University of Molise, Valahia University of Targoviste, University of Split

- Physics (lecture: 7 credits, laboratory: 0 credits)
  - Mechanics (2 credits)
  - Thermodynamics (1 credit)
  - Electromagnetism (2 credits)
  - Light and Optics (1 credit)
  - Modern Physics (1 credit)
- Literatures:
  - R.A. Serway, J.S. Faughn, *College Physics,* Fifth Edition, Saunders College Publishing, 2000
  - D. Haliday, R. Resnick, J. Walker, *Fundamentals of Physics,* Sixth Edition, John Wiley & Sons, 2001
  - M. Dželalija, http://www.pmfst.hr/~mile/physics
- Professor:
  - Prof. Mile Dželalija, University of Split, Croatia
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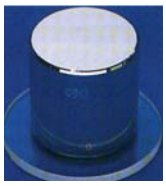
Split, 2001.

- Physics
  - is concerned with the basic principles of the Universe
  - is one of the foundations on which the other sciences are based
  - is tipical experimental science
  - The beauty of physics lies in the simplicity of its fundamental theories
  - The theories are usually expressed in mathematical form
- Mechanics
  - is the first part of this lecture
  - Sometimes referred to as classical mechanics or Newtonian mechanics
  - is concerned with the effects of forces on material objects
  - The first serious attempts to develop a theory of motion were made by Greek astronomers and philosophers
  - A major development in the theory was provided by Isac Newton in 1687 when he published his Principia
  - Today, mechanics is of vital inportance to students from all disciplines

The International System of Units

- To report the result of a measurement of a certain physical quantity, a unit for the quantity must be defined.
- In 1960 an international committee agreed on a system of standards, called SI system.
- The SI units of length, mass, and time are the meter (m), kilogram (kg), and second (s)
  - The meter as the SI unit of length
    - 1799, defined as 1/10000000 of the distance from the Equator to the North Pole
    - 1983, redefined as the distance traveled by light in vacuum during a time interval of 1/299792458 second (this establishes that the speed of light is 299792458 m/s)
  - The kilogram as the SI unit of mass
    - definied as the mass of a specific platinum-iridium alloy cylinder



  - The second as the SI unit of time
    - Before 1960, the second was defined as 1/86400 of average length of solar day in the year 1990.
    - The second is now defined as 9192631700 times the period of oscilation of radiation from the cesium atom

Examples

Some Lengths	(m)
Diameter of the Universe	$1 \cdot 10^{26}$
Distance to the nearest star (Proxima Centauri)	$4 \cdot 10^{16}$
Mean distance from Earth to Moon	$4 \cdot 10^{8}$
Mean radius of the Earth	$6 \cdot 10^{6}$
Length of a soccer field	$1 \cdot 10^{2}$
Size of the smallest dust particles	$1 \cdot 10^{-4}$
Size of cells of most living organisms	$1 \cdot 10^{-5}$
Diameter of a hydrogen atom	$1 \cdot 10^{-10}$
Diameter of an atomic nucleus	$1 \cdot 10^{-14}$
Diameter of a proton	$1 \cdot 10^{-15}$

Some Masses	(kg)
Universe	$1 \cdot 10^{52}$
Milky Way Galaxy	$7 \cdot 10^{41}$
Sun	$2 \cdot 10^{30}$
Earth	$6 \cdot 10^{24}$
Human	$7 \cdot 10^{1}$
Mosquito	$1 \cdot 10^{-5}$
Bacterium	$1 \cdot 10^{-15}$
Hydrogen atom	$1.7 \cdot 10^{-27}$
Electron	$9 \cdot 10^{-31}$

Some Time Intervals	(s)
Age of the Universe	$5 \cdot 10^{17}$
Age of the Earth	$1 \cdot 10^{17}$
Average age of student	$3 \cdot 10^{7}$
One day	$8.64 \cdot 10^{4}$
Time between normal heartbeat	$8 \cdot 10^{-1}$
Period of typical radio waves	$1 \cdot 10^{-6}$
Period of visible light waves	$2 \cdot 10^{-15}$

Prefixes for SI Units

- As a convinience when dealing with very large or very small measurements, we use the prefixes, which represents a certain power of 10, as a factor.
- Attaching a prefix to an SI unit has the effect of multiplying by the associated factor.
- For examples, we can express
  - a particular time interval as

$$2.35 \cdot 10^{-9}\ \mathrm{s} = 2.35\ \mathrm{ns}$$

  - a particular length as

$$7.2 \cdot 10^{3}\ \mathrm{m} = 7.2\ \mathrm{km}$$

  - a particular mass as

$$5 \cdot 10^{-6}\ \mathrm{kg} = 5 \cdot 10^{-6} \cdot 10^{3}\ \mathrm{g} = 5\ \mathrm{mg}$$

- The most commonly used prefixes are:
  - kilo, mega, and giga
  - centi, mili, micro, and nano

Symbol	Prefix	Factor
Y	yotta	10^{24}
Z	zetta	10^{21}
E	exa	10^{18}
P	peta	10^{15}
T	tera	10^{12}
G	giga	10^{9}
M	mega	10^{6}
k	kilo	10^{3}
h	hecto	10^{2}
da	deka	10^{1}
d	deci	10^{-1}
c	centi	10^{-2}
m	milli	10^{-3}
μ	micro	10^{-6}
n	nano	10^{-9}
p	pico	10^{-12}
f	femto	10^{-15}
a	atto	10^{-18}
z	zepto	10^{-21}
y	yocto	10^{-24}

Order-of-magnitude Calculations

- We often need to change the units in which the physical quantity is expressed. We do so multiplying the original measurement by a conversion factor.
  - For example,
    - to convert 2 min to seconds, we have

$$2\ \mathrm{min} = 2\ \mathrm{min} \cdot \frac{60\ \mathrm{s}}{\mathrm{min}} = 120\ \mathrm{s}$$

    - or, 15 in to centimeters (1 in = 2.54 cm)

$$15\ \mathrm{in} = 15\ \mathrm{in} \cdot \frac{2.54\ \mathrm{cm}}{\mathrm{in}} = 38.1\ \mathrm{cm}$$

- Order-of-magnitude Calculations
  - Sometimes it is useful to estimate an answer to a problem in which little information is given. This answer can then be used to determine whether or not a more precise calculation is necessery.
  - When it is necessery to know a quantity only within a factor of 10, we refer to the order of magnitude of the quantity.
  - For example,
    - the mass of a person might be 75 kg.
    - We would say that the person's mass is on the order of 10^2 kg.

- The micrometer (1 μm) is often called the micron. How many microns make up 1 km?
  - $$\frac{1\,\text{km}}{1\,\mu\text{m}} = \frac{10^3\ \text{m}}{10^{-6}\ \text{m}} = 10^9$$
- Earth is approximately a sphere of radius 6370 km. What are its circumference in meters, its surface area in square kilometers, its volume in cubic kilometers?
  - $$l = 2\pi R = 2\cdot 3.14\cdot 6370\ \text{km} \approx 40000\ \text{km} = 4\cdot 10^4\cdot 10^3\,\text{m} = 4\cdot 10^7\,\text{m}$$
    $$\text{A} = 4\pi \text{R}^2 = 4\cdot 3.14\cdot (3.67\cdot 10^3\ \text{km})^2 = 509.6\cdot 10^6\,\text{km}^2 \approx 5.1\cdot 10^8\,\text{km}^2$$
    $$\text{V} = \frac{4}{3}\pi R^3 = \frac{4}{3}\cdot 3.14\cdot (6.37\cdot 10^3\ \text{km})^3 = 1082.1\cdot 10^9\,\text{km}^3 \approx 1.08\cdot 10^{12}\,\text{km}^3$$
- One gallon of paint (volume = 0.00378 m³) covers an area of 25 m². What is the thickness of the paint on the wall?
  - $$d = \frac{V}{A} = \frac{0.00378\,\text{m}^3}{25\,\text{m}^2} \approx 1.5\cdot 10^{-4}\ \text{m}$$

- Estimate the number of times your heart beats in a month. (Approximate time between normal hearbeat is 0.8 s)
  - $$n = \frac{1\,\text{month}}{0.8\,\text{s}} = \frac{1\cdot 30\cdot 24\cdot 3600\,\text{s}}{0.8\,\text{s}} \approx 3\cdot 10^6$$
- Estimate the number of breaths taken during an average life span of 70 years.
  - $$\text{n} = \frac{70\ \text{y}}{3\,\text{s}} = \frac{70\cdot 365\cdot 24\cdot 3600\,\text{s}}{3\,\text{s}} \approx 10^9$$
- You can obtain a rough estimate of the size of a molecule by the following simple experiment. Let a droplet of oil spread out on a smooth water surface. The resulting oil slick will be approximately one molecule thick. Given an oil droplet of mass 0.9 mg and density 918 kg/m³ that spreads out into a circle of radius 41.8 cm on the water surface, what is the diameter of an oil molecule?
  - $$V = \frac{m}{\rho} = r^2\pi d$$
    $$d = \frac{m}{\pi\rho r^2} = \frac{9\cdot 10^{-7}\ \text{kg}}{3.14\cdot 918\,\text{kg/m}^3\cdot (0.418\,\text{m})^2} = 1.7\cdot 10^{-9}\,\text{m}$$

The part of mechanics that describes motion without regard to its causes is called kinematics. Here we will focus on one dimensional motion.

- Position
  - To describe the motion of an object, one must be able to specify its position at all time using some convinient coordinate system.
    - For example, a particle might be located at x=+5 m, which means that it is 5 m in the positive direction from the origin. If it were at x=-5 m, it would be just as far from the origin but in the opposite direction.
  - Position is an example of a vector quantity, i.e. the physical quantity that requires the specification of both direction and magnitude. By contrast, a scalar is quantity that has magnitude and no direction.
- Displacement
  - A change from one position x_1 to another x_2 position is called displacement

$$\Delta x = x_2 - x_1$$

  - For examples, if the particles moves from +5 m to +12 m, then

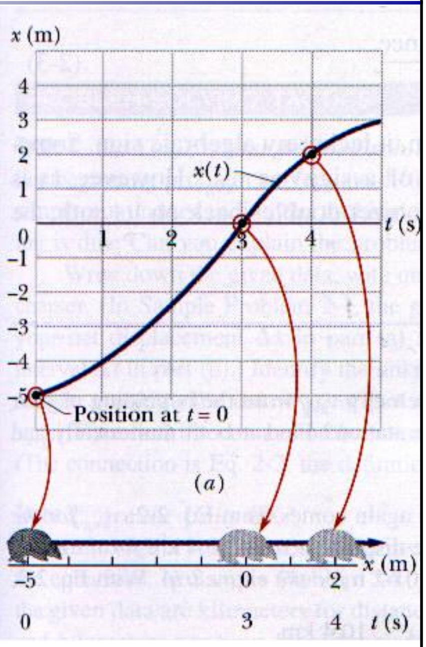
$$\Delta x = (+12\text{ m}) - (+5\text{ m}) = +7\text{ m}$$

    - The + sign indicates that the motion is in the positive direction. The plus sign for vector quantity need to be shown, but a minus sign must always be shown.
  - The displacement is a vector quantity.

- A compact way to describe position is with a graph of position plotted as a function of time.
- For example, an armadillo is first noticed when it is at the position –5 m. It moves toward x=0 m passes through that point at t=3 s , and then moves to increasingly larger positive values of x.
- Several quantities are associated with the phrase "how fast". One of them is the average velocity which is the ratio of the displacement that occurs during a particular time interval to that interval:

$$\langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- A common unit of velocity is the meter per second (m/s).

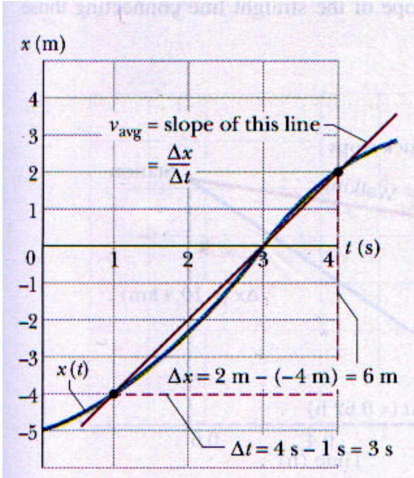


- On a graph of x versus t, average velocity is the slope of the stright line that connects two particular positions. For the time interval 1 s to 4 s, the average velocity is $\langle v\rangle$=(+6 m)/(3 s)= +2 m/s.
- Like displacement average velocity has both magnitude and direction (it is another vector quantity).
- Average speed
  - is a different way of describing "how fast" a particle moves, and involves the total distance covered independent of direction:

$$\langle s\rangle = \frac{\text{total distance}}{\Delta t}$$

  - is a scalar quantity
  - For the given example

$$\langle s\rangle = \frac{\text{total distance}}{\Delta t} = \frac{6\,\text{m}}{3\,\text{s}} = 2\,\text{m/s}$$



- The instantaneous velocity is obtained from the average velocity by shrinking the time interval closer and closer to 0. As Δt dwindles, the average velocity $\langle v\rangle$ approaches a limiting value, which is the velocity v at that instant:

$$v = \lim_{\Delta t\to 0}\langle v\rangle = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}$$

- For example, assume you have been observing a runner racing along a track, as given in one table. In another table there are calculated values of the time intervals. displacements, and average velocities. With some degree of confidence we can state that the instantaneous velocity of the runner was +2 m/s at the time 0.00 s.

t (s)	x (m)
0.00	0.00
0.01	+0.02
0.10	+0.21
0.20	+0.44
0.50	+1.25
1.00	+3.00
2.00	+8.00

t_1 to t_2 (s)	Δt (s)	Δx (m)	$\langle v\rangle$ (m/s)
0.00 to 2.00	2.00	+8.00	+4.00
0.00 to 1.00	1.00	+3.00	+3.00
0.00 to 0.50	0.50	+1.25	+2.50
0.00 to 0.20	0.20	+0.44	+2.20
0.00 to 0.10	0.10	+0.21	+2.10
0.00 to 0.01	0.01	+0.02	+2.00

- The instantaneous speed, which is a scalar quantity, is defined as the magnitude of the instantaneous velocity.

- Does an automobile speedometer measure average or instantaneous speed?

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  It measures the average speed, but the time interval is generally very short, so that the average speed is close to the instantaneous speed.

- An athlete swims the length of a 50 m pool in 20 s and makes the return trip to the starting position in 22 s. Determine her average velocities and speeds in: (a) the first half of the swim, (b) the second half of the swim, and (c) the round trip.

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  (a) $\langle v_f \rangle = \frac{+50\ \text{m}}{20\ \text{s}} = +2.5\ \text{m/s}$ $\quad \langle s_f \rangle = \frac{50\ \text{m}}{20\ \text{s}} = 2.5\ \text{m/s}$

  (b) $\langle v_s \rangle = \frac{-50\ \text{m}}{22\ \text{s}} = -2.27\ \text{m/s}$ $\quad \langle s_s \rangle = \frac{50\ \text{m}}{22\ \text{s}} = 2.27\ \text{m/s}$

  (c) $\langle v_r \rangle = \frac{0\ \text{m}}{42\ \text{s}} = 0\ \text{m/s}$ $\quad \langle s_r \rangle = \frac{100\ \text{m}}{42\ \text{s}} = 2.38\ \text{m/s}$

- When a particle's velocity changes, the particle is said to accelerate. For motion along an axis, the average acceleration over a time interval is

$$\langle a \rangle = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

where the particle has velocity v_1 at the time t_1 and then velocity v_2 at time t_2

- The instantaneous acceleration (or simply acceleration) is defined as the limit of the average acceleration as the time interval goes to zero

$$a = \lim_{\Delta t \to 0} \langle a \rangle = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

- Acceleration is vector quantity.
- The common unit of acceleration is the meter per second per second (m/s²).
- The acceleration at a certain time equals the slope of the velocity-time graph at that instant of time.

- In many types of motion, the acceleration is either constant or approximately so. In that case the instantaneous acceleration and average acceleration are equal

$$a = \langle a \rangle = \frac{v_2 - v_1}{t_2 - t_1}$$

For convenience, let $t_1 = 0$ and t_2 be any arbitrary time t
Also, let $v_1 = v_0$ (the initial velocity) and $v_2 = v$ (the velocity at arbitrary time)
With this notation we have

$$v = v_0 + at$$

- In a similar manner we can have

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

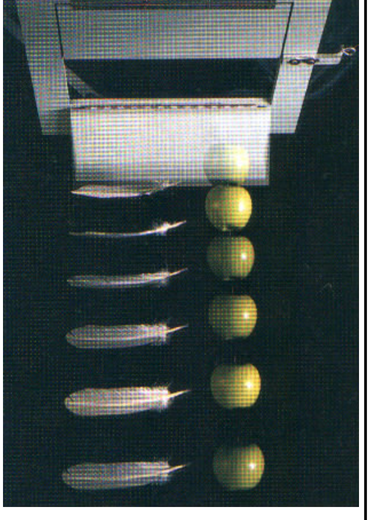
where x_0 is the position of the particle at initial time.

- Finally, from this two equations we can obtain expression that does not contain time

$$v^2 = v_0^2 + 2a(x - x_0)$$

- These equations may be used to solve any problem in one-dimensional motion with constant acceleration.

- All objects dropped near the surface of the Earth in the absence of air resistance fall toward the Earth with the same nearly constant acceleration.
- We denote the magnitude of free-fall accelerarion as *g*.
- The magnitude of free-fall acceleration decreases with increasing altitude. Furthemore, slight variations occur with latitude. At the surface of the Earth the magnitude is approximately 9.8 m/s². The vector is directed downward toward the center of the Earth.
- Free-fall acceleration is an important example of straight-line motion with constant acceleration.
- When air resistance is negligible, even a feather and an apple fall with the same acceleration, regardless of their masses.



- At t=0, a particle moving along an x axis is at position –20 m. The signs of the particle's initial velocity and constant acceleration are, respectively, for four situations: (a) +,+; (b) +,-; (c) -,+; (d) -,-.
  In which situation will the particle: (a) undergo a momentary stop, (b) definitely pass through the origin (given enough time), (c) definitely not pass through the origin?

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad v = v_0 + at$$

(a) undergo a momentary stop: +,- and -,+.
Initial velocity and constant acceleration must have opposite sign.

(b) definitely pass through the origin (given enough time): +,+ and -,+.
Constant acceleration and initial position must have opposite sign.

(c) definitely not pass through the origin: -,-.
Constant acceleration and initial position must have the same sign, and initial velocity must not have opposite sign with high magnitude.

- To measure your reaction time, have a friend hold a ruler vertically between your index finger and thumb. Note the position of the ruler with respect to your index finger. Your friend must release the ruler and you must catch it (without moving your hand downword). Repeat the measure and average your results and calculate your reaction time t. (For most people, the reaction time is at best about 0.2 s.)

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  The ruler falls through a distance

$$d = \frac{1}{2} g t^2, \quad g = 9.8 \mathrm{m/s^2}$$

- A car traveling initially at +7.0 m/s accelerates at the rate of +0.8 m/s^2 for an interval of 2.0 s. What is its velocity at the end of the acceleration?

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$$v = v_0 + at = +7.0 \text{ m/s} + 0.8 \text{ m/s}^2 \cdot 2.0 \text{ s} = 8.6 \text{ m/s}$$

- Jules Verne in 1865 proposed sending men to the Moon by firing a space capsule from a 220-m-long cannon with final velocity of 10.97 km/s. What would have been the unrealistically large acceleration expirienced by the space travelers during launch?

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  $d = 220$ m
  $v = 10.97$ km/h $= 10.97 \cdot 10^3$ m/s
  $a =?$

$$\begin{aligned} v^2 &= v_0^2 + 2a(x - x_0) \\ &= 2ad \\ a &= \frac{v^2}{2d} = \frac{(10.97 \cdot 10^3 \text{ m/s})^2}{2 \cdot 220 \text{ m}} \\ &= 2.7 \cdot 10^5 \text{ m/s}^2 \\ &\approx 2.8 \cdot 10^4 \, g \end{aligned}$$

- A renger in a national park is driving at 60 km/h when a deer jumps into the road 50 m ahead of the vehicle. After a reaction time of t_1, the ranger applies the brakes to produce an acceleration of $a = -3$ m/s^2. What is the maximum reaction time allowed if she is to avoid hitting the deer?

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  $v_0 = +60$ km/h $= +16.7$ m/s
  $l = 50$ m
  $a = -3$ m/s^2
  $t_1 =?$

$$\begin{aligned} t_2 &= \frac{\Delta v}{a} = \frac{-v_0}{a} \\ &= 5.56 \text{ s} \\ l_1 &= v_0 t_1 \\ l_2 &= v_0 t_2 + \frac{1}{2} a t_2^2 \\ l &= l_1 + l_2 = v_0(t_1 + t_2) + \frac{1}{2} a t_2^2 \\ t_1 &= \frac{l - \frac{1}{2} a t_2^2 - v_0 t_2}{v_0} \\ &= 0.22 \text{ s} \end{aligned}$$

- A peregrine falcon dives at a pigeon. The falcon starts downward from rest and falls with free-fall acceleration. If the pigeon is 76 m below the initial position of the falcon, how long does it take the falcon to reach the pigeon? Assume that the pigeon remains at rest.

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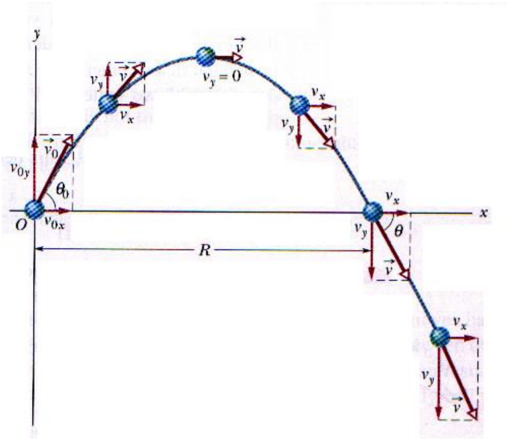
  $l = 76$ m
  $t = ?$

$$l = \frac{1}{2} g t^2$$
$$t = \sqrt{\frac{2l}{g}} = \sqrt{\frac{2 \cdot 76 \text{ m}}{9.8 \text{ m/s}^2}} = 3.9 \text{ s}$$

- In one-dimensional motion the vector nature of some physical quantities was taken into account through the use of positive (+) and negative (-) signs.
- In two-dimensional motion there are an infinity possibilities for the vector directions. So, we must make use of vectors.
- Position: $\vec{r} = x\hat{i} + y\hat{i} + z\hat{i}$
- Displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$.
- Average velocity: $\langle\vec{v}\rangle = \frac{\Delta\vec{r}}{\Delta t} = v_x\hat{i} + v_y\hat{i} + v_z\hat{i}$, where $v_x = \frac{\Delta x}{\Delta t}$, $v_y = \frac{\Delta y}{\Delta t}$, $v_z = \frac{\Delta z}{\Delta t}$.
- Average acceleration: $\langle\vec{a}\rangle = \frac{\Delta\vec{v}}{\Delta t} = a_x\hat{i} + a_y\hat{i} + a_z\hat{i}$, where $a_x = \frac{\Delta v_x}{\Delta t}$, $a_y = \frac{\Delta v_y}{\Delta t}$, $a_z = \frac{\Delta v_z}{\Delta t}$.
- Instantaneous velocity (instantaneous acceleration) is definied as the limit of the average velocity (average acceleration) when Δt goes to zero.

- We next consider a special case of two-dimensional motion: a particle moves in a vertical plane with some initial velocity $\vec{v}_0$ but its acceleration is always the free-fall acceleration, which is downword. Such motion is called projectile motion. (It might be a golf ball in flight.)
- Initial velocity can be written as

$$\begin{aligned}\vec{v}_0 &= v_{x0}\hat{\imath} + v_{y0}\hat{\jmath} \\ &= v_0\cos\theta_0\hat{\imath} + v_0\sin\theta_0\hat{\jmath}\end{aligned}$$



- The horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.
- The acceleration in the x direction is 0 (air resistance is neglected), so v_{0x} remains constant, and horizontal position of the projectile is:

$$x = x_0 + v_{x0}t.$$

- The acceleration in the y direction is $-g$ and we have

$$\begin{aligned}y &= y_0 + v_{y0}t - \frac{1}{2}gt^2 \\ v_y &= v_{y0} - gt.\end{aligned}$$

- We can find the equation of the projectile's path (its trajectory) by eliminating t

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$$

For simplicity we let $x_0 = 0$ and $y_0 = 0$.

- A 2.00-m-tall basketball player wants to make a goal from 10 m from the basket, as in Figure. If he shots the ball at a 45^0 angle, at what initial speed must he throw the basketball so that it goes through the hoop without striking the backboard?

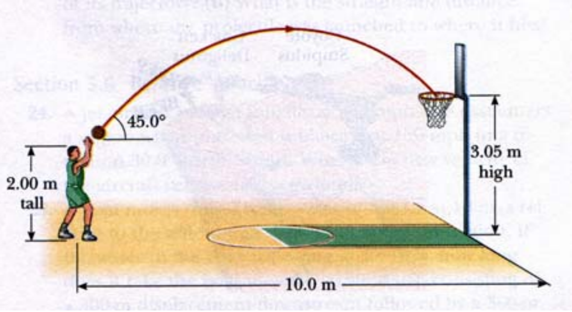
$\vdash$

$x = 10$ m
$y = 3.05 \text{ m} - 2 \text{ m} = 1.05$ m
$\theta_0 = 45^0$

$v_0 = ?$

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$$

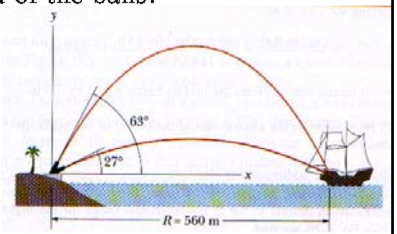
$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2(x\tan\theta_0 - y)}} = 10 \text{ m/s}$$



- Soft drinks are commonly sold in aluminium containers. Estimate the number of such containers thrown away each year consumers in your country. Approximately how many tons of aluminium does this represent?
- Estimate your age in seconds.
- Estimate the volume of gasoline used by all cars in your country each year.
- One cubic meter (1 m^3) of aluminium has a mass of $2.7 \cdot 10^3$ kg, and 1 m^3 of iron has a mass of $7.86 \cdot 10^3$ kg. Find the radius of an aluminium sphere whose mass is the same as that of an iron sphere of radius 2 cm. (Note: Density is defined as the mass of an object devided by its volume $\rho = m/V$.)
- A hamburger chain advertises that it has sold more than 50 billion hamburgers. Estimate how many head of cattle were required to furnish the meat.
- Estimate your average speed and average velocity for the whole day.

- A person walks first at a constant speed of 5 m/s along a stright line from point A to point B and then back along the line from B to A at a constant speed of 3 m/s. What is her average speed over the entire trip and what is her average velocity over the entire trip?
  (A: 3.75 m/s; 0 m/s)
- A ball thrown vertically upward is caught by the thrower after 2 s. Find the initial velocity of the ball and the maximum height it reaches.
  (A: 9.8 m/s; 4.9 m)
- A parachutist with a camera, both descending at a speed of 10 m/s, releases that camera at an altitude of 50 m. How long does it take the camera to reach the ground, and what is the velocity of the camera just before hits the ground?
  (A: 2.33 s; -32.9 m/s)

- The record for a ski jump is 180 m set in 1989. Assume the jumper comes off the end of the ski jump horizontally and falls 90 m vertically before contacting the ground. What was the initial horizontal speed of the jumper?
  (A: 42 m/s)
- Indiana Jones is trapped in a maze. To find his way out, he walkes 10 m, makes 90^0 right turn, walks 4 m, makes another 90^0 turn, and walks 7 m. What is his displacement from his initial position?
  (A: 5 m)
- A pirat ship 560 m from a fort defending the harbor entrance of an island. A defense cannon, located at see level, fires a balls at the angles of 63^0 and 27^0. What is the initial speed of the balls?
  (A: 82 m/s)



- Classical mechanics describes the relationship between the motion of an object and the force acting on it.
- There are conditions under which classical mechanics does not apply. Most often these conditions are encountered when dealing with objects whose size is comparable to that of atoms or smaller and/or which move at speed of light.
  - If the speeds of the interacting objects are very large, we must replace Classical mechanics with Einstein's special theory of relativity, which hold at any speed, including those near the speed of light.
  - If the interacting bodies are on the scale of atomic structure, we must replace Classical mechanics with quantum mechanics.
  - Classical mechanics is a special case of these two more comprehensive theories. But, still it is a very important special case because it applies to the motion of objects ranging in size from the very small to astronomical.
- The concept of force
  - **Force** is an interaction that can cause deformation and/or change of motion of an object
  - Force is a vector quantity
  - Fundamental forces are:
    - gravitational
    - electromagnetic
    - weak nuclear
    - strong nuclear

- Before Newton formulated his mechanics, it was thought that some influence was needed to keep a body moving at constant velocity. A body was thought to be in its "natural state" when it was at rest.
- Galileo was the first to take a different approach. He concluded that it is not the nature of an object to stop once set in motion. This approach to motion was later formalized by Newton in a form that has come to be known as **Newton's first law of motion**:

  "An object at rest remains at rest, and an object in motion continues in motion with constant velocity, unless it experiences a net external force."
- Newton's first law says that when the net external force on an object is zero, its acceleration is zero.

- **Inertial Reference Frames**
  - Newton's first law is not true in all reference frames, but we can always find reference frames in which it is true. Such frames are called **inertial reference frames**, or simply inertial frames.
  - A inertial reference frame is one in which Newton's laws hold.
  - Other frames are noninertial frames.

Isaac Newton

Mass and Inertia

- The tendency of an object to resist any attempt to change its motion is called the **inertia** of the object.
- **Mass** is a measurement of inertia.
- The greater the mass of a body, the less it accelerates under the action of an applied force.
- Mass is a scalar quantity that obeys the rules of ordinary arithmetic.
- Mass is an intrinsic characteristic of a body – that is a characteristic that automatically comes with the existance of the body. It has no definition. The mass of a body is the characteristic that relates a force on the body to the resulting acceleration.
- The SI unit of mass is the kilogram (1 kg).
- For example, if a given force acting on a 3-kg mass produces an acceleration of 4m/s², the same force applied to a 6-kg mass will produce an acceleration of only 2 m/s².
- Inertia is the principle that underlies the operation of seat belts and air begs.

Newton's Second Law

- Newton's first law explains what happens to an object when the net force acting on it is zero. Newton's second law answers the question of what happens to an object that has nonzero force acting on it.
- From observations, we can conclude that the acceleration of an object is directly proportional to the net force acting on it, and these observation are summarized in **Newton's second law**:

  "The net force on a body is equal to the product of the body's mass and the acceleration of the body."

  In equation form:

$$\sum \vec{F} = m\vec{a}$$

  where $\sum \vec{F}$ represents the vector sum of all external forces acting on the object, m is its mass, and $\vec{a}$ is the acceleration of the object.

  - This equation is a vector equation, and it is equivalent to three component equations:

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$$

- The SI unit of force is the newton (1 N = 1 kgm/s²).

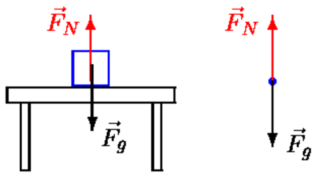
- The Gravitational Force and Weight
  - The force exerted by the Earth on an object is the gravitational force $\vec{F}_g$
  - This force is directed approximately toward the center of the Earth and its magnitude varies with location.
  - The magnitude of the gravitational force is called the **weight** of the object

$$W = F_g = mg$$

where g is the magnitude of the free-fall acceleration.

  - Weight is not an inherent property of an object.

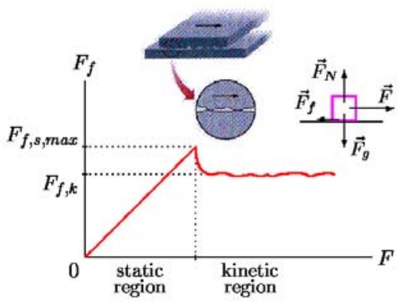
- The Normal Force
  - If you stand on a mattress, Earth pulls you downward, but you are stationary. The reason is that the mattress, because it deformes, pushes up on you. This force from the mattress is called a normal force $\vec{F}_N$
  - The normal force is perpendicular to the surface.



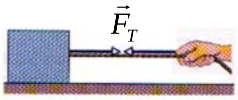
- The Frictional Force
  - If we slide or attempt to slide a body over a surface, the motion is resisted by a bonding between the body and the surface. The resistence is considered to be a single force called the frictional force $\vec{F}_f$
  - This force is very important in our everyday lives. They allow us to walk or run and are necessery for the motion of wheeled vehicles.
  - The frictional force is directed along the surface, opposite the direction of the intended motion.
  - For an object in motion the frictional force we call kinetic frictional force; otherwise static frictional force.
  - Both, kinetic and static frictional force are proportional to the normal force acting on the object

$$F_{f,s,\max} = \mu_s F_N \quad F_{f,k} = \mu_k F_N$$

where μ_s, μ_k are coefficients of static and kinetic friction.

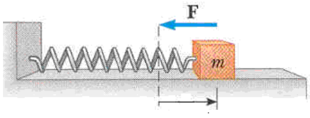


- The Tension Force
  - When a cord is attached to a object and pulled taut, the cord pulls on the object with the force $\vec{F}_T$ called a tension force.
  - The force is directed away from the object and along the cord.



- The Spring Force
  - A good approximation for many springs, the force $\vec{F}$ from a spring is proportional to the displacement $\vec{d}$ of the free end from its position when the spring is in the relaxed state

$$\vec{F} = -k\vec{d}$$



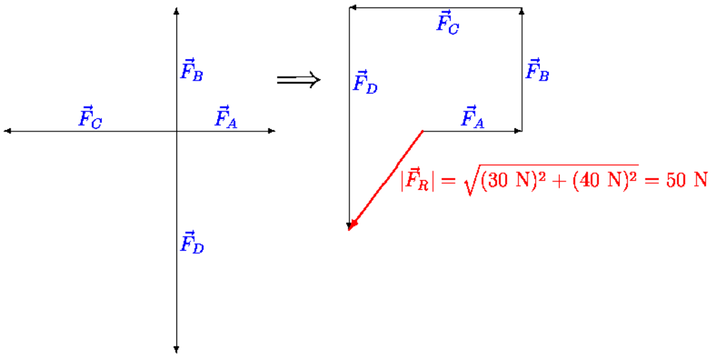
  - This is known as Hooke's law.
  - The minus sign indicates that the spring force is always opposite in direction from the displacement of the free end.
  - The constant k is called the spring constant
    - It is a measure of the stiffness of the spring. The larger k is, the stiffer the spring.
  - Note that a spring force is a variable force.

- A force is exerted on an object when that object comes into contact with some other object.
- Newton recognized that forces in nature always exist in pair. He described this type of situation in terms of his **third law of motion**:
  "When two objects interact, the force on the object from each other are always equal in magnitude and opposite in direction."

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

- For example, the force acting on a freely falling projectile is the force of the Earth on the projectile, and the magnitude of this force is *mg*. The reaction to this force is the force of the projectile on the Earth. The reaction force must accelerate the Earth toward the projectile. However, because the Earth has such a large mass, its acceleration due to this reaction force is negligibly small.

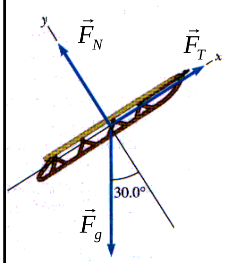
- Four forces act on an object, given by $\vec{F}_A = 40$ N east, $\vec{F}_B = 50$ N north, $\vec{F}_C = 70$ N west, and $\vec{F}_D = 90$ N south. What is the magnitude of the net force on the object?



- A child holds a sled at rest on a frictionless, snow-covered hill. If sled weighs 77 N, find the force exerted on the rope by child and the force exerted on the sled by the hill.

Applying the condition for equilibrium ($\vec{a} = 0$) to the sled, we find that

$$\sum F_x = F_T - F_g \sin 30^0 = 0$$
$$F_T = F_g \sin 30^0 = (77\text{ N}) \sin 30^0 = 38.5\text{ N}$$
$$\sum F_y = F_N - F_g \cos 30^0 = 0$$
$$F_N = F_g \cos 30^0 = 66.7\text{ N}.$$



What happens to the normal force as the angle of incline increase?

It decreases.

When is the magnitude of the normal force equal to the weight of the sled?

When the sled is on a horizontal surface and the applied force is either zero or along the horizontal.

- You are playing with your younger sister in the snow. She is sitting on a sled and asking you to slide her across a flat, horizontal field. You have a choice of pushing her from behind, by applying a force at 30^0 below the horizontal or attaching a rope to the front of the sled and pulling with a force at 30^0 above the horizontal. Which would be easier for you and why?

  It is easier to attack the rope and pull. In this case, there is a component of your applied force that is upward. This reduces the normal force between the sled and the snow. In turn, this reduces the friction force between the sled and the snow, making it easier to move. If you push from behind, with a force downward component, the normal force is larger, the friction force is larger, and the sled is harder to move.

- An object has only one force acting on it. Can it be at rest? Can it have an acceleration?

  If a single force acts on it, the object must accelerate. If an object accelerates, at least one force must act on it.

- An object has zero acceleration. Does this mean that no forces act on it?

  If an object has no acceleration, you cannot conclude that no forces act on it. In this case, you can only say that the net force on the object is zero.

- Is it possible for an object to move if no net force acts on it?

  Motion can occur in the absence of a net force. Newton's first law holds that an object will continue to move with a constant speed and in a straight line if there is no net force acting on it.

