorage lengths. The reduction can be effected by the adoption of coefficients of α_2 and α_3 as indicated in Table 8.2 of EC2. By the required provision of transverse reinforcements in the Code, 10% reduction of lap length or anchorage length can generally be achieved.

Simulation of Curves for Shrinkage and Creep Determination – K_j and K_m

For ease of computation, the charts in Figures 3.5 and 3.2 of the Code for K_j and K_m are converted into equations so that their values can be computed in spread sheets.

K.1 Simulation of K_j values

Figure 3.5 of the Code is expanded and intermediate lines are added for reading more accurate values. The intermediate values are scaled off from the expanded figure and listed in Table K-1 as follows (h_e = 50 mm which is seldom used is ignored) :

h_e = 100 mm		h_e = 200 mm		h_e = 400 mm		h_e = 800 mm	
Days	K_j	Days	K_j	Days	K_j	Days	K_j
2	0.09	6	0.09	16.6	0.065	60	0.065
3	0.108	7	0.095	20	0.08	70	0.075
4	0.125	8	0.1	30	0.115	80	0.084
5	0.145	9	0.105	40	0.145	90	0.092
6	0.165	10	0.112	50	0.165	100	0.099
7	0.185	11	0.12	60	0.185	200	0.17
8	0.2	12	0.13	70	0.2	300	0.22
9	0.213	13	0.138	80	0.22	400	0.265
10	0.225	14	0.145	90	0.235	500	0.31
20	0.33	20	0.18	100	0.25	600	0.35
30	0.4	30	0.23	200	0.375	700	0.386
40	0.45	40	0.275	300	0.46	800	0.42
50	0.5	50	0.31	400	0.54	900	0.45
60	0.543	60	0.345	500	0.6	1000	0.48
70	0.57	70	0.37	600	0.64	2000	0.73
80	0.6	80	0.4	700	0.67	3000	0.83
90	0.625	90	0.425	800	0.7	4000	0.888
100	0.645	100	0.445	900	0.72	5000	0.923
200	0.775	200	0.61	1000	0.74	6000	0.95
300	0.827	300	0.7	2000	0.87	7000	0.97
400	0.865	400	0.75	3000	0.935	8000	0.98
500	0.892	500	0.79	4000	0.97		
600	0.91	600	0.81	5000	0.99		
700	0.927	700	0.84				
800	0.937	800	0.855				
900	0.945	900	0.87				
1000	0.955	1000	0.883				
1500	0.975	2000	0.955				

Table K-1 – tabulated Values for K_j

Curves are plotted accordingly in Microsoft Excel as shown in the following Figure K-1 :

