Relativity predicts a variable G

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Abstract. It is shown that relativity predicts a variable G. The proof starts by considering a dimensionless particle in an empty universe. Then two particles, three particles, and an infinite set of particles are studied. This allows to calculate space-time structure for any realistic energy distribution. The proof uses the interchange of limits theorem, and ad hoc sequences of energy distributions. With only one particle, the result is a singularity everywhere if the universe is empty outside of the particle. Those singularities disappear completely with three particles. Then this calculation is done for any realistic energy distribution. An equation of G is given naturally in the process. This equation is a correct approximation in most of the realistic energy distributions. The fundamental principles building Einstein equation are still valid. But it is shown that the G anthropocentric solar system constant must be replaced by a variable value, which is weaker in strong matter density environments, and greater in low matter density environments. It means that the surrounding effect arises, it was introduced by previous works [1, 2] And this effect was shown to solve the gravitational mysteries of today in astrophysics and in cosmology. Under a unifying relevant assumption, a solution is also given to the Yang-Mills Millennium problem.

1 Introduction

The purpose of the present document is to show that relativity predicts a variable G. It is also to describe a solution to the Millennium Problem [3, 4].

This study follows from previous works. In particular in [2], it was already shown that the relativistic speed of the quarks implies that G is a variable. Here it is shown that it's a theoretical prediction of relativity, independent of the speed of the quarks or any other experimental information about energy distributions.

The proof will start by considering a dimensionless particle in an empty universe. Then space-time structure can be calculated for several particles up to any infinite set of particles. The interchange of limits theorem will be used.

There are many ways to motivate this study. In the list below the most important are presented first.

1) Solving the Yang-Mills Millennium problem.

2) Solving today's gravitational mysteries.

3) Searching for a link between energy and first metric derivative.

4) Solving the slight caveats of General Relativity (GR) [2].

5) Searching for a theoretical justification of Newton's law.

6) Replacing the anthropocentric solar system value of G by a more general and sounded value.

7) Retrieving the information lost in the construction of the stress-energy tensor.

8) Studying the behaviour of the concept of normal frames.

The latter item, normal frames, is a usual key concept in GR. But one of its property has been dismissed in the literature. Working on this motivation will reveal that Special Relativity (SR) is more than an algebraic rule of GR. It allows to describe the local space-time deformation by energy. And it will be the first step for addressing item 3). At the beginning of the present research, only items 2) and 3) were present. Item 4) is by itself a list of several motivations. Items 1) 3) 5) 6) and 8) will be addressed directly in the present document. Item 7) will be addressed indirectly. Item 2) has been addressed in [1]. Item 4) has been addressed in [2] which can be considered as a previous version of the present document.

It will be proven that relativity predicts a variable equivalent G. This variation will be driven by the surrounding effect [1] which, in its weaker version, is the following. Gravitational force increases in low matter density environments, and decreases in high matter density environments. Then an equation of G will be given naturally in the process. Finally, the Yang-Mills Millennium problem will be addressed.

2 Mathematic reminder: interchange of limits theorems

Norms are equivalent in finite dimension, hence this is true for the four dimensional space-time of GR. By other means, the determination of space-time structure is a function of energy distribution. And this function is a continuous function. Indeed, if any amount of energy at any space-time location is decreased to 0, then the effect of this amount of energy on space-time structure decreases also to 0. This continuity around the 0 value is the result of conservation of energy principle. The restriction to the 0 value is possible because any space-time structure can be seen as flat euclidean almost locally by chosing an ad hoc system of frames. A rigorous mathematical description of this is given in appendix 1. One formulation of the interchange of limits theorem is the following. If any f_n sequence of energy distributions over space-time tends uniformly to some limit energy distribution, if S is the function giving space-time structure from an energy distribution, then the following equation arises.

$$S\Big(lim_u(f_n)\Big) = lim_u\Big(S(f_n)\Big) \tag{1}$$

In equation (1), lim_u means the uniform limit, for the f_n sequence, and for the space-time structure.