- What force causes an automobile to move?

  The force causing an automobile to move is the force of friction between the tires and the roadway as the automobile attempts to push the roadway backward.

- What force causes a propeller airplane to move?

  The force driving a propeller airplane forward is the reaction force of the air on the propeller as the rotating propeler pushes the air backword.

- What force causes a rowboat to move?

  In a rowboat, the rower pushes the water backword with the oars. The water pushes forward on the oars and hence the boat.

- In a tug-of-war between two athletes, each pulls on the rope with a force of 200 N. What is the tension in the rope?

  The tension in the rope is the maximum force that occurs in both directions. In this case, then, because both are pulling with a force of 200 N, the tension is 200 N.

- Identify the action-reaction pairs in the following situations: a man takes a step; a gust of wind strikes a window.

  As a man takes a step, the action is the force his foot exerts on the Earth; the reaction is the force of the Earth on his foot. As a gust of wind strikes a window, the action is the force exerted on the window by the air molecules; the reaction is the force on the air molecules exerted by the window.

- Suppose you are driving a car at a high speed. Why you should you avoid alamming on your brakes when you want to stop in the shortest possible distance?

  The brakes may lock and the car will slide farther than it would if the wheels continued to roll because the coefficient of kinetic friction is less than the coefficient of static friction. Hence, the force of kinetic friction is less than the maximum force of static friction.

- An object has a mass of 6 kg and acceleration of 2 m/s². What is the magnitude of the resulting force acting on it?

$$F = ma = (6\,\text{kg})\cdot(2/\text{s}^2) = 12\,\text{N}$$

- The force of the wind on the sails of a sailboat is 390 N north. The water exerts force of 180 N east. If the boat has a mass of 270 kg, what are the magnitude and direction of its acceleration?

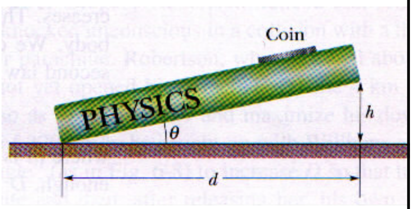
$$F = \sqrt{F_{wind}^2 + F_{water}^2} = \sqrt{(390\ \text{N})^2 + (180\ \text{N})^2} = 429.5\ \text{N}$$

$$a = \frac{F}{m} = 1.59\ \text{m/s}^2$$

$$\theta = \arctan\frac{F_{wind}}{F_{water}} = 65.2^0\ \text{north of east}$$

- A coin of mass m at rest on a book that has been tilted at an angle θ with the horizontal. By experiment, when θ is increased to 15^0, the coin is on the verge of sliding down the book, which means that even a slight increase beyond 15^0 produces sliding. What is the coefficient of static friction μ_s between the coin and the book?

$$F_{s,max} = \mu_s F_N$$
$$F_N = mg\cos\theta$$
$$F_{s,max} = mg\sin\theta$$
$$\mu_s mg\cos\theta = mg\sin\theta$$
$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = \tan 15^0 = 0.27$$



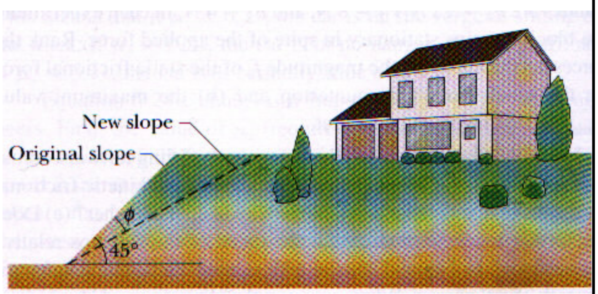
- A house is bilt on the top of a hill with a nearby 45° slope. An engineering study indicates that the slope angle should be reduced because the top layers of soil along the slope might slip past the lower layers. If the static coeficcient of friction between two such layers is 0.5, what is the least angle ϕ through which the present slope should be reduced to prevent slipping?

  (A: $\phi > 18.4°$)

$$\mu = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

$$\theta = \arctan\mu = 26.6^0$$

$$\phi = 45^0 - \theta = 18.4^0$$



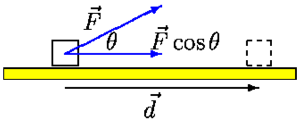
The concept of energy is one of the most important in the world of science. In everyday use, the term energy has to do with the cost of fuel for transportation and heating, electricity for lights and appliances, and the foods we consume.

Energy is present in the Universe in a variety of forms, including mechanical energy, chemical energy, electromagnetic energy, nuclear energy, and many others.

Here we are concerned only with mechanical energy, and begin by defining work.

- Work

We see an object that undergoes a displacement of $\vec{d}$ along a stright line while acted on by a constant force, $\vec{F}$, that makes an angle of θ with $\vec{d}$.



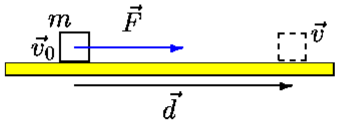
The work W done on an object by a constant force $\vec{F}$ during a displacement is defined as the product of the component of the force along the direction of displacement and the magnitude of the displacement.

$$W = (F\cos\theta)d$$

- As an example of the distinction between this definition of work and our everyday understanding of the word, consider holding a heavy book at arm's length. After 5 minutes, your tired arms may lead yout to think that you have done a considerable amout of work. According to our definition, however, you have done no work on the book. Your muscles are continuosly contracting and relaxing while the book is being supported. Thus, work is being done on your body, but not on the book.
- A force does no work on a object if the object does not move.
- The sign of the work depends on the angle θ between the force and displacement.
- Work is a scalar quantity, and its units is joule (1 J = 1 Nm)
- For example, a man cleaning his apartment pulls the canister of a vacuum cleaner with a force of magnitude 50 N at an angle 30°. He moves the vacuum cleaner a distance of 3 m. Calculate the work done by the force.

$$W = F\cos\theta\, d = (50\ \text{N})(\cos 30^0)(3\ \text{m})$$
$$= 130\ \text{J}$$

Figure shows an object of mass m moving to the right under the action of a constant net force, $\vec{F}$.



The work done by $\vec{F}$ is

$$W = Fd = (ma)d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

The quantity $\frac{1}{2}mv^2$ has a special name in physics: kinetic energy. Any object of mass m and speed v is defined to have a kinetic energy E_k, of

$$E_k = \frac{1}{2}mv^2$$

We see that it is possible to write W as $W = E_{k,2} - E_{k,1}$.

- A car with mass of 1400 kg has a net forward force of 4500 N applied to it. The car starts from rest and travels down a horizontal highway. What are its kinetic energy and speed after it has traveled 100 m? (Ignore friction and air resistance.)

  The work done by the net force on the car is

$$W = Fd = (4500\text{ N})(100\text{ m}) = 4.5 \cdot 10^5\text{ J}$$

  This work all goes into changing the kinetic energy of the car, thus the final of the kinetic energy is also $E_k = 4.5 \cdot 10^5$ J.

  The speed of the car can be found from

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2(4.5 \cdot 10^5\text{ J})}{1400\text{ kg}}} \\ &= 25.4\text{ m/s} \end{aligned}$$

Let we examine the work done by a gravitational force.

As an object falls freely in a gravitational fields, the field exerts a force on it, doing positive work on it and thereby increasing its kinetic energy.

Consider an object of mass m at an initial height h_1 above the ground. As the object falls, the only force that does work (we neglect air resistance) on it is the gravitational force, $m\vec{g}$.

The work done by the gravitational force as the object undergoes a downward from the position of h_1 to h_2

$$W_g = mgh_1 - mgh_2.$$

We define the quantity mgh to be the gravitational potential energy $E_{p,g}$

$$E_{p,g} = mgh.$$

- Conservative forces
  - A force is **conservative** if the work it does on an object moving between two points is independent of the path the object takes between the points. In other words, the work done on an object by a conservative force depends only on the initial and final positions of the object.
  - The gravitational force is conservativ.
- Nonconservative forces
  - A force is **nonconservative** if it leads to a dissipation of mechancal energy.
  - If you moved an object on a horizontal surface, returning it to the same location and same state of motion, but found it necessary to do net work on the object, then something must have dissipated the energy transferred to the object. That dissipative force is recognized as friction between object and surface.
  - Friction force is a nonconservative force.

- Conservative principles play a very important role in physics, and conservation of energy is one of the most important.
- Let us assume that the only force doing work on the system is conservative. In this case we have

$$W = E_{p1} - E_{p2} = E_{k2} - E_{k1}$$

or

$$E_{k1} + E_{p1} = E_{k2} + E_{p2}$$

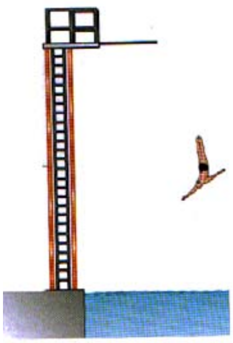
- The total mechanical energy in any isolated system of objects remains constant if the objects interact only through conservative forces.
- This is equivalent to saying that, if the kineic energy of a conservative system increases by some amount, the potential energy of the system must decrease by the some amount.
- If the gravitational force is the only force doing work on an object, then the total mechanical energy of the object remains constant

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

- A 77-kg diver drops from a board 10 m above the water surface. Use conservation of mechanical energy to find his speed 5 m above the water surface.

⊢

Conservation of mechanical energy gives



$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

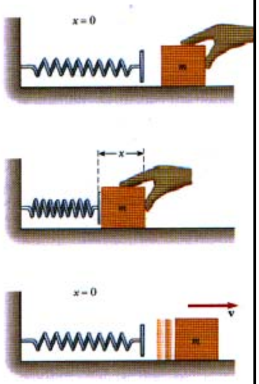
$$0 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

$$v_2 = \sqrt{\frac{2mg(h_1 - h_2)}{m}}$$

$$= \sqrt{\frac{2(77\ \text{kg})(9.8\ \text{m/s}^2)(10\ \text{m} - 5\ \text{m})}{77\ \text{kg}}}$$

$$= 9.9\ \text{m/s}^2$$

Elastic potential energy



- The concept of potential energy is of tremendous value in descriptions of certain types of mechanical energy. One of these is the motion of a mass attached to a streched or compressed spring.
- In order to compress the spring, we must exert on the block a force of

$$\vec{F} = -k\vec{x}$$

  where k is a constant for a particular spring called the spring constant.
- The force increases linearly with position.
- It is possible to find the work done by the applied force. This work is stored in the compressed spring as elastic potential energy

$$E_{p,e} = \frac{1}{2}kx^2$$

- The elastic potential energy stored in the spring is zero when the spring is in equilibrium (x=0).
- Note that energy is stored in the spring when it is streched as well.

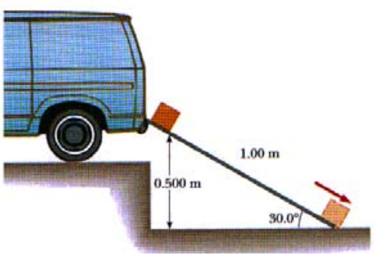
Work and nonconservative forces

In realistic situations, nonconservative forces such as friction are usually present. In such situations, the total mechanical energy of the system is not constant. The work done by all nonconservative forces equals the change in mechanical energy of the system $W_{nc} = (E_{k,2} + E_{p,2}) - (E_{k,1} + E_{p,1})$.

- A 3-kg crate slides down a ramp at a loading dock. The crate experinces a constant frictional force of magnitude 5 N. Determine the speed of the crate at the bottom of the ramp.

  ⊢

$$E_{k,1} = 0, \quad E_{p,1} = mgh_1, \quad E_{k,2} = \frac{1}{2}mv_2^2, \quad E_{p,2} = 0$$



$$\begin{aligned} W_{nc} &= E_{k,2} - E_{p,1} \\ -F_f d &= \frac{1}{2}mv_2^2 - mgh_1 \\ v_2 &= \sqrt{\frac{2(mgh_1 - F_f d)}{m}} = 2.54 \text{ m/s} \end{aligned}$$

From the practical viewpoint, it is interesting to know not only the amount of energy transferred to or from a system, but also the rate at which the transfer occured.

Power is defined as the time rate of energy transfer.

If an external force is applied to an object and if the work done by this force is W in the time interval Δt, then the average power $\bar{P}$ during this time interval is defined as the ratio of the work to the time interval:

$$\bar{P} = \frac{W}{\Delta t}$$

The units of power in SI system are joules per second, which are also called watts (1 W).

Note, that a kilowatt-hour is a unit of energy, not power.

When you pay your electric bill, you are buying energy.

For example, an electric bulb rated at 100 W would "consume" 3.6×10^5 J of energy in 1 h, or 0.1 kWh (kilowatt-hour).

- A whale has a length of about 18 m. Assume that its leap from the water carries him about half his hight out of the water and that all the upward surge is achieved solely by its speed at the instant of leaving the water. How fast was it going as it left the water?

⊢

$$\begin{aligned} m\frac{v_1^2}{2} &= mgh_2 \\ v_1 &= \sqrt{2gh_2} \\ &= \sqrt{2(9.8\ \mathrm{m/s^2})(9\ \mathrm{m})} \\ &= 13.3\ \mathrm{m/s} \end{aligned}$$

- Can the kinetic energy of an object be negative? ⊢ No.
- If the speed of a particle is doubled, what happens to its kinetic energy?

  ⊢

$$E_{k,2} = \frac{1}{2}mv_2^2 = \frac{1}{2}m(2v_1)^2 = 4\frac{1}{2}mv_1^2 = 4E_{k,1}$$

- Which has the greater kinetic energy, a 1000-kg car traveling at 50 km/h or a 500-kg car traveling at 100 km/h?

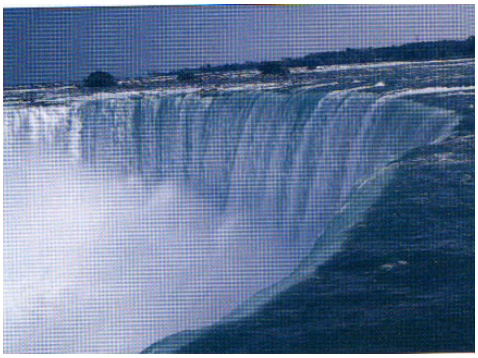
  ⊢

$$\begin{aligned} v_1 &= 50\ \mathrm{km/h} = 50 \cdot \frac{1000\ \mathrm{m}}{3600\ \mathrm{s}} = 13.9\ \mathrm{m/s} \\ E_{k,1} &= \frac{1}{2}m_1v_1^2 = 9.6 \cdot 10^4\ \mathrm{J} \\ v_2 &= 100\ \mathrm{km/h} = 100 \cdot \frac{1000\ \mathrm{m}}{3600\ \mathrm{s}} = 27.8\ \mathrm{m/s} \\ E_{k,2} &= \frac{1}{2}m_2v_2^2 = 1.9 \cdot 10^5\ \mathrm{J} \end{aligned}$$

- Water flows over a section of Niagara Falls at the rate of $1.2 \cdot 10^6$ kg per second and falls 50 m. How much power is generated by the falling water?

  ⊢

$$\begin{aligned} P &= \frac{W}{t} \\ &= \frac{mgh}{t} = 5.9 \cdot 10^8\ \mathrm{W} \end{aligned}$$



- A 70-kg man normally uses about 10^7 J per day. The exact amount depending on his physical activity. Find his metabolic rate P_m, i.e. the rate of energy use, $P_m = E/t$

  ⊢

$$P_m = \frac{E}{t} = \frac{10^7\ \mathrm{J}}{86400\ \mathrm{s}} = 116\ \mathrm{W}$$

- The metabolic rate of a person engaged in a particular activity is measured determining the amount oxygen consumed, which reacts with carbohydrates, fats, and protein in the body, releasing an average of about $2 \cdot 10^4$ J of energy for each liter of oxygen consumed. How much oxygen in one minute does a person consume while sleeping ($P_m = 75$ m)?

  ⊢

$$\begin{aligned} E &= P_m t = (75\ \mathrm{W})(60\ \mathrm{s}) = 4500\ \mathrm{J} \\ V &= \frac{E}{(2 \cdot 10^4\ \mathrm{J/l})} = \frac{4500\ \mathrm{J}}{(2 \cdot 10^4\ \mathrm{J/l})} = 0.225\ \mathrm{l} \end{aligned}$$

- The center of mass of a body or a system of bodies is the point that moves as though all of the mass were concentrated there and all external forces were applied there.
- **System of particles**

  If n particles are distributed in three dimensions, the center of mass must be identified by three coordinates. They are

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i \qquad z_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

M is the total mass of the system

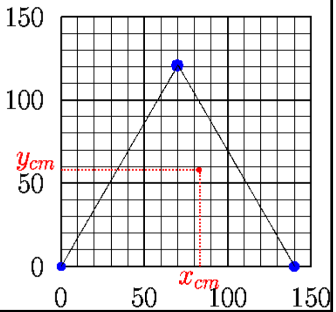
$$M = m_1 + m_2 + m_3 + \ldots + m_n = \sum_{i=1}^{n} m_i$$

and x_i, y_i, z_i are coordinates of i-th particle position.

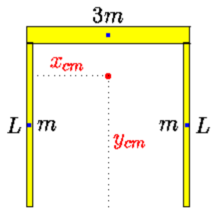
- Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg from an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this three-particle system?

The three particles have following coordinates: 1.2 kg: (0,0); 2.5 kg: (140 cm, 0); 3.4 kg: (70 cm, 121 cm). The total mass of the system is $M = m_1 + m_2 + m_3 = 7.1$ kg. The coordinates of the center of mass are

$$\begin{aligned} x_{cm} &= \frac{1}{M}\sum_{i=1}^{3} m_i x_i \\ &= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ cm})}{7.1\text{ kg}} \\ &= 83\text{ cm} \\ y_{cm} &= \frac{1}{M}\sum_{i=1}^{3} m_i y_i \\ &= \frac{(1.2\text{ kg})0 + (2.5\text{ kg})(0) + (3.4\text{ kg})(121\text{ cm})}{7.1\text{ kg}} \\ &= 58\text{ cm} \end{aligned}$$



- Three thin rods, each of length L, are arranged in an inverted U. The two rods on the arms of the U each have mass m; the third rod has mass $3m$. Where is the center of mass of the assembly?



$$x_{cm} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{(m)(0) + (3m)(L/2) + (m)(L)}{5m} = \frac{1}{2}L$$

$$y_{cm} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{(m)(L/2) + (3m)(L) + (m)(L/2)}{5m} = \frac{4}{5}L$$

- The linear momentum of an object of mass m moving with a velocity $\vec{v}$ is defined as the product of the mass and velocity

$$\vec{p} = m\vec{v}$$

Momentum is a vector quantity, with its direction matching that of the velocity.

- Often we will work with the components of momentum. For two-dimensional motion, these are

$$p_x = mv_x \qquad p_y = mv_y$$

- Newton didn't write the second law as $\vec{F} = m\vec{a}$ but as

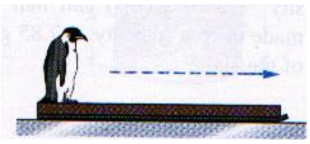
$$\vec{F} = \frac{\text{change in momentum}}{\text{time interval}} = \frac{\Delta\vec{p}}{\Delta t}$$

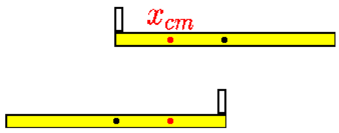
where Δt is the time interval during which the momentum changes $\Delta\vec{p}$. This expression is equivalent to $\vec{F} = m\vec{a}$ for an object of constant mass.

- A penguin stands at the left edge of a uniform sled of length L, which lies on frictionless ice. The sled and penguin have equal masses. (a) Where is the center of mass of the sled? (b) How far and in what direction is the center of the sled from the center of mass of the sled-penguin system?

  The penguin then waddles to the right edge of the sled, and the sled slides on the ice. (c) Does the center of mass of the sled-penguin system move leftward, rightward, or not at all?

  (a) at the center of the sled;
  (b) $x_{cm} = (m \cdot 0 + mL/2)/(2m) = L/4$;
  (c) not at all (no net external force).





- Newton's second law $\vec{F} = (\Delta\vec{p})/(\Delta t)$ can be written as

$$\vec{F}\Delta t = \Delta\vec{p}$$

  The term $\vec{F}\Delta t$ is called the impulse of the force $\vec{F}$ for the time interval Δt. We see that the impulse of the force acting on an object equals the change in momentum of that object.

- To change the momentum of an object we shuld consider the impulse, that is, the amount of force and the time of contact.

- For example, think what you do when you jump from a high position to the ground. As you strike the ground, you bend you knees. If you were to land on the ground with your legs locked, you would receive a painful shock in your legs as well as along your spine. The lending is much less painful if you bend your knees. By bending your knees, the change in momentum occurs over a longer time interval than with the knees locked. Thus, the force on the body is less than with the knees locked.

- Now consider a system of n particles, each with its own mass, velocity, and linear momentum. The particle may interact with each other, and external force may act on them as well. The system as whole has a total linear momentum $\vec{p}$ as a sum of the individual particles' linear momentum

$$\begin{aligned} \vec{p} &= \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_n \\ &= m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n = \sum_{i=1}^{n} m_i\vec{v}_i \\ &= M\vec{v}_{cm}, \end{aligned}$$

where $M = m_1 + \ldots + m_n$ is the total mass of the system, and

$$\vec{v}_{cm} = \frac{1}{M}(m_1\vec{v}_1 + \ldots + m_n\vec{v}_n)$$

the velocity of the center of mass.

- It is possible to prove that equation that governs the motion of the center of mass of a system of particles is

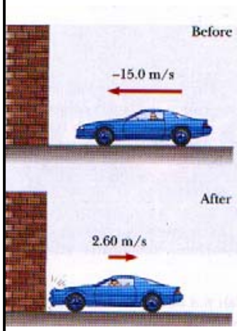
$$\vec{F}_{net} = M\vec{a}_{cm},$$

where $\vec{F}_{net}$ is the net force of all external forces that act on the system (not internal forces), $\vec{a}_{cm}$ is the acceleration of the center of mass. We assume that no mass enters or leaves the system (the system is closed). This equation gives no information about the acceleration of any other point of the system. This equation is equivalent to three equations involving components, $F_x = Ma_{cm,x}$, $F_x = Ma_{cm,x}$, $F_x = Ma_{cm,x}$.

- For a nonclosed system of particles it is possible to derive the following expression

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$$

which is generalization of the single-particle Newton's second law.



- In a particular crash test, a $1.5 \cdot 10^3$ kg automobile collides a wall. The initial and final velocities of the automobile are $v_i = -15.0$ m/s and $v_f = +2.6$ m/s, respectively. If the collision lasts for 0.15 s, find the impulse due to the collision and the average force exerted on the automobile.

The initial and final linear momenta of the automobile are

$$p_i = mv_i = (1.5 \cdot 10^3 \text{ kg})(-15.0 \text{ m/s}) = -2.25 \cdot 10^4 \text{ kg m/s}$$
$$p_f = mv_f = (1.5 \cdot 10^3 \text{ kg})(2.6 \text{ m/s}) = +0.39 \cdot 10^4 \text{ kg m/s}$$

Hence, the impulse, which equals the change in momentum, is

$$\bar{F}\Delta t = p_f - p_i = (+0.39 \cdot 10^4 \text{ kg m/s}) - (-2.25 \cdot 10^4 \text{ kg m/s}) = +2.64 \cdot 10^4 \text{ kg m/s}$$

And average force is

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{+2.64 \cdot 10^4 \text{ kg m/s}}{0.15 \text{ s}} = +1.76 \cdot 10^5 \text{ N}$$

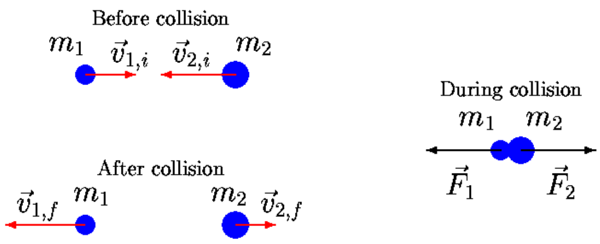
- Suppose that the net external force acting on a system of particle iz zero (the system is isolated) and that no particles leave or enter the system (the system is closed). From the previous we have

$$\Delta\vec{p} = 0 \quad \text{or} \quad \vec{p} = const.$$

In words we say, if no net external force acts on a system of particles, the total linear momentum $\vec{p}$ of the system cannot change. This result is called the law of conservation of linear momentum. It means that the total linear momentum at some initial time is equal to the total one at some later time.

- Depending on the forces acting on a system, linear momentum might be conserved in one or two directions but not in all directions. However, we see that if the component of the net external force on a closed system is zero an axis, then the component of the linear momentum of the system along that axis cannot change.

- A collision is an isolated event in which two or more bodies exert relatively strong forces on each other for a relatively short time.
- We must be able to distinguish times that are before, during, and after a collision. During the collision the force on object with mass m_1 due to m_2 is equal in magnitude and opposite in direction to the force on m_2 due to m_1. The momentum of each object changes as a result of the collision, but the total momentum of the system remains constant. We can say for any type of collision, the total momentum of the system just before collision equals the total momentum just after collision.



- We see that the total momentum is always conserved for any type of collision. However, the total kinetic energy is generally not conserved, because some of kinetic energy is converted to thermal energy or internal potential energy when the objects deform.
- An inelastic collision is a collision in which momentum is conserved, but kinetic energy is not.
- A perfectly inelastic collision is an inelastic collision in which the two objects stick together after the collision, so that their final velocities are the same and the momentum of the system is conserved.
- An elastic collision is one in which both momentum and kinetic energy are conserved.

  Elastic and perfectly inelastic collisions are limiting cases. Most actual colisions fall into a category between them.

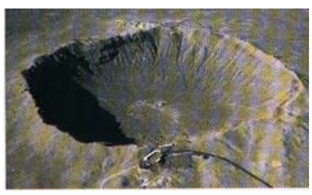
- A 80-kg man stands in the middle of a frozen pond of radius 5 m. He is unable to get to the other side because of a lack of friction between his shoes and the ice. To overcome this difficulty, he throws his 1.2-kg coat horizontally toward the nord shore, at a speed of 5 m/s. How long does it take him to reach the south shore?

$$0 = m_c v_c - m_m v_m$$

$$v_m = \frac{m_c v_c}{m_m} = \frac{(1.2 \text{ kg})(5 \text{ m/s})}{80 \text{ kg}} = 0.075 \text{ m/s}$$

$$t = \frac{d}{v_m} = \frac{5 \text{ m}}{0.075 \text{ m/s}} = 66.67 \text{ s}$$

- Meteor Crater in Arizona is thought to have been formed by the impact of a meteor with Earth some 20000 years ago. The mass of the meteor is estimated at $5 \cdot 10^{10}$ kg, and its speed at 7200 m/s. What speed would such a meteor give Earth in a head-on collision? (Mass of Earth is $6 \cdot 10^{24}$ kg)



$$p_m + p_E = const.$$

$$m_m v_m + 0 = (m_m + m_E)v$$

$$v = \frac{m_m}{m_m + m_E} v_m$$

$$\approx \frac{m_m}{m_E} v_m = \frac{5 \cdot 10^{10} \text{ kg}}{6 \cdot 10^{24} \text{ kg}} 7200 \text{ m/s} = 6 \cdot 10^{-11} \text{ m/s}$$

- The blocks slide without friction. (a) What is the velocity $\vec{v}$ of the 1.6 kg block after the collision? (b) Is the collision elastic?

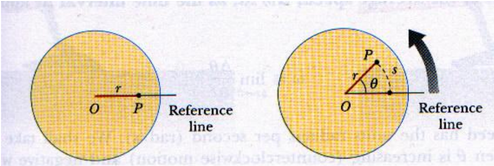
$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

$$\begin{aligned} v_{1,f} &= \frac{m_1 v_{1,i} + m_2 v_{2,i} - m_2 v_{2,f}}{m_1} \\ &= \frac{(1.6\ \text{kg})(5.5\ \text{m/s}) + (2.4\ \text{kg})(2.5\ \text{m/s}) - (2.4\ \text{kg})(4.9\ \text{m/s})}{1.6\ \text{kg}} \\ &= +1.9\ \text{m/s} \end{aligned}$$

$$\begin{aligned} \frac{m_1 v_{1,i}^2}{2} + \frac{m_2 v_{2,i}^2}{2} &\stackrel{?}{=} \frac{m_1 v_{1,f}^2}{2} + \frac{m_2 v_{2,f}^2}{2} \\ 24.4\ \text{J} + 7.5\ \text{J} &= 2.888\ \text{J} + 28.812\ \text{J} \end{aligned}$$

The collision is elastic.

- We wish to examine the rotation of a rigid body about a fixed axis. A rigid body is a body that can rotate with all its parts locked together and without any change in its shape. We deal with the angular equivalents of the linear quantities: position, displacement, velocity, and acceleration.



The angular position θ of some fixed line in the body, perpendicular to the rotation axis, is the angle of the line relative to the fixed direction. Measured in radians (rad) it is defined as

$$\theta = \frac{s}{r}$$

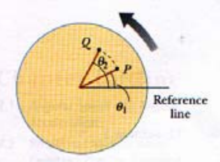
where s is the length of arc along a circle and between the reference line and the fixed line in the body; r is the radius of that circle. ($1\ \text{rad} = 180^0/\pi \approx 57.3^0$)

- The rotor on a helicopter turns at an angular speed of 320 revolutions per minute. Express this in radians per second.

$$320\frac{\text{rev}}{\text{min}} = 320\frac{2\pi\ \text{rad}}{60\ \text{s}} = 10.7\pi\ \text{rad/s} = 33.6\ \text{rad/s}$$

- A wheel has a radius of 4.1 m. How far (path length) does a point on the circumference travel if the wheel is rotated through angle of 30^0 and 30 rad, respectively?

$$\begin{aligned} s_1 &= r\theta = (4.1\ \text{m})(30^0\frac{\pi\ \text{rad}}{180^0}) = 2.14\ \text{m} \\ s_2 &= r\theta = (4.1\ \text{m})(30\ \text{rad}) = 123\ \text{m} \end{aligned}$$



- If the body rotates about the rotational axis changing the fixed line in the body from θ_1 to θ_2, the body undergoes an angular displacement $\Delta\theta$ given by

$$\Delta\theta = \theta_2 - \theta_1$$

The average angular velocity $\bar{\omega}$ of a rotating rigid body is the ratio of the angular displacement to the time interval

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

By analogy with linear velocity, instantaneous angular velocity, ω, is defined as the limit of the average speed as the time interval approaches zero

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Angular velocity has the units rad/s. We shall take ω to be positive when θ increasing and negative when θ is decreasing. The magnitude of an angular velocity is called the angular speed.

- If the angular velocity of a rotating body is not constant, then the body has an angular acceleration. Let ω_1 and ω_2 be its angular velocities at times t_1 and t_2, respectively. The average angular acceleration of the rotating body in the interval from t_1 to t_2 is defined as

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

The instantaneous angular acceleration is the limit of the average angular acceleration as the time interval Δt approaches zero

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

Angular acceleration has the units rad/s^2.

When a rigid object rotates about a fixed axis every portion of the object has the same angular velocity and the same angular acceleration. This is what makes these variables so useful for describing rotational motion.

- We developed a set of kinematic equations for linear motion under constant acceleration. The same procedure can be used to derive a similar set of equations for rotational motion under constant angular acceleartion. The resulting equations for rotational kinematics, with the corresponding equations for linear motion, are as follows

Rotational Motion	Linear Motion
$\omega = \omega_0 + \alpha t$	$\omega = v_0 + at$
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x = x_0 + v_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	$v^2 = v_0^2 + 2a(x - x_0)$

Variables θ_0, x_0, ω_0, and v_0 are all initial angular position, linear position, angular velocity, and linear velocity, respectively. Note the one-to-one correspondence between the rotational equations involving the angular variables θ, ω and α and the equations of linear motion involving the variables x, v, and a.

- When a rigid body rotates around an axis, each particle in the body moves in its own circle around that axis. Since the body is rigid, all the particles make one revolution in the same amount of time; that is, they all have the same angular displacement, angular velocity, and angular acceleration.

  The linear variables for a particular point in a rotating body are relate to the angular variables by the perpenducular distance r

$$v = \omega r \quad \text{(the magnitude of tangential velocity)}$$

$$a_t = \alpha r \quad \text{(tangential component of acceleration)}$$

$$a_c = \frac{v^2}{r} = \omega^2 r \quad \text{(radial component of acceleration (or centripetal))}$$

The radial component a_c of linear acceleration (or centripetal acceleration) is present whenever the angular velocity of the body is not zero. The tangential component a_t is present whenever the angular acceleration is not zero.

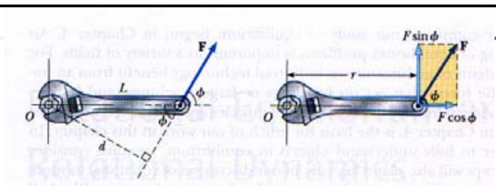
- A compact disc is designed such as the read head moves out from the center of the disc, the angular speed of the disc changes so that the linear speed at the position of the head will always be at a constant value of about 1.3 m/s. Find the angular speed of the disc when the read head is at a distance of 5 cm from the center.

$$\omega_1 = \frac{v}{r_1} = \frac{1.3 \text{ m/s}}{0.05 \text{ m}} = 26 \text{ rad/s}$$

- A machine part rotates at an angular velocity of 0.6 rad/s; its value is then increased to 2.2 rad/s at an angular acceleration of 0.7 rad/s^2. Find the angle through which the part rotates before reaching this final velocity.

$$\omega = \omega_0 + \alpha t \qquad \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

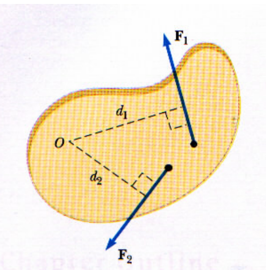
$$\theta = \frac{1}{2}\frac{\omega^2 - \omega_0^2}{\alpha} = \frac{1}{2}\frac{(2.2 \text{ rad/s})^2 - (0.6 \text{ rad/s})^2}{0.7 \text{ rad/s}^2} = 3.2 \text{ rad}$$



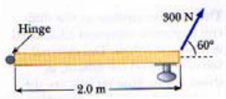
- Consider the wrench pivoted about the axis O. The applied force acts at an angle of ϕ.

  The tendency of a force to ratate a body about some axis is measured by a quantity called the torque. The torque has the magnitude

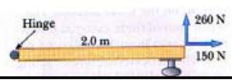
$$\tau = Fr\sin\phi$$



  If two or more forces are acting on an object, then each has a tendency to produce a rotation about the pivot O. We shall use the convention that the sign of torque resulting from a force is positive if its turning tendency is counterclockwise and negative if its turning tendency is clockwise.



- As an example, find the torque produced by the 300-N force applied at an angle 60^0 to the door, as in figure.



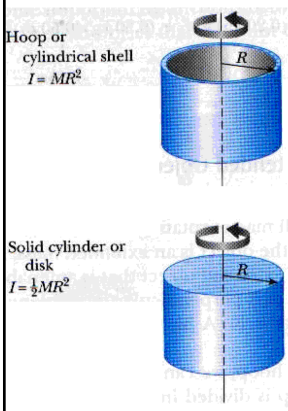
$$\tau = Fr\sin\phi = (300 \text{ N})(2 \text{ m})\sin 60^0 = 520 \text{ Nm}$$

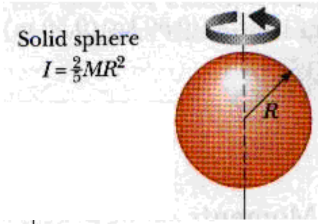
- We define the moment of inertia of a body with respect to the axis of rotation as follows

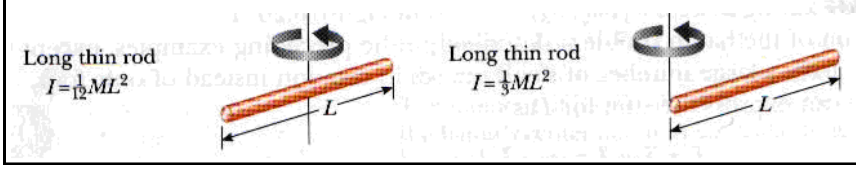
$$I = \sum_i m_i r_i^2$$

where m_i is the mass of the ith particle and r_i is the perpendicular distance of the ith particle from the given rotation axis. The sum is taken over all the particles in the body.

The moments of inertia about an axis through its center for some common shapes are given without proof (integral calculus are required).







- It is possible to prove that the angular acceleration of an object is proportional to the net torque acting on it. The proportional constant between them is the moment of inertia.

$$\tau_{net} = I\alpha.$$

It is important to note that this equation is rotational counterpart to Newton's second law $F_{net} = ma$. We now see that force and mass in linear motion correspond to torque and moment of inertia in rotational motion.

If we define the product of the angular velocity and the moment of inertia

$$L = I\omega$$

as the angular momentum of the object, then we can write

$$\tau_{net} = \frac{\text{change in angular momentum}}{\text{time interval}} = \frac{\Delta L}{\Delta t}$$

- When the net external torque acting on the system is zero, we see from the Newton's second law for rotation that the rate of change of the system's angular momentum is zero

$$\Delta L = 0.$$

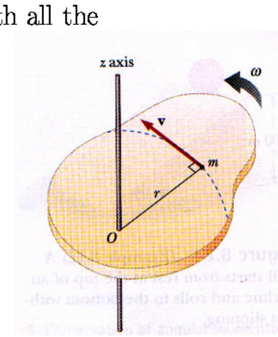
The angular momentum of a system is conserved when the net external torque acting on the system is zero. That is, when $\tau_{net} = 0$, the initial angular momentum equals the final angular momentum. The magnitude of angular velocity increases when the skater pulls her arms in close to her body, demonstrating that angular momentum is conserved.