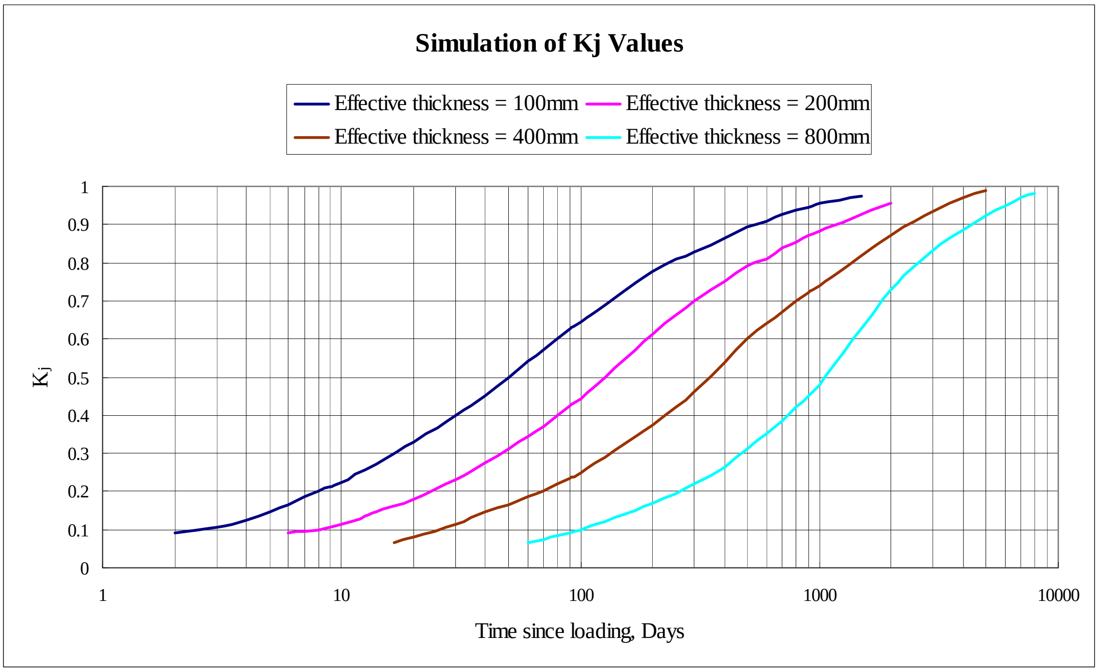


Figure K-1 – Re-plotted Chart for K_j

These curves are divided into parts and polynomial equations (x denote days) are simulated by regression done by the Excel as follows :

(i) Effectiveness thickness $h_e = 100$ mm

for $2 \le x \le 10$

$K_j = -1.5740740764\times10^{-6}x^6 + 7.1089743699\times10^{-5}x^5 - 1.2348646738\times10^{-3}x^4 + 1.0396454943\times10^{-2}x^3 - 4.4218106746\times10^{-2}x^2 + 1.0785366750\times10^{-1}x - 1.4422222154\times10^{-2}$;

for $10 < x \le 100$

$K_j = -8.2638888726\times10^{-12}x^6 + 2.9424679436\times10^{-9}x^5 - 4.1646100361\times10^{-7}x^4 + 2.9995170408\times10^{-5}x^3 - 1.1964688098\times10^{-3}x^2 + 3.0905446162\times10^{-2}x + 9.3000049487\times10^{-3}$

for $100 < x \le 1000$

$K_j = -9.9999999553\times10^{-18}x^6 + 3.7871794729\times10^{-14}x^5 - 5.7487179303\times10^{-11}x^4 + 4.4829720169\times10^{-8}x^3 - 1.9268813492\times10^{-5}x^2 + 4.6787198128\times10^{-3}x + 3.3059999890\times10^{-1}$

(ii) Effectiveness thickness $h_e = 200$ mm

for $1 \le x \le 10$

$K_j = -5.5555555584\times10^{-7}x^6 + 1.9230769236\times10^{-5}x^5 - 2.3632478631\times10^{-4}x^4 + 1.1888111887\times10^{-3}x^3 - 1.8372455154\times10^{-3}x^2 + 5.1966197721\times10^{-3}x + 5.0666667394\times10^{-2}$

for $10 < x \le 100$

$K_j = -6.0905886799\times10^{-12}x^6 + 2.0287559012\times10^{-9}x^5 -$

$2.6706836340\times10^{-7}x^4 + 1.7840233064E\times10^{-5}x^3 - 6.6454331705\times10^{-4}x^2 + 1.7736234727\times10^{-2}x - 1.3696178365\times10^{-2}$

for $100 < x \le 1000$

$K_j = -4.1666665317\times10^{-19}x^6 + 4.6185897038\times10^{-15}x^5 - 1.2899038408\times10^{-11}x^4 + 1.6179152071\times10^{-8}x^3 - 1.0631842073\times10^{-5}x^2 + 3.8848713316\times10^{-3}x + 1.4793333214\times10^{-1}$

(iii) Effectiveness thickness $h_e = 400$ mm

for $1 \le x \le 16.6$

$K_j = 1.4187214466\times10^{-6}x^4 - 3.5464080361\times10^{-5}x^3 + 3.3384218737\times10^{-4}x^2 - 2.2688256448\times10^{-5}x + 2.7836053347\times10^{-2}$

for $16.6 < x \le 100$

$K_j = -1.5740740764\times10^{-6}x^6 + 7.1089743699\times10^{-5}x^5 - 1.2348646738\times10^{-3}x^4 + 1.0396454943\times10^{-6}x^3 - 4.4218106746\times10^{-2}x^2 + 1.0785366750\times10^{-1}x - 1.4422222154\times10^{-2}$

for $100 < x \le 1000$

$K_j = -9.3749999678\times10^{-18}x^6 + 3.1193910157\times10^{-4}x^5 - 4.0436698591\times10^{-11}x^4 + 2.6279902314\times10^{-8}x^3 - 9.8112164735\times10^{-6}x^2 + 2.8475810022\times10^{-3}x + 4.1166665811\times10^{-2}$

for $1000 < x \le 5000$

$K_j = -8.3333333334\times10^{-16}x^4 + 1.4166666667\times10^{-11}x^3 - 9.6666666667\times10^{-8}x^2 + 3.3333333333\times10^{-4}x + 4.9000000000\times10^{-1}$