The continuity of the determination of space-time structure as a function of energy distribution has another interesting consequence. It is possible to imagine thought experiments in which energy is increased or decreased progressively. And this can be done without violation of the conservation of energy principle. Indeed, GR is a mathematical framework which can, and sometimes must be watched independently of reality.

3 A privileged frame in relativity

3.1 Definition and properties

In relativity there exists a privileged frame. Moreover, the boost which is associated with the motion of matter in this frame, describes the evolution of this privileged frame. This can be reminded with a thought experiment, avoiding then any complicated and tedious calculation. It is done in appendix 2. The conclusion is that it exists a privileged frame in any space-time event (using the extension of identification which is presented in appendix 2). And for any particle located in a given x event, this privileged frame exists in x and is transformed by the particle, using the boost which is associated with the four-momentum of the particle. Roughly speaking for the understanding, let's write that this boost is calculated in the "old privileged frame", that is, the one "just before the particle", and that it transforms this old privileged frame into the new one, that is, the one "just after the particle". Another definition is that the old one describes space-time locally without the existence of the particle, and the new one does it just after the existence of the particle. Moreover, the identification of the new privileged frame from the old one can be done progressively, using the continuity of the function giving space-time structure from energy distribution. For this identification, the following scenario is unrealistic but mandatory. The energy of the particle is increased progressively from 0 to its real value. Conservation of energy principle is violated but it does not matter. The reasoning is only mathematical, exploiting the features of the GR mathematical framework.

Also a rescaling of time and space unit occurs after the boost. This will be shown further. But in the present document, it will be written abusively that the local deformation is described by a boost. Indeed, only the context will allow to deduce if a rescaling occurs also, or not. There will be more information about that in appendix 4.

This concept of privileged frame is central in GR. It is referenced in the literature as the "normal frame". It has been associated here with the rule governing its evolution with respect to matter and with motion of matter. The link between local space-time deformation and matter is given by this concept and this rule. This link is local and implies only the first degree of derivation of the metric. It should be possible to induce from that the second degree of derivation, and then compare the result with Einstein equation. Whatever the result is, a new view of space-time deformation by energy arises.

3.2 Example

A simple example is a P particle in motion along a straight line in a static universe filled with a constant matter density. Without P this universe results in a flat Minkowskian universe. The old $R_{0\ system}$ system of privileged frames is represented by the same and constant R_0 frame, a frame for which universe is at rest. This is called the "frame of fixed stars" in old literature. The $R'_{0\ system}$ new system of frames is the real existing one. It results from the existence of P.

This can be refined by assuming a progressive appearance of P in space and time. Then a continuous set of privileged frames is constructed progressively, from $R_{0\ system}$ to $R'_{0\ system}$.

For a gravitational wave (GW), the same definition applies: the "old privileged frame" is the one which would be the privileged one if the GW was not existing, the "new priviled frame" takes this GW existence into account.

4 Fundamental assumption

From now on in the present document, it will be assumed the following assumption.

Assumption (I): a GW propagates at the speed of light.

The relevance of this assumtion (I) is well described in the literature [5].

5 Space-time structure around a uniformly moving relativistic particle

Now the aim is to describe the local space-time deformation of a GW generated by a particle moving at the speed of light. The usual following assumption will be assumed. The P particle is moving at the speed of light along a D straight line in a empty universe previously structured by a flat Minkowskian space-time.

The local space-time deformation of the GW will be studied at the location where the GW deformation is the greatest. Being only local, the studied deformation is transforming the time and space axis into new ones. That is, the old privileged frame is transformed into the new one. Therefore this local deformation is described by a linear transform.

Whatever the R inertial frame is chosen, this trajectory will always be a straight line and the speed of the particle will always be the speed of light. Moreover, the property of physics are the same whatever is the chosen inertial frame. Therefore the space-time structure generated by this particle will always be the same whatever the inertial frame is chosen. It means that in R at any given time the locations in which the deformation is the greatest is always the same space cone centered on D.