- We defined the kinetic energy of a particle moving through space with a velocity v as the quantity $\frac{1}{2}mv^2$. Analogously, a body rotating about some axis with an angular velocity ω has rotational kinetic energy given by

$$E_{k,r} = \frac{1}{2}I\omega^2$$

To prove this, consider a rigid plane body rotating. the body consists of many small particles. All this particles rotate in circular paths about the axis. The total kinetic energy of the body is the sum of all the kinetic energies associated with all the particles making up the body

$$\begin{aligned} E_{k,r} &= \Sigma(\frac{1}{2}m_i v_i^2) = \Sigma(\frac{1}{2}m_i r_i^2\omega^2) \\ &= \frac{1}{2}(\sum m_i r_i^2)\omega^2 \\ &= \frac{1}{2}I\omega^2 \end{aligned}$$



- Consider a ball of mass m tied to a string of length r and being whirled in a horizontal circular path. Let us assume that the ball moves with constant speed. Because the velocity changes its direction continuosly during the motion, the ball experiences a centripetal acceleration directed toward the center of motion, with magnitude

$$a_c = \frac{v_t^2}{r}.$$

The strings exerts a force on the ball that makes a circular path. This force is directed along the length of the string toward the center of the circle with magnitude of

$$F_c = m\frac{v_t^2}{r}$$

This force we call centripetal force. Note that a centripetal force is not a new kind of force. The name indicates the direction of the force. It can, in fact, be a frictional force, a gravitational force, or any other force.

- In 1687 Newton published his work on the universal law of gravitation, which states that
Every particle in the Universe attracts any other particle with a gravitational force. If the particles have masses m_1 and m_2 and their centers are separated by the distance r, the magnitude of the gravitational force between them is

$$F = G\frac{m_1 m_2}{r^2}$$

where G is a universal constant called the constant of universal gravitation

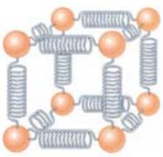
$$G = 6.673 \cdot 10^{-11}\ \mathrm{Nm/kg^2}.$$

Assuming that Earth is a uniform sphere of mass M_E, for the magnitude of gravitational acceleration a_g we find

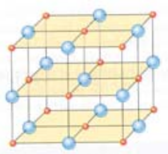
$$a_g = \frac{GM_E}{R_E^2} = \frac{(6.673 \cdot 10^{-11}\ \mathrm{Nm/kg^2})(6 \cdot 10^{24}\ \mathrm{kg})}{(6.37 \cdot 10^6\ \mathrm{m})^2} = 9.87\ \mathrm{m/s^2}$$

- An object executes circular motion with a constant speed whenever a net force of constant magnitude acts perpendicular to the velocity. What happens to the speed if the force is not perpendicular to the velocity?
  - An object can move in a circle even if the total force on it is not perpendicular to its velocity, but then its speed will change. Resolve the total force into an inward radial component and a tangential component. If the tangential force is forward, the object will speed up, and if the tangential force acts backward, it will slow down.
- An object moves in a circular path with constant speed. Is the object's velocity constant? Is its acceleration constant? Explain.
  - As an object moves in its circular path with constant speed, the direction of the velocity vector changes. Thus, the velocity of the object is not constant. The magnitude of its acceleration remains constant, and is equal to v^2/r. The acceleration vector is always directed toward the center of the circular path.

- Matter is normally classified as being in one of three states: solid, liquid, or gaseous.
- Often this classifiation system is extended to include a fourth state, refered to as a plasma. When matter is heated to high temperatures, many of the electrons surrounding each atom are freed from the nucleus. The resulting substance is a collection of free, electrically charged particles. Such a highly ionized substance containing equal amounts of positive and negative charges is a plasma. Plasmas exist inside stars, for example.
- Everyday expirience tells us that a solid has definite volume and shape. We also know that a liquid has a definite volume but no definite shape. Finally, a gas has neither definite volume nor definite shape.
- All matter consists of some distribution of atoms or molecules.



- The atoms in a solid are held by forces, that are mainly electrical, and vibrate about specific equilibrium positions. A vibrating atom can be viewed as being bound in its equilibrium position by springs attached to neighboring atoms.



- Solids can be classified as being either crystalline or amorphous. A crystaline solid is one in which the atoms have an ordered structure. For example, in the sodiumic hloride crystal (common table salt). In an amorphous solid, such as glass, the atoms are arranged randomly.

- For any given substance, the liquid state exists at a higher temperature than the solid state. The intermolecular forces in a liquid are not strong enough to keep the molecules in fixed positions. When an attempt is made to compress solids and liquids, strong repulsive atomic forces act internally to resist compression.

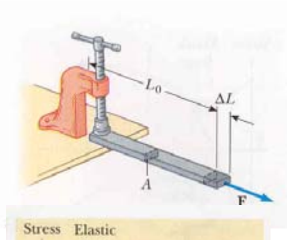
- In reality, all objects are deformable. That is, it is possible to change the shape or/and size of object through the application of external force. When the forces are removed, the object tends to return to its original shape and size. It means that the deformation exhibits an elastic behaviour.
- The elastic properties of solids are discussed in terms of stress and strain. Stress is related to the force causing a deformation; strain is a measure of the degree of deformation.

  It is found that, for sufficient small stresses, stress is proportional to strain, and the constant of proportionality depends on the material being deformed and the nature of the deformation. We call this proportionality constant the elastic modulus

$$\text{Elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

Consider a long bar of cross-sectional area A and length L_0, clamped at one end. When an external force $\vec{F}$ is applied along the bar, perpendicularly to the cross-section, internal forces in the bar resist the distortion. The bar attains an equilibrium in which its length is grater than L_0 and the external force is balanced by internal forces. In such a situation the bar is said to be stressed.

We define tensile stress as the ratio of the magnitude of the external force to the cross-sectional area. The SI unit is newton per square meter (N/m^2), with a special name, the pascal (1 Pa).

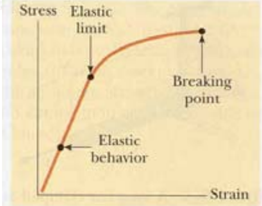


The tensile strain in this case is defined as the ratio of the change in length ΔL to the original length L.

We define Young's modulus, Y as

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_0}$$

Applying a sufficiently great stress it is possible to exceed the elastic limit.



Another type of deformation occurs when a body is subjected to a force tangential to one of its faces while the opposite face is held fixed. In this situation, the stress is called a shear stress.

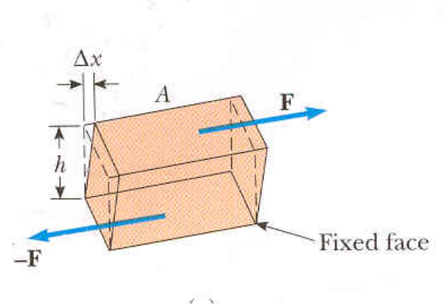
We define a shear stress as the ratio of the magnitude of the external force of the tangential force to the area of the face being sheared.

The shear strain is the ration $\Delta x/h$, where Δx is the horizontal distance the sheared face moves and h is the height of the object.

The shear modulus, S is defined as

$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$

Note that the SI units of shear modulus are pascal (Pa).



Bulk modulus characterizes the response of a substance to uniform squeezing. A body subject to this type of deformation undergoes a change in volume but no change in shape.

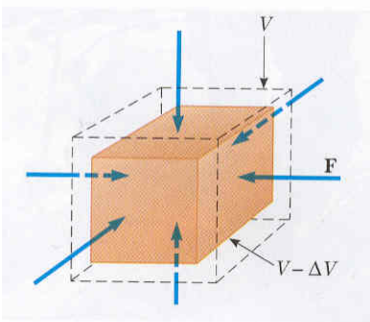
The volume stress is defined as the ratio of the magnitude of the change in the normal force to the area. The volume strain is equal to the change in volume devided by the original volume.

The bulk modulus, B is defined as

$$B = \frac{\text{volume stress}}{\text{volume strain}} = -\frac{F/A}{\Delta V/V}$$

Note that a negative sign is included in this defining equation so that B is positive.

The reciprocal of the bulk modulus is called the copmressibility of the material.



The density of a substance of uniform composition is defined as its mass per unit volume

$$\rho = \frac{m}{V}$$

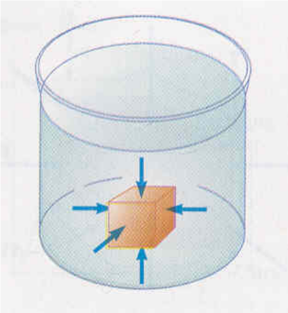
The SI units of density are kilograms per cubic meter (kg/m^3).

The only stress that can exist on an object submerged in a fluid is one that tends to compress the object. The force exerted by the fluid on the object is always perpendicular to the surfaces of the object.

The pressure, p of the fluid is defined as the ratio of the magnitude of the force to area

$$p = \frac{F}{A}$$

Pressure has units of pascals.



If a fluid is at rest, all portitions of the fluid must be in static equilibrium. Furthermore, all points at the same depth must be at the same pressure.

It is possible to get that the pressure, p, at the depth of h below the surface of a liquid open to the atmosphere is greter than atmospheric pressure p_0 by the amount $\rho g h$

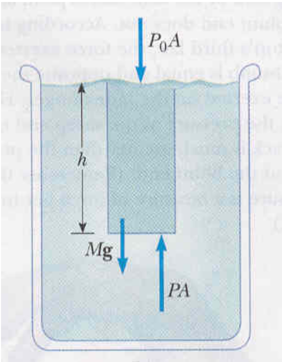
$$p = p_0 + \rho g h$$

Normal atmosferic pressure at sea level is

$$p_0 = 1.01 \cdot 10^5 \text{ Pa}.$$

Example: Calculate the absolute pressure at an ocean depth of 1000 m. Assume that the density of water is 10^3 kg/m^3 and that $p_0 = 1.01 \cdot 10^5$ Pa.

$$\begin{aligned} p &= p_0 + \rho g h \\ &= 1.01 \cdot 10^5 \text{ Pa } + (10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(10^3 \text{ m}) \\ &\approx 9.9 \cdot 10^6 \text{ Pa}. \end{aligned}$$



Archimedes's principle can be stated as follows:
Any body completely or partially submerged in a fluid is buoyed up by a force whose magnitude is equal to the weight of the fluid displaced by the body

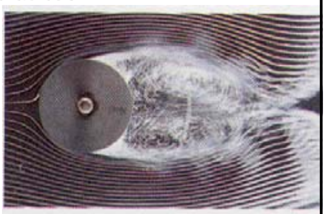
$$F_B = \rho_f V g,$$

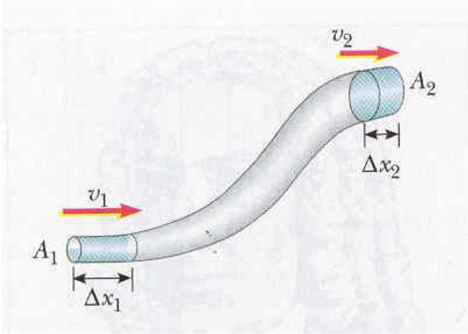
where ρ_f is the density of the fluid, V is the volume of the displaced fluid, and $g = 9.8\ \mathrm{m/s^2}$ is the magnitude of free-fall acceleration.
This upward force we call the buoyant force. This force acts vertically upward through what was the center of gravity of the fluid before the fluid was displaced.

When a fluid is in motion, its flow can be characterized in one of two ways. The flow is said to be streamline, or laminar, if every particle that passes a particular point moves exactly along the smooth path followed by particles that passes that point earlier.
In contrast, the flow of a fluid becomes irregular, or turbulent, above a certain velocity or under any conditions that can cause abrupt changes in velocity. Irregular motions of the fluid, called eddy currents, are characteristic in turbulent flow.

Many features of fluid motion can be understood by considering the bahaviour of an ideal fluid, which satisfies the following conditions:

1. The fluid is nonviscous (there is no internal friction force)
2. The fluid is incompressible (its density is constant)
3. The fluid motion is steady (velocity, density, and pressure at each point in the fluid do not change in time)
4. The fluid moves without turbulence (no eddy currents)





For a fluid flowing through a pipe of nonuniform size, the equation of continuity we can express as

$$A_1 v_1 = A_2 v_2$$

where A_1 and A_2 is the cross-sectional areas, and v_1 and v_2 are the speeds of the fluid at two given locations.

We see that the product of any cross-cestional area of the pipe and the fluid speed at that cross-section is a constant. Therefore, the speed is high where the tube is constricted and low where the tube has a larger diameter.

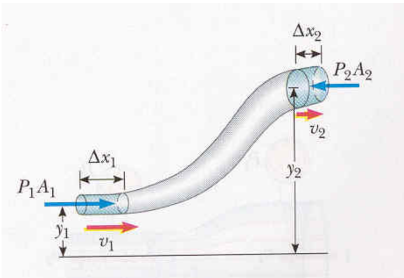
As a fluid moves through a pipe of varying cross-section and elevation, the pressure changes along the pipe. In 1738 the Swiss physicist Daniel Bernoulli derived a fundamental expression that relates pressure to fluid speed and elevation.

Bernoulli's equation is often expressed as

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

where p_1 and p_2 are the pressure at two given points; v_1 and v_2 are speeds; h_1 and h_2 are highes of the given points.

Bernoulli's equation states that the sum of the pressure p, the kinetic energy per unit volume $\frac{1}{2}\rho v^2$, and the potential energy per unit volume $\rho g h$ has the same value at all points along a streamline.



- A typical silo on a farm has many bands wrapped around its perimeter. Why is the spacing between succesive bands smaller at the lower portions of the silo?

  If you think of the grain stored in the silo as a fluid, the pressure the grain exerts on the walls of the silo increases with increasing depth just as water pressure in a lake increases with increasing depth. Thus, the spacing between bands is made smaller at the lower portions to overcome the larger outward forces on the walls in these regions.

- Will a ship ride higher in an inland lake or in the ocean? Why?

  According to Archimedes's principle, the magnitude of the buoyant force on the ship is equal to the weight of the water displaced by the ship. Because the density of salty ocean water is greater than fresh lake water, less ocean water needs to be displaced to enable the ship to float. Thus, the boat floats higher in the ocean than in the inland lake.

- The heels on a pair of woman's shoes have radii of 0.5 cm at the bottom. If 30 % of the weight of a 50-kg woman is supported by each heel, find the stress on each heel.

$$\text{stress} = \frac{F}{A} = \frac{0.30 \cdot mg}{r^2\pi} = \frac{0.30(50\text{ kg})(9.8\text{ m/s}^2)}{(0.5\cdot 10^{-2}\text{ m})^2\cdot 3.14} = 1.87\cdot 10^6\text{ Pa}$$

- For safety in climbing, a mountaineer uses a nylon rope that is 50 m long and 1 cm in diameter. When suporting a 90-kg climber, the rope elongates 1.6 m. Find its Young's modulus.

$$Y = \frac{F/A}{\Delta l/l_0} = \frac{mg/((d/2)^2\pi)}{\Delta l/l} = \frac{(90\text{ kg})(9.8\text{ m/s}^2)/((0.005\text{ m})^2\cdot 3.14)}{(1.6\text{ m})/(50\text{ m})} = 3.5\cdot 10^8\text{ Pa}$$

- The four tires of an automobile are inflated to a gauge pressure of $2.0 \cdot 10^5$ Pa. Each tire has an area of 0.024 m^2 in contact with the ground. Determine the weight of the automobile.

$$p = \frac{mg}{A} = \frac{W}{A}$$
$$W = pA = (2.0 \cdot 10^5\ \mathrm{Pa})(4 \cdot (0.024\ \mathrm{m}^2)) = 1.9 \cdot 10^4\ \mathrm{N}$$

- Water is to be pumped to the top of the Empire State Building, which is 365 m high. What gauge pressure is needed in the water line at the base of the building to raise the water to this height?

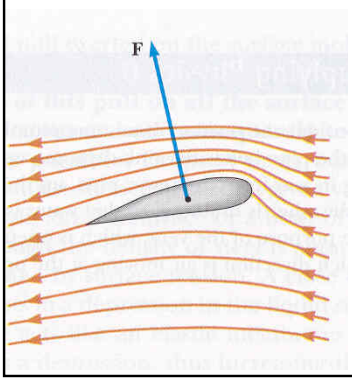
$$p = \rho_w g h = (10^3\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})(365\ \mathrm{m}) = 3.58 \cdot 10^6\ \mathrm{Pa}$$

- The density of ice is 920 $\mathrm{kg/m^3}$, and that of seawater 1030 $\mathrm{kg/m^3}$. What fraction of the total volume of an iceberg is exposed?

$$m_i g = F_B$$
$$\rho_i g V = \rho_s g (V - V_{ex})$$
$$\frac{V_{ex}}{V} = 1 - \frac{\rho_i}{\rho_s} = 1 - \frac{920\ \mathrm{kg/m^3}}{1030\ \mathrm{kg/m^3}} = 0.107$$

- What is the net upward force on an airplane wing of area 20 m^2 if the speed of air flow is 300 m/s across the top of the wing and 280 m/s across the bottom? (The density of air is 1.29 $\mathrm{kg/m^3}$)

$$p_b + \rho\frac{v_b^2}{2} = p_t + \rho\frac{v_t^2}{2}$$
$$p_b - p_t = \rho\frac{v_t^2}{2} - \rho\frac{v_b^2}{2} = \frac{\rho}{2}(v_t^2 - v_b^2)$$
$$F_{net} = (p_b - p_t)A = \frac{\rho}{2}(v_t^2 - v_b^2)A$$
$$= \frac{1.29\ \mathrm{kg/m^3}}{2}((300\ \mathrm{m/s})^2 - (280\ \mathrm{m/s})^2)(20\ \mathrm{m^2})$$
$$= 1.5 \cdot 10^5\ \mathrm{N}$$

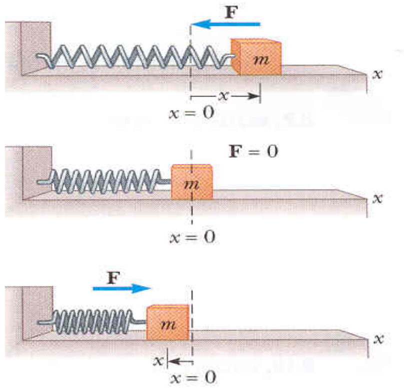


Simple Harmonic Motion

Simple harmonic motion occurs when the net force along the direction of motion is a Hooke's law type of force. That is, when the net force is proportional to the displacement and in the opposite direction. (For example, a mass attached to a spring)

Let we define few terms:

The **amplitude, *A*,** is the maximum distance traveled by an object away from its equilibrium position. In the absence of friction, an object attached to a spring continues in simple harmonic motion and reaches a maximum displacement equal to the amplitude on each side of the equilibrium position during each cycle.



The **period, *T*,** is the time it takes the object to execute one complete cycle of motion.

The **frequency, *f*,** is the number of cycles or vibrations per unit of time.

Elastic Potential Energy

We have worked with kinetic energy and gravitational potential energy. Here we will consider elastic potential energy.

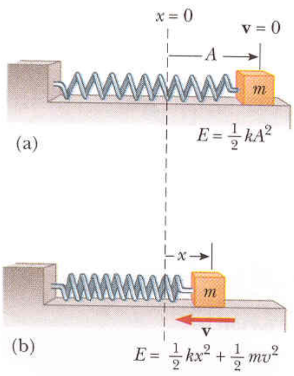
An object has potential energy by virtue of its shape or position. As we learned an object of mass m at height h above the ground has gravitational potential energy equal to mgh. This means that the object can do work after it is released. Likewise, a compressed spring has potential energy by virtue of its shape. In this case, the compressed spring can move an object and thus do work on it.

The energy stored in a stretched or compressed spring or other elastic material is called **elastic potential energy, $E_{p,e}$,** given by

$$E_{p,e} = \frac{1}{2}kx^2$$

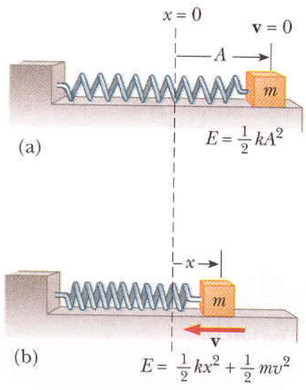
where k is a positive konstant, and x displacement from its unstretched position.

Note that energy is stored in an elastic material only when it is either stretched or compressed.



Conservation of energy provides a simple method of deriving an expression for the velocity of a mass attached to a spring undergoing periodic motion as a function of position. Let the mass be initially at its maximal extension, and then is releaced from rest. As the mass moves toward the origin to some new position x part of potential energy is transformed into kinetic energy. We can equate energies at initial and some final position:

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$



Solving, we get speed as a function of position

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)}$$

This expression shows us that the speed is a maximum at $x = 0$ and zero at the extreme positions $x = A$.

The period, T, represents the time required for one complete trip forth and back (we also say the complete cycle), and for the mass attached to the spring is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Recall that the frequency, f, is the number of cycle per unit of time. The symmetry in the units of period and frequency should lead you to see that the period and frequency must be related inversely as

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The units of frequency are s^{-1} or hertz (Hz).
We define angular frequency, ω, as

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

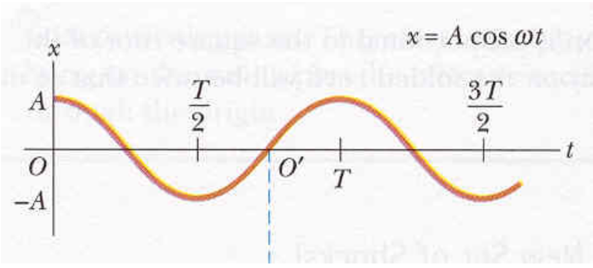
We can better understand and visualize many aspects of simple harmonic motion along s straight line by looking at their relationnships to uniform circular motion.

We find that as the turntable rotates with constant angular speed, the shadow of the ball moves back and forth with simple harmonic motion. The x coordinate of the ball is

$$x = A\cos(\omega t)$$

This equation represents the position of an object moving with simple harmonic motion as a function of time.

The curve should be familiar to you from trigonometry.



A simple pendulum is another mechanical system that exhibits periodic motion. It consists of a small bob of mass m suspended by a light string of length L fixed at its upper end.

The net force on the mass is proportional to $\sin\theta$ rather than to θ

$$F_t = -mg\sin\theta$$

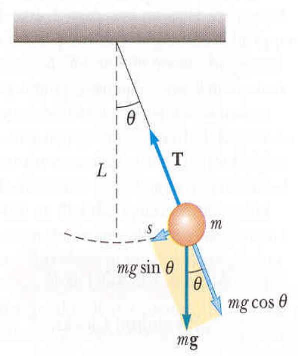
Thus, in general, the motion of a pendulum is not simple harmonic. However, for small angles, less than about 15 degrees, $\sin\theta$ and θ are approximately equal. Therefore, for the motion with small angles, the net force can be written as

$$F_t = -mg\theta$$

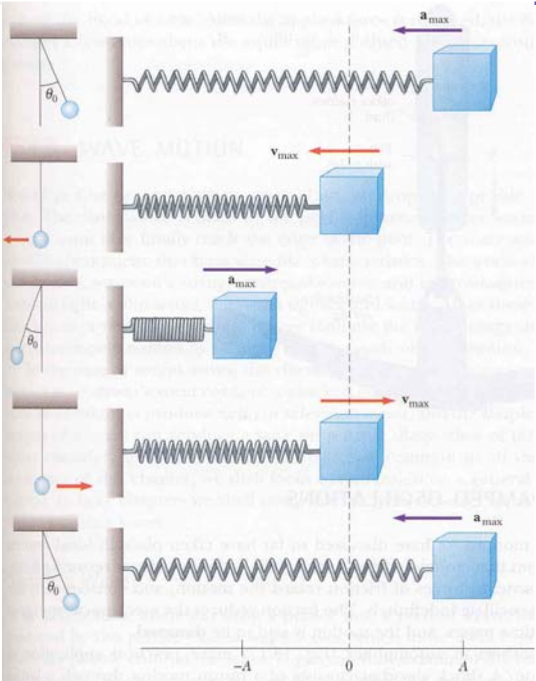
Recalling the period of a mass-spring system, we see that the period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

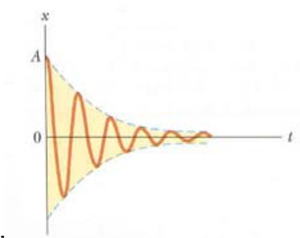
(Application: pendulum clocks, prospecting for oil)



Motion of a Pendulum ..

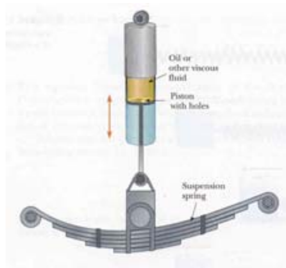


Damped Oscillations



The vibrating motions we have discussed so far have taken place in ideal systems — that is, systems that oscillate indefinitely.

In many real systems, forces of friction retard the motion, which reduces the mechanical energy of the system as time passes, and the motion is said to be **damped**. Damped motion varies depending on the friction forces. The amplitude of vibration decreases in time, and we say that the system has underdamped oscillation. If the friction force is increased the mass returns to equilibrium and does not oscillate. In this case, the system is said to be critically damped. If the friction force is made grater still, the system is said to be overdamped. In this case the time required to reach equilibrium is greater than at critical dampin.



Shock absorbers in automobiles make practical application of damped motion. The shock absorbers are designed to be slightly underdamped.

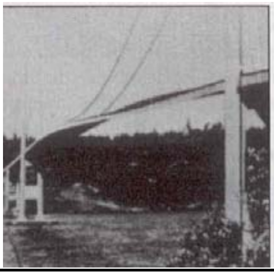
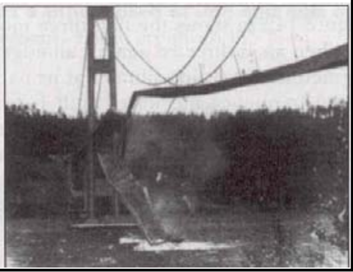
Resonance

We learned that the energy of a damped oscillator decreases in time because of friction. It is possible to compensate for this energy loss by applying an external force that does positive work on the system.

For example, suppose a mass-spring system, having some natural frequency of vibration, is pushed back and forth with a periodic force whose frequency is f. The system vibrates at the frequency of the driving force. This type of motion is referred to as a forced vibration. Its amplitude reaches a maximum when the frequency of the driving force equals the natural frequency of the system, called the resonant frequency of the system. Under this condition, the system is said to be in **resonance**.

Resonance vibrations occur in a wide variety of circumstances, as you can see on the figures.



Wave Motion

There are a wide variety of physical phenomena that have wave-like characteristics. The world is full of waves: sound waves, waves on strings, earthquake waves, electromagnetic waves. All of these waves have as their source a vibrating object.

Thus, we shall use the terminology and concepts of simple harmonic motion as we move into the study of wave motion. In the case of sound waves, the vibrations that produce waves arise from such source as a person's vocal cords or a plucked guitar string. The vibrations of electrons in an antenna produce radio or television waves.

For example, when we observe a water wave, what we see is a rearrangement of the water's surface. Without the water there would be no wave. A wave travelling on a string would not exist without the string. Sound waves travel through air as a result of pressure variations from point to point. Therefore, we can consider a wave to be the motion of a disturbance. (We will discuss later electromanetic waves which do not require a medium)

Mechanical waves require: a source of disturbance, a medium that can be disturbed, and physical mechanism through which adjacent portions of the medium can influence each other.

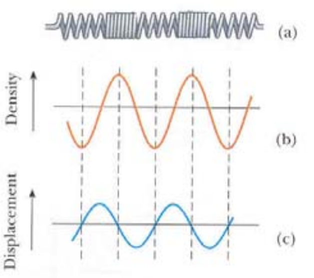
All waves carry energy and momentum.

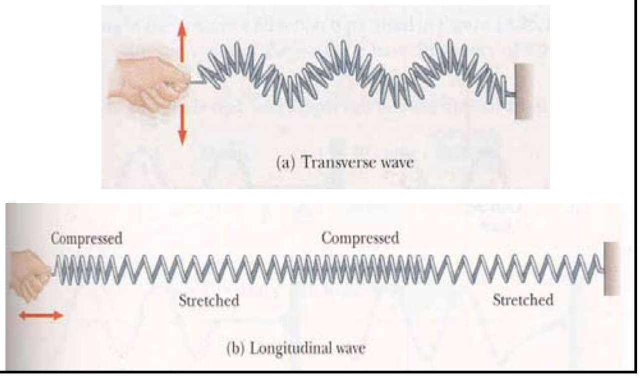
Types of Waves

One of the simplest ways to demonstrate wave motion is to flip one end of a long rope that is under tension. The bump (called a pulse) travels to thr right with a definite speed. A disturbance of this type is called a traveling wave.

As a pulse travels along the rope, each segment of the rope that is disturbed moves perpendicularly to the wave motion. A travelling wave such as this is called a **transverse wave**.

In another class of waves, called **longitudinal waves**, the particles of the medium undergo displacements parallel to the direction of wave motion. Sound waves in air, for instance, are longitudinal. Their disturbence corresponds to a series of high- and low-pressure regions that may travel through air or through any material with a certain speed.





(a) Transverse wave

(b) Longitudinal wave

Wavelength

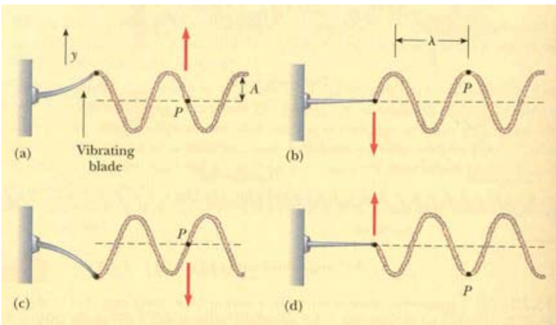
Figure illustrates a method of producing a wave on a very long string. One end of the string. Each particle of the string (such as P) oscillates vertically in the y direction with simple harmonic motion.

The maximum distance the string moves above or below the equilibrium value is called the amplitude, A, of the wave. For the waves we work with, the amplitudes at the crest and the trough will be identical.

Figure illustrates another characteristic of a wave. The distance between two successive point that behave identically is called **wavelength, λ**.

A wave will advance a distance of one wavelength λ in a time interval equal to one period T of the vibration. Thus,

$$v = \frac{\lambda}{T} = f\lambda$$



It is easy to understand why the wave speed depends on the tension in the string. If a string under tension is pulled sideways and released, the tension is responsible for accelerating a particular segment back toward its equilibrium position. The acceleration and wave speed increase with increasing tension in the string. Likewise, the wave speed is inversely dependent on the mass per unit length of the string. Thus, wave speed is directly dependent on the tension and inversely dependent on the mass per unit length. The exact relationship of the wave speed, v, the tension, F_T and the mass per per length, μ, is

$$v = \sqrt{\frac{F_T}{\mu}}$$

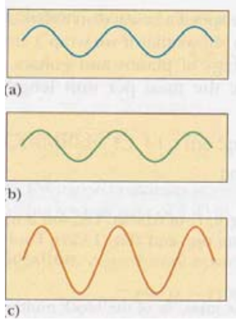
We san increase the speed of a wave on a streched string by increasing the tension in the string. If we wrap a string with a metalic winding, as is done to the bass strings of pianos and guitars, we decrease the speed of a transmitted wave.

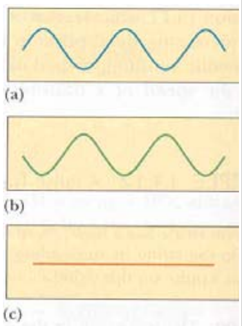
Many interesting wave phenomena in nature are impossible to describe with a single moving wave. Instead, one must analyze what happens when two or more waves pass through the same region of space. For such analyses one can use the superposition principle: if two or more traveling waves are moving through a medium, the resultant wave is found by adding together the displacement of the individual waves point by point.

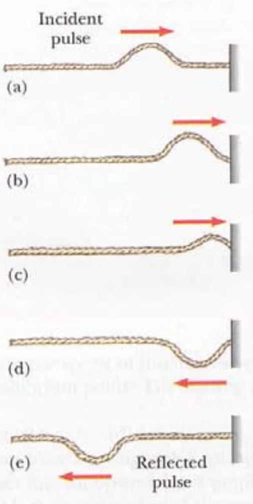
One consequence of the superposition principle is that two traveling waves can pass through each other without being destroyed or even altered.

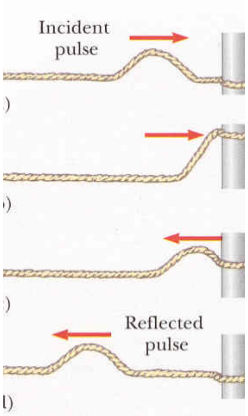
If two waves having the same frequency and amplitude are in phase, the resultant wave when they combine has the same frequency as the individual waves but twice their amplitude. Waves coming together like this are said to be in phase and to undergo constructive interference.

When two waves with the same frequency and amplitude are inverted (180° out of phase) one to the other, the result when they combine is complete cancellation. Waves undergo destructive interference.









Whenever a traveling wave reaches a boundary, part or all of the wave is reflected. For example, consider a pulse traveling on a string that is fixed at one end. When the pulse reaches the wall, it is reflected. Note, that the reflected pulse is inverted (according to Newtons's third law).

In a case in which the pulse arrives at the string's end, which is attached to a ring of negligible mass that is free. Pulse is reflected, but it is not inverted.

- A mass-spring system undergoes simple harmonic motion with an amplitude A. Does the total energy change if the mass is dubled but the amplitude is not changed? Are the kinetic and potential energies at a given point in its motion affected by the change in mass? Explain.

  NO! Because the total energy $E = \frac{1}{2}kA^2$, changing the mass while keeping A constant has no effect on the total energy. When the mass is at displacement x from equilibrium, the potential energy is $\frac{1}{2}kx^2$, independent of mass, and the kinetic energy is $E - \frac{1}{2}kx^2$. The larger mass must move slower to have the same kinetic energy. At the particular instant in time, both kinetic and potential energy would change as the mass is increased.

- A grandfather clock depends on the period of a pendulum to keep correct time. Suppose the clock is calibrated correctly and then the temperature of the room in which it resides increases. Does the clock run slow, fast, or correctly? (A metal expands when its temperature increases.)

  As the temperature increases, the length of the pendulum will increase. due to thermal expansion. With a longer length, the period of the pendulum will increase. Thus, it will take longer to exceute each swing, so that each second according to the clock will take longer than an actual second. Thus, the clock will run slow.

- What is the total distance traveled by a body executing simple harmonic motion in a time equal to its period if its amplitude is A?

  It traveles a distance of $4A$.

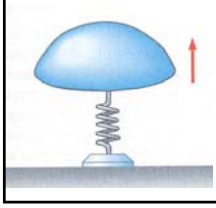
- Determine whether or not the following quantities can be in the same direction for a simple harmonic oscillator: displacement and velocity, velocity and acceleration, displacement and acceleration.

  There are times when both the displacement and the velocity are in the same direction.
  There are also times when the velocity and the acceleration are in the same direction.
  The displacement and the acceleration are always in opposite directions.

- A 0.4-kg mass is attached to a spring with a spring constant of 160 N/m so that the mass is allowed to move on a horizontal frictionless surface. The mass is released from rest when the spring is compressed 0.15 m. Find the force on the mass and its acceleration at this instant.

$$F = -kx = -(160 \text{ N/m})(0.15 \text{ m}) = -24 \text{ N}$$
$$a = \frac{F}{m} = \frac{-24 \text{ N}}{0.4 \text{ kg}} = -60 \text{ m/s}^2$$

- A child's toy consists of a piece of plastic attached to a spring. The spring is compressed against the floor a distance of 2 cm, and the toy is released. If the toy has a mass of 100 g and rises to a maximum height of 60 cm, estimate the spring constant.



$$\frac{1}{2}kx^2 = mgh$$
$$k = \frac{2mgh}{x^2} = \frac{2(0.1 \text{ m})(9.8 \text{ m/s}^2)(0.6 \text{ m})}{(0.02 \text{ m})^2} = 2.94 \cdot 10^3 \text{ N/m}$$

- A mass of 0.4 kg connected to a light spring with a spring constant of 19.6 N/m oscillates on a frictionless horizontal surface. If the spring is compressed 4 cm and released from rest, determine the maximum speed of the mass, the speed of the mass when the spring is compressed 1.5 cm, and the speed of the mass when the spring is streched 1.5 cm.