(iv) Effectiveness thickness $h_e = 800$ mm

for $3 \le x \le 60$

$K_j = 9.5889348301\times10^{-12}x^5 - 1.5604725262\times10^{-8}x^4 + 1.8715280898\times10^{-6}x^3 - 7.5635030550\times10^{-5}x^2 + 1.8805930655\times10^{-3}x + 1.4981311831\times10^{-2}$

for $60 < x \le 100$

$K_j = -5.4210108624\times10^{-20}x^4 + 1.3010426070\times10^{-17}x^3 - 5.0000000012\times10^{-6}x^2 + 1.6500000000\times10^{-3}x - 1.6000000000\times10^{-2}$

for $100 < x \le 1000$

$K_j = -3.9583333158\times10^{-18}x^6 + 1.4818910202\times10^{-14}x^5 - 2.1967147366\times10^{-11}x^4 + 1.6383442558\times10^{-8}x^3 - 6.5899851301\times10^{-6}x^2 + 1.8249511657\times10^{-3}x - 3.1900000544\times10^{-2}$

K.2 Simulation of K_m values

Values of Figure 3.2 of the Code for Ordinary Portland Cement are read, Excel chart is plotted and polynomial equations are simulated as :

for $1 \le x \le 7$

$K_m = 8.3333333333\times10^{-3}x^2 - 1.3333333333\times10^{-1}x + 1.925$

for $7 < x \le 28$

$K_m = 7.3129251701\times10^{-4}x^2 - 4.4642857143\times10^{-2}x + 1.6766666667$

for $28 < x \le 90$

$K_m = 3.8967199783 \times 10^{-5} x^2 - 8.6303876389 \times 10^{-3} x + 1.2111005693$

for $90 < x \le 360$

$K_m = 2.3662551440 \times 10^{-6} x^2 - 1.9722222222 \times 10^{-3} x + 9.0833333333 \times 10^{-1}$

The chart for K_m is plotted as follows in Figure K-2

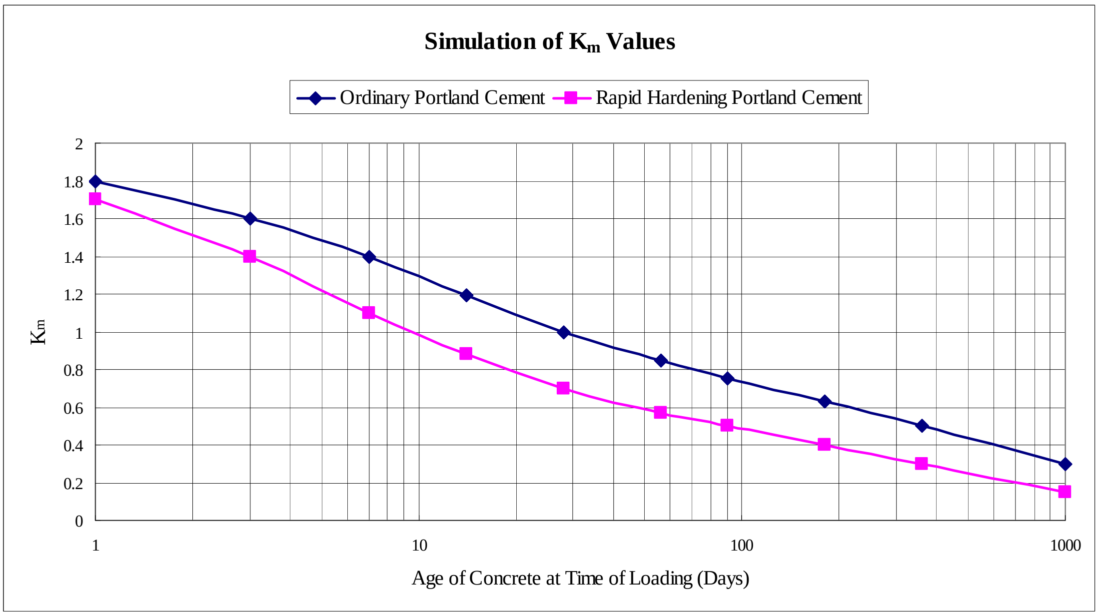


Figure K-2 – Re-plotted Chart for K_m