More precisely, let's study in R the GW generated by P in the E_C event which is a given C space location pertaining to D, and a given t_C time. This GW propagates from C at the c speed along the space directions which are perpendicular to D, starting from C. Let's choose an E_N space-time event composed of the N space point and the t_N time in R, such that CN is perpendicular to D and E_N receives the GW propagation starting from E_C. During the same $t_N - t_C$ time interval, P moves from C to some C' space point pertaining to D. Also, when P is located in H, which is the middle of C and C', the GW starts another propagation from H in the direction of CN. This propagation is located in M at t_N, such as M is the middle of N and C'. The propagations along CN and HM are done at the c speed. Therefore this GW propagation is also along the CM line at the $c/\sqrt{2}$ speed. This is the speed of the enveloppe of the GW along its trajectory. The spatial vector of this propagation speed is normal to the enveloppe. The enveloppe is a cone centered on D. This cone is "isosceles". In other words, there is $CN = CC'$ and $HM = HC'$.

Now let's study the deformation generated by the GW. First of all, in C the local deformation which is generated by P is the b_C boost associated with the speed of light in the same direction as P. This is proven in appendix 5.

Along the D line, exactly the same boost propagates itself, at the speed of light. This is proven in appendix 7. Here, since the P trajectory is the same D line, of course this can't be noticed. But it will be noticed as soon as P will deviate from this D trajectory.

Outside of D, the local GW deformation is still described by the b_C boost. Indeed, this is the initial deformation in C. As such it is the deformation which is propagated. Also, this is coherent with the propagation speed of the enveloppe. More details are available in appendix 8 and 9.

Outside of the cone, space-time structure is determined by the following equation (the Ricci tensor is null). This is because, as it will be seen further, Einstein equation can still be assumed valid in vacuum.

$$R_{\mu}^{\nu}(g_{\mu\nu}) = 0 \tag{2}$$

$R_{\mu}^{\nu}()$ is the function giving the Ricci tensor from the metric, and $g_{\mu\nu}$ is the metric.

6 Space-time structure around a relativistic particle moving along a circle

Now let's assume that the previous P particle is forced to move along a circle. Then the previous cone transforms into a more complicated geometric figure. But infinitely far from the circle, therefore asymptotically, the enveloppe of the GW propagation at constant time is a sequence of spheres centered in O, the center of the circle. Asymptotically those spheres inflate themselves around O at the speed of light. The rays of these spheres are separated by the same d constant value which is the circle circonference. Now the deformation which is propagated asymptotically is a boost associated with the speed of light. This speed is the vector which is normal to the propagation sphere, in the direction which goes out of the sphere. Also, this deformation propagates with this speed. This is proven in appendix 10.

If the ray of the circle decreases progressively and tends to the 0 value, it means that the circle tends to its limit which is a dimensionless particle located in O. Then the enveloppe of the GW transforms itself into the previous sequence of spheres, but also d tends to 0. The final result is a sequence of spheres which are infinitely close to each other, all centered on O.

The interchange of limits theorem can be applied to this Dirac distribution of matter, namely here, the dimensionless particle located in O. It can be applied because the circle-like trajectories of microscopic particles are energy distributions which tend to this Dirac distribution. If any doubt exists because the microscopic particle is in a rotating motion around O, then it is possible to add another microscopic particle sharing the same energy and moving along the same circle-like trajectory in the opposite direction. This would cancel the whole rotating motion without changing the final result which is the following. Finally this distribution tends to the Dirac distribution centered in O, when d tends to 0.

$$\underset{n \to +\infty}{lim_u} (S_n) = m\delta \tag{3}$$

S_n is this energy distribution made of a circle-like trajectory of a virtual relativistic microscopic particle in an empty universe, this circle being centered on P. m is the mass of P, which is assumed to be located in the center of the δ Dirac distribution. Equation (4) tells that S_n tends to the Dirac distribution of P since the ray of the S_n circle tends to 0.

Therefore let's apply the interchange of limit theorem. The space-time structure generated by a 3D Dirac distribution centered in O is the limit of the previously described sequence of spheres, limit when d tends to 0, as written in the following equation.

$$S(m\delta) = S\left(\underset{n \to +\infty}{lim_u} (S_n)\right) = \underset{n \to +\infty}{lim_u} S(S_n) \tag{4}$$

Those limit deformations are described in any M space point by the boost associated with a speed of light which is oriented along OM. Let's write R, an inertial frame attached to P (in which P is at rest). M belongs to a sphere centered in O which contains all the deformations arriving at the same time in R. Moreover those spheres are now infinitely closed to each other. It means that each space-time event of the universe is modified by this singular boost. This is the space-time structure generated by the 3D Dirac distribution.