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)}$$
$$v_{max} = \sqrt{\frac{19.6 \text{ N/m}}{0.4 \text{ kg}}((0.04 \text{ m})^2 - 0^2)} = 0.28 \text{ m/s}$$
$$v_{com.} = \sqrt{\frac{19.6 \text{ N/m}}{0.4 \text{ kg}}((0.04 \text{ m})^2 - (-0.015 \text{ m})^2)} = 0.26 \text{ m/s}$$
$$v_{str.} = \sqrt{\frac{19.6 \text{ N/m}}{0.4 \text{ kg}}((0.04 \text{ m})^2 - (+0.015 \text{ m})^2)} = 0.26 \text{ m/s}$$

- The motion of an object is described by the equation

$$x = (0.3\ \text{m})\cos[(\frac{\pi}{3}\ \text{Hz})t]$$

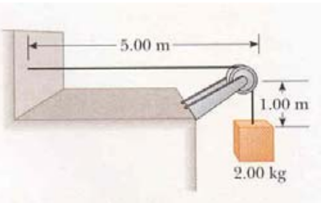
Find the position of the object at $t = 0$ and $t = 0.6$ s, the amplitude of the motion, the frequancy of the motion, and the period of the motion.

$$\begin{aligned} x &= A\cos(\omega t) \\ x(t = 0\ \text{s}) &= (0.3\ \text{m})\cos[(\frac{\pi}{3}\ \text{Hz})(0\ \text{s})] = 0.3\ \text{m} \\ x(t = 0.6\ \text{s}) &= (0.3\ \text{m})\cos[(\frac{\pi}{3}\ \text{Hz})(0.6\ \text{s})] = 0.24\ \text{m} \\ A &= 0.3\ \text{m} \\ \omega &= \frac{\pi}{3}\ \text{Hz} \\ f &= \frac{\omega}{2\pi} = \frac{\frac{\pi}{3}\ \text{Hz}}{2\pi} = \frac{1}{6}\ \text{Hz} \\ T &= \frac{1}{f} = \frac{1}{\frac{1}{6}\ \text{Hz}} = 6\ \text{s} \end{aligned}$$

- A simple 2-m-long pendulum oscillates in a location where g = 9.8 m/s^2. How many complete oscillations does it make in 5 min?

$$\begin{aligned} T &= 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{2\ \text{m}}{9.8\ \text{m/s}^2}} = 2.84\ \text{s} \\ n &= \frac{t}{T} = \frac{300\ \text{s}}{2.84\ \text{s}} = 105.6 \qquad \text{(105 complete oscilations)} \end{aligned}$$

- A uniform string has a mass of 0.3 kg and a length of 6 m. Tension is maintained in the spring by suspending a 2-kg block from one end. Find the speed of a pulse on this string.

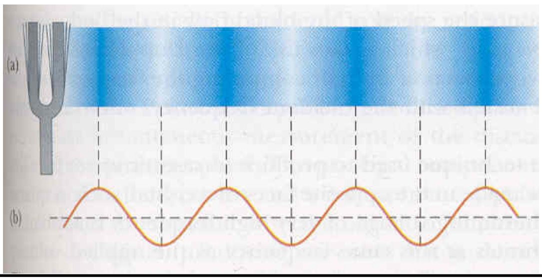


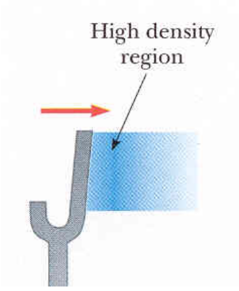
$$\begin{aligned} F_T &= mg = (2\ \text{kg})(9.8\ \text{m/s}^2) = 19.6\ \text{N} \\ \mu &= \frac{m_s}{l} = \frac{0.3\ \text{kg}}{6\ \text{m}} = 0.05\ \text{kg/m} \\ v &= \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{19.6\ \text{N}}{0.05\ \text{kg/m}}} = 19.8\ \text{m/s} \end{aligned}$$

Sound waves are the most important example of longitudinal waves. Any sound wave has its source in a vibrating object. Musical instruments produce sounds in a variety of ways. For example, the sound from a clarinet is produced by a vibrating reed, the sound from a piano by a vibrating strings, and the sound from a singer by vibrating vocal folds.

Sound waves are longitudinal waves traveling through a medium, such as air. In order to investigate how sound waves are produced, we focus our attention on the tuning fork, a common device for producing pure musical notes. A tuning fork consists of two metal prongs that vibrate when struck. Their vibration disturbs the air near them.

A region of high molecular density and high air pressure is called a compression. A region of lower than normal density is called a rarefaction. As the tuning fork vibrates, a series of condensations and rarefactions moves outward, away from the fork. The crests of the wave correspond to condensations, and the troughs to rarefactions.





General motion of air molecules near a vibrating object is back and forth between regions of compression and rarefaction. Back-and-forth molecular motion in the direction of the disturbance is characteristic of a longitudinal wave.

Sound waves fall into three categories covering different ranges of frequencies.

Audiable waves are longitudinal waves that lie within the range of sensitivity of the human ear, approximately 20 Hz to 20000 Hz.

Infrasonic waves are longitudinal waves with frequencies below the audible range. Earthquarke waves are an example.

Ultrasonic waves are longitudinal waves with frequencies above the audible range for humans. For example, certain types of whistles produce ultrasonic waves. Some animals, such as dogs, can hear the waves emitted by these whistles, even though humans cannot.

The speed of a sound wave in a liquid or gas depends on the medium's compressibility and inertia. If the fluid has a bulk modulus of B and an equilibrium density of ρ, the speed of sound is

$$v = \sqrt{\frac{B}{\rho}}$$

The speed of a longitudinal wave in a solid rode is

$$v = \sqrt{\frac{Y}{\rho}}$$

where Y is the Young's modulus of the solid, and ρ is the density of the solid.
The speed of sound also depends on the temperature of the medium. For example traveling through air, the relationship between the speed of sound and tepmerature θ in degrees Celsius is

$$v = (331 \text{ m/s})\sqrt{1 + \frac{\theta}{273}}$$

As the tines of a tuning fork move back and forth through the air, they exert a force on a layer of air and cause i to move. In other words, the tines do work on the layer of air.

We define the **intensity, I,** of a wave to be the rate at which energy flows through a unit area, A, perpendicularly to the direction of travel of the wave.

$$I = \frac{1}{A}\frac{\Delta E}{\Delta t}$$

It can be written in an alternative form

$$I = \frac{\text{power}}{\text{area}} = \frac{P}{A}$$

where P is the sound power passing through A. The intensity has units of watts per square meter.

The fintest sounds the human ear can detect at a frequency of 1000 Hz have intensity of about 10^{-12} W/m^2. This intensity is called the **threshold of hearing**. The laudest sounds the ear can tolerate have an intensity of about 1 W/m^2, which is called the **threshold of pain**.

The human ear can detect a wide range of intensities, with the loudest tolerable sounds having intensities about 10^{12} times greater than those of the faintest detectable sounds. However, the most intense sound is not perceived as being 10^{12} times louder than the faintest sound.

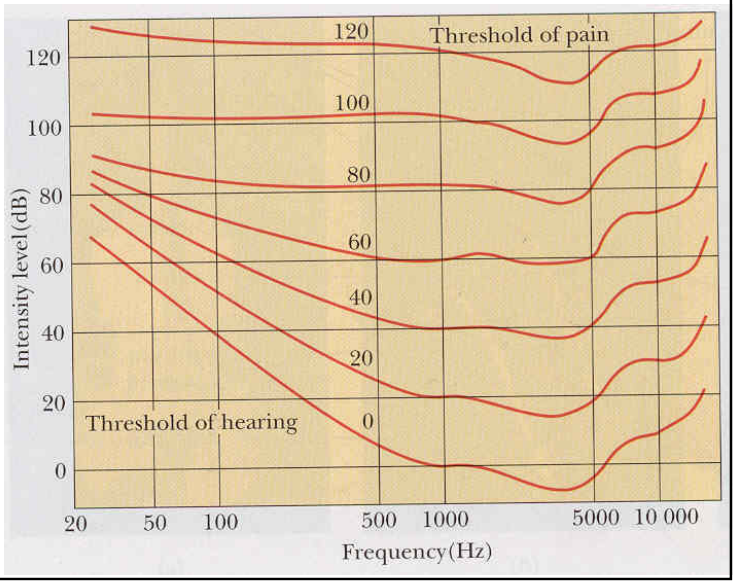
The relative intensity of a sound is called the **intensity level, β,** and is defined as

$$\beta = 10 \log \left(\frac{I}{I_0} \right)$$

where $I_0 = 10^{-12}\ \mathrm{W/m^2}$ is the reference intensity, and I is any intensity. β is measured in decibels (dB).

On this scale, the threshold of pain corresponds to an intensity level of $\beta = 120$ dB. Nearby jet airplanes can create intensity levels of 150 dB. The electronically amplified sound heard at rock concerts can be at levels of up to 120 dB, the threshold of pain. Recent evidence suggests that noice pollution may be contributing factor to high blood pressure, anxiety, and nervousness.

Figure shows the frequency response curves of an average human ear for sounds of equal loudness, ranging from 0 to 120 dB. To interpret this series of graphs, take the bottom curve as the threshold of hearing. The easiest frequencies to hear are around 3300 Hz.



If a car is moving while its horn is blowing, the frequency of the sound you hear is higher as the vehicle approaches you and lower as it moves away from you. This is one example of the Doppler effect. When the source and observer are moving toward each other, the observer hears a frequency higher than the frequency of the source in the absence of relative motion. When the source and observer are moving away from each other, the observer hears a frequency lower the source frequency. Doppler effect is a phenomenon common to all waves, not only to sound waves.

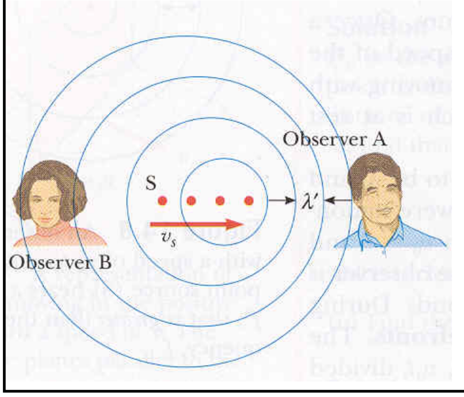
One finds the following general relationship for the observer frequency

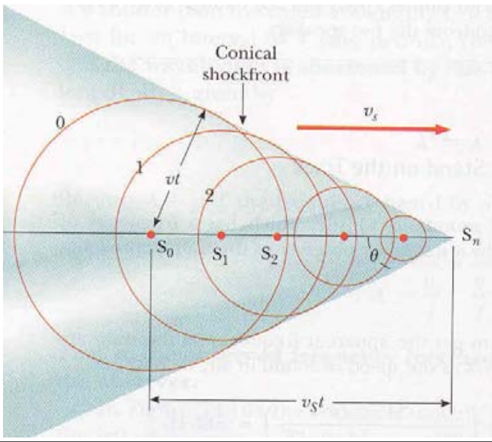
$$f_o = f_s \left(\frac{v \pm v_o}{v \mp v_s} \right)$$

The upper signs ($+v_o$ and $-v_s$) refer to motion of one toward the other, and the lower signs ($-v_o$ and $+v_s$) refer to motion of one away the other.

Shock waves are produced when the source speed exceeds the wave velocity. Figure describes this situation graphically. The circles represent spherical wavefronts emitted by the source at various times during its motion. The conical wavefront produced is known as a shock wave.

A shock wave carries a great deal of energy concentrated on the surface of the cone, with the great pressure variations. Shock waves are unplesent to hear and can damage buildings when, for example, aircraft fly supersonically at low altitudes.

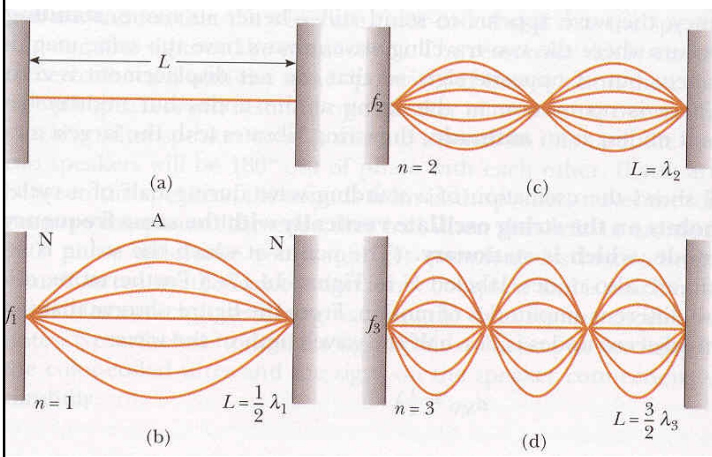




Standing waves can be set up in a streched string by connecting one end of the string to a stationary clamp and connecting the other end to a vibrating object. In this situations, traveling waves reflect from the ends, creating waves traveling in both directions on the string. The incident and reflected waves combine according to the superposition principle. If the string is vibrated at exactly the right frequency, the wave appears to stand. A node occurs where the two traveling waves always have the same magnitude of displacement but of opposite sign, so that the net displacement is zero at this point. But midway between two nodes, at an antinude, the string vibrates with the largest amplitude. Note, that the ends of the string must be nodes because these points are fixed. The characteristic frequencies of standing waves in a streched string of length L are

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} \qquad n = 1, 2, 3, \ldots$$

where F_T is the tension in the string, μ is its mass per unit length.



- Find the speed of the sound in water, which has a bulk modulus of about $2.1 \cdot 10^9$ Pa and a density of about 10^3 kg/m^3.

$$v_w = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.1 \cdot 10^9 \text{ Pa}}{10^3 \text{ kg/m}^3}} \approx 1500 \text{ m/s}$$

- If a solid bar is struck at one end with a hammer, a longitudinal pulse propagates down the bar. Find the speed of sound in a bar of aluminium, which has a Young's modulus of $7 \cdot 10^{10}$ Pa and a density of $2.7 \cdot 10^3$ kg/m^3.

$$v_{Al} = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7 \cdot 10^9 \text{ Pa}}{2.7 \cdot 10^3 \text{ kg/m}^3}} \approx 5100 \text{ m/s}$$

- Determine the intensity level of a sound wave with an intensity of $5 \cdot 10^{-7}$ W/m^2.

$$\beta = 10 \log\left(\frac{I}{I_0}\right) = 10 \log\left(\frac{5 \cdot 10^{-7} \text{ W/m}^2}{10^{-12} \text{ W/m}^2}\right) = 57 \text{ dB}$$

- A noise grinding machine in a factory produces a sound intensity of $1 \cdot 10^{-5}$ W/m^2. Find the intensity level of this machine, and calculate the new intensity level when a second, identical machine is added to the factory.

$$\beta_1 = 10 \log\left(\frac{I}{I_0}\right) = 10 \log\left(\frac{1 \cdot 10^{-5} \text{ W/m}^2}{10^{-12} \text{ W/m}^2}\right) = 70 \text{ dB}$$

$$\beta_2 = 10 \log\left(\frac{2I}{I_0}\right) = 10 \log\left(\frac{2 \cdot 10^{-5} \text{ W/m}^2}{10^{-12} \text{ W/m}^2}\right) = 73 \text{ dB}$$

- A train moving at a speed of 40 m/s sounds its whistle, which has a frequency of 500 Hz. Determine the frequency heard by a stationary observer as the train approaches the observer. (Take 340 m/s as the speed of sound in air.)

$$\begin{aligned} f_o &= f_s \frac{v}{v - v_s} \\ &= (500\ \text{Hz}) \frac{340\ \text{m/s}}{(340\ \text{m/s}) - (40\ \text{m/s})} = 567\ \text{Hz} \end{aligned}$$

Determine the frequency heard by the stationary observer as the train recedes from the observer.

$$\begin{aligned} f_o &= f_s \frac{v}{v + v_s} \\ &= (500\ \text{Hz}) \frac{340\ \text{m/s}}{(340\ \text{m/s}) + (40\ \text{m/s})} = 447\ \text{Hz} \end{aligned}$$

- An ambulance travels down a highway at a speed of 120 km/h, its siren emitting sound at a frequency of 400 Hz. What frequency is heard by a passenger in a car traveling at 90 km/h in the opposite direction as the car approaches?

$$\begin{aligned} v_s &= 120\ \text{km/h} = 33.3\ \text{m/s} \\ v_o &= 90\ \text{km/h} = 25.0\ \text{m/s} \\ f_o &= f_s \frac{v + v_o}{v - v_s} \\ &= (400\ \text{Hz}) \frac{340\ \text{m/s} + (25\ \text{m/s})}{(340\ \text{m/s}) - (33.3\ \text{m/s})} = 476\ \text{Hz} \end{aligned}$$

What frequency is heard as the car moves away from the ambulance?

$$\begin{aligned} f_o &= f_s \frac{v - v_o}{v + v_s} \\ &= (400\ \text{Hz}) \frac{340\ \text{m/s} - (25\ \text{m/s})}{(340\ \text{m/s}) + (33.3\ \text{m/s})} = 338\ \text{Hz} \end{aligned}$$

- Find the first four harmonics of a 1-m-long string if the string has a mass per unit length of $2 \cdot 10^{-3}$ kg/m and is under a tension of 80 N.

$$
\begin{aligned}
f_n &= \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} \qquad n = 1, 2, 3, 4 \\
f_1 &= \frac{1}{2(1\ \mathrm{m})}\sqrt{\frac{80\ \mathrm{N}}{2 \cdot 10^{-3}\ \mathrm{kg/m}}} = 100\ \mathrm{Hz} \\
f_2 &= 200\ \mathrm{Hz} \\
f_3 &= 300\ \mathrm{Hz} \\
f_4 &= 400\ \mathrm{Hz}
\end{aligned}
$$

We now move to a new branch of physics, thermal physics. We shall find that quantitative descriptions of thermal phenomena require careful definitions of the concepts of temperature, heat, and internal energy.

In order to understant the concept of temperature, it is useful to define thermal contact and thermal equilibrium.

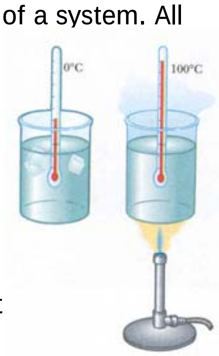
Two objects are in **thermal contact** with each other if energy can be exchanged between them. **Thermal equilibrium** is the situation in which two objects in thermal contact with each other case to have any exchange of energy.

Zeroth law of thermodynamics:

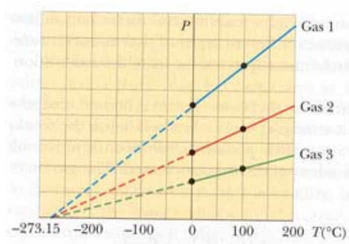
If bodies A and B are separately in thermal equilibrium with a third body, C, then A and B will be in thermal equilibrium with each other if placed in thermal contact.

This statement, insignificant and obvious as it may seem, is easily proved experimentally and is very important because it makes it possible to define temperature. We can think of temperature as the property that determines whether or not an object will be in thermal equilibrium with other objects. Two objects in thermal equilibrium with each other are at the same temperature.

Thermometers are devices used to measure the temperature of a system. All thermometers make use of a change in some physical property with temperature. One common thermometer in everyday use consists of a mass of liquid (usually mercury or alchocol) that expands into a glass capillary tube when heated. In this case the physical property is the change in volume of a liquid. The thermometer can be calibrated by placing it in thermal contact with some natural systems (E.g. mixture water and ice). It is defined to have a temperature of zero degrees Celsius (0°C). Another commonly used system is a mixture of water and steam at atmospheric pressure (100°C).



In a gas thermometer, the temperature readings are nearly independent of the type of gas used, so long as the gas pressure is low and the temperature is well above the point at which the gas liquifies. If the curves in Figure are extended back toward negative temperatures, in every case the pressure is zero when the temperature is -273.15°C. This significant temperature is used as the basis for the Kelvin scale, which sets –273.15°C as its zero point (0 K). The size of a kelvin is identical to the size of a degree on the Celsius scale: $T(K) = T(°C) + 273.15$



The phenomenon known as thermal expansion plays an important role in numerous applications. For example, thermal expansion joints must be included in buildings, concrete highways, and bridges to compensate for changes in demensions with temperature variations.

The overall thermal expansion of an object is a consequence of the change in the average separation between its constituent atoms or molecules. At ordinary temperatures, the atoms vibrate about their equilibrium positions with an amplitude of about 10^{-11} m, and the average spacing between the atoms is about 10^{-10} m. As the temperature of the solid increases, the atoms vibrate with greater amplitudes and the average separation between them increases. Consequently, the solid as a whole expands.

The length of some object increases by Δl for the change in temperature ΔT. Experiments show that when ΔT is small enough, Δl is proportional to ΔT and the initial length l_0 of the object

$$\Delta l = \alpha l_0 \Delta T$$

where α is called the average coefficient of linear expansion for a given material.

Because the linear dimensions of an object change with temperature, if follows that surface area and volume also change with temperature.

It is possible to get similar expression for change in the area A of an object

$$\Delta A = \gamma A_0 \Delta T$$

where the quantity $\gamma = 2\alpha$ is called the average coefficient of area expansion. A_0 is the initial area of the object.

Similarly, we can have it for change in volume of an object

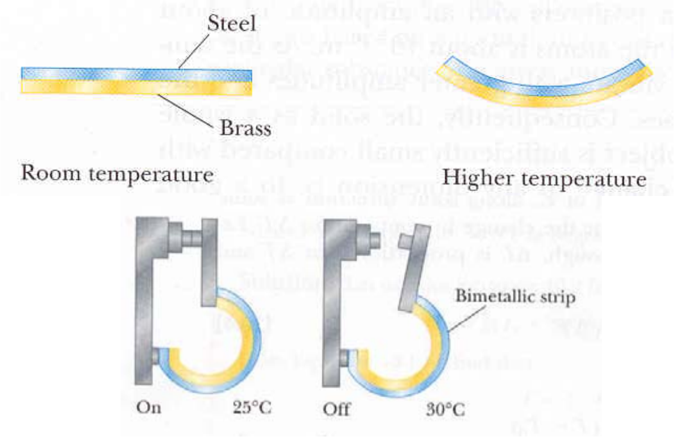
$$\Delta V = \beta V_0 \Delta T$$

where $\beta = 3\alpha$ is the average coefficient of volume expansion.

As table indicates, each substance has its own characteristic coefficients of expansion.

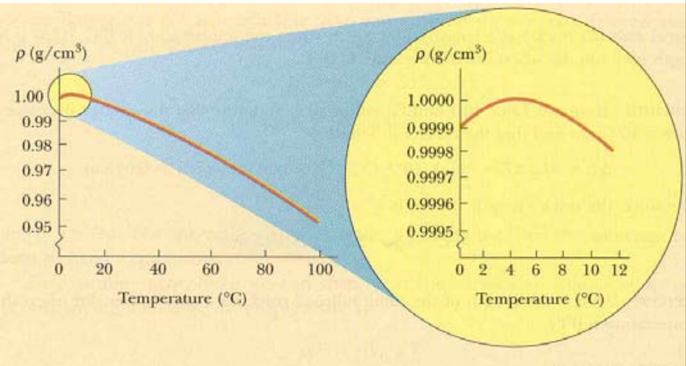
Material	Average Coefficient of Linear Expansion [(°C)$^{-1}$]	Material	Average Coefficient of Volume Expansion [(°C)$^{-1}$]
Aluminum	24×10^{-6}	Ethyl alcohol	1.12×10^{-4}
Brass and bronze	19×10^{-6}	Benzene	1.24×10^{-4}
Copper	17×10^{-6}	Acetone	1.5×10^{-4}
Glass (ordinary)	9×10^{-6}	Glycerin	4.85×10^{-4}
Glass (Pyrex®)	3.2×10^{-6}	Mercury	1.82×10^{-4}
Lead	29×10^{-6}	Turpentine	9.0×10^{-4}
Steel	11×10^{-6}	Gasoline	9.6×10^{-4}
Invar (Ni-Fe alloy)	0.9×10^{-6}	Air	3.67×10^{-3}
Concrete	12×10^{-6}	Helium	3.665×10^{-3}

- When the temperatures of a brass rod and a steel rod of equal length are raised by the same amount from some common initial value, the brass rod expands more than the steel rod because brass has a larger coefficient of expansion than steel. A simple device that utilizes this principle, called a bimetallic strip, is found in practical devices such as thermostats. The strip is made by securely bonding two different metals together. As the temperature of the strip increases, the two metals expand by different amounts and the strip bends, as in figure.



Liquids generally increase in volume with increasing temperature and have volume expansion coefficients about ten times greater than those of solids. Water is an exception to this rule, as we can see from its density-versus-temperature curve. As the temperature increases from 0°C to 4°C, water contracts and thus its density increases. Above 4°C, water expands with increasing temperature. The density of water reaches its maximum value 4 degrees above the freezing point.

When the atmospheric temperature is between 4°C and 0°C the surface water expands as it cools, becoming less dense than the water below it. The mixing process stops, and eventually the surface water freezes. As the water freezes, the ice remains on the surface because ice is less dense than water. The ice continues to build up on the surface, and water near the bottom of the pool remains at 4°C. If this did not happen, fish and other forms of marine life would not survive.



It is useful to know how temperature T, pressure p, volume V, and mass m of a gas are related. In general, the equation that interrelates these quantities, called the equation of state, is very complicated. However, if the gas is maintained at a very low pressure or low density (ideal gas), the equation of state is found experimentally to be quite simple.

It is convinient to express the amount of gas in a given volume in terms of the number of moles, n. Recall that one mole of any substance is that mass of the substance that contains Avogadro's number, $6.022 \cdot 10^{23}$, of molecules. The number of moles of a substance is related to its mass, m, as $n = m/M$, where M is the molar mass.

Equation of state for an ideal gas is

$$pV = nRT$$

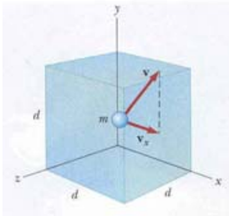
where R is the same for all quantities, called the universal gas constant $R = 8.31\ \mathrm{J/(mol\ K)}$.

We discusses the properties of an ideal gas, using such quantities as pressure, volume, number of moles, and temperature. We shall find that pressure and temperature can be understood on the basis of what is happening on the atomic scale. We use the kinetic theory of gases to show that the pressure a gas exerts on the walls of its container is a consequance of the collisions of the gas molecules with the walls.

We make the following assumptions of molecular model for an ideal gas:

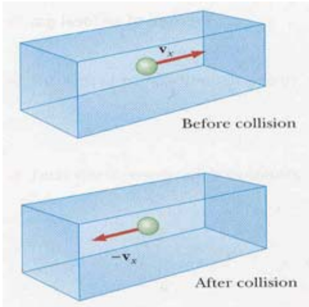
- The number of molecules is large, and the average separation between them is large compared with their dimensions. This means that the molecules occupy a negligible volume in the container.
- The molecules obey Newton's laws of motion, but as a whole they move randomly. Any molecule can move equally in any direction.
- The molecules undergo elastic collisions with each other and with the walls of the container. Thus, in the collisions kinetic energy is constant.
- The forces between molecules are negligible except during a collision.
- The gas under consideration is a pure substance. That is, all molecules are identical.

To derive an expression for the pressure of N molecules of an ideal gas in container of volume V, we suppose that the container is a cube with edges of length d. We focus our attention on one of these molecules, of mass m. As the molecule collides elastically with any wall, its velocity is reversed. Applying the Newton's second law one can find the pressure exerted on the wall:



$$
\begin{aligned}
p &= \frac{F}{A} = \frac{F}{d^2} = \frac{1}{3}\frac{N}{d^3} m\bar{v^2} = \frac{1}{3}\frac{N}{V}(m\bar{v^2}) \\
&= \frac{2}{3}\frac{N}{V}(\frac{1}{2}m\bar{v^2})
\end{aligned}
$$

This results shows that the pressure is proportional to the number of molecules per unit volume and to the average translational kinetic energy of the molecules, $(\frac{1}{2}m\bar{v^2})$. With this simplified model of an ideal gas, we have arrived at an important result that relates the macroscopic quantity of pressure to an atomic quantity, the average value of the translational kinetic energy. Thus, we have a key link between the atomic world and the large-scale world.



The expression for pressure we can write as

$$pV = \frac{2}{3}N(\frac{1}{2}m\bar{v^2})$$

and the equation of state for an ideal gas as:

$$pV = nRT = \frac{N}{N_A}RT = Nk_BT,$$

where we use alternative method for calculating the number of moles $n = N/N_A$ ($N_A = 6.02 \cdot 10^{23}$ molecules/mol is Avogadro's number). $k_B = \frac{R}{N_A} = 1.38 \cdot 10^{-23}$ J/K is Boltzmann's number. Equating the right sides of these expressions, we find that

$$T = \frac{2}{3k_B}(\frac{1}{2}m\bar{v^2}).$$

Temperature is a direct measure of average molecular kinetic energy. The total translational kinetic energy of N molecules of gas is

$$E = N(\frac{1}{2}m\bar{v^2}) = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$$

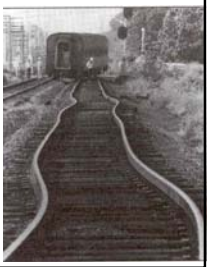
- What is the temperature 10^0C in kelvins?

$$T = 10^0\text{C} + 273.15\ \text{K} \approx 283\ \text{K}$$

- A pan of water is heated from 25^0C to 80^0C. What is the change in its temperature on the Kelvin scale and on Celsius scale?

$$\Delta T_C = 80^0\text{C} - 25^0\text{C} = 55^0\text{C}$$
$$\Delta T = 353\ \text{K} - 298\ \text{K} = 55\ \text{K}$$

- A steel railroad track has a length of 30 m when the temperature is 0^0C. What is its length on a hot day when the temperature is 40^0C?



$$\Delta l = \alpha l_0 \Delta T = [11 \cdot 10^{-6} (^0\text{C})^{-1}](30\ \text{m})(40^0\text{C}) = 0.013\ \text{m}$$
$$l = l_0 + \Delta l = 30\ \text{m} + 0.013\ \text{m} = 30.013\ \text{m}$$

- A hole of cross-cestion area 100 cm^2 is cut in a piece of steel at 20^0C. What is the area of the hole if the steel is heated from 20^0C to 100^0C?

A hole in a substance expands in exactly the same way as would a piece of the substance having the same shape as the hole.

$$\Delta A = \gamma A_0 \Delta T = [2 \cdot 11 \cdot 10^{-6} (^0\text{C})^{-1}](100\ \text{cm}^2)(80^0\text{C}) = 0.18\ \text{cm}^2$$
$$A = A_0 + \Delta A = 100\ \text{cm}^2 + 0.18\ \text{cm}^2 = 100.18\ \text{m}^2$$

- Verify that one mole of any gas at standard temperature (0^0C) and pressure ($1\ \text{atm} = 1.013 \cdot 10^5$ Pa) occupies a volume of 22.4 l.

$$V = \frac{nRT}{p} = \frac{(1\ \text{mol})(8.31\ \text{J}/(\text{mol K}))(273\ \text{K})}{1.013 \cdot 10^5\ \text{Pa}} = 22.4 \cdot 10^{-3}\ \text{m}^3 = 22.4\ \text{l}$$

- A ideal gas occupies a volume of 100 cm^3 at 20^{0}C and pressure of 10^5 Pa. Determine the number of moles of gas in the container.

$$n = \frac{pV}{RT} = \frac{(10^5\ \text{Pa})(100 \cdot 10^{-6}\ \text{m}^3)}{(8.31\ \text{J/(mol K)})(293\ \text{K})} = 4.1 \cdot 10^{-3}\ \text{mol}$$

- Pure helium gas is admitted into a leakproof cylinder containing a movable piston. The initial volume, pressure, and temperature are 15 l, 2 atm, and 300 K. If the volume is decreased to 12 l, and the pressure increased to 3.5 atm, find the final temperature of the gas. (Assume that helium behaves as an ideal gas.)
  Because no gas escapes from the cylinder, the number of moles remains constant. Therefore, use of $pV = nRT$ at the initial and final points gives

$$\frac{p_iV_i}{T_i} = \frac{p_fV_f}{T_f}$$

$$T_f = \frac{p_fV_f}{p_iV_i}T_i = \frac{(3.5\ \text{atm})(12\ \text{l})}{(2.0\ \text{atm})(15\ \text{l})}(300\ \text{K}) = 420\ \text{K}$$

- A tank contains 2 mol of helium gas at 20^{0}C. Assume that the helium behaves as an ideal gas. Find the total internal energy of the system.

$$E = \frac{3}{2}nRT = \frac{3}{2}(2\ \text{mol})(8.31\ \text{J/(mol K)})(293\ \text{K}) = 7.3 \cdot 10^3\ \text{J}$$

  What is the average kinetic energy per molecules?

$$\frac{1}{2}m\bar{v^2} = \frac{3}{2}k_BT = \frac{3}{2}(1.38 \cdot 10^{-23}\ \text{J/K})(293\ \text{K}) = 6.1 \cdot 10^{-21}\ \text{J}$$

- Two spheres are made of the same metal and have the same radius, but one is hollow and the other is solid. The spheres are taken through the same temperature increase. Which sphere expands more?
  A cavity in a material expands in exactly the same way as if the cavity were filled with material. Thus, both spheres will expand by the same amount.

- Common thermometers are made of a mercury column in a glass tube. Based on the operation of these common thermometers, which has the larger coefficient of linear expansion, glass or mercury? (Don't answer this by looking in a table.)

  Mercury must have the larger coefficient of expanssion. As the temperature of a thermometer rises, both the mercury and the glass expand. If they both had the same coefficient of linear expansion, the mercury and the cavity in the glass would both expand by the same amount, and there would be no apparent movement of the end of the mercury column relative to the calibration scale on the glass. If the glass expanded more than the mercury, the reading would go down as the temperature went up. (Now, we can look in a table and find that the coefficient for mercury is about 20 times as large as for glass, so that the expanssion of the glass can sometimes be ignored.)

- Some picknickers stop at a store to buy food, including bags of potato chips. They drive up into the mountains to their picnic site. When they unload the food, they notice that the chip bags are puffed up like balloons. Why did this happen?

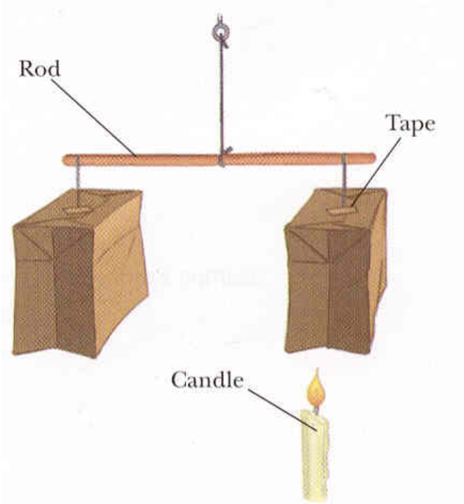
  The chip bags contain a sealed sample of air. When the bags are taken up the mountain, the external atmospheric pressure on the bags is reduced. As a result, the difference between the pressure of the air inside the bags and the reduced pressure outside results in a net force pushing the plastic of the bag outward.

- A gold ring has an inner diameter of 2.168 cm at a temperature of 15^0C. Determine its inner diameter at 100^0C. ($\alpha_{\rm gold} = 1.42 \cdot 10^{-5} (^0{\rm C})^{-1}$)

$$\begin{aligned}\Delta d &= \alpha d_0 \Delta T = (1.42 \cdot 10^{-5} (^0{\rm C})^{-1})(2.168 \text{ cm})(85^0{\rm C}) \\ &= 2.6 \cdot 10^{-3} \text{ cm}\end{aligned}$$

$$\begin{aligned}d &= d_0 + \Delta d = (2.168 \text{ cm}) + (2.6 \cdot 10^{-3} \text{ cm}) \\ &= 2.171 \text{ cm}\end{aligned}$$

- Tie and tape two inverted empty paper bags to the ends of a rod as in the figure. Balance the setup. Then place a candile under one of the bags and note what happens. Why does this system become unbalanced? What do your results tell you concerning the density of warm air versus the density of cold air?



Heat is defined as energy that is transfered between a system and its environment because of a temperature difference between them. The SI unit of heat is the same as for energy, joule, (J).
Because of early misunderstanding about heat the unusual units in which heat was measured had already been developed. One of the most widely used is the calorie (cal), defined as the heat required to raise the temperature of 1 g of water from 14.5^0C to 15.5^0C. A related unit is the kilocalorie (kcal), 1 kcal = 1000 cal. Heat is most often measured in joules 1 cal = 4.186 J.

Example: A student eats a dinner rated at 2000 kcal. He wishes to do an equivalent amount of work by lifting a 50-kg mass. How many times must he raise the weight to expend this much energy? Assume that he raises the weight a distance of 2 m each time and that no work is done when the weight is dropped to the floor.

The work done in lifting the weight n times is $W = nmgh$. Thus,

$$n = \frac{W}{mgh} = \frac{2 \cdot 10^6 \cdot 4.186 \text{ J}}{(50 \text{ kg})(9.8 \text{ m/s}^2)(2 \text{ m})} = 8540 \text{ times}$$

(It assumes perfect conversion of chemical energy into mechnical.)

The quantity of heat energy required to raise the temperature of a given mass of a substance by some amount varies from one subatsnce to another. For example, the heat required to raise the temperature of 1 kg of water by 1^0C is 4186 J, but for copper is only 387 J. Every substance has a unique value for the amount of heat required to change the temperature of 1 kg of it by 1^0C.

Suppose that a quantity, Q, of heat is transfered to a substance of mass m, thereby changing its temperature by ΔT. The **specific heat,** c, of the substance is defined as

$$c = \frac{Q}{m\Delta T}$$

From this definition we can express the heat transferred between a system of mass m and its surroundings for the temperature change of ΔT as

$$Q = mc\Delta T$$

When ΔT and Q are negative, heat flows out of the system.

Specific Heats of Some Materials at Atmospheric Pressure

Substance	J/kg·°C
Aluminum	900
Beryllium	1820
Cadmium	230
Copper	387
Germanium	322
Glass	837
Gold	129
Ice	2090
Iron	448
Lead	128
Mercury	138
Silicon	703
Silver	234
Steam	2010
Water	4186

The fact that the specific heat of water is higher than that of land is responsible for the pattern of air flow at a beach. During the day, the Sun adds roughly equal amounts of energy to beach and water, but the lower specific heat of sand causes the beach to reach a higher temperature than the water. As a consequence, the air above the land reaches a higher temperature than that over the water, and cooler air from above the water is drawn in to displace this rising hot air, resulting in a breeze from water to land during the day. Because the hot air gradually cools as it rises, it subsequently sinks, setting up the circulating pattern shown in figure. During the night, the land cools more quickly than the water, and the circulating pattern reverses itself because the hotter air is now over the water. The offshore and onshore breezes are certainly well known to sailors.

