Dimensionless Groups in Chemical Engineering

Dimensionless Groups or numbers – i.e. relationships with no units of measurement – are often used by process engineers. The equations given below are quoted in metric units, however provided consistent units are used, the dimensionless number remains unchanged. Numerous dimensionless groups are used in chemical engineering. This article introduces some of the more common ones

Reynolds Number (Re)

The Reynolds Number, Re, is arguably the most commonly used dimensionless group in chemical engineering. It gives a measure of the ratio of inertial and viscous forces in fluid flow. It is often used to determine if the flow is either laminar or turbulent:

– In laminar flow, viscous forces dominate. The flow paths are smooth, streamline and constant.

– In turbulent flow, inertial forces dominate. The flow regime is unstable, generating eddies and vortices. The Reynolds Number can be calculated using the following equation:

$$Re = \frac{\rho . u . L}{\mu}$$

Where

ρ - Fluid density (kg/m3)

u - Fluid velocity (m/s)

L - Characteristic dimension (m)

m - Dynamic viscosity (Pa.s)

The units used to calculate Reynolds Number are unimportant – provided that they are consistent.

In pipes (or circular cross-section) the characteristic dimension can be taken as the pipe diameter. For pipe flow, laminar flow is usually found at Reynolds Numbers below about 2000, and turbulent flow is found above about 4000. At Reynolds number between about 2000 and 4000, the flow regime is transitional – i.e. rapidly changing between laminar and transitional flow.

Prandtl Number (Pr)

The Prandtl number, Pr, is the ratio of kinematic viscosity to the thermal diffusivity. It is used in many calculations involving heat transfer in flowing fluids, as it gives a measure of the relative thickness of the thermal and momentum boundary layers. It can be calculated using the following equation:

$$\mathrm{Pr} = \frac{C_p.\mu}{k}$$

Where

Cp - Fluid Specific Heat Capacity (J/kg.K)

m - Dynamic viscosity (Pa/s)

k - Thermal conductivity (W/m.K)

It should be noted that the Prandtl number is dependent on the fluid physical properties alone. Consequently, it is often found in physical properties. For many gases (with the notable exception of hydrogen), the Prandtl number has a value of 0.6 to 0.8 over a wide range of conditions.

Nusselt Number (Nu)

The Nusselt Number, Nu, is the ratio of convective to conductive heat transfer in a fluid over a given length, l:

$$Nu = \frac{h.l}{k}$$

Where

h	-	Heat Transfer Coefficient (W/m2.K)
l	-	Characteristic length (m)
k	-	Thermal conductivity (W/m.K)

For heat transfer in pipes, the characteristic length is the pipe diameter.

At a Nusselt number of around one means that convection and conduction are about equal. Typically, this occurs in laminar conditions. As the Nusselt number becomes larger, convective heat transfer becomes relatively more important – this occurs as the flow becomes more turbulent. The mass transfer equivalent of the Nusselt number is the Sherwood Number

Sherwood Number (Sh)

The Sherwood Number, Sh, is a measure of the ratio of convective and diffusive mass transfer in a fluid. It is analogous to the Nusselt Number in heat transfer:

$$Sh = \frac{h_D.l}{D}$$

Where

hD - Mass Transfer Coefficient (m/s)

l - Characteristic length (m)

k - Molecular Diffusivity (m2/s)

Froude Number (Fr)

The Froude Number,Fr, is a measure of the ratio of the inertial and gravitational forces and can be expressed as:

$$Fr = \frac{v}{(g.l)^{1/2}}$$

Where

v - Velocity (m/s)

g - Acceleration due to gravity (m/s2)

l - Characteristic length (m)

It is often used to analyse fluid flow problems where there is a free surface. For example, in agitated vessels, Fr governs the formation of free surface vortices:

Grashof Number (Gr)

The Grashof Number, Gr, is a ratio of the buoyancy and viscous forces. It can be expressed as:

$$Gr = \frac{\beta . g . \Delta T . l^3 . \rho^2}{\mu^2}$$

Where

b	-	Volumetric coefficient of thermal expansion (-)
g	-	Acceleration due to gravity (m/s2)
DT	-	Temperature difference (K)
l	-	Characteristic length (m)
r	-	Fluid Density (kg/m3)
m	-	Dynamic Viscosity (Pa.s)

It is used to calculate heat transfer in natural convection, where fluid velocity depends on buoyancy.

Mach Number (Ma)

The Mach Number, Ma, is the ratio of the fluid velocity to the velocity of sound in that medium. It can be expressed as: $Ma = \frac{u}{a}$ Where

u - Fluid velocity (m/s)

a - Speed of sound in fluid medium

In Chemical Engineering, the Mach Number is commonly used in calculations involving high velocity gas flow.

Schmidt Number (Sc)

Schmidt Number, Sc, is the ratio of kinematic viscosity to the diffusivity. It can be expressed as:

$$Sc = \frac{\mu}{\rho.D}$$

Where

m - Dynamic viscosity (Pa/s)

r - Fluid Density (kg/m3)

D - Diffusivity (m2/s)

It characterises mass transfer in a flowing fluid