Hence the result is radically different from what is told by today's literature. Let's remind that with today's literature the space-time deformation here is the one which corresponds exactly to Newton's law occuring in solar system. But what has been proven here is that this deformation is a singularity everywhere.

This huge difference will explain why G is not a constant but a variable.

7 Partial resolution of the issue of Mach's principle

This resulting space-time structure might be argued to be wrong because it is not realistic. But the correct argument is the opposite. This description appears to be more correct than the one which is given in the literature because the distribution was supposed irrealistic in the first place. Indeed, a Dirac distribution of matter is already by itself, irrealistic since it assumes an empty universe outside of the center of the Dirac distribution. For that reason, only an irrealistic result would be searched for.

Moreover, this GR prediction is in perfect compatibility with Mach's principle [6]. Let's remind briefly the Mach's principle problem in GR. For example the issue appears in the case of a static spherically symmetric universe. A particle is located in the center of this spherical symmetry. If ρ is the matter density filling the universe, then one can distinguish two assumptions. The first one is $\rho > 0$, the second is $\rho = 0$. Close to the particle, ρ appears insignificant in both cases. Therefore, there, the spacetime deformations will be approximately the same for the two assumptions. But in the first assumption it is possible to find an inertial frame, R, at rest with the particle, which is not in rotation with respect to the universe. In R, there are no fictitious forces such as centrifugal forces. But in the second assumption it is not possible to find such a frame. Supposing that R is at rest with respect to the object is not enough. It is not possible to know if R remains inertial or not. One cannot say if in R it will appear fictitious forces or not.

The new GR prediction solves this problem. Now space-time structure becomes singular everywhere for the second case. An answer is given: in each frame at rest with the particle, no fictitious force might ever appear, since those singularities would dissolve them completely. Therefore, with this new, correctly irrealistic prediction, GR becomes more Machian.

8 Two particles

Now let's add another particle, apart from the first one. So there are two particles, at different locations, and out of them the universe is still empty. Everything is still supposed to be static, that is, the two particles are at rest in some given inertial frame.

There are still singularities, but they are located only along the straight line containing the particles's locations, but only on those points which are not between the particles's locations. They are described by the boost associated with the speed of light in the direction moving away from the particles.

Outside of the particles and of those singularities, space-time structure is first determined by conservation of GR Lagrangian in vacuum. It is simpler to say that the Ricci tensor is null in vacuum (equation (2)). This is a second degree of derivation differential equation. An integration constant is still required at the end of the calculation. Under today's version of GR, G solar system constant value and the mass of each particle are used for this. Now the determination of this integration constant remains to be proven. Hints and clues for this determination are given in appendix 3. Although this determination would be an improvement, nevertheless it is not mandatory for the study of the present document.

Also, asymptotically the space-time deformation is singular. Indeed, this asymptotic deformation is the same as the previous asymptotic deformation of the distribution with only one particle. And of course each particle still generates locally a space-time singularity. If the masses of the particles are not equal then a tough calculation is required. Let's suppose that they are equal. Then, the calculation is simpler since there is the perpendicular symmetry with respect to the mediating plane between the particles. This picture is still hugely different from what is written in today's literature.

9 Three or more than three particles

Adding a third particle to the scene will disolve those singularities which are located on the straight line containing the particles. As usual it is supposed that the third particle is at rest with respect to the other particles.

With three or more particles, space-time structure is still determined by equation (2). And under the assumption that the rest mass of the particles are equal, symmetry considerations might help to calculate space-time structure. For the determination of the integration constant, the same arguments apply as in paragraph 8.

10 Space-time structure for any distribution of energy

The studied distribution of energy is a realistic one: an infinite set of dimensionless particles. Indeed modeling reality this way is often a good approximation in astrophysics, from planets to cosmology scales, and in particle physics, because of the sparse nature of matter. Of course a more general distribution of energy might be studied. Notably a uniformly continuous distribution of energy would possibly still allow the use of the interchange of limits theorem. But this is out of the scope of the present document.