A similar effect produces rising layers of air, called thermals, that can help eagles to soar higher and hang gliders to stay in flight longer.

Situations in which mechanical energy is converted to thermal energy occur frequently. In problems using the procedure called calorimetry, only the transfer of thermal energy between the system and its surroundings is considered.

One technique for measuring the specific heat of a solid or liquid is simply to heat the substance to some known temperature, place it in a vessel containing water of known mass and temperature, and measure the temperature of the water after equilibrium is reached.

Suppose that m_x is the mass of a substance whose specific heat we wish to measure, c_x its specific heat, and T_x its initial temperature. Let m_w, c_w, and T_w represent the corresponding values for the water. If T is the final equilibrium temperature after everything is mixed. The heat gained by the water must equal the heat lost by the substance (conservation of energy)

$$m_w c_w (T - T_w) = m_x c_x (T_x - T)$$

Solving it, one can have specific heat c_x of a substance.

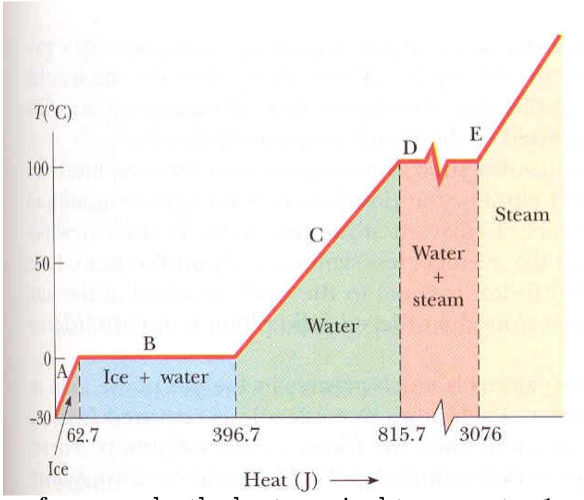
A substance usually undergoes a change in temperature when heat is transferred between it and its surroundings. There are situations, however, in which the flow of heat does not result in a change in temperature. This is a case whenever the substance undergoes a physical alteration from one form to another, referred to as a **phase change**. Some common phase changes are solid to liquid (melting), liquid to gas (boiling), and a change in crystalline structure of a solid. Every phase change involves a change in internal energy.
The heat required to change the phase of a given mass, m, of a pure substance is

$$Q = mL$$

where L is called the **latent heat** of the substance and depends on the nature of the phase change as well as on the properties of the substance. **Latent heat of fusion, L_f**, is the term used when the phase change is from solid to liquid, and **latent heat of vaporization, L_v**, is the term used when the phase change is from liquid to gas. For example, the latent heat of fusion for water at atmospheric pressure is $3.33 \cdot 10^5$ J/kg, and the latent heat of vaporization for water is $2.26 \cdot 10^6$ J/kg. A latent heats of different substances vary considerably.

Latent heat of fusion and vaporization

Substance	Melting Point (°C)	Latent Heat of Fusion J/kg	(cal/g)	Boiling Point (°C)	Latent Heat of Vaporization J/kg	(cal/g)
Helium	−269.65	5.23×10^3	(1.25)	−268.93	2.09×10^4	(4.99
Nitrogen	−209.97	2.55×10^4	(6.09)	−195.81	2.01×10^5	(48.0)
Oxygen	−218.79	1.38×10^4	(3.30)	−182.97	2.13×10^5	(50.9)
Ethyl alcohol	−114	1.04×10^5	(24.9)	78	8.54×10^5	(204)
Water	0.00	3.33×10^5	(79.7)	100.00	2.26×10^6	(540)
Sulfur	119	3.81×10^4	(9.10)	444.60	3.26×10^5	(77.9)
Lead	327.3	2.45×10^4	(5.85)	1750	8.70×10^5	(208)
Aluminum	660	3.97×10^5	(94.8)	2450	1.14×10^7	(2720)
Silver	960.80	8.82×10^4	(21.1)	2193	2.33×10^6	(558)
Gold	1063.00	6.44×10^4	(15.4)	2660	1.58×10^6	(377)
Copper	1083	1.34×10^5	(32.0)	1187	5.06×10^6	(1210)



Consider, for example, the heat required to convert a 1-g block of ice at -30^0C to steam (water vapor) at 120^0C. Figure indicates the experimental results.

Part A: The ice is changing from -30^0C to 0^0C. The heat added is

$$Q_A = m_i c_i \Delta T = (10^{-3}\ \text{kg})(2090\ \text{J/kg}^0\text{C})(30^0\text{C}) = 62.7\ \text{J}$$

B: The ice-water mixture remains at 0^0C (even though heat is being added) until all the ice melts to become water at 0^0C. The heat is

$$Q_B = mL_f = (10^{-3}\ \text{kg})(3.33 \cdot 10^5\ \text{J/kg}) = 333\ \text{J}$$

C: The heat is being used to increase temperature of the water

$$Q_C = m_w c_w \Delta T = (10^{-3}\ \text{kg})(4.190 \cdot 10^3\ \text{J/kg}^0\text{C})(100^0\text{C}) = 419\ \text{J}$$

D: At 100^0C, another phase change occurs (water to steam). Just as in Part B, the heat required for that is

$$Q_D = mL_V = (10^{-3}\ \text{kg})(2.26 \cdot 10^6\ \text{J/kg}) = 2260\ \text{J}$$

E: The heat is being used to increase temperature of the steam is

$$Q_E = m_w c_w \Delta T = (10^{-3}\ \text{kg})(2.01 \cdot 10^3\ \text{J/kg}^0\text{C})(20^0\text{C}) = 40.2\ \text{J}$$

The total heat is therefore 3115 J. Conversely, to cool one gram of steam at 120^0C down to -30^0C, we must remove about 3115 J of heat.

Phase changes can be described in terms of rearrangements of molecules when heat is added to or removed from a substance. Consider first the liquid-gas phase change. The molecules in a liquid are close together, and the forces between them are stronger than those between the more widely separated molecules of a gas. Therefore, work must be done on the liquid against these attractive molecular forces in order to separate the molecules. The latent heat of vaporization is the amount of energy that must be added to the liquid to accomplish this.

Similarly, at the melting point of a solid, we imagine that the amplitude of vibration of the atoms about their equilibrium positions becomes great enough to allow the atoms to pass the barriers of adjacent atoms and move to their new positions. The new locations are, on the average, less symmetrical and therefore have higher energy. The latent heat of fusion is equal to the work required at the molecular level to transform the mass from the ordered solid phase to the disordered liquid phase.

The average distance between atoms is much greater in the gas phase than in either the liquid or the solid phase. Each atoms or molecule is removed from its neighbors, without the compensation of attractive forces to new neighbors. Therefore, it is not surprising that more work is required at the molecular level to vaporize a given mass of substance than to melt it. Thus the latent heat of vaporization is much greater than the latent heat of fusion (see Table: Latent Heat).

There are three ways in which heat energy can be transferred from one location to another: **conduction, convection, and radiation**. Regardless of the process, however, no net heat transfer takes place between a system and its surroundings when the two are at the same temperature.

Each of the methods of heat transfer can be examined by considering the ways in which you can warm your hands over an open fire. If you insert a copper rod into flame, the temperature of the metal in your hand increases rapidly. **Conduction**, the process by which heat is transferred from the flame through the copper rod to your hand, can be understood by examining what is happening to the atoms of the metal. As the flame heats the rod, the copper atoms near the flame begin to vibrate with greater and greater amplitudes. These vibrating atoms collide with their neighbors and transfer some of their energy in the collisions.

The rate of heat conduction depends on the properties of the substance being heated. Metals are good conductors of heat because they contain large numbers of electrons that are relatively free to move through the metal and transport energy from one region to another. In these conductors heat conduction takes place both via the vibration of atoms and via the motions of free electrons.

Heat Transfer Rate

If Q is the amount of heat transferred from one location on an object to another in the time Δt, the **heat transfer rate, H** is defined as

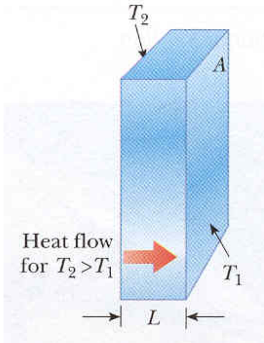
$$H = \frac{Q}{\Delta t}$$

Note that H is expressed in watts.

The conduction of heat occurs only if a difference in temperature exists between two parts of the conducting medium. Consider a slab of thickness L and cross-sectional area A. Suppose that one face is maintained at a temperature of T_2 and the other face is held at a lower temperature, T_1. The rate of flow of heat is given by

$$H = kA\frac{T_2 - T_1}{L}$$

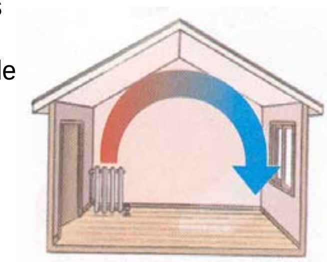
where k is a constant called the **thermal conductivity** of the material.



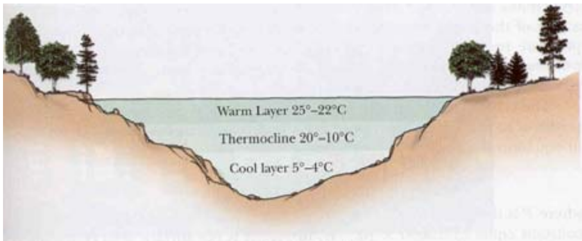
Heat Transfer by Convection

The air directly above the flame is heated and expands. As a result, the density of the air decreases and the air rises. This warmed mass of air heats your hands as it flows by. Heat transferred by the movement of a heated substance is said to have been transferred by **convection**. When the movement results from differences in density, as it does in air around a fire, it is referred to as natural convection. When the heated substance is forced to move by a fan or pump, as in some heating systems, the process is called forced convection.

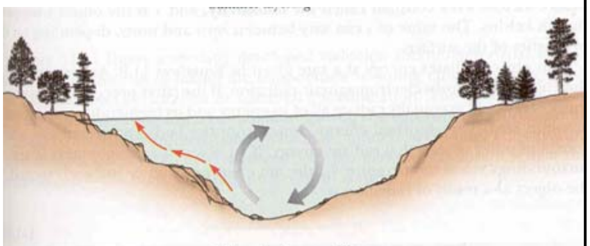
The circulating pattern of air flow at a beach is an example of convection. Convection process occurs when a room is heated by a radiator. The warm air expands and rises to the ceiling because of its lower density. The denser regions of cooler air from above replace the warm air. Your automobile engine is maintained at a safe operating temperature by a combination of conduction and forced convection.



The algal blooms often seen in temperate lakes and ponds during spring or autumn are caused by convection currents in the water. During the summer, bodies of water develop temperature gradients such that an upper warm layer of water is separated from a lower cold layer by a buffer zone called a thermocline. In the spring or autumn, the temperatures changes in the water break down this thermocline, setting up convection currents that mix the water. This mixing process transports nutrients from the bottom to the surface. The nutrient-rich water forming at the surface can cause a rapid, temporary increase in the population of algae.



Summer layering of water


Fall and spring upwelling

The third way of transferring heat is through **radiation**. You have must likely experienced radiant heat when sitting in front of a fireplace. The hands that are placed to one side of the flame are not in physical contact with flame, and therefore conduction cannot account for the heat transfer. Furthermore, convection is not important in this situation because the hands are not above the flame in the path of convection currents. The important process in this case is the radiation of heat energy.

All objects continuously radiate energy in the form of electromegnetic waves, which we shall discuss later. Electomagnetic radiation associated with the loss of heat energy from an object at a temperature of a few hundred kelvins is referred to as infrared radiation.

The surface of the Sun is at a few thousand kelvins and most strongly radiates visible light. Approximately 1340 J of sunlight energy strikes 1 m^2 of the top of the Earth's atmosphere every second. Some of this energy is reflected back into space, and some is absorbed by the atmosphere, but enough arrives at the surface of the Earth.

The rate at which an object emits radiant energy is proportional to the fourth power of its absolute temperature. This is known as **Stefan's law** and is expressed as

$$P = \sigma A e T^4$$

where P is power radiated by the object in watts, $\sigma = 5.67 \cdot 10^{-8}$ $\mathrm{W/m^2K^4}$ is a constant, A is the surface area of the object, e is a constant called the **emissivity**, and T is the object's temperature. The value of e can vary between 0 and 1, depending on the properties of the surface.

An object radiates energy, and at the same time the object also absorbs electromagnetic radiation. When an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate, and so its temperature remains constant.

An ideal absorber is defined as an object that absorbs all of the energy incident on it. Its emissivity is equal to 1. Suach an object is called **black body**. An ideal obsorber is also an ideal radiator of energy. In contrast, an object with an emissivity equal to zero reflects all the incident energy and so is perfect reflector.

During the day, sunlight passes into the greenhouse and is absorbed by the walls, earth, and plants. This absorbed visible light is subsequently re-radiated as infrared radiation, which causes the temperature of the interior to rise. In addition, convection currents are inhibited in a greenhouse.

A phenomenon known as the greenhouse effect can also play a major role in determining the Earth's temperature. Earth's atmosphere is a good transmitter of visible radiation and a good absorber of infrared radiation. Carbon dioxide in the Earth's atmosphere allows incoming visible radiation from the Sun to pass through more easily than infrared radiation. The visible light that reaches the Earth's surface is absorbed and re-radiated as infrared light, which in turn is absorbed by the Earth's atmosphere.

At present, about 350 billions tons of carbon dioxide are released into the atmosphere each year. Most of this gas results from human activities such as the burning of fosil fuels, the cutting of forests, and manufacturing processes. Other greenhouse gases are also increasing in concentration in the atmoshpere. One of these is methane (cows and termites are major producers), nitrous oxide, and sulfur dioxide (automobile and industrial pollution).

Whether the increasing greenhouse gases are responsible or not, there is convincing evidence that global warming is certainly underway.

- A 0.05-kg ingot of metal is heated to 200^{0}C and then dropped into a beaker containing 0.4-kg of water that is initially at 20^{0}C. If the final equilibrium temperature of the mixed system is 22.4^{0}C, find the specific heat of the metal.

$$m_x c_x (T_x - T) = m_w c_w (T - T_w)$$
$$c_x = \frac{(0.4\text{ kg})(4186\text{ J/kg}^0\text{C})(22.4^0\text{C} - 20^0\text{C})}{(0.05\text{ kg})(200^0\text{C} - 22.4^0\text{C})}$$
$$= 453\text{ J/kg}^0\text{C}$$

The ingot is most likely iron.

- If 10 W of power is supplied to 1 kg of water at 100^{0}C, how long will it take for the water to completely boil away?

$$t = \frac{Q}{P} = \frac{mL_v}{P}$$
$$= \frac{(1\text{ kg})(2.26 \cdot 10^6\text{ J/kg})}{10\text{ W}}$$
$$= 2.26 \cdot 10^5\text{ s} = 62.8\text{ h}$$

- Find the amount of heat transferred in 1 h by conduction through a concrete wall 2 m high, 3.65 m long, and 0.2 m thick if one side of the wall is held at 20^{0}C and the other side is at 5^{0}C.

$$A = (2\text{ m})(3.65\text{ m}) = 7.3\text{ m}$$
$$\frac{Q}{\Delta t} = kA\frac{T_2 - T_1}{L}$$
$$Q = (1.3\text{ J/sm}^0\text{C})(7.3\text{ m}^2)(3600\text{ s})\frac{15^0\text{C}}{0.2\text{ m}} = 2.6 \cdot 10^6\text{ J}$$

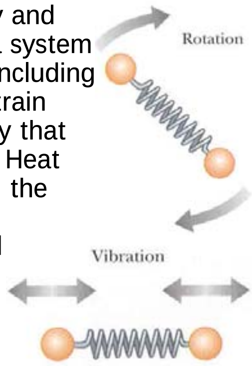
- A tile floor may feel uncomfortable cold to your bare feet, but a carpeted floor in an adjoining room at the same temperature feels warm. Why?

  The tile is a better conductor of heat than carpet. Thus, energy is conducted away from your feet more rapidly by the tile than by the carpeted floor.

- A solar collector is thermally insulated, so conduction is negligible in comparison with radiation. On a cold but sunny day the temperature outside is -20^0C, and the Sun irradiates the collector with a power per unit area of 300 W/m^2. Treating the collector as a black body (emissivity = 1), determine its interior temperature after the collector has achieved a steady-state condition (radiating energy as fast as it is received).

$$\begin{aligned} P_{absorbs} &= P_{radiates} \\ P_{Sun} + \sigma A e T_0^4 &= \sigma A e T_c^4 \\ T_c^4 &= T_0^4 + \frac{P_{Sun}}{\sigma A e} \\ &= (253 \text{ K})^4 + \frac{(300 \text{ W/m}^2)A}{(5.67 \cdot 10^{-8} \text{ W/m}^2\text{K}^4)A(1)} \\ &= 9.4 \cdot 10^9 \text{ K}^4 \\ T_c &= 311 \text{ K} = 38^0\text{C} \end{aligned}$$

A major distinction must be made between internal energy and heat. Internal energy is all of the energy belonging to a system while it is stationary (neither translating nor rotating), including heat as well as nuclear energy, chemical energy, and strain energy. Thermal energy is the portion of internal energy that changes when the temperature of the system changes. Heat transfer is caused by a temperature difference between the system and its surrundings. We shawed that the thermal energy of a monoatomic ideal gas is associated with the motion of its atoms. In this special case, the thermal energy is simply kinetic energy on a microscopic scale. In general, thermal energy includes other forms of molecular energy, such as rotational energy and vibrational kinetic and potential energy.



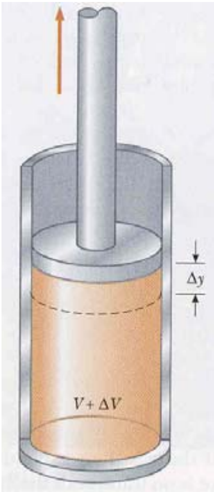
The work done on (or by) a system is a measure of the energy transferred between the system and its surroundings. When a person does work on a system, energy is transferred from the person to the system. It makes no sense to talk about the work of a system – one shuld refer only to the work done on or by a system when some process has occurred in which energy has been transferred to or from the system. Likewise, it makes no sense to use the term heat unless energy has been transferred as a result of a temperature difference.

The state of a system can be described with the use of such variables as pressure, volume, temperature, and internal energy. The number of macroscopic variables needed to characterize a system depends on the system's nature. For a homogeneous system, usually only two variables are needed. It is important to note that a macroscopic state of an isolated system can be specified only if the system is in thermal equilibrium internally.

Consider a gas contained by a cylinder fitted with a movable piston. In equilibrium, the gas occupies a volume of V and exerts a uniform pressure, p, on the cylinder walls and piston. If the piston has cross-sectional area A, the force exerted by the gas on the piston is $F = pA$. As the piston moves up a distance of Δy, the work done on the piston by the gas is

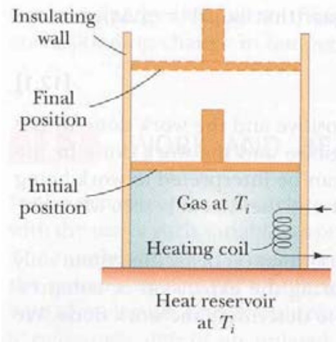
$$W = F\Delta y = pA\Delta y = p\Delta V$$

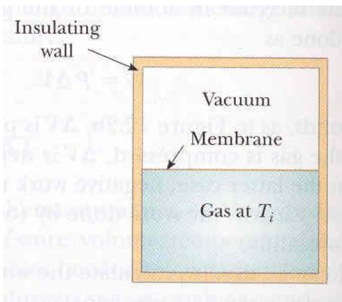
The work done by the system is zero when the volume remains constant. The work done by a system depends on the process by which the system goes from the initial to the final state.



The heat transferred into or out of a system is also found to depend on the process. This can be demonstrated by the situations depicted in figures. In each case, the gas has the same initial volume, temperature, and pressure, and is assumed to be ideal. In one figure, the gas expands slowly by absorbing heat from a reservoir at the same temperature. In other figure, the gas expands rapidly into an evacuated region after a membrane separating it from that region is broken. In both cases temperatures remain constant.

The initial and final states of the ideal gas in one figure are identical to the initial and final states in other figure, but processes are different. In the first case, heat is transfered slowly to the gas, and the gas does work on the piston. In the second case, no heat is transferred, and the work done is zero. Therefore, we conclude that heat transfer, like work, depends on the initial, final, and intermediate states of the system.





When we principle of conservation of energy was first introduced, it was started that the mechanical energy of a system is constant in the absence of nonconservative forces, such as friction. That mechanical model did not encompass changes in the internal energy of the system. We now broaden our scope for all kind of processes.

For system that undergoes a change from an initial state to a final state, one can find that $Q - W$ is the same for all processes connecting the initial and final states. We conclude that the quantity $Q - W$ is determined completely by the initial and final states of the system, and we call it the change in the internal energy of the system. If we represent the internal energy function with U, than the change in internal energy, $\Delta U = U_f - U_I$, can be expressed as

$$\Delta U = Q - W$$

This equation is known as **the first law of thermodynamics**.

For an isolated system, no heat transfer takes place and the work done is zero. Hence, the internal energy remains constant.

For a cyclic process (originates and ends at the same state) the change in the internal energy must again be zero. Therefore, the heat added to the system must equal the work done during the cycle.

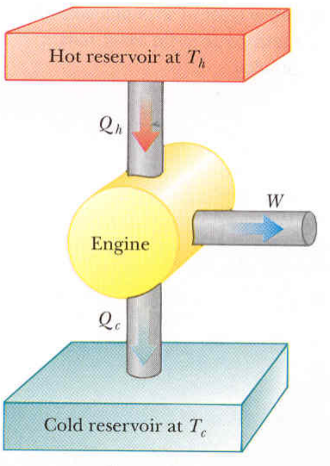
A heat engine is a device that converts thermal energy to other useful forms, such as mechanical energy. A heat engine carries some working substance through cyclic process during which (1) heat is absorbed from a source at high temperature, (2) work is done by the engine, and (3) heat is expelled by the engine to a reservoir at a lower temperature. The engine absorbs a quantity of heat Q_h, does a work W, and gives heat Q_c to the cold reservoir. Because the working substance goes through cycle, the work W done equals the net heat flowing into it, $Q_h - Q_c$

$$W = Q_h - Q_c$$

The thermal efficiency, e, of a heat engine is the ratio of the net work done to the heat absorbed at the higher temperature during one cycle

$$e = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

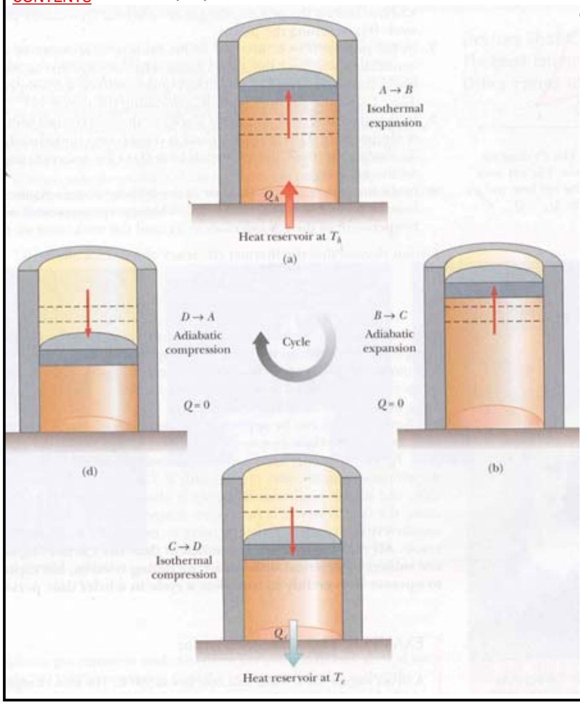
The **second law of thermodynamics** can be stated as follows: It is impossible to convert a heat engine that, operating in a cycle, produces no other effect than the absorption of heat from a reservoir and the preformance of an equal amount of work.



A reversible process is one in which every state between the initial and final states is an equilibrium state, and that can be reversed in order to be followed exactly from the final state back to the initial state. A process that does not satisfy these requirements is irreversible.

All natural processes are known to be irreversible. But some are almost reversible. If a real process occurs very slowly so that the system is virtually always in equilibrium, the process can be considered reversible.

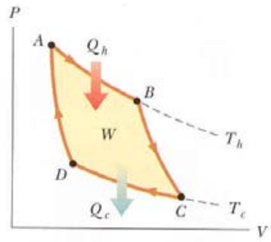
A general characteristic of a reversible process is that no dissipative effects that convert mechanical energy to thermal energy, such as turbulence or friction, can be present. In reality, such effects are impossible to eliminate completely, and hence it is not surprising that real processes in nature are irreversible.



Sadi Carnot showed that a heat engine operating in an ideal, reversible cycle (called Carnot cycle) between two reservoirs is the most efficient engine possible.

No real engine operating between two heat reservoirs can be more efficient than a Carnot engine, operating between the same two reservoirs.

The cycle consists of two adiabatic and two isothermal processes, all reversible.



(1) First process is an isothermal expansion at temperature T_h. During the process, the gas absorbs heat Q_h from the reservoir and does work W_{AB} in raising the pistion. **(2)** Than the gas expands adiabatically; that is no heat enters or leaves the system. During the process, the temperature falls from T_h to T_c, and gas does work W_{BC} in raising the piston. **(3)** Next, the gas is placed in thermal contact with a heat reservoir at temperature T_c and is compressed isothermally at temperature T_c. The gas expels heat Q_c to the reservoir, and the work done on the gas is W_{CD}. **(4)** In the final stage, the gas is compressed adiabatically. The temperature of the gas increases to T_h and the work done on the gas is W_{DA}.

Thermal efficiency of a Carnot engine is

$$e_c = 1 - \frac{T_c}{T_h}$$

All real engines are less efficient than the Carnot engine because they are subject to practical difficulties, including friction, but especially the need to operate irreversibly to complete a cycle in a brief time period.

The concept of temperature is involved in the zeroth law of thermodynamics, and the concept of internal energy is involved in the first law. Temperature and internal function are both state functions. Another state function related to the second law of thermodynamics is the **entropy function, S**.

Consider a reversible process between two equilibrium states. If ΔQ_r is the heat absorbed or expelled by the system, the change of entropy, ΔS, between two equilibrium states is given by the heat transferred, δQ_r, divided by the absolute temperature, T, of the system

$$\Delta S = \frac{\Delta Q_r}{T}$$

where subscript r emphasizes that the definition applies only to reversible processes. When a heat is absorbed, the entropy increases. Note, that the change in entropy is defined, but not entropy.

It was found that the entropy of the Universe increases in all natural processes. This is another way of stating the second law of thermodynamics.

Entropy can also be interpreted in terms of probabilities.

Boltzmann found an alternative method for calculating entropy through use of the relation

$$S = k_B \ln W$$

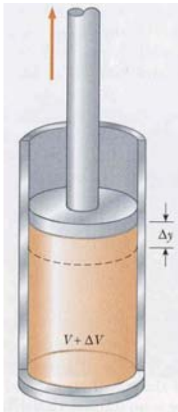
where $k_B = 1.38 \cdot 10^{-23}$ J/K is Boltzmann's constant and W is a probability that the system has a particular configuration. ("ln" is abreviation for the natural logarithm)

Grade of energy.
Various forms of energy can be converted to thermal energy, but the reverse transformation is never complete. In general, if two kinds of energy can be completely interconverted, we say that they are the same grade. However, if form A can be completely converted to form B and the reverse is never complete, then form A is a higher grade of energy than form B. For example, kinetic energy of the ball is of higher grade than the thermal energy contained in the ball and the wall after the collision.

All real processes where heat transfer occurs, the energy available for doing work decreases.

- In the system shown in figure, the gas in the cylinder is at a pressure of 8000 Pa and the piston has an area of 0.1 m^2. As heat is slowly added to the gas, the piston is pushed up a distance of 4 cm. Calculate the work done on the surroundings by the expanding gas. Assume that the presure remains constant. (Any process in which the pressure remains constant is called an isobaric process.)

$$\Delta V = A\Delta y = (0.1\ \mathrm{m}^2)(4\cdot 10^{-3}\ \mathrm{m}) = 4\cdot 10^{-3}\ \mathrm{m}^3$$
$$W = p\Delta V = (8000\ \mathrm{Pa})(4\cdot 10^{-3}\ \mathrm{m}^3) = 32\ \mathrm{J}$$



If 42 J of heat is added to the system during the expansion, what is the change in internal energy of the system?

$$\Delta U = Q - W = 42\ \mathrm{J} - 32\ \mathrm{J} = 10\ \mathrm{J}$$

If 42 J of heat is added to the system with the piston clamped in a fixed position, what is the work done by the gas? What is the change in its internal energy?

$$W = 0$$
$$\Delta U = Q - W = 42\ \mathrm{J} - 0\ \mathrm{J} = 42\ \mathrm{J}$$

- Water with a mass of 2 kg is held at constant volume in a container while 10000 J of heat is slowly added by a flame. The container is not well insulated, and as a result 2000 J of heat leaks out to the surroundings. What is the temperature increase of the water? (A process that takes place at constant volume is called an isovolumetric process.)

$$\Delta T = \frac{Q}{mc} = \frac{10000\ \mathrm{J} - 2000\ \mathrm{J}}{(2\ \mathrm{kg})(4.186\cdot 10^3\ \mathrm{J/kg^0C})} = 0.96^0\mathrm{C}$$

- Find the efficiency of an engine that introduces 2000 J of heat during the combustion phase and loses 1500 J at exhaust.

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{1500\ \mathrm{J}}{2000\ \mathrm{J}} = 0.25\ (\text{or } 25\ \%)$$

If an engine has an efficiency of 20 % and loses 3000 J at exhaust and to the cooling water, how much work is done by the engine?

$$Q_h = \frac{Q_c}{1-e} = \frac{3000\ \mathrm{J}}{1-0.2} = 3750\ \mathrm{J}$$
$$W = Q_h - Q_c = 3750\ \mathrm{J} - 3000\ \mathrm{J} = 750\ \mathrm{J}$$

- A steam engine has a boiler that operates at 500 K. The heat changes water to steam, which drives the piston. The temperature of the exhaust is that of the outside air, about 300 K. What is the maximum thermal efficiency of this steam engine?

$$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{300\ \mathrm{K}}{500\ \mathrm{K}} = 0.4$$

Determine the maximum work the engine can perform in each cycle of operation if it absorbs 200 J of thermal energy from the hot reservoir during each cycle.

$$W = eQ_h = 0.4(200\ \mathrm{J}) = 80\ \mathrm{J}$$

- The highest theoretical efficiency of a gasoline engine, based on the Carnot cycle, is 30 %. If this engine expels its gases into the atmosphere, which has a temperature of 300 K, what is the temperature in the cylinder immediatelly after combustion?

$$T_h = \frac{T_c}{1-e_c} = \frac{300\ \mathrm{K}}{1-0.3} = 430\ \mathrm{K}$$

Actual gasoline engines operate on a cycle significantly different from the Carnot cycle and therefore have lower maximum possible efficiency.

- Calculate the change in entropy when 300 g of lead melts at 327^0C. Lead has a latent heat of fusion of $2.45 \cdot 10^4$ J/kg.

$$Q = mL_f = (0.3\ \mathrm{kg})(2.45 \cdot 10^4\ \mathrm{J/kg}) = 7.35 \cdot 10^3\ \mathrm{J}$$

$$\Delta S = \frac{Q}{T} = \frac{7.35 \cdot 10^3\ \mathrm{J}}{600\ \mathrm{K}} = 12.3\ \mathrm{J/K}$$

- A large, cold object is at 273 K, and a large hot object is at 373 K. Show that it is impossible for a small amount of heat energy, say 8 J, to be transferred from the cold object to the hot object without decreasing the entropy of the isolated system and hence violating the second law. Assume that during the heat transfer the two systems undergo no significant temperature change.

$$\Delta S_h = \frac{Q_h}{T_h} = \frac{8\text{ J}}{373\text{ K}} = 0.0214\text{ J/K}$$
$$\Delta S_c = \frac{Q_c}{T_c} = \frac{-8\text{ J}}{273\text{ K}} = -0.0293\text{ J/K}$$
$$\Delta S = \Delta S_c + \Delta S_h = 0.0214\text{ J/K} - 0.0293\text{ J/K} = -0.0079\text{ J/K}$$

This is in violation of the law that the entropy of an isolated system always increases in natural processes. That is, the spontaneous transfer of heat from a cold object to a hot object cannot occur.

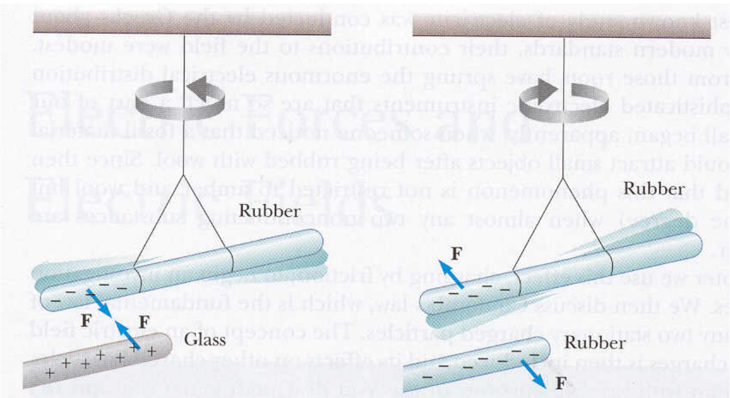
Suppose that 8 J of heat were transferred from the hot to the cold object. What would be the net change in entropy?

$\Delta S = 0.0079$ J/K.

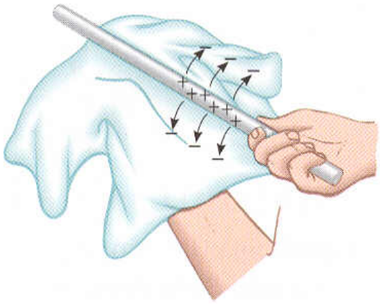
- What is wrong with the statement: “Given any two bodies, the one with the higher temperature contains more heat”?
  Heat is energy in the process of being trensferred, not a form of energy that is held or contained. Correct statement would be: (1) “Given any two objects in thermal contact, the one with the higher temperature will transfer heat to the other.” or (2) “Given any two objects of equal mass, the one with the higher products of absolute temperature and specific heat contains more internal energy.”
- A thermodinamic process occurs in which the entropy of a system changes by –10 J/K. According to the second law of thermodynamics, what can you conclude about the entropy change of the environment?
  The environment must have an entropy change of +10 J/K or more.

A number of simple experiments demonstrate the existance of electrostatic forces. For example, after running a plastic comb through your hair, you will find that the comb attracts bits of paper.
When materials behave in this way, they are said to have become **electrically charged**. You can give your body an electric charge by sliding across a cat seat. You can then feel, and remove, the charge on your body by lightly touching another person. Under the right conditions, a visible spark can be seen when you touch, and a slight tingle is felt by both parties.

Experiments also demonstrate that there are two kinds of **electric charge**, which Benjamin Franklin named **positive** and **negative**. A rubber rod that has been rubbed with fur is suspended by a piece of string. When a glass rod that has been rubbed with silk is brought near the rubber rod, the rubber rod is attracted toward the glass rod. If two charged rubber rods (or two charged glass rods) are brought near each other, the force between them is repulsive. This observation demonstrates that the rubber and glass have different kinds of charge (on the glass rod is called positive, and on the rubber rod negative).



We now know that origin of charge is atom. Nature's basis carrier of positive electricity is the proton located in the nucleus of an atom, and protons never moved from one material to another. Thus, when an object becomes charged, it does so because it has either gained or lost nature's basic carrier of negative electricity, the **electron**.



An important characteristic of charge is that **electric charge is always conserved**. One object gains some amount of negative charge while the other loses an equal amount of negative charge and hence is left with a positive charge. For example, when a glass rod is rubbed with silk, the silk obtains a negative charge that is equal in magnitude to the positive charge on the glass rod as negatively charged electrons are transferred from the glass to the silk in the rubbing process. Likewise, when rubber is rubbed with fur, electrons are transferred from the fur to the rubber.

In 1909 Robert Millikan discovered that the charge is quantizied. This means that **charge occurs as discrete bundles** in nature. Thus, an object may have a charge of $\pm e$, $\pm 2e$, and so on. An electron has a charge of $-e$. The value of e is now known to be $1.6 \cdot 10^{-19}$ C

It is convinient to classify substances in terms of their ability to conduct electric charge.

Conductors are materials in which electric charges move freely, and insulators are materials in which electric charges do not move freely.

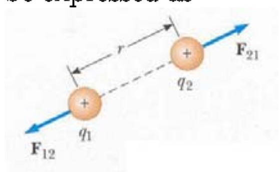
Glass and rubber are **insulators**. When such materials are charged by rubbing, only the rubbed area becomes charged, and there is no tendency for the charge to move into other regions of the material. In contrast, materials such as copper, aluminium, silver, or gold are good conductors. When such materials are charged in some small region, the charge readily distributes itself over the entire surface of the material.