Hence let's apply the interchange of limits theorem, to this set of Dirac distributions of matter, namely here, the dimensionless particles of the universe. Let's remind that for each P_i particle, where i is the particle's number, i from 0 to infinity, there is a S_n^i sequence of energy distributions. For each i and n, S_n^i is a distribution made of a circle-like trajectory of a virtual relativistic microscopic particle in an empty universe, this circle being centered on P_i. For each i, i from 0 to infinity, the circle's ray of S_n^i tends to 0 and S_n^i tends to the Dirac distribution of the P_i particle, when n tends to infinity.

$$\underset{n\to+\infty}{lim_u}\left(S_n^i\right) = m_i\delta_i \tag{5}$$

In equation (5), δ_i is the Dirac space distribution centered on the P_i particle, and m_i is the mass of P_i. Although lim_u is a uniform convergence, nevertheless from the view point of the set of P_i, this is a simple convergence. This simple convergence for each P_i can be transformed into a uniform convergence for the whole set of P_i. It suffices for that to choose, for each n, whatever i is, the same S_n^i circle's ray. For example the value of $1/n$ can be chosen for the circle's rays of S_{n+1}^i, in the system of privileged frames of space-time structure generated by S_n^i.

The final picture is the following. Space-time structure is the result of all the microscopic GWs. Those microscopic GWs are those generated by the virtual microscopic relativistic particles representing the real P_i particles in the above study. From now on in the present document, those GWs generated by those microscopic virtual particles will be called "virtual gravitational waves" (VGW). Considering only the limit distributions ($n \to +\infty$), the set of the S_n^i is equal to the set of the $m_i\delta_i$. Then, each space-time event receives a VGW from each real particle in the universe. This gives a clue for calculating space-time structure in a different way.

11 Calculating space-time structure with virtual gravitational waves

11.1 Four-momentum equation

The same distribution of energy is still assumed. In any x space-time event, the four-momentums of all the GWs propagating in x add themselves. The fundamental reason for this is conservation of energy principle. This is shown in appendix 11.

This is true for the VGWs as shown above, but let's study how any kind of GWs combine themselves when they encounter in x. This will be studied, generally, for any kind of GWs, and then applied for the VGWs.

The resulting equation has been described in [2] and is the following.

$$D^\mu(x) = \Sigma_{n=0}^{\infty} 1_w(x, y_n) f(x, y_n) C^\mu(y_n) \tag{6}$$

Equation (6) shows the calculation of the resulting four-momentum in x. For n from 0 to infinity, each y_n event represents a space-time location in which the P_n particle is possibly propagating a GW in x. The $1_w(x, y_n)$ is equal to 1 if x and y_n events are connected by a null geodesic and if x is located after y_n along this geodesic. It means that the GW generated in y_n is received in x. Considering only the limit distributions of equation (5), $1_w(x, y_n)$ is always equal to 1 and can be supressed from the equations. This will proven further. $f(x, y_n)$ is a scalar positive function. It is assumed to be equal to 1 if y_n is equal to x. This allows to retrieve the rule of local space-time deformation generated by a particle. It expresses the attenuation of the GW energy which is emitted from y_n. $C^\mu(y_n)$ is a four-vector which contains the information of the energy of the GW in y_n. Later on, it will be shown that $C^0(y_n)$ is not the effective energy of the GW, but is proportional to its square root. Nevertheless, in order to avoid a heavy reading, the words ”four-momentum” and ”energy” will be used respectively for $C^\mu(y_n)$ and $C^0(y_n)$. The context will allow to understand if those are effective energy or contributions to equation (6). And this ”contribution” word will mean the $1_w(x, y_n)f(x, y_n)C^\mu(y_n)$ terms which are in the sum of the rhs of equation (6).

Equation (6) can be used for calculating the four-momentum describing the local space-time deformation generated by any of the following objects.

- A single particle located in x,
- a GW propagating in x,
- many GWs encountering in x.