Semiconductors are third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors. Silicon and germanium are well-known semiconductors that are widely used in the fabrication of a variety of electronic devices.

Coulomb's Law

In 1785 Charles Coulomb established the fundamental law of electric force between two stationary charged particles. From the experimental observations, the magnitude of the electric force between two charges separated by a distance of r can be expressed as

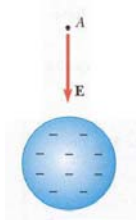
$$F = k\frac{|Q_1||Q_2|}{r^2}$$



where $k = 9 \cdot 10^9\ \mathrm{Nm^2/C^2}$ is the Coulomb constant ($k = \frac{1}{4\pi\varepsilon_0}$, where $\varepsilon_0 = 8.85 \cdot 10^{-12}\ \mathrm{C^2/Nm^2}$ is the permitivity of free space); $|Q_1|$ and $|Q_2|$ are the magnitudes of the charges on the two particles. Electric force is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.
The mathematical form of the Coulomb force is the same as that of the gravitational force. However, there are some important differences between electric and gravitational forces. Electric forces can be either attractive or repulsive, but gravitational forces are always attractive.
The SI unit of charge is the **coulomb (C)**.

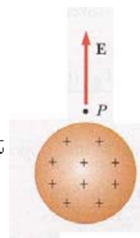
The Electric Field

The question about action on a distance can be answered by saying that one charge sets up an **electric field** in the space surrounding it. When another charged object enters this electric field, forces of an electrical nature arise.
An electric field is a vector quantity, and has both magnitude and direction.



The magnitude of the electric field, due to the point-like charge Q, at the distance r from that charge is

$$E = k\frac{|Q|}{r^2}$$



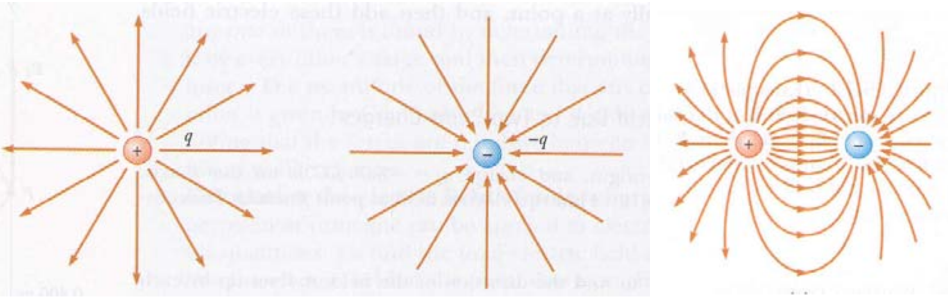
If Q is positive, the electric field, due to this charge, is radialy outward from Q. If Q is negative, the field is directed toward Q.
Thus, the electric force on the charge q due to an electric field $\vec{E}$ at the position of the charge q is

$$\vec{F} = q\vec{E}$$

A convinient aid for visualizing electric field patterns is to draw lines pointing in the direction of the electric field vector at any point. These lines, called **electric field lines**, are related to the electric field in any region of space in the following manner:

- The electric field vector is tangent to the electric field lines at each point.
- The number of lines per unt area through a surface perpendicular to the lines is proportional to the strength of the electric field in a given region.

Thus the magnitude of electric field is large when the field lines are close together and small when they are far apart.



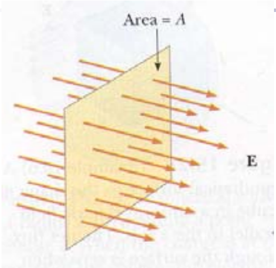
A good **electric conductor**, such as copper, contains charges (electrons) that are not bound to any atom and are free to move about within the material. When no net motion of charge occurs within a conductor, the conductor is said to be in **electrostatic equilibrium**.

It is possible to see that an isolated conductor (one that is insulated from ground) has the following properties:

- the electric field is **zero** everywhere **inside** the conductor.
- Any excess charge on an isolated conductor resides entirely on its surface.
- The electric field just outside a charged conductor is prependicular to the conductor's surface.
- On an irregularly shaped conductor, the charge tends to accumulate at locations where the radius of curvature of the surface is smallest. That is, at a sharp point.

- Why is it safe to stay inside an automobile during a lightning storm?

  Although many people believe that this is safe because of the insulating characteristics of the rubber tires, this is not true. Lighting is able to penetrate a centimeter of rubber. The safety of remaining in the car is due to the fact that charges on the metal shell of the car will reside on the outer surface of the car. Thus an occupant in the automibile touching the inner surfaces is not in danger.

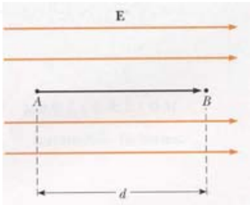


Consider an electric field that is uniform in both magnitude and direction. The electric field lines penetrate a surface of area A, which is perpenducular to the field. We define the magnitude of the electric flux Φ as

$$\Phi = EA$$

When the area is constructed such that a closed surface is formed, we say that flux lines passing into the interior of the volume are negative and those passing out of the interior of the volume are positive.

Gauss's Law: The net electric flux through any closed surface (gaussian surface) is equal to the net charge inside the surface devided by $\varepsilon_0 = 8.85 \cdot 10^{-12}$ C^2/Nm^2.



Because the Coulomb force is conservative, it is possible to define a electrical potential energy function associated with this force.

Imagine a small positive charge Q placed at point A in a uniform electric field of magnitude E. As the charge moves from point A to point B under the influence of the electric force exerted on it, QE, the work done on the charge by the electric force is

$$W = Fd = QEd$$

where d is the distance between A and B.

By definition, the work done by the conservative force equals the negative of the charge in **potential energy,** ΔE_p

$$\Delta E_p = -QEd$$

This is valid only for the case of a uniform electric field.

More practical importance in the study of electricity is the concept of **electric potential**.

Potential diference, ΔV, between two points A and B, is defined as the change in potential energy of a charge Q, moved from A and B, devided by the charge Q

$$\Delta V = V_B - V_A = \frac{\Delta E_p}{Q}.$$

Because electrical energy is a scalar quantity, electric potential is also scalar quantity. The SI units of electric potential are joules per coulomb, called volts

$$1\ \mathrm{V} = 1\ \mathrm{J/C}$$

In the case of a uniform electric field, $\vec{E}$, the potential difference (between two points) is

$$\Delta V = -Ed$$

where d is the distance between the points.

In electric circuits a point of zero electric potential is often defined by grounding (connecting to Earth) some point in the circuit. It is possible to define the electric potential due to a point charge at a point in space. In this case, the point of zero electric potential is taken to be at an infinite distance from the charge. With this choice it is possible to show that the electric potential created by a point charge Q at any distance r from the charge is given by

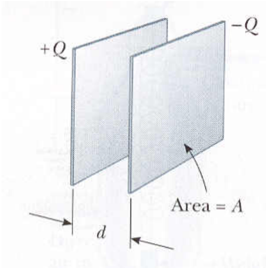
$$V = k\frac{Q}{r}$$

The electric potential of two or more charges is obtained by applying the superposition principle. That is, the total electric potential at some point due to several point charges is the algebraic sum of the electric potentials due to the individual charges.

Now, we can express the electrical potential energy of pair of charges Q_1 and Q_2 as potential created by charge Q_1 times charge Q_2

$$E_p = k\frac{Q_1 Q_2}{r}$$

A **capacitor** is a device used in a variety of electric circuits. For example, to tune the frequency of radio receivers, eliminate sparking in automobile ignition systems, or store short-term energy in electronic flash units.



It consists of two parallel metal plates separated by a distance of *d*. When used in an electric circuit, the plates are connected to the positive and negative terminals of some voltage source. When this connection is made, electrons are pulled of one of the plates, leaving it with a charge of +*Q*, and other plates with −*Q*.

The **capacitance**, *C*, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors

$$C = \frac{Q}{\Delta V}$$

Large capacitence is needed to store a large amount of charge for a given applied voltage. Capacitence has SI units coulombs per volt, called farads 1F=1C/V. The farad is a very large unit of capacitance. In practice, most typical capacitors have capacitance ranging from microfarads to picofarads.

For example, the capacitance of a parallel-plate capacitor whose plates are separated by air is

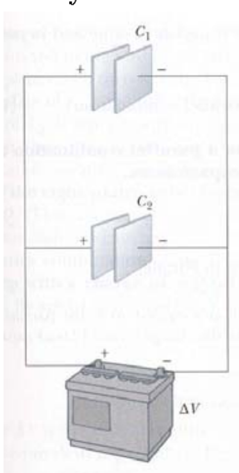
$$C = \varepsilon_0 \frac{A}{d}, \quad (\varepsilon_0 = 8.85 \cdot 10^{-12}\ \mathrm{C^2/Nm^2}\ \text{is permitivity of free space})$$

where *A* is the area of one of the plates and *d* is the distance of the plates.

Two or more capacitors can be combined in circuits in several ways. The equivalent capacitance of certain combinations can be calculated.

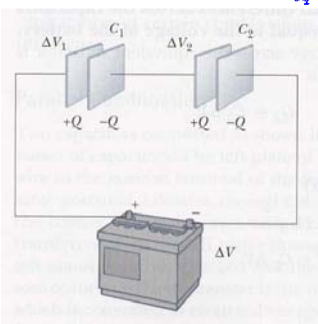
Parallel Combination of capacitors

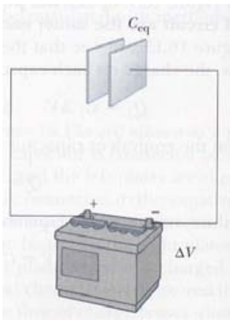
$$C_{eq} = C_1 + C_2 + \ldots$$



Series Combination of capacitors

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$





Almost everyone who works with electronic equipment has at some time verified that a capacitor can store energy.

It is possible to show that the energy stored in the capacitor can be expressed as

$$E = \frac{1}{2} Q \Delta V$$

From the definition of capacitance, we find $Q = C\Delta V$, hence, we can express the energy stored as

$$E = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

or

$$E = \frac{1}{2} Q \Delta V = \frac{Q^2}{2C}$$

This can be applied to any capacitor. In practice, there is a limit to the maximum energy that can be stored, because electrical breakdown ultimately occurs between the plates at a sufficiently large value of ΔV.

A dielectric is an insulating material, such as glass, rubber or waxed paper. When a dielectric is inserted between the plates of a capacitor, the capacitance increases. If the dielectric completely fills the space between the plates, the capacitance is multiplied by the factor κ, called the dielectric constant

$$C = \kappa C_0 \qquad (C_0 \text{ is the capacitance in the absence of a dielectric})$$

Material	Dielectric Constant, κ
Vacuum	1.000 00
Air	1.000 59
Bakelite	4.9
Fused quartz	3.78
Pyrex glass	5.6
Polystyrene	2.56
Teflon	2.1
Neoprene rubber	6.7
Nylon	3.4
Paper	3.7
Strontium titanate	233
Water	80
Silicone oil	2.5

- Find the electric force on a proton placed in an electric field of $2 \cdot 10^4$ N/C, that is directed along the positive x axis.

  Because the charge on a proton is $+e = 1.6 \cdot 10^{-19}$ C, the electric force acting on the proton is

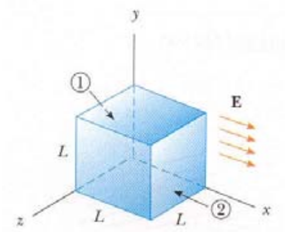
  $$F = eE = (1.6 \cdot 10^{-19}\ \mathrm{C})(2 \cdot 10^4\ \mathrm{N/C}) = 3.2 \cdot 10^{-15}\ \mathrm{N}$$

  where the force is in the positive x direction.

- Find a uniform electric flux oriented in the x direction. Find the net electric flux through the surface of a cube of edges L oriented as in figure.

  The net flux can be evaluated by summing up the fluxes through each face of the cube. First, note that the flux through four of the faces is zero, because electric field is parallel to the area on these surfaces. For surfaces 1 and 2 that lie in the yz-plane are

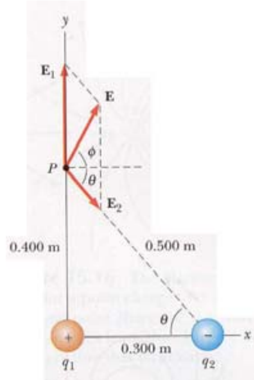
  $$\begin{aligned}\Phi_1 &= -EA = -EL^2\\ \Phi_2 &= EA = EL^2\\ \Phi &= \Phi_1 + \Phi_2 = -EL^2 + EL^2 = 0\end{aligned}$$



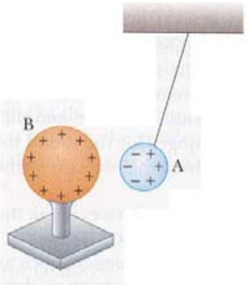
- Charge $q_1 = 7 \cdot 10^{-6}$ C is at the origin, and charge $q_2 = -5 \cdot 10^{-6}$ C is on the x axis, 0.3 m from the origin. Find the magnitude of the electric field at point P which has coordinates on the y axis 0.4 m from the origin.

  The magnitudes of $\vec{E}_1$ and $\vec{E}_2$ are

  $$\begin{aligned}E_1 &= k\frac{|q_1|}{r_1^2} = (9 \cdot 10^9\ \mathrm{Nm^2/C^2})\frac{7 \cdot 10^{-6}\ \mathrm{C}}{(0.4\ \mathrm{m})^2} = 3.93 \cdot 10^5\ \mathrm{N/C}\\ E_2 &= k\frac{|q_2|}{r_2^2} = (9 \cdot 10^9\ \mathrm{Nm^2/C^2})\frac{5 \cdot 10^{-6}\ \mathrm{C}}{(0.5\ \mathrm{m})^2} = 1.80 \cdot 10^5\ \mathrm{N/C}\\ E_x &= E_{1,x} + E_{2,x} = 0 + \frac{3}{5}E_2 = 1.08 \cdot 10^5\ \mathrm{N/C}\\ E_y &= E_{1,y} + E_{2,y} = E_1 - \frac{4}{5}E_1 = 2.49 \cdot 10^5\ \mathrm{N/C}\\ E &= \sqrt{E_x^2 + E_y^2} = 2.72 \cdot 10^5\ \mathrm{N/C}\end{aligned}$$



- If a suspended object A is attracted to object B, which is charged, can we conclude that object A is charged?
  No. Object A might have a charge opposite in sign to that of B, but it also might be a neutral conductor. In the later case, object B causes object A to be polarized, pulling charge of one sign to the near face of A and pushing an equal amount of charge of the opposite sign to the far face. Then the force of attraction exerted on B by the induced charge on the near side of A is slightly larger than the force of repulsion exerted on B by the induced charge on the far side of A. Therefore, the net force on A is toward B.

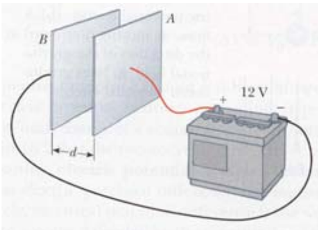


- If a metal object receives a positive charge, does its mass increase, decrease, or stay the same? What happens to its mass if the object receives a negative charge?
  An object's mass decreases very slightly (immeasurably) when it is given a positive charge, because it loses electrons. When the object is given a negative charge, its mass increases slightly because it gains electrons.
- In fair weatherm there is an electric field at the surface of the Earth, pointing down into the ground. What is the electric charge on the ground in this situations?
  Electric field lines start on positive charges and end on negative charges. Thus, if the fair weather field is directed into the ground, the ground must have a negative charge.

- Figure illustrates a situation in which a constant electric filed can be set up. A 12-V battery is connected between two parallel metal plates separated by 0.003 m. Find the magnitude of the electric field.

$$E = -\frac{V_B - V_A}{d} = -\frac{-12\ \mathrm{V}}{0.003\ \mathrm{m}} = 4000\ \mathrm{V/m}$$

  The direction of this field is from the positive plate to the negative plate.



- A 3-μF capacitor is connected to a 12-V battery. What is the magnitude of the charge on each plate of the capacitor?

$$Q = C\Delta V = (3 \cdot 10^{-6}\ \mathrm{F})(12\ \mathrm{V}) = 36\ \mu\mathrm{C}$$

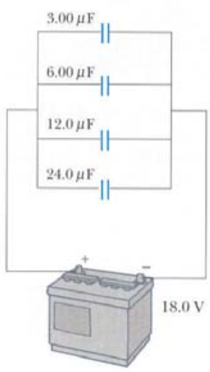
- A parallel-plate capacitor has an area of 2 cm^2 and a plate separation of 1 mm. Find its capacitance.

$$C = \varepsilon_0 \frac{A}{d} = (8.85 \cdot 10^{-12}\ \mathrm{C^2/Nm^2}) \frac{2 \cdot 10^{-4}\ \mathrm{m^2}}{10^{-3}\ \mathrm{m}} = 1.77\ \mathrm{pF}$$

Exercises .....

- Determine the capacitance of the single capacitor that is equivalent to the parallel combination of capacitors in figure.

$$\begin{aligned} C_{eq} &= C_1 + C_2 + C_3 + C_4 \\ &= 3\ \mu\text{F} + 6\ \mu\text{F} + 12\ \mu\text{F} + 24\ \mu\text{F} = 45\ \mu\text{F} \end{aligned}$$

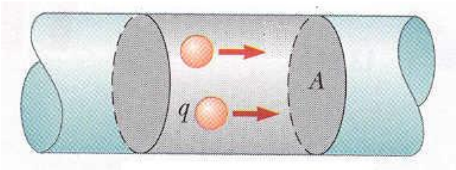


Find the capacitance of the equivalent capacitor if they are connected in series with the battery.

$$\begin{aligned} \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \\ &= \frac{1}{3\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}} + \frac{1}{12\ \mu\text{F}} + \frac{1}{24\ \mu\text{F}} \\ C_{eq} &= 1.6\ \mu\text{F} \end{aligned}$$

- Find the amount of energy stored in a 4-μF capacitor when it is connected across a 120-V battery.

$$E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(5 \cdot 10^{-6}\ \text{F})(120\ \text{V})^2 = 0.036\ \text{J}$$



Whenever electric charges of like signs move, an electric current is said to exist. The current is the rate at which charge flows through this surface. If ΔQ is the amount of charge that passes through this area in a time of Δt, the current, I, is equal to the ratio of the charge to the time interval

$$I = \frac{\Delta Q}{\Delta t}$$

The SI unit of current is the ampere, 1 A = 1 C/s. The current has the same direction as the flow of positive charge. In a common conductor, such as copper, the current is due to the motion of the negatively charged electrons. Therefore, when we speak of current in such a conductor, the direction of the current is opposite the direction of flow of electrons.

When a potential difference (voltage) ΔV, is applied across the ends of a metallic conductor, the current in the conductor is found to be proportional to the applied voltage. If the proportionality is exact, we can write

$$\Delta V = IR$$

where the proportionality constant R is called the **resistance** of the conductor. Resistance has the SI units volts per ampere, called ohms, Ω. For many materials, including most metals, experiments show that the resistance is constant over a wide range of applied voltages. This statement is known as **Ohm's law**.

Ohm's law is an empirical relationship that is valid only for certain materials, called ohmic materials.
The resistance R of an ohmic conductor is proportional to its length, l, and inversely proportional to its cross-sectional area, A,

$$R = \rho \frac{l}{A}$$

where ρ is called the resistivity of the material.

The **resistivity**, and hence the resistance, of a conductor depends on a number of factors. One of the most important is the temperature of the metal. For most metals, resistivity increases with increasing temperature.

Good electric conductors have very low resistivity, and good insulators have very high resistivity. Table lists the resistivities of a variety of materials at 20°C.

Material	Resistivity ($\Omega \cdot m$)
Silver	1.59×10^{-8}
Copper	1.7×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.82×10^{-8}
Tungsten	5.6×10^{-8}
Iron	10.0×10^{-8}
Platinum	11×10^{-8}
Lead	22×10^{-8}
Nichrome[b]	150×10^{-8}
Carbon	3.5×10^{5}
Germanium	0.46
Silicon	640
Glass	$10^{10} - 10^{14}$
Hard rubber	$\approx 10^{13}$
Sulfur	10^{15}
Quartz (fused)	75×10^{16}

If a battery is used to establish an electric current in a conductor, chemical energy stored in the battery is continuously transformed into thermal energy in the resistor.

It is possible to show that the power dissipated in the resistor is

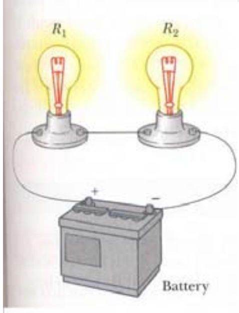
$$P = I\Delta V$$

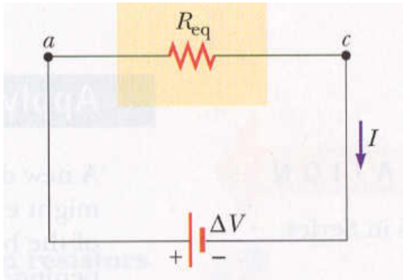
Using the fact that $\Delta V = IR$ for a resistor, we can express the power dissipated by the resistor in the alternative form

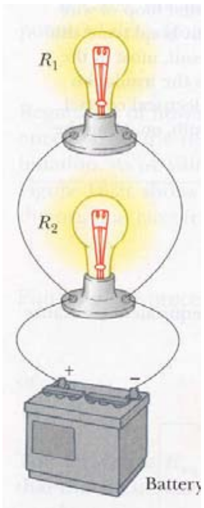
$$P = I^2 R = \frac{(\Delta V)^2}{R}$$

Regardless of the ways in which you use electrical energy in your home, you ultimately must pay for it. The unit of energy used by electric companies to calculate consumption, the kilowatt-hour

$$1 \text{ kWh} = 3.6 \cdot 10^6 \text{ J}$$







Series combination of resistors

$$R_{eq} = R_1 + R_2 + \ldots$$

The equivalent resistance of a series combinations is greater than any individual resistance.

Parallel combination of resistors

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

The equivalent resistance of a parallel combinations is less than any individual resistance.

- Electrical devices are often rated with a voltage and a current (for example, 120 V, 5 A). Batteries, however, are only rated with a voltage (for example, 1.5 V). Why?
  An electrical appliance has a given resistance. Thus, when it is attached to a power source with a known potential difference, a definite current will be drawn. The device can be labeled with both the voltage and the current. Batteries, however, can be applied to a number of devices. Each device will have a different resistance, so the current from the battery will vary with the device. As a result, only the voltage of the battery can be specified.
- Why is it possible for a bird to sit on a high-voltage wire without being electrocuted?
  The bird is resting on a wire of a fixed potential. In order to be electrocuted, a potential difference is required. There is no potential (very low) difference between the bird's feet.

- All devices are required to have identifying plates that specify their electrical characteristics. The plate on a certain steam iron states that the iron carries a current of 5 A when connected to a 220-V source. What is the resistance of the steam iron?

  From Ohm's law, we find that the resistance to be

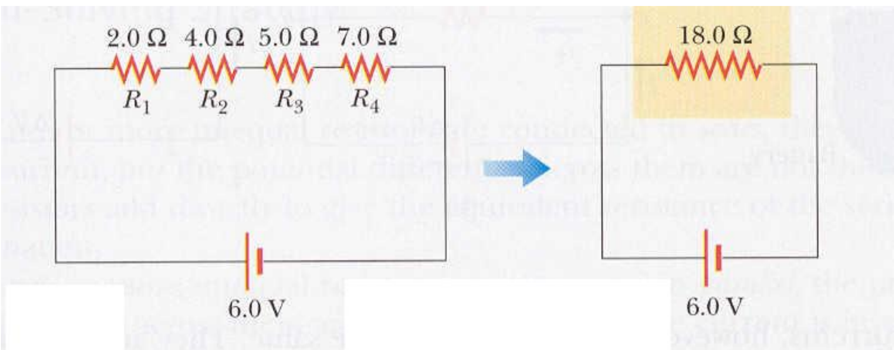
$$R = \frac{\Delta V}{I} = \frac{220\text{ V}}{5\text{ A}} = 44\ \Omega$$

- An electric heater is operated by applying a potential difference of 50 V to a nichrome wire of total resistance 8 Ω. Find the current by the wire and the power rating of the heater.

$$I = \frac{\Delta V}{R} = \frac{50\text{ V}}{8\ \Omega} = 6.25\text{ A}$$
$$P = I^2 R = (6.25\text{ A})^2 (8\ \Omega) = 313\text{ W}$$

- Four resistors are arranged as shown in figure. Find the equivalent resistance and the current in the circuit.

$$R_{eq} = R_1 + R_2 + R_3 + R_4 = 2\ \Omega + 4\ \Omega + 5\ \Omega + 7\ \Omega = 18\ \Omega$$
$$I = \frac{\Delta V}{R_{eq}} = \frac{6\text{ V}}{18\ \Omega} = 0.33\text{ A}$$



Most people have had experience with some form of **magnet**. Iron objects are most strongly attracted to the ends of magnet, called its poles. One end is called the **north pole** and the other the **south pole**. The names come from the behaviour of a magnet in the presence of the Earth's magnetic field (north pole points to the north of the Earth).

Magnetic poles also exert attractive or repulsive forces on each other similar to the electrical forces between charged objects. Like poles repel each other and unlike poles attract each other.

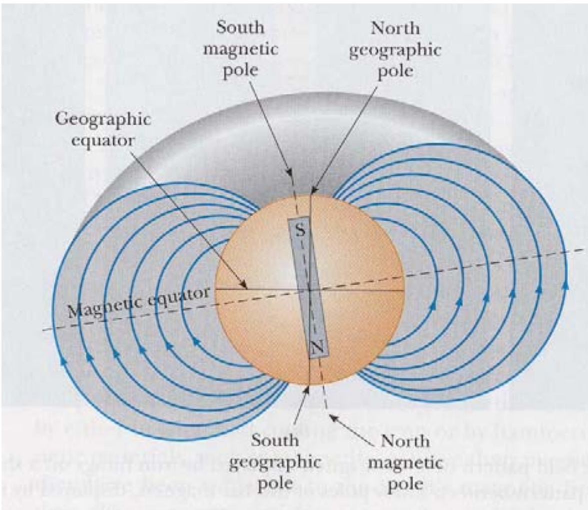
Electric charges can be isolated, but magnetic poles cannot. Magnetic poles always occur in pairs.

Magnetism can be induced in some materials. For example, if a piece of unmagnetized iron is placed near a strong permanent magnet, the piece of iron eventually becomes magnetized. Iron is easily magnetized but also tend to lose their magnetism easily. In contrast, cobalt and nickel are difficult to magnetize but tend to retain their magnetism.

Recall that an electric field surrounds any electric charge. The region of space surrounding a **moving charge** also includes a **magnetic field**.

The geografic north pole corresponds to a magnetic south pole, and the geografic south pole corresponds to a magnetic north pole.

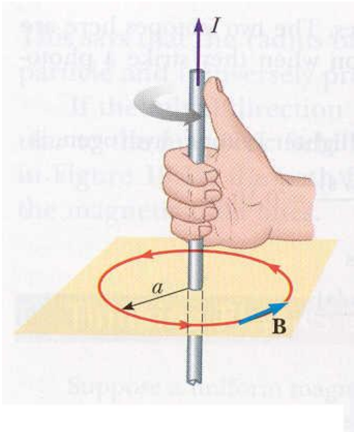
If a compass needle is suspended in bearings that allow it to rotate in the vertical plane as well as in the horizontal plane, the needle is horizontal with respect to the Earth's surface only near the Equator. As the device is moved northward the needle rotates so that it points more and more toward the surface of the Earth. Finally, at a point just north of Hudson Bay in Canada, the north pole of the needle points directly downward. It is about 2000 km from the Earth's geografic north pole and varies with time. Thus it is only approximately correct to say that a compass needle point north.



Stationary charged particle does not interact with a static magnetic field. However, when moving through a magnetic field a charged particle experiences a magnetic force. This force has its maximum value when the charge moves perpendicularly to the magnetic field lines. It becomes zero when the particle moves along the field lines.

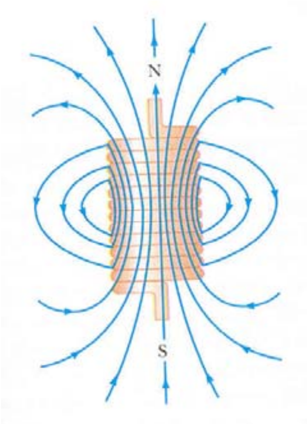
The SI unit if magnetic field is the tesla (T). For example, the Earth's magnetic field near its surface is abut 0.00005 T.

From a simple experiment it is possible to demonstrates that a current-carrying conductor produces a magnetic field (first found by Oersted, 1820).



If a long straight wire is bent into a coil of several closely spaced loops, the resulting device is a solenoid. This device is important in many applications because it acts as a magnet only when it carries a current. The magnitude of the magnetic field *B* inside a solenoid increases with the current *I* and is proportional to the number of coils per unit length *N/l*

$$B = \mu_0 \frac{N}{l} I, \qquad \mu_0 = 4\pi \cdot 10^{-7} \text{ Tm/A (permeability of free space)}$$



- Why does the picture on a television screen become distorted when a magnet is brought near the screen? (You should not do this at home on a color television set, because it may permanently affect the television picture quality.)
  The magnetic field of the magnet produces a magnetic force on the electrons moving toward the screen that produce the image. This magnetic force deflects the electrons to regions on the screen other than the ones to which they are supposed to go. The result is a distorted image.
- Can you use a compass to detect the currents in wires in the walls near light switches in your home?
  A compass would not detect currents in wires near light switches for two reasons. Because the cable to the light switch contains two wires, with one carrying current to the switch and the other away from the switch, the net magnetic field would be very small and fall off rapidly. The second reason is that the current is alternating at 50 Hz. As a result, the magnetic field is oscillating at 50 Hz, also. This frequency would be too fast for the compass to follow, so the effect on the compass reading would average to zero.

In 1865 James Clerk Maxwell provided a mathematical theory that showed a close relationship between electric and magnetic phenomena. His theory predicted that electric and magnetic fields can move through space as waves. The theory he developed is based on the following:

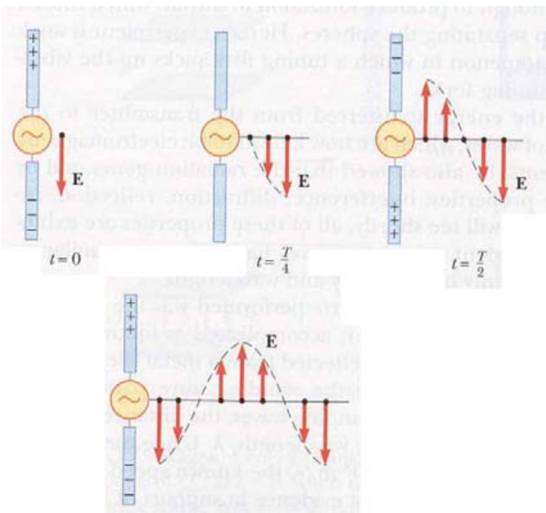
- Electric field lines originate on positive charges and terminate on negative charges.
- Magnetic field lines always form closed loops.
- A varying magnetic field induces an electric field.
- Magnetic fields are generated by moving charges (or currents) or by a varying electric fields.

The waves sent out by the oscillating charges are fluctuating electric and magnetic fields, and so they are called **electromagnetic waves**, traveling through empty space with a speed of about 300 000 000 m/s.

In 1887, Heinrich Hertz was the first to generate and detect electromagnetic waves in a laboratory setting.

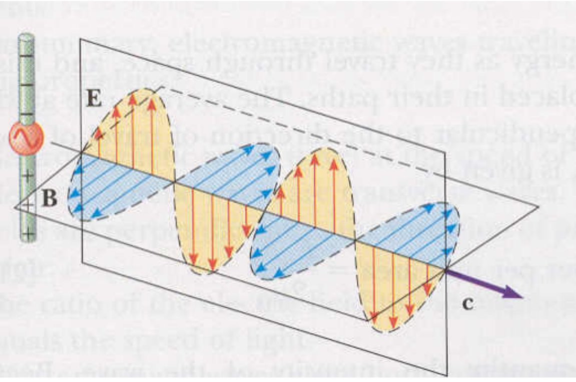
Electromagnetic waves are radiated by any circuit carrying an alternating current. The fundamental mechanism responsible for this radiation is the acceleration of a charged particle. Whenever a charged particle undergoes an acceleration, it must radiate energy.

An alternating voltage applied to the wires of an antenna forces an electric charge in the antenna to oscillate. This is a common technique for accelerating charged particles and is the source of the radio waves emitted by the broadcast antenna of a radio station. As the charges continue to oscillate between the rods, the electric field moves away from the antenna at the speed of light.



Because the oscillating charges create a current in the rods, a magnetic field is also generated. The magnetic field lines circle the antenna and are perpendicular to the electric field at all points. Both fields are perpendicular to the direction of motion of the wave. Hence, we see that an electromagnetic wave is a **transverse wave**.

At great distance from the antenna, the strengths of he electric and magnetic fields become very weak.



Electromagnetic waves travel with the speed of light. In fact, it can be shown that the speed of an electromagnetic wave is related to the permeability and permittivity of the medium through which it travels. For free space it is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 2.9979 \cdot 10^8 \text{ m/s}$$

where c is speed of light, $\mu_0 = 4\pi \cdot 10^{-7}$ Ns^2/C^2 is the permeability constant of vacuum, and $\varepsilon_0 = 8.85 \cdot 10^{-12}$ C^2/Nm^2 is the permittivity of vacuum.

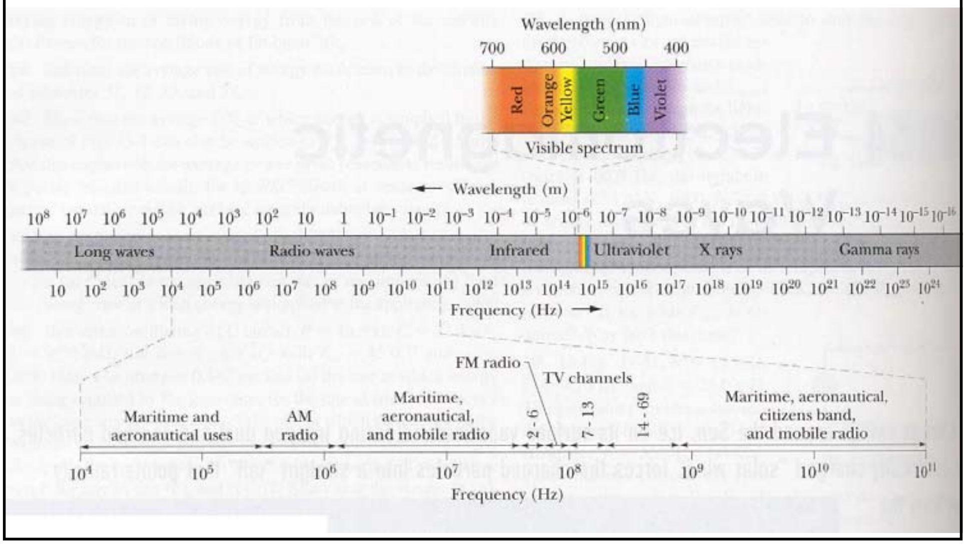
It can be shown also that the magnitude of the electric to the magnetic field in an electromagnetic wave equals the speed of light

$$\frac{E}{B} = c.$$

Electromagnetic waves carry both energy and momentum as they travel through space.

All electromagnetic waves travel through vacuum with a speed of *c*. Hence, their frequency, *f*, and wavelength, λ, are related by the expression

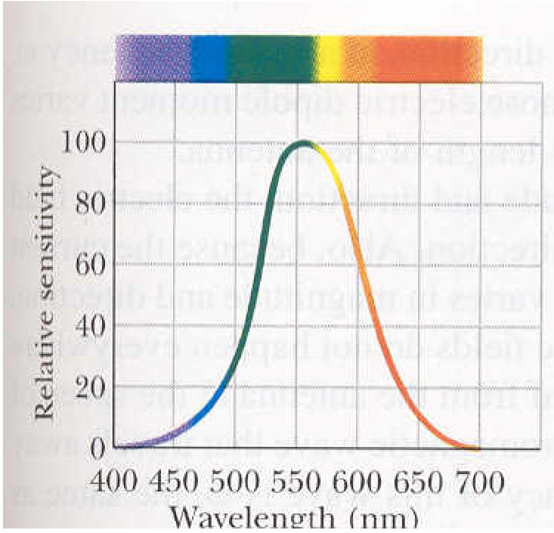
$$c = \lambda f$$



The types of electromagnetic waves are (there are no sharp division between one kind of wave and the next):

- **Radio waves**, are the result of charges accelerating through conduction wires. They are used in radio and television communication systems.
- **Microwaves**, have wavelengths ranging between about 1 mm and 30 cm, and are generated by electronic devices. They are well suited for the radar systems used in aircraft navigation. Microwave ovens are an interesting domestic application.
- **Infrared waves** (sometimes called heat waves), produced by hot bodies and molecules, have wavelengths ranging from about 1 mm to the longest wavelength of visible light, 700 nm. They are readily absorbed by most materials. The infrared energy absorbed by a substance appears as heat. This is because the energy agitates the atoms of the object, increasing their vibrational or translational motion, and the result is a temperature rise. Physical therapy and infrared photography are some practical applications.

- **Visible light**, the most familiar form of electromagnetic waves, may be defined as the part of the spectrum that is detected by a human eye. Light is produced by the rearrangement of electrons in atoms and molecules. The wavelength of visible light are classified as colors ranging from violed, 400 nm, to red, 700 nm. The eye's sensitivity is a function of wavelength and is greatest at a wavelength of about 560 nm (yellow-green).



- **Ultraviolet light** (UV) covers wavelengths ranging from about 400 nm to 0.6 nm. The Sun is an important source of ultraviolet light (which is the main cause of suntans). Most of the ultraviolet light from the Sun is absorbed by atoms in the upper atmosphere, or stratosphere. This is fortunate, because UV light in large quantities has harmful effects on humans. One important constituent of the stratosphere is ozone from reactions of oxygen with ultraviolet radiation. This ozone shield converts lethal high-energy ultraviolet radiation to heat, which warms the stratosphere.
- **X-rays** are electromagnetic waves with wavelengths from about 10 nm to 0.1 pm. The most common source of x-rays is the acceleration of high-energy electrons bombarding a metal target. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because x-rays damage or destroy living tissues and organisms, care must be taken to avoid unnecessary exposure and overexposure.
- **Gamma rays** are emitted by radioactive nuclei. They are highly penetrating and cause serious damage when absorbed by living tissues. Those working near such radiation must be protected by garments containing heavily absorbing materials, such as layers of lead.

- The human eye is sensitive to electromagnetic waves that have wavelengths in the range from 400 nm to 700 nm. What range of frequencies of electromagnetic radiation can the eye detect?

$$f_1 = \frac{c}{\lambda_1} = \frac{3 \cdot 10^8 \text{ m/s}}{700 \text{ nm}} = 4.3 \cdot 10^{14} \text{ Hz}$$
$$f_2 = \frac{c}{\lambda_2} = \frac{3 \cdot 10^8 \text{ m/s}}{400 \text{ nm}} = 7.5 \cdot 10^{14} \text{ Hz}$$

- What are the wavelength ranges in the FM (frequency modulation) radio band, 88 – 108 MHz

$$\lambda_1 = \frac{c}{f_1} = \frac{3 \cdot 10^8 \text{ m/s}}{88 \text{ MHz}} = 3.4 \text{ m}$$
$$\lambda_2 = \frac{c}{f_2} = \frac{3 \cdot 10^8 \text{ m/s}}{108 \text{ MHZ}} = 2.8 \text{ m}$$

The Nature of Light

Until the beginning of the 19th century, **light** was considered to be a **stream of particles**, emitted by a light source, that stimulated the sence of sight on entering the eye. That was proposed by Newton.

During Newton's lifetime Christian Huygens proposed another theory – **wave theory of light**. The wave theory did not receive immediate acceptance because there were not clear experimental evidence and also due to Newton's great reputation as a scientist. The first clear demonstration of the wave nature of light was provided in 1801 by Thomas Young, who showed that light exhibits **interference behavior**. That is, for example, at certain points in the vicinity of two sources, light waves can combine and cancel each other by destructive interference.

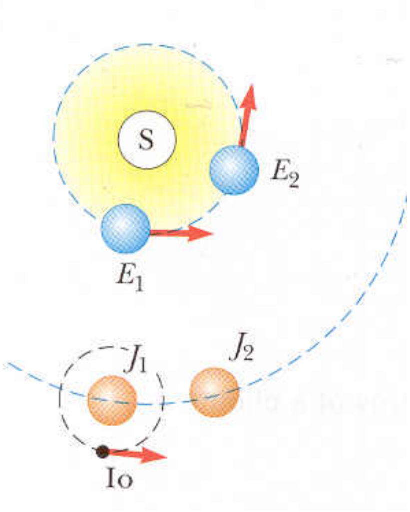
The most important development concerning the theory of light was the work of Maxwell, who in 1865 predicted that light was a form of **electromagnetic wave**. Althought his theory explained most known properties of light, some subsequent experiments could not be explained by the assumption that light was a wave. The most striking of these was the photoelectric effect (clean metal surfaces emit charges when exposed to ultraviolet light).

In 1905 Einstein formulated **theory of light quanta** and explained the photoelectric effect. He conclused that light is composed of corpuscles (photons) with energy proportional to the frequency of the electromagnetic wave, $E=hf$, where h is Planck's constant.

Thus, light must have a **dual nature**. That is, in some cases **light acts as a wave and** in others **as a particle**, but **never acts as both** in the same experiments.

The Speed of Light

Light travels so fast that early attempts to measure its speed were unsuccessful. The first known successful estimate of the speed of light was made in 1675 by Ole Roemer. His technique involved astronomical observations of one of the moons of Jupiter, Io. He estimated the speed of light to be about 210 000 km/s.



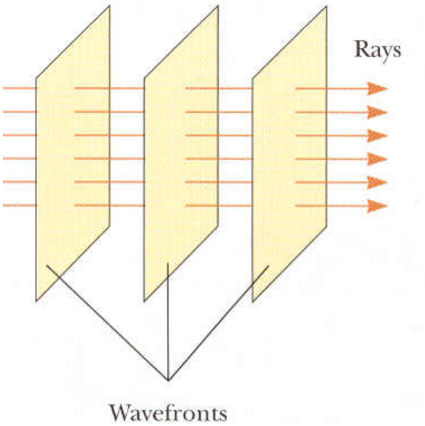
Later, in 1849, Fizeau arrived at a value of 310 000 m/s.

A recent value of the speed of light in a vacuum, obtained using a laser technique, is **299 792 458 m/s**.

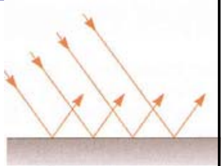
Firstly, our discussion of light will be concerned with what happens when light passes through some optic materials or reflects from them (lenses, mirrors, etc). Explanations of such phenomena can be done by geometrical analysis of light rays. That part of optics is often called **geometric optics**.

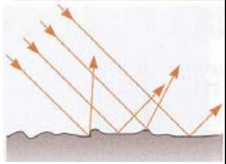
First property of light, inside geometric optics, can be understood based on common experience: **light travels in a straight line path until it encounters a boundary between two different materials**. When light strikes a boundary it either is **reflected** from the boundary, **passes** into the material on the other side of the boundary, or **partially** does **both**.

We use the ray approximation to represent beams of light.



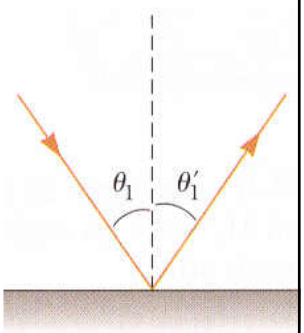
When a light ray traveling in a transparent medium encounters a boundary leading into a second medium, part (or total) of the incident ray is reflected back into first medium. Reflection of light from a smooth surface is called **specular reflection**. If the reflecting surface is rough, the surface reflects the rays in a variety of directions. Reflections from any rough surface is known as **diffuse reflection**. We will concern ourselves only with specular reflection, and we use term **reflection** to mean specular reflection.





Experiments show that **the angle of reflection equals the angle of incidence**, that is,

$$\theta_1^{'} = \theta_1$$



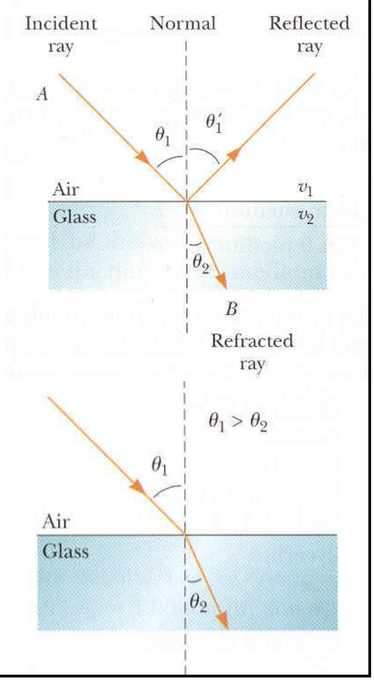
When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, part of the ray enters the second medium, and is said to be **refracted**.

Experiments show that **the angle of refraction depends on the properties of the two media and on the angle of incidence** as

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{(called Snell's law)}$$

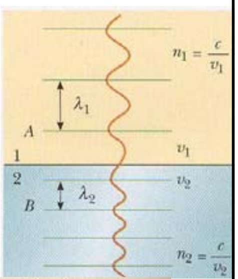
where n_1 and n_2 are **indices of refraction of two mediums**, defined as

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}$$



From the definition, we see that the index of refraction is a dimensioless number that is **greater than 1**, because speed of light in any medium is less than speed of light in vacuum. (For a vacuum index equals 1.)

It is possible to show that, as light travels from one medium to another, **its wavelength changes but its frequency remains constant**.

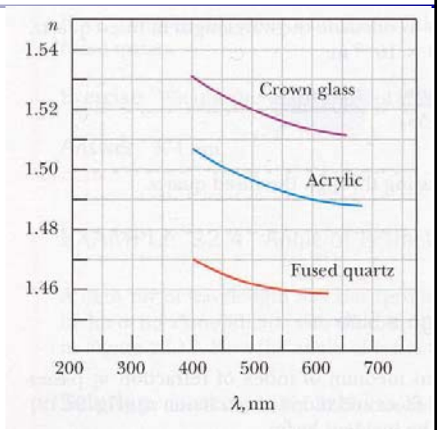


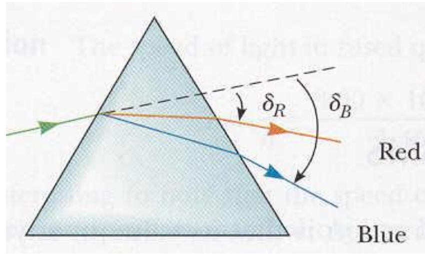
Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		**Liquids at 20°C**	
Diamond (C)	2.419	Benzene	1.501
Fluorite (CaF_2)	1.434	Carbon disulfide	1.628
Fused quartz (SiO_2)	1.458	Carbon tetrachloride	1.461
Glass, crown	1.52	Ethyl alcohol	1.361
Glass, flint	1.66	Glycerine	1.473
Ice (H_2O) (at 0°C)	1.309	Water	1.333
Polystyrene	1.49		
Sodium chloride (NaCl)	1.544	**Gases at 0°C, 1 atm**	
Zircon	1.923	Air	1.000 293
		Carbon dioxide	1.000 45

Dispersion of Light

An important property of the index of refraction is that its value in anything but vacuum depends on the wavelength of light. This phenomena is called **dispersion**. Using Snell's law we can see that light of different wavelengths is bent at different angles when incident on a refracting material. Blue light (~470 nm) bends more than red light (~650 nm) when passing into a refracting material.

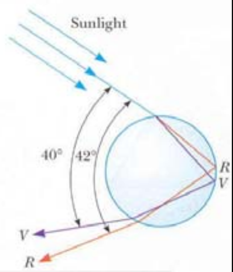
Suppose a beam of white light (a combination of all visible wavelengths) is incident on a prism. Because of dispersion, the rays that emerge from the second face of the prism fan out in a series of colors known as a visible **spectrum**.

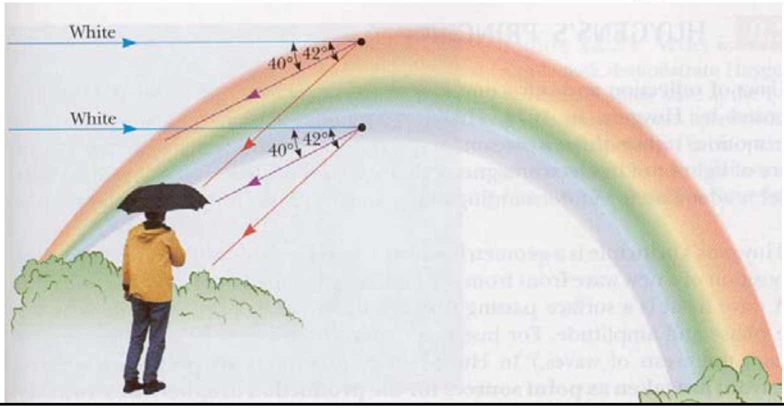




The Rainbow

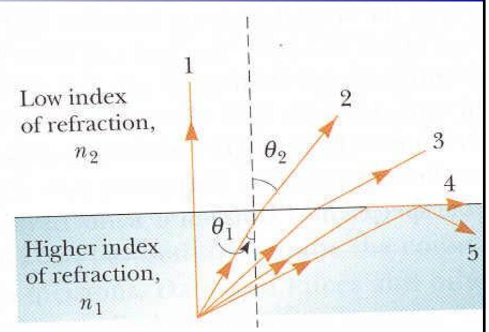
The dispersion of light into a spectrum is demonstrated most vividly in nature through the formation of a **rainbow**, often seen by an observer positioned between the Sun and a rain shower. A ray of light passing overhead strikes a drop of water in the atmosphere and is refracted and reflected as shown in figure. The small angular difference between the returning violet and red rays causes us to see the bow.

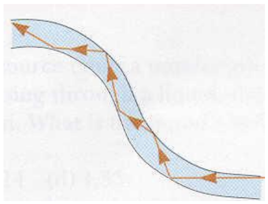




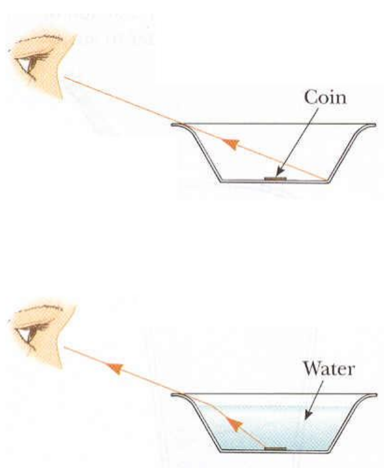
An interesting effect called **total internal reflection** can occur **when light attempts to move from a medium with a high index of refraction to one with a lower index of refraction**. At some particular angle of incidence, called the **critical angle**, the refracted light ray moves parallel to the boundary. For angles of incidence greater than the critical angle, the beam is entirely reflected at the boundary.

Interecting applications are submarine periscopes and fiber optics (in medicine and telecommunications)





- Tape a coin to the bottom of a large opaque bowl as in figure. Look at the coin from the side and move backwards until you can no longer see the coin. Remain still at that position, and have a friend pour water into the bowl until it is full as in figure. Nore that you are now able to see the coin because the light is refracted at the water-air interface.



- Why do astronomers looking at distant galaxies talk about looking backward in time?

  Light travels through a vacuum at a speed of 3000 000 km/s. Thus an image we see from a distant star or galaxy must have been generted some time ago.

- Find the speed of light in water ($n = 1.333$).

$$n = \frac{c}{v}$$

$$v = \frac{c}{n} = \frac{3 \cdot 10^8 \text{ m/s}}{1.333} = 2.25 \cdot 10^8 \text{ m/s}$$

- A beam of light enters a layer of water at an angle of 36^0 with the vertical. What is the angle between the refracted ray and the vertical?

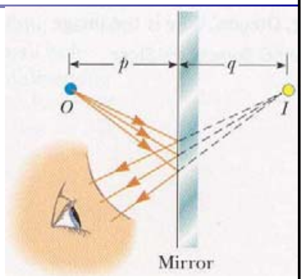
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1}{1.333}\sin 36^0 = 0.44095$$

$$\theta_2 = \arcsin(0.44095) = 26.2^0$$

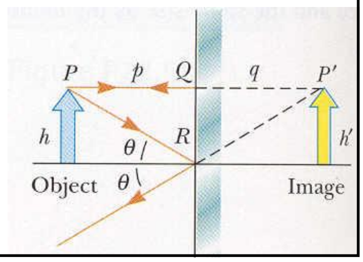
Flat Mirrors

A distance, *p*, of a point light source (called **object**) in front of a flat mirror is called the **object distance**. Light rays leave the source and are reflected from the mirror. After reflection, the reys diverge (spread apart), but they appear to the viewer to come from a point behind the mirror, called the **image** of the object. Images are formed at the point at which rays of light actuylly intersect or at which they appear to originate.
A distance, *q*, of the image is called **image distance**.



Images are classified as real or virtual. A **real image** is one in which light actually intersects, or passes through, the image point; a **virtual image** is one in which the light does not pass through the image point but appears to come (diverge) from that point. The image formed by the flat miror in the figure is a virtual image. **Real images can be displayed on a screan, but virtual images cannot**.

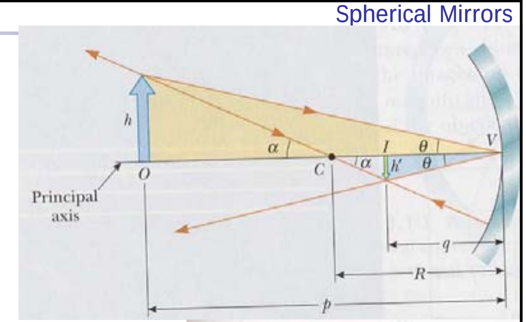
The image formed by an object placed in front of a flat mirror is as far behind the mirror as the object is in front of the mirror, *p=q*.



Spherical Mirrors

A spherical mirror has the shape of a segment of a sphere. A spherical mirror with light reflecting from its inner concave surface is called a **concave mirror**. The mirror has radius of curvature *R*, and its center of curvature is at point C.

A spherical mirror that light is reflected from the outer convex surface is called **convex mirror**.



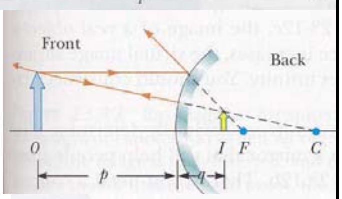
Using simple algebra it is possible to get the expression called the **mirror equation**

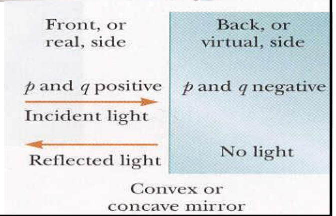
$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

where *f* is the **focal length** defined as $f = R/2$.

For the **magnification** of the mirror we can find

$$M=\frac{\text{image height}}{\text{object height}}=\frac{h'}{h}=-\frac{q}{p}$$



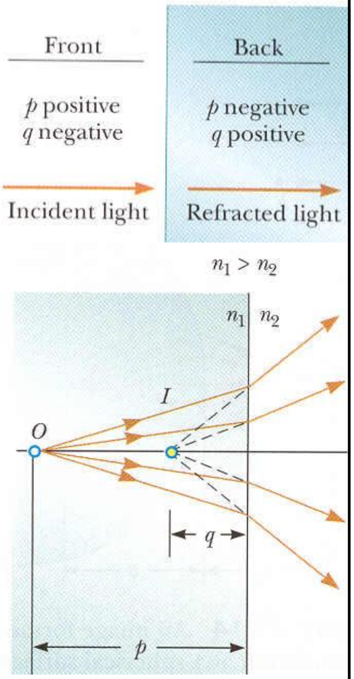


Using Snell's law of refraction and simple geometrical techniques it is possible to show that, if the refracting surface is flat, the object distance and image distance are related by the equation

$$\frac{n_1}{p} = -\frac{n_2}{q} \quad (n_1, n_2 \text{ are indices of refraction})$$

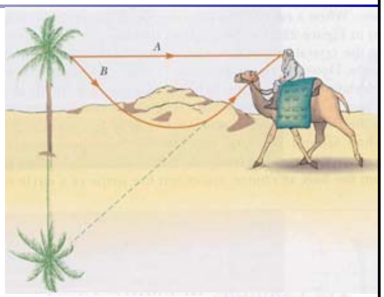
Forthermore, the magnification of a refracting surface is

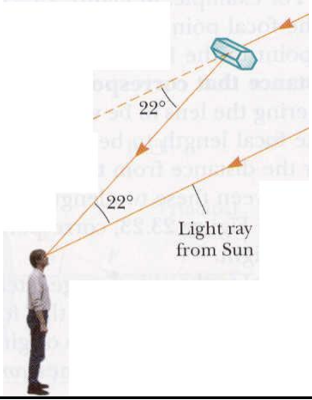
$$M = \frac{\text{image height}}{\text{object height}} = \frac{h'}{h} = -\frac{n_1 q}{n_2 p}$$



The **mirage** is phenomenon of nature produced by refraction in the atmosphere. A mirage can be observed when the ground is so hot that the air directly above it is warmer than the air at higher elevations. The desert is a region in which such circumstances prevail, but mirages are also seen on heated roadways during the summer. The layers of air at different heights above the Earth have different densities and different refractive indicies.

A **halo** around Moon is another phenomenon of nature. These halos are most commonly seen on winter nights because an abundance of ice crystals in the sky is necessary for their production. When a light ray passes through, it is deflected by an angle of about 22°.



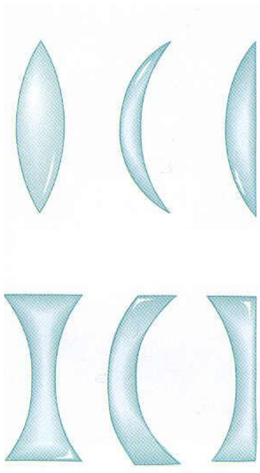


A typical thin **lens** consists of a piece of glass or plastic, ground so that each of its two refracting surfaces is a segment of either a sphere or a plane. Lenses are commonly used to form images by refraction in optical instruments, such as cameras, microscopes, and telescopes. The equation that relates object and images distances for a lens

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

is virtually identical to the mirror equation.

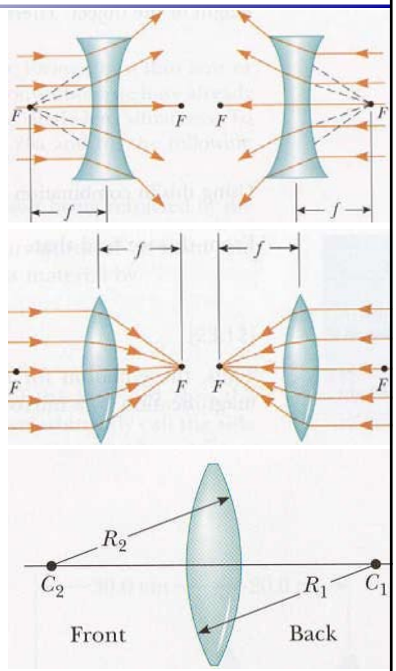
Lenses can be placed in two groups. The lenses that are thicker at the center than at the tim are called **converging lenses**, and those that are thinner at the center than at the rim are **diverging lenses**.



The **focal length, *f*,** is defined as the image distance that corresponds to an infinite object distance. Note that a converging lens has a positive focal length, and a diverging has a negative focal length. Hence the names positive and negative are often given to those lenses.

The focal length for a lens in air is related to the curvatures of its front and back surfaces and to the index of refraction of the lens material by

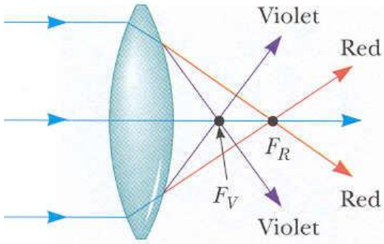
$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

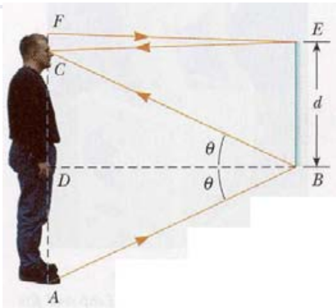


One of the basic problems of lenses is the imperfect quality of the images. The departures of real (imperfect) images from the ideal predicted by the simple theory are called **aberrations**. Two common types of abrreations are **spherical** and **chromatic** aberrations.

Spherical aberration results from the fact that the focal points of light rays far from the principal axis of a spherical lens (or mirror) are different from the focal points of rays passing near the axis. In the case of mirrors one can minimize spherical aberration by employing a parabolic rather than spherical surface (but they are very expensive).

The fact that different wavelenghts of light refracted by a lens focus at different points rise to chromatic aberration. When light passes through a lens, for example, voilet light rays are refracted more than red light rays. Chromatic aberration can be greatly reduced by the use of a combination of converging and diverging lenses made from two different types of glases.





- A 1.8-m tall man stands in front of a mirror in hopes of seeing his full height, no more and no less. If his eyes are 0.1 m from the top of his head, what is the minimum height of the mirror?

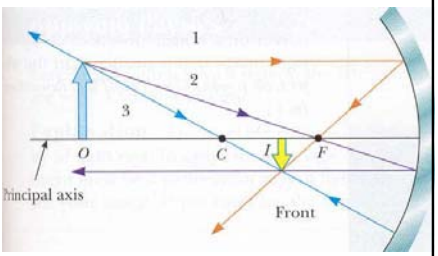
  From the figure we have

$$AD = DC = \frac{1}{2}AC = \frac{1}{2}(1.8\text{ m} - 0.1\text{ m}) = 0.85\text{ m}$$

$$\frac{1}{2}CF = \frac{1}{2}(0.1\text{ m}) = 0.05\text{ m}$$

$$d = FA - AD - \frac{1}{2}CF = 1.8\text{ m} - 0.85\text{ m} - 0.05\text{ m} = 0.9\text{ m}$$

- Assume that a certain concave spherical mirror has a focal length of 10 cm. Locate the images for object distance of 25 cm. Describe the image.



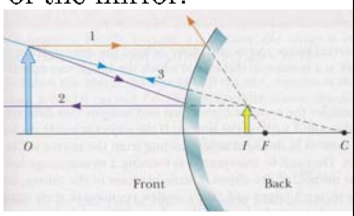
$$\frac{1}{p}+\frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{25\text{ cm}}+\frac{1}{q} = \frac{1}{10\text{ cm}}$$
$$q = 16.7\text{ cm}$$

The magnification is

$$M=-\frac{q}{p}=-\frac{16.7\text{ cm}}{25\text{ cm}}=-0.667$$

Thus the image is smaller than the object. Furthermore, the image is inverted because M is negative. Finally, because q is positive, the image is on the front side of the mirror and is real.

- An object 3 cm high is placed 20 cm from a convex mirror with a focal length of 8 cm. Find the position of the final image, the height of the image, and the magnification of the mirror.



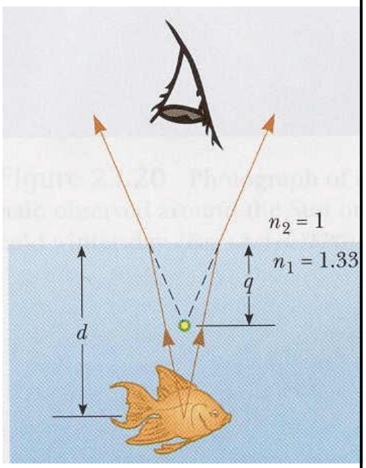
$$\frac{1}{p}+\frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{20\text{ cm}}+\frac{1}{q} = \frac{1}{-8\text{ cm}}$$
$$q = -5.7\text{ cm}$$
$$M = -\frac{q}{p}=-\frac{-5.71\text{ cm}}{20\text{ cm}}=0.286$$
$$h' = Mh=(0.286)(3\text{ cm})=0.86\text{ cm}$$

The negative value of q indicates that the image is virtual. The image is upright because M is positive.

- A small fish is swimming at a depth of 1 m below the surface of a pond. Find the apparent depth of the fish as viewed from directly overhead and the magnification.

$$q = -\frac{n_2}{n_1}p = -\frac{1}{1.33}1 \text{ m} = -0.75 \text{ m}$$
$$M = -\frac{n_1 q}{n_2 p} = 1$$

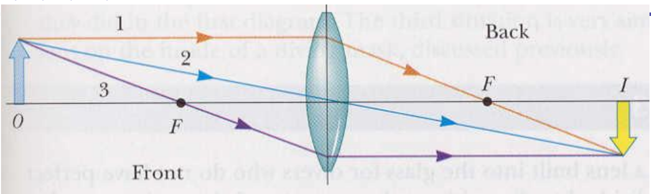
The negative value of q indicates that the image is virtual.



- The biconvex lens has an index of refraction of 1.5. The radius of curvature of the front surface is $R_1 = 10$ cm, and that of the back surface is $R_2 = -15$ cm. Find the focal length of the lens.

From the sign conventions we find that $R_1 = +10$ cm and $R_2 = -15$ cm. Thus, using the lens maker's equation, we have

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5-1)\left(\frac{1}{10 \text{ cm}} - \frac{1}{-15 \text{ cm}}\right)$$
$$f = 12 \text{ cm}$$



- A converging lens of focal length 10 cm forms images of object placed 30 cm from the lens. Find the image distance and describe the image.

$$\frac{1}{p}+\frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{30\text{ cm}}+\frac{1}{q} = \frac{1}{10\text{ cm}}$$
$$q = 15\text{ cm}$$
$$M = -\frac{q}{p} = -\frac{15\text{ cm}}{30\text{ cm}} = -0.5$$

The image is reduced in height by one half, and the negative sign for M tells us that the image is inverted.

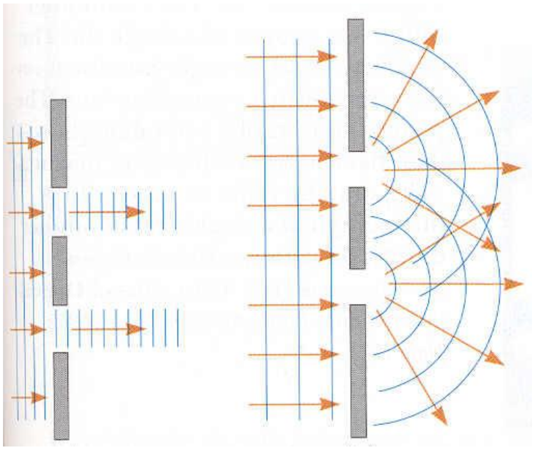
Our discussion of light has been concerned with what happens when light passes through a lens or reflects from a mirror. Explanations of such phenomena rely on a geometric analysis of light rays. That part of optics is called geometric optics. **Interference, diffraction**, and **polarization** are phenomena that cannot be adequatly explained with ray optics, but the wave theory leads us to satisfying description. That part of optics we call **wave optics**.

In our discussion of interference of mechanical waves (Part 7), we found that two waves could add together either constructively or descructively. In **conscructive interference**, the amplitude of the resultant wave is greater than that of either of the individual waves, whereas in **descructive interference**, the resultant amplitude is less than that of either individual wave. Electromagnetic waves also undergo interference. Furthermore, all interference associated with electromagnetic waves arises from the combining of the electric and magnetic fields that constitute the individual waves.

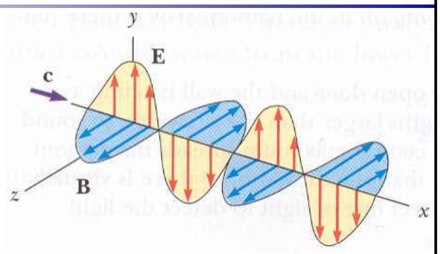
Interference effects in light waves are not easy to observe because of the short wavelengths involved (about 400-750 nm). For sustained interference between two sources of light to be observed, the sources must contain a constant phase with respect to each other (must be coherent), and must have identical wavelengths.

If the light truly traveled in straight-line paths after passing through the slits, the waves would not overlap and no interference pattern would be seen. But, the light deviates from a straight-line path and enters the region that would otherwise be shadowed. This divergence of light from its initial line of travel is called **diffraction**.

In general, diffraction occurs when waves pass through small opening, around obstacles, or by sharp edges. This phenomena cannot be explained within the framework of geometric optics, which says that light rays traveling in straight lines.

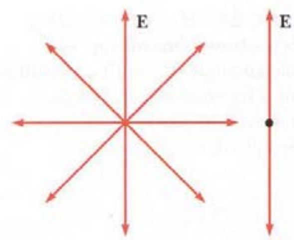


We described the transverse nature of electromagnetic waves. The electric and magnetic field vectors asociated with an electromagnetic wave are at right angles to each other and also to the direction of wave propagation.



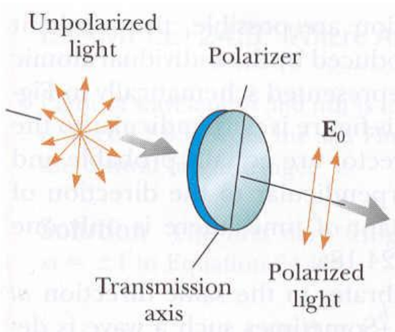
An ordinary beam of light consists of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation of electric field vector, corresponding to the direction of atomic vibration. However, because all directions of vibration are possible, the resultant electromagnetic wave is a superposition of waves produced by the individual atomic sources. The result is an **unpolarized** light wave.

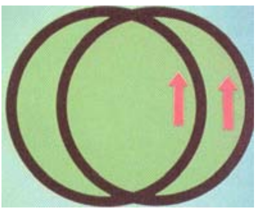
A wave is said to be lineraly **polarized** if electric field vector is in the same direction at all times at a particular point.

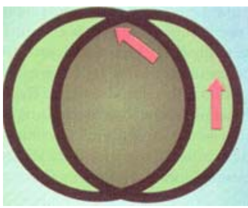


The most common technique for polarizing light is to use a material that transmits waves whose electric field vectors are in a plane parallel to a certain direction and absorbs those waves whose electric field vectors are in directions perpendicular to that direction.

In 1932, Land discovered a material, called **polaroid**, that polarizes light through selective absorption by oriented molecules. This material is fabricated in thin sheets of long-chain hydrocarbons. The molecules absorb light whose electric field vector is perpendicular to their lengths.



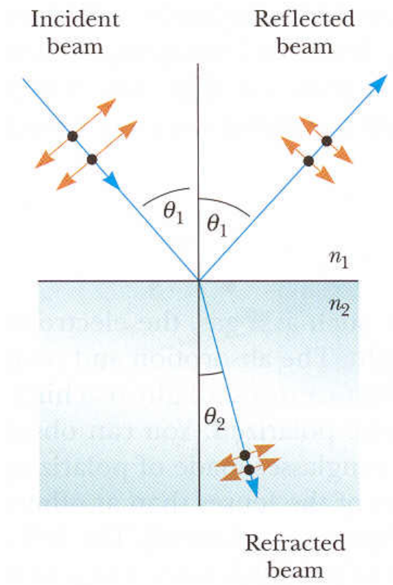




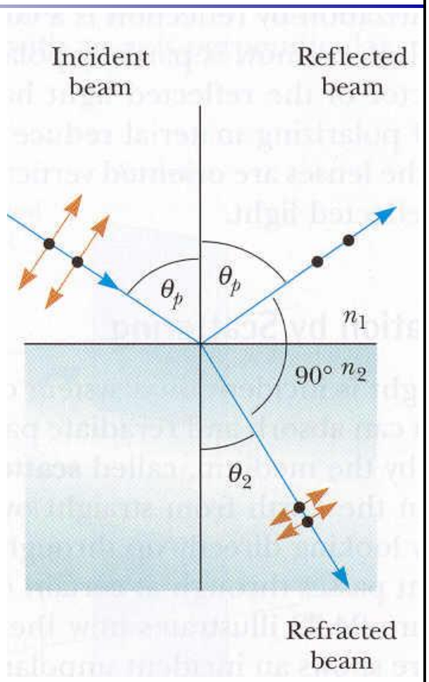
When an unpolarized light beam is reflected from a surface, the reflected light is completely polarized, partially polarized, or unpolarized, depending on the angle of incidence. If the angle of incidence is either 0° or 90°, the reflected beam is unpolarized. However, for angles of incidence between 0° or 90°, the reflected light is polarized to some extent. For one particular angle of incidence, the reflected beam is completely polarized. This angle is called the **polarizing angle** (or Brewster's angle), and it is valid

$$\tan\theta_p = n$$

where n is relative index of refraction. This occurs when the angle between the **reflected and refracted beams is 90°.**



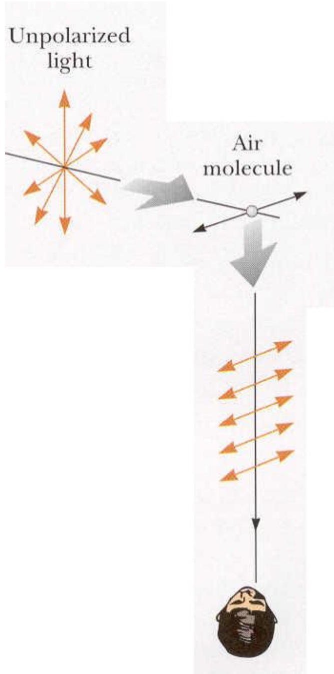
Polarization by reflection is a common phenomena. Sunlight reflected from water, glass, or snow is partially polarized. If the surface is horizontal, the electric field vector of the reflected light has a strong horizontal component. Sunglasses made of polarizing material reduce the glare of reflected light.



Polarization by Scattering

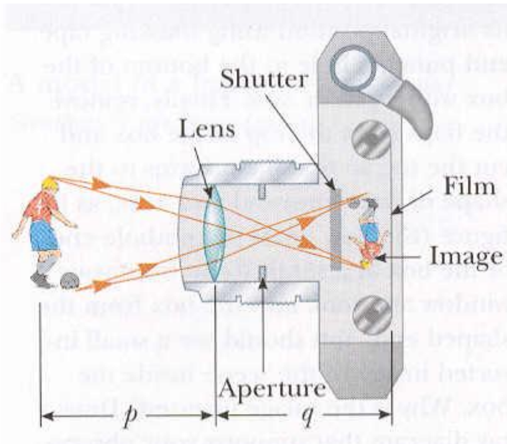
When light is incident on a system of particles, such as a gas, the electrons in the medium can absorb and reradiate part of light. The absorption and reradiation of light by the medium, called **scattering**, is what causes sunlight reaching an observer on the Earth from straight overhead to be polarized. You can observe this effect by looking directly up through a pair of sunglasses made of polarizing glass. Less light passes through at certain orientations of the lenses than at others.