A remark about equation (6) is the question of wether an infinite number can result from this equation or not. For example if an infinite universe is filled with a constant and uniform distribution of particles, then the result is infinite if the f attenuation function decreases less than $1/r^3$. This problem is similar to the Olbers's paradox problem [7]. But a more practical solution can be found here. There is no need to understand the universe expansion and horizon. When translating this equation (6) into a gravitational model such as surrounding [1], a solution is found. Indeed, in surrounding, the fitting of the model with experimental data forces this sum of equation (6) to be translated into a finite value. A practical approach here is simply to ignore the possibility of divergence of this equation, and to fix this issue later on, when working on gravitational models.

11.2 From the four-momentum to the boost

Let's follow the natural calculations. The distribution of energy of paragraph 10 is still assumed. Let's write the resulting four-momentum of equation (6).

$$D^\mu(x) = \gamma \frac{E}{c}\left(1, \frac{v}{c}, 0, 0\right) \tag{7}$$

E and v are respectively the energy at rest of the four-momentum and its speed in a frame. It has been used $\gamma = 1/\sqrt{1 - v^2/c^2}$. This equation (7) has been written in a $R_0(O; ct, x, y, z)$ frame, at rest with the universe, which is an old privileged frame with respect to the GWs of equation (7). R_0 is such that v is along the Ox line. It is possible to find such a frame. Then, from $D^\mu(x)$ is calculated the local space-time deformation which is generated. This is done [2] by using the boost described in R_0 by the following equation.

$$B^{\mu}_{\nu}(x) = \gamma \begin{pmatrix} 1 & -\frac{v}{c} & 0 & 0 \\ -\frac{v}{c} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8}$$

This boost is directly deduced from the four-momentum of equation (7).

11.3 From the boost to the metric

Now it is possible to derive the space-time metric from $B^{\mu}_{\nu}(x)$.

The distribution of energy of paragraph 10 is still assumed, but now the universe is filled with a constant matter density. It means, notably, that the grid of particles has cells which are small enough for allowing such an approximation.

Let's write R_0, a frame at rest with the universe. Since the universe is filled with a constant matter density, space-time structure is flat Minkowskian everywhere and R_0 represents a whole system of frames such that R_0 is the privileged frame everywhere. Now let's assume that a P particle is added to the scene, located in O at $t = 0$, at rest in R_0. Let's write x the event composed by a given M space point, $M \neq O$ at $t = 0$ in R_0. R_0 can be assumed to be the privileged frame in x, before adding P to the scene. Let's call R'_0 the privileged frame after adding P to the scene. Therefore, R_0 is the "old privileged frame", and R'_0 is the "new privileged frame", with respect to the existence of P and in the M space location. Let's write x' the first event in M when R_0 has been transformed into R'_0, along the time of R_0. R'_0 is obtained by transforming R_0, in x, using the $B^{\mu}_{\nu}(x)$ boost.

From R_0 to R'_0 it can also be generated a successive continuous sequence of privileged frames, starting with R_0 and ending with R'_0. For that it suffices to add slowly the P particle energy from 0 to its real value. (Let's remind that avoiding the conservation of energy principle is allowed in a thought experiment).

Then, it is required to rescale the lengths of the "boosted" time and space axis. The boosted time and space axis are the time and space axis which have been modified by the boost, in their states after the boost. The rescaling is done in such a way that the resulting time line described successively by those successive infinitesimal steps is a geodesic. Equivalently this constraint is that the privileged frame must be inertial. This is detailed by the following equations, relating X'^{ν} the coordinates after the boost, to X^{μ} the coordinates in R_0, and then relating $X"^{\rho}$ the final rescaled coordinates in R'_0 to X'^{ν}.

$$X'^{\nu}(x') = B^{\nu}_{\mu}(x) X^{\mu}(x) \tag{9}$$

$$X"^{\rho}(x') = S^{\rho}_{\nu}(x') X'^{\nu}(x) \tag{10}$$

$$g_{\alpha\beta}(x) = B^{\rho}_{\alpha}(x) S^{\mu}_{\rho}(x') g_{\mu\nu}(x') S^{\nu}_{\kappa}(x') B^{\kappa}_{\beta}(x) \tag{11}$$

$S^{\mu}_{\rho}(x')$ is a symetric linear map which has the ability of being diagonalized in R'_0. Its value is determined by the constraint above (the time line of the set of successive privileged frames must be a geodesic). Equations (9) (10) and (11) show how $g_{\mu\nu}(x')$ the new metric is deduced from $g_{\alpha\beta}(x)$ the old one, due to the action of $B^{\nu}_{\mu}(x)$, which results from the $D^{\mu}(x)$ added energy of P. Of course equation (11) can be inverted, using the inverse mixed tensors $(B^{-1})^{\beta}_{\kappa}(x)$ and $(S^{-1})^{\rho}_{\mu}(x')$ of, respectively, $B^{\nu}_{\mu}(x)$ and $S^{\mu}_{\rho}(x')$. It results the following equation.

$$g_{\mu\nu}(x') = (S^{-1})^{\rho}_{\mu}(x')(B^{-1})^{\alpha}_{\rho}(x)g_{\alpha\beta}(x)(B^{-1})^{\beta}_{\kappa}(x)(S^{-1})^{\kappa}_{\nu}(x') \quad (12)$$

Equations (9) (10) and (11) have been obtained by studying the spherically symmetric static case. In the Schwarzschild metric, a P_1 free falling particle, having a negligeable mass, being at rest when located infinitely far from the center of the symmetry, follows a time line which is transformed by those equations [10]. Those equations are still valid in a more general case. This is proven in appendix 12. Indeed, their construction follows the rule of the privileged frame being modified by the boost associated with local matter motion, and this privileged frame is inertial. A more rigorous demonstration might be given. But in the scope of the present document, only the particular spherically symmetric static case is required.

It is already known that $g_{\mu\nu}(x)$ is a diagonal matrix in the R_0 frame and that $g_{\mu\nu}(x')$ is a diagonal matrix in the R'_0 frame. Using equation (7), since the direction of the boost is along the x space axis, only time and x space dimensions are modified by the metric evolution. If the $(S^{-1})^{\rho}_{\nu}(x')$ rescaling is written with an α time rescaling and with a β space rescaling, then, using equation (12) and $\alpha\beta = 1$ usual convention, the resulting metric shows $g_{00}(x') = \alpha^2$ and $g_{11}(x') = -\alpha^{-2}$. This allows to check and understand the involved mechanism of equations (9) (10) and (11).

As a conclusion, space-time structure is calculated first by calculating a one and only four-momentum in x which contains the information about the local deformation in x generated by P. From it the final local space-time deformation is determined. This determination is deducing the speed associated with this four-momentum, and then the boost associated with this speed. This boost describes the final space-time deformation occuring locally in x. Taking equation (6) into acount, and the VGWs of paragraph 10, the final local space-time deformation in x is described by the boost which is associated with the four-velocity which is the barycentric operation of all the four-velocity of the VGWs propagating in x. This barycentric operation uses the total energy of each VGW as its weight. The attenuation of the propagation of the VGWs follows the rule of Ricci tensor being null in vacuum. From this boost is calculated the metric. This is done in each space-time event. Therefore, the global space-time structure is calculated. The question of the stability of this self-induced mechanism arises. But if the universe is static, with this mechanism, space-time structure converges into a stable structure. Indeed if the universe is static, then a thought experiment can be done in which the energy distribution is constructed by adding progressively the particles one after the other. Also each of them can be added progressively, their energy at rest being increased progressively. Each of those successive energy distributions are static. As usual in GR, space-time structure for each of them can be described with the system of the normal frames, system of frames in which space-time appears virtually flat Minkowskian. This system of frames is the system of old privileged frames. In this system, in any x space-time event, the sum of the momentum of the VGWs occuring in x is null. This is valid also for the limit of these energy distributions and the limit of those system of frames.

11.4 An equation of G

The context and notations of paragraph 11.3 are used. The $R_0(O; ct, x, y, z)$ frame is still used, the P particle is located in O, the x event is any possible space-time event. For trying the construction of an