Bees and homing pigeons use the polarization of sunlight as a navigational aid.

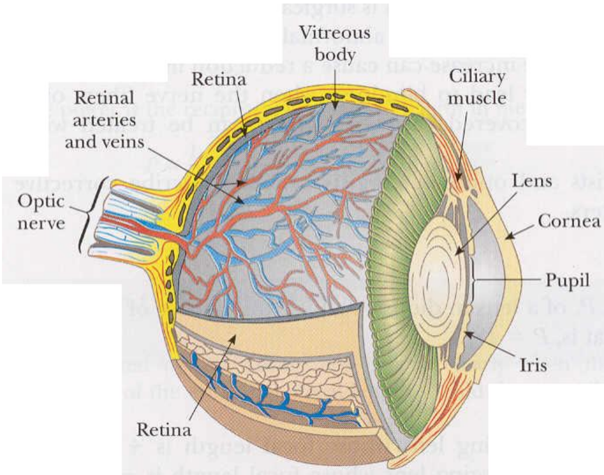


The Camera

The single-lens photographic **camera** is a simple optical instrument whose essential features are shown in figure. It consists of a light-tight box, a converging lens that produces a real image, and a film behind the lens to receive the image. For proper focusing, which leads to sharp images, the lens-to-film distance will depend on the object distance as well as on the focal length of the lens. The shutter, located behind the lens, is a mechanical device that is opened for selected time intervals. With this arrangement, moving objects can be photographed with the use of short exposure times, and dark scenes with the use of long exposure times.Typical shutter times are 1/30, 1/60, 1/125, and 1/250 s.



Like the camera, the **eye** gathers light and produces a sharp image. The front of the eye is covered by a transparent membrane called the cornea. Inward from the cornea are a clear liquid region (the aqueous humur), a variable aperture (the iris surrounding the pupil), and the crystalline lens. Most of the refraction occurs in the cornea, because the liquid medium surrounding the lens has an average index of refraction close to that of the lens. The iris, the colored portion of the eye, is a muscular diaphragm that regulates the amount of light entering the eye by dilating the pupil (increasing its diameter) in light of low intensity and contracting the pupil in high-intensity light.



Light entering the eye is focused by the cornea-lens system onto the back surface of the eye, called the retina. The surface of the retina consists of millions of sensitive receptors called rods and cones. When stimulated by light, these structures send impulses via the optic nerve to the brain, where a distinct image of an object is perceived.

The eye focuses on a given object by varying the shape of the pliable crystalline lens through an amazing process called accommodation. An important component in accommodation is the ciliary muscle, which is attached to the lens. It is evident that there is a limit to accomodation, because objects that are very close to the eye produce blurred images. The **near point** is the smallest distance for which the lens will produce a sharp image on the retina. This distance usually increases with age.

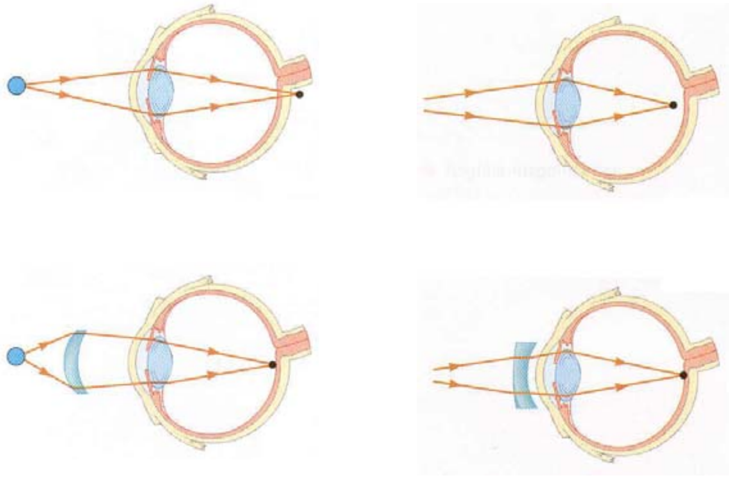
Defects of the Eye

An eye can have several abnormalities that keep it from functioning properly. When the relaxed eye produces an image of a nearby object behind the retina, the abnormality is known as **hyperopia**, and the person is said to be **farsighted**. With this defect, distant object are seen clearly but near object are blurred. Either the hyperopic eye is too short or the ciliary muscle cannot change the shape of the lens enough to focus the image properly. The condition can be corrected with a converging lens.

Another condition, known as **myopia**, or **nearsightedness**, occurs when a distant object is focused in front of the retina. This can be corrected with a diverging lens.

A common eye defect is **astigmatism**, in which light from a point source produces a line image on the retina. This occurs when the cornea or the lens are not perfectly spherical. A cylindrical lens is used to correct this.

The Power



The **power, *P*,** of a lens in diopters equals the inverse of the focal length in meters. That is, $P=1/f$.

For example, a converging lens whose focal length is +20 cm has a power of 1/(+0.2 m)=+5 diopters, and a diverging lens whose focal length is –40 cm has a power of 1/(-0.4 m)=-2.5 diopters.

- The near point of an eye is 50 cm. What focal length must a corrective lens have to enable the eye to see clearly an object 25 cm away? What is the power of this lens?

  The thin-lens equation enables us to solve this problem. We have placed an object at 25 cm, and we want the lens to form an image at the closest point that eye can see clearly. This corresponds to the near point, 50 cm.

$$\begin{aligned} \frac{1}{p}+\frac{1}{q} &= \frac{1}{f} \\ \frac{1}{25\ \mathrm{cm}}+\frac{1}{-15\ \mathrm{cm}} &= \frac{1}{f} \\ f &= 50\ \mathrm{cm} \\ P &= \frac{1}{f}=\frac{1}{0.5\ \mathrm{m}}=2\ \mathrm{diopters} \end{aligned}$$

- A particular nearsighted person cannot see objects clearly when they are beyond 50 cm (the far point of the eye). What focal length should the prescribed lens have to correct this problem?

  For an object at infinity, the purpose of the lens in this instance is to place the image at a distance at which it can be seen clearly.

$$\begin{aligned} \frac{1}{p}+\frac{1}{q} &= \frac{1}{f} \\ \frac{1}{\infty}+\frac{1}{-50\ \mathrm{cm}} &= \frac{1}{f} \\ f &= -50\ \mathrm{cm} \\ P &= \frac{1}{f}=\frac{1}{-0.5\ \mathrm{m}}=-2\ \mathrm{diopters} \end{aligned}$$

Most of our everyday experiences and observations deal with objects that move at speeds much lower than the speed of light. Newtonian mechanics and the early ideas on space and time were formulated to describe the motion of such objects. Although Newtonian mechanics works very well at low speeds, it fails when applied to particles whose speeds approach that of light.

In 1905, Einstein published his **special theory of relativity** which covers such phenomena. This theory is based on two postulates:

- The laws of physics are the same in all inertial reference systems.
- The speed of light in a vacuum is always measured to be 300 000 km/s, and the measured value is independent of the motion of the observer or of the motion of the source of light.

Within the framework of Einstein's postulates of relativity, it is found that momentum is not conserved if the classical definition of momentum, $p=mv$, is used. However, according to the principle of relativity, momentum must be conserved in all reference systems. The correct relativistic equation for **momentum** that satisfies these conditions is

$$p = \frac{mv}{\sqrt{1-v^2/c^2}}$$

where v is the velocity of the particle.

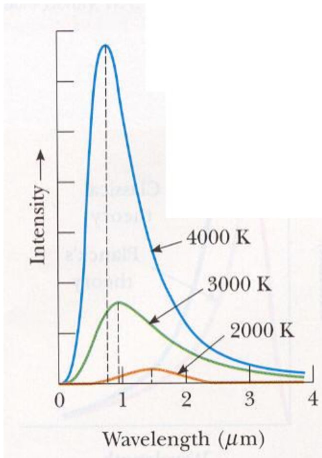
It is also found that the minimum energy of some object is

$$E=mc^2$$

called the **rest energy**, where m is mass of the object and c is speed of the light. This famous mass-energy equivalence equation shows that **mass is one possible manifestation of energy**. It shows that a small mass corresponds to an enormous amount of energy.

A modern version of mechanics called **quantum physics** was highly successful in explaining the behaviour of atoms, molecules, and nuclei. The earliest and most basic ideas of quantum theory were introduced by Planck. An extensive study of quantum theory is beyond the scope of this course, as well as relativistic theory. We will only underly ideas of quantum theory.

An object at any temperature is known to emit radiation (Stefan's law, Part 10). The spectrum of the radiation depends on the temperature and properties of the object. At low temperatures, the wavelengths of the thermal radiation are mainly in the infrared region and hence not observable by the eye. As the temperature of an object increases, the object eventually begins to glow red. At sufficiently high temperatures, it appears to be white. With increasing temperature, the peak of the distribution shifts to shorter wavelengths. This shift was found to obey the following relationship, called **Wien's law**



$$\lambda_{max} T = 2.898 \cdot 10^{-3}\ \mathrm{mK}$$

Early attempts to use classical ideas to explain the blackbody radiation failed. In 1900 Planck developed a formula for blackbody radiation that was in complete agreement with experiments. He made assumption that submicroscopic electric oscillators can emit discrete units of light energy that are called **photons** with energy of

$$E = hf \qquad (h = 6.626 \cdot 10^{-34}\ \text{Js is Planck's constant})$$

f is the frequency of an electric oscillator (and light emitted).

The key point in Planck's theory is the radical assumption of quantizied energy states.

Experiments showed that, when light is incident on certain metallic surfaces, electrons are emitted from the surface. This phenomenon is known as the **photoelectric effect**. According to photoelectric effect equation (introduced by Einstein) the kinetic energy for those liberated electrons is

$$E_k = hf - W$$

where W is called the **work function** of the metal. The work function represents the minimum energy with which an electron is bound in the metal.

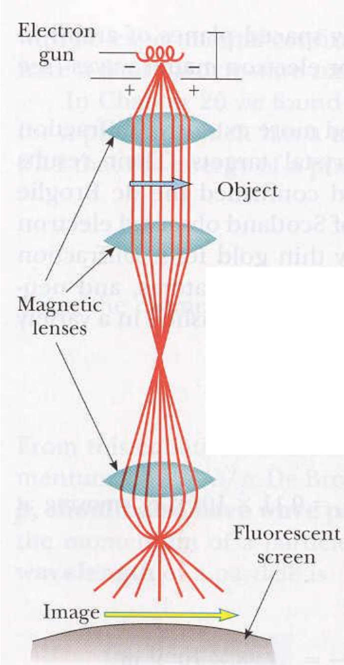
Many practical devices in our everyday lives depend on the photoelectric effect. For example, a use familiar to everyday is that of turning street lights on at night and off in the morning. A photoelectric control unit in the base of the light activates a switch to turn off the streetlight when ambient light of the correct frequency falls on it.

In 1923 Louis de Broglie postulated that because photons have wave and particle characteristics, perhaps all forms of matter have wave as well as particle properties. He suggested that material particles, of momentum p, should also have **wave properties** and a corresponding wavelength λ

$$\lambda = \frac{h}{p}$$

This proposal was first regarded as pure speculation. But in 1927 Davisson and Germer succeeded in measuring the wavelength of electron.

A practical device that relies on the wave characteristics of electrons is the **electron microscope**, which is in many respects similar to an ordinary compound microscope. One important difference is that the electron microscope has a much greater resolving power because electrons can be accelerated to high momentum, giving them a very short wavelength. **Any sort of microscopes is capable of detecting details that are comparable in size to the wavelength of the radiation used to illuminate the object**. The wavelengths of electrons typically are about 100 times shorter than those of the visible light used in optical microscopes. As a result, electron microscopes are able to distinguish details about 100 times smaller.



If you were to mesuring the position and velocity of a particle at any instant, you would always be faced with reducing the experimental uncertainties in the measurements as much as possible.

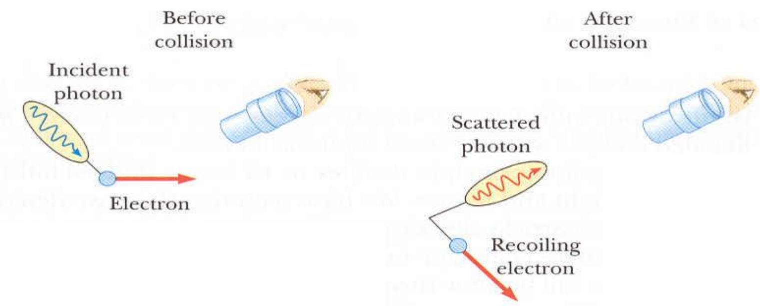
According to classical mechanics, there is no fundamental barrier to an ultimate refinement of the apparatures or experimental procedures.

Quantum theory predicts, however, that **it is impossible to make simultaneous measurements of a particle's position and velocity with infinite accuracy**. This statement, known as **uncertainty principle**, was first derived by Heisenberg in 1927.

Suppose that Δx and Δp_x represent the uncertainty in the measured values of the particle's position and momentum along the x axis at some instant. The uncertainty principle says that the product $\Delta x\, \Delta p_x$, is never less than a number of the order of Planck's constant h

$$\Delta x \Delta p_x \geq \frac{h}{2\pi}$$

That is, it is physically impossible to measure simultaneously the exact position and exact momentum of a particle.



- What is the Sun's surface temperature if the peak wavelength in its radiation is 500 nm?

  From Wien's law we have

$$\begin{aligned} \lambda_{max} T &= 2.898 \cdot 10^{-3}\text{mK} \\ T &= \frac{2.898 \cdot 10^{-3}\ \text{mK}}{500 \cdot 10^{-9}\ \text{m}} \\ &= 5800\ \text{K} \end{aligned}$$

- Calculate the energy of a photon having a wavelength in the x-ray range, 5 nm.

$$\begin{aligned} E_\gamma &= hf = h\frac{c}{\lambda} \\ &= (6.626 \cdot 10^{-34}\ \text{Js})\frac{3 \cdot 10^8\ \text{m/s}}{5 \cdot 10^{-9}\ \text{m}} = 3.98 \cdot 10^{-17}\ \text{J} \end{aligned}$$

◦ Suppose optical radiation ($\lambda = 500$ nm) is used to determine the position of an electron to within the wavelength of the radiation. What will be the resulting uncertainty in the electron's velocity?

From the uncertainty principle we have

$$
\begin{aligned}
\Delta x \Delta p_x &\geq \frac{h}{4\pi} \\
\Delta x \Delta (m v_x) &\geq \frac{h}{4\pi} \\
m \Delta x \Delta v_x &\geq \frac{h}{4\pi} \\
\Delta v_x &\geq \frac{h}{4\pi m \Delta x} \\
\Delta v_x &\geq \frac{6.626 \cdot 10^{-34}\ \mathrm{Js}}{43.14 (9.1 \cdot 10^{-31}\ \mathrm{kg})(500 \cdot 10^{-9}\ \mathrm{m})} \\
\Delta v_x &\geq 116\ \mathrm{m/s}
\end{aligned}
$$

Early Models of the Atom

The model of the atom in the days of Newton was a tiny, hard, indestrucible sphere.

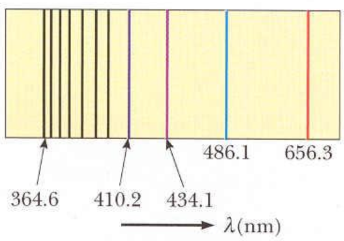
Thomson suggested a model of the atom as a volume of positive charge with electrons embedded throughout the volume.

Rutherford assumed that the positive charge in an atom was concentrated in a region that was small relative to the size of the atom, called the **nucleus**. Any electrons belonging to the atom were assumed to be in the volume outside the nucleus, moving in the same manner as the planets orbit the Sun.

Using the simplest atom, hydrogen, Bohr proposed a model of the hydrogen atom based on a clever combination of classical and early quantum concepts. His basic assumption – that atoms exist in discrete quantum states of well-defined energy – was a bold break with classical ideas. In spite of its successes, Bohr's specific model of the hydrogen atom was inconsistent with the uncertainty principle and was replaced by the probability density model derived fom Schrödinger's work.

Atomic Spectra

If a voltage applied between metal electrodes in the tube (filled with some gas), the tube emits light whose color is characteristic of the gas in the tube. When the emitted light is analyzed with a spectrometer, a series of lines is observed. Such a series of spectral lines is commonly referred to as an **emission spectrum**. The wavelengths contained in a given line spectrum are characteristic of the element emitting the light. Because no two elements emit the same line spectrum, this phenomenon represents a marvelous and reliable technique for identifying elements in a substance.

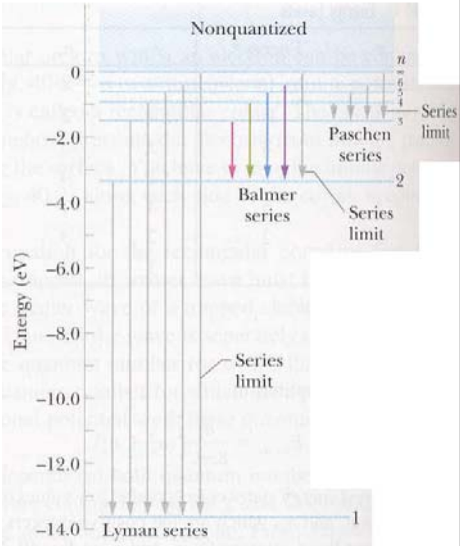


An element can also absorb light at specific wavelengths, known as **absorption spectrum**. The absorption spectrum consists of a series of dark lines superimposed on the otherwise continuous spectrum. Each line in the absorption spectrum of a given element coincides with a line in the emission spectrum of the element.

One of the first great achievements of quantum mechanics was the solution of the wave equation for the hydrogen atom. Three **quantum numbers** emerged from the solution of the wave equation

- principal quantum number, n=1,2,3,...
- orbital quantum number, l=0,1,2,…,n-1
- orbital magnetic quantum number, m_l =-l,…,l

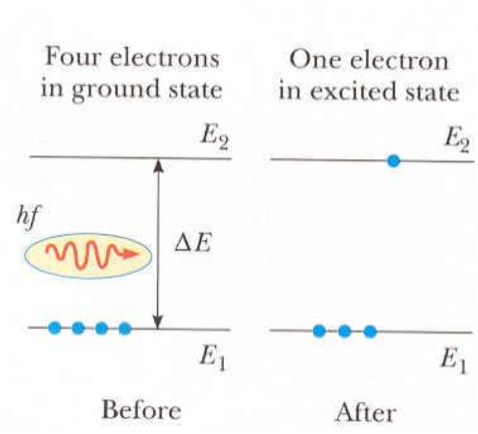
It was later found that another quantum number, m_s, the spin magnetic quantum number had to be introduced with two values, +½, -½.



The state of an electron in an atom is specified by four quantum numbers, that we introduced (n, l, m_l, m_s). These quantum numbers can be used to describe all the electronic states of an atom regardless of the number of electrons in its structure. Obvoious question that arises is, how many electrons in an atom can have a particular set of quantum numbers. Pauli answered this in statetements known as the **exclusion principle**: **no two electrons in an atom can ever be in the same quantum state; that is, no two electrons in the same atom can have the same set of quantum numbers**.

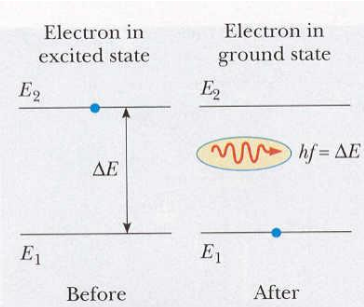
Hydrogen has only one electron, which, in its ground state, can be described by either of two sets of quantum numbers: 1,0,0,+½ or 1,0,0,-½. The electronic configuration of this atom is designated as $1s^1$. The notation 1s refers to a state for which n=1 and l=1, and the superscript indicates that one electron is present in this level. Neutral helium has two electrons. The quantum numbers are 1,0,0,+½ and 1,0,0,-½, with configuration $1s^2$.

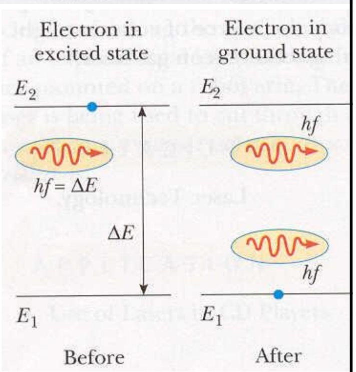
An atom will emit radiation only at certain frequency that corresponds to the energy separation between the various allowed states. When light is incident on the atom, only those photons whose energy, *hf*, matches the energy separation ΔE between two levels can be absorbed by the atom (**stimulated absorption process**). As a result, some atoms are raised to various allowed higher energy levels, called excited states.



Once an atom is in an excited state, there is a constant probability that it will jump back to a lower energy level by emitting a photon. This process is known as **spontaneous emission**.

A third process that is important in lasers, **stimulated emission**, was predicted by Einstein in 1917. Suppose an atom is in the excited state and a photon with energy *hf* =ΔE is incident on it. The incoming photon increases the probability that the excited electron will return to the ground state and thereby emit a second photon having the same energy *hf*. These photons can stimulate other atoms to emit photons in a chain of similar processes. The many photons produced in this fashion are the source of the intense, coherent light in a **laser**.





All nuclei are composed of two types of particles: **protons** and **neutrons**. In describing some of the properties of nuclei, such as their charge, mass, and radius, we make use of the following quantities:

- the **atomic number, Z**, which equals the number of protons in the nucleus
- the **neutron number, N**, which equals the number of neutrons in the nucleus
- the **mass number, A**, which equals the number of nucleons in the nuclus. (Nucleon is a generic term used to refer to either a proton or a neutron.)

The symbol we use to represent nuclei is $^{A}_{Z}\mathrm{X}$, where X represents the chemical symbol for the element. The subscript Z can be omited because the chemical symbol determine Z.

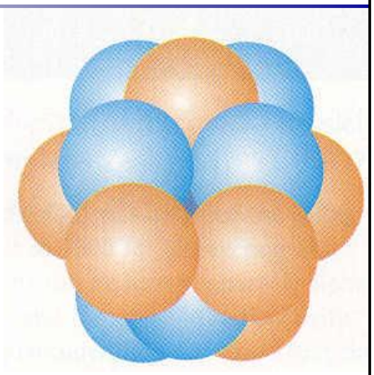
The nuclei of all atoms of a particular element must contain the same number of protons, but they may contain different numbers of neutrons. Nuclei that are related in this way are called **isotopes**. The isotopes of an element have the same Z value but different N and A values.

The proton carries a single positive charge, $+e$, where $e = 1.6 \cdot 10^{-19}$ C and the neutron is electrically neutral.

The masses of the proton and the neutron are almost equal, $1.67 \cdot 10^{-27}$ kg and about 2000 times as massive as the electron.

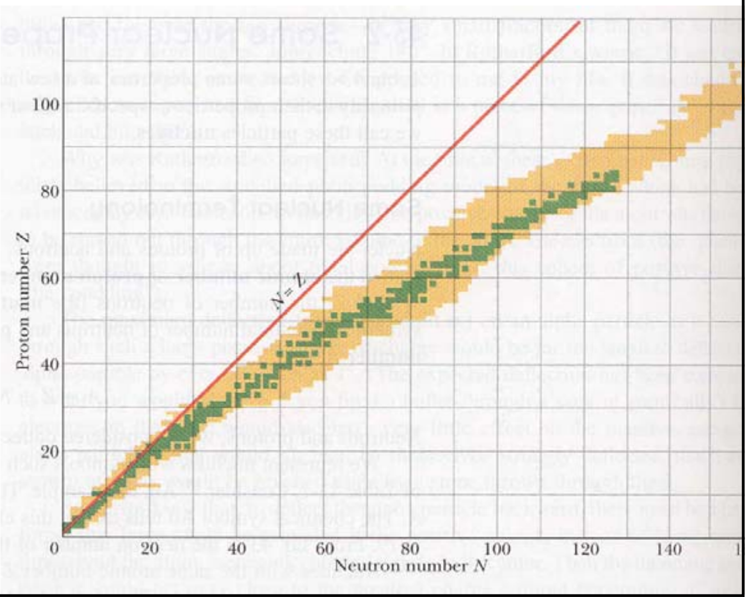
It is convinient to define the unified **mass unit**, u, in such a way that the mass of one atom of the isotope ^{12}C is exactly 12u, where $u = 1.67 \cdot 10^{-27}$ kg

The very large repulsive electrostatic forces between protons should cause the nucleus to fly apart. However, nuclei are stable, because of the presence of another, short-range force, the nuclear force. This is an attractive force that acts between all nuclear particles. The protons attract each other via the nuclear force, and at the same time they repel each other through the coulomb force. The nuclear force also acts between pairs of neutrons and between neutrons and protons.



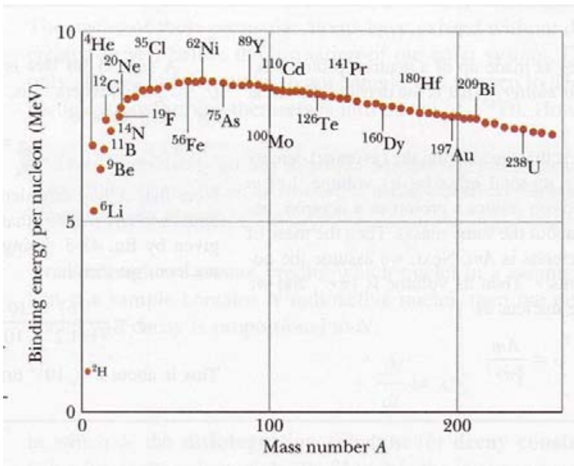
The nuclear force dominates the coulomb force within the nucleus, and this strong nuclear force is nearly independent of charge.

There are about 260 stable nuclei; hundreds of others have been observed but are unstable. Light nuclei are most stable if they contain equal numbers of protons, but heavy nuclei are more stable if N>Z. Elements that contain more than 83 protons do not have stable nuclei.



The total mass of a nucleus is always less than the sum of the masses of its nucleons. Because mass is another manifestation of energy, the total energy of the bound system (the nucleus) is less than the combined energy of the separated nucleons. This difference in energy is called the **binding energy** of the nucleus and can be thought of as the energy that must be added to a nucleus to break it apart into its components. Therefore, in order to separate a nucleus into protons and neutrons, energy must be put into the system.

It is interesting to examine a plot of binding energy per nucleon E/A as a function of mass number for various stable nuclei. Nuclei with mass numbers greater or less than about 60 are not as strongly bound as those with about A=60.



In 1896 Becquerel accidentally discovered that uranium salt crystals emit an invisible radiation that can darken a photographic plate even if the plate is covered to exclude light. This spontaneous emission of radiation was soon called **radioactivity**.

Three types of radiation can be emitted by a radioactive: **alpha (α) rays**, in which the emitted particles are He nuclei; **beta (β) rays**, in which the emitted particles are either electrons or positrons; and **gamma (γ) rays**, in which high-energy photons are emitted.

The three types of radiation have quite different penetrating powers, Alpha particles barely penetrate a sheet of paper, beta particles can penetrate a few milimeters of aluminium, and gamma rays can penetrate several centimeters of lead.

If a radioactive sample contains N radioactive nuclei at some instant, it is found that the number of nuclei, ΔN, that decay in a small time interval δt is proportional to N

$$\Delta N = -\lambda N \Delta t$$

where λ is a constant called the **decay constant**. The negative sign signifies that N decreases with time. The value of λ for any isotope determines the rate at which that isotope will decay. **The decay rate, or activity**, R, of a sample is defined as the number of decays per second

$$R = \left| \frac{\Delta N}{\delta t} \right| = \lambda N$$

A general decay curve for a radioactive sample varies with time according to the expression

$$N = N_0 e^{-\lambda t}$$

where N is the number of radioactive nuclei present at time t. N_0 is the number present at time $t = 0$, and $e = 2.718$ is the base of the natural logarithms.

Another parameter that is useful for characterizing radioactive decay is the **half-life, $T_{1/2}$**. The half-life of a radioactive substance is the time it takes half of a given number of radioactive nuclei to decay. Setting $N = N_0/2$ and $t = T_{1/2}$ in above equation we get

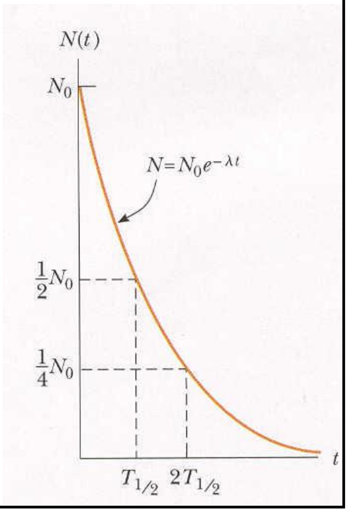
$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

The unit of activity is the curie (Ci), defined as

$$1\ \text{Ci} = 3.7 \cdot 10^{10}\ \text{decays/s}$$

but the SI unit of activity is the becquerel (Bq)

$$1\ \text{Bq} = 1\ \text{decays/s}$$



If a nucleus emits an alpha particle ($^4_2$He), it loses two protons and two neutrons. It can be written symbolically as

$$^A_Z\text{X} \longrightarrow {}^{A-4}_{Z-2}\,\text{Y} + {}^4_2\,\text{He}$$

where X is called the **parent nuclei** and Y **daughter nuclei**. For example, ^{226}Ra is alpha emitter

$$^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\,\text{Rn} + {}^4_2\,\text{He}$$

The half-life for ^{226}Ra is 1600 years.

When one element changes into another, as happens in alpha decay, the process is called **spontaneous decay**, or **transmutation**. As a general rule, the sum of the mass numbers A must be the same on both sides of the equation, and also the sum of the atomic numbers Z.

When a radioactive nucleus undergoes beta decay, the daughter nucleus has the same number of nucleons as the parent nucleus, but the atomic number is changed by 1

$$ {}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z+1}\mathrm{Y} + e^{-} + \bar{\nu} \qquad \text{or} \qquad {}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z-1}\mathrm{Y} + e^{+} + \nu $$

where $\bar{\nu}$ indicates antineutrino and ν neutrino (both electrically neutral and have little or no mass); e^{+} indicates positron and e^{-} electron.

A typical beta decay event is

$$ {}^{14}_{6}\mathrm{C} \longrightarrow {}^{14}_{7}\mathrm{N} + e^{-} + \bar{\nu} $$

Very often a nucleus that undergoes radioactive decay is left in an excited energy state. The nucleus can then undergo a second decay to a lower energy state, perhaps to the ground state, by emitting one or more photons. The photons emitted in such a de-excitation process are called **gamma rays**.

The beta decay of ^{14}C is commonly used to date organic samples. Cosmic rays (high-energy particles from outer space) in the upper atmosphere cause nuclear reactions that create ^{14}C from ^{14}N. In fact, the ratio of ^{14}C to ^{12}C in the carbon dioxide molecules of our atmosphere has a constant value of about $1.3 \cdot 10^{-12}$ as determined by measuring carbon ratios in tree rings. All living organisms have the same ratio of ^{14}C to ^{12}C because they continuosly exchange carbon dioxide with their surroundings. When an organism dies, however, it no longer absorbs ^{14}C from atmosphere, and so the ratio of ^{14}C to ^{12}C decreases as the result of the beta decay of ^{14}C. It is therefore possible to determine the age of a material by measuring its activity per unit mass as a result of the decay of ^{14}C. Using carbon dating, samples of wood, charcoal, bone, and shell have been identified as having lived from 1000 to 25000 years ago (^{14}C has half-life of 5730 years). This knowledge has helped scientists and researchers to reconstruct the history of living organisms during this time span.

Radiation absorbed by matter can cause severe damage. The degree and type of damage depend on several factors, including the type and energy of the radiation and the properties of the absorbing material. Radiation damage in biological organisms is primarily due to ionization effects in cells. The normal function of a cell may be disrupted when highly reactive ions or radicals are formed as the result of ionizing radiation. Cells that do survive the radiation may become defective, which can lead to cancer.

In biological systems, it is common to separate radiation damage into two categories: **somatic** and **genetic damage**. Somatic damage is radiation damage to any cells except the reproductive cells. Such damage can lead to cancer at high radiation levels or seriously alter the characteristics of specific organisms. Genetic damage affects only reproductive cells. Damage to the genes in reproductive cells can lead to defective offspring.

Several units are used to quantify radiation exposure and dose. The **rad** (radiation absorbed dose) is defined as that amount of radiation that deposits 0.01 J of energy into 1 kg of absorbing material.

Although the rad is a perfectly good physical unit, it is not the best unit for measuring the degree of biological damage produced by radiation. This is because the degree of damage depends not only on the dose but also on the type of radiation. For example, a given dose of alpha particles causes about ten times more biological damage than equal dose of x-rays.

The **RBE** (relative biological effectivness) factor is defined as the number of rad of x-radiation or gamma radiation that produces the same biological damage as 1 rad of the radiation being used.

The **rem** (roentgen equivalent in man) is defined as the product of the dose in rad and the RBE factor

(Dose in rem) = (dose in rad) x (RBE)

According to this definition, q rem of any two radiation will produce the same amount of biological damage. From table, we see that a dose of 1 rad of fast neutrons represents an effective dose of 10 rem and that 1 rad of x-radiation is equivalent to a dose of 1 rem.

Radiation	RBE Factor
X-rays and gamma rays	1.0
Beta particles	1.0–1.7
Alpha particles	10–20
Slow neutrons	4–5
Fast neutrons and protons	10
Heavy ions	20

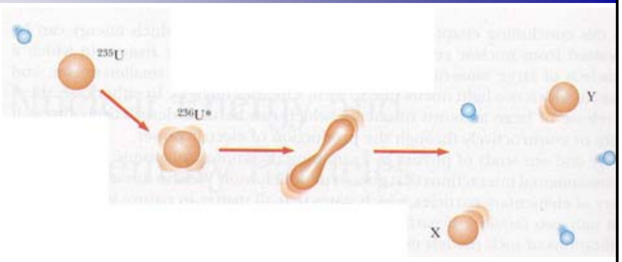
Low-level radiation from natural sources, such as cosmic rays and radioactive rocks and soil, delivers to each of us a dose of about 0.13 rem/year. The upper limit of radiation dose (recommended) is 0.5 rem/year. An acute whole-body dose of 500 rem results in a mortality rate of about 50 %.

- Can carbon-14 dating be used to measure the age of a stone?
  Carbon dating cannot generally be used to estimate the age of a stone, because the stone was not alive to take up carbon from the environment. Only the ages of artifacts that were alive can be estimated with carbon dating.

- What fraction of a radioactive sample has decayed after two half-lives have elapsed?
  After the first half-life, half the original sample remains. After the second half-life a quarter of the original sample remains. Thus, three quarters of a radioactive sample has decayed after two half-lives.

- A person whose mass is 75 kg is exposed to a whole-body dose of 25 rad. How many joules of energy are deposited in the person's body?
  E = (75 kg)(25)(0.01 J/kg) = 18.75 J.

Nuclear fission occurs when a heavy nucleus, such as U-235, splits, or fissions, into two smaller nuclei. In such a reaction, the total mass of the products is less than the original mass of the heavy nucleus.

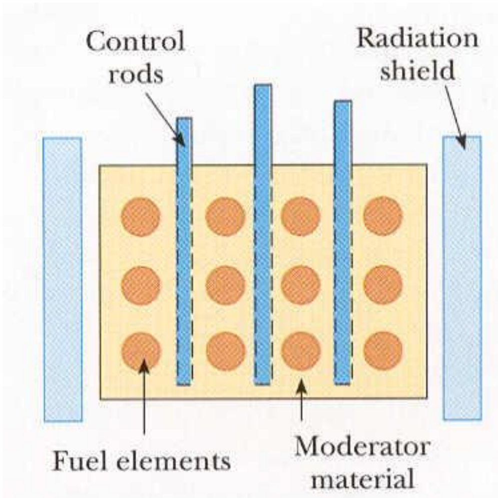


A typical reactions of this type is

$$^{1}_{0}n + ^{235}_{92}\mathrm{U} \rightarrow ^{236}_{92}\mathrm{U} \rightarrow ^{141}_{56}\mathrm{Ba} + ^{92}_{36}\mathrm{Kr} + 3\,^{1}_{0}n$$

The fission fragments, barium and krypton, and the released neutrons have a great deal of kinetic energy following the fission event. Neutrons that are emitted can in turn trigger other nuclei to undergo fission, with the possibility of a chain reaction. If the chain reaction is not controlled, it could result in a violent explosion, with the release of an enormous amount of energy.

The basic design of a nuclear reactor is shown in figure. The fuel elements consists of enriched uranium. Moderator substance regulate neutron energies slowing the neutrons down. Control rods are made of materials such as cadmium that are very efficient in absorbing neutrons. The rods controled the average number of neutrons from each fission event that will cause another event. This average number has to be 1.



Binding energy for light nuclei is much smaller than the binding energy for heavier nuclei. When two light nuclei combine to form a heavier nucleus, the process is called **nuclear fusion**. Because the mass of the final nucleus is less than the masses of the original nuclei, there is a loss of mass accompanied by a release of energy. Although fusion power plants have not yet been developed, a great worldwide effort is under way to harness the energy from fusion reactions in the laboratory.

The hydrogen bomb, first exploded in 1952, is an example of an uncontrolled fusion.

All stars generate their energy through fusion processes. About 90 % of the stars, including the **Sun**, fuse hydrogen. The Sun radiates energy at the rate of 390 YW (yotta watt) and has been doing so for several billion years. The fusion in the Sun is a multistep process in which hydrogen is burned into helium. There is enough hydrogen to keep the Sun going for about 5 billion year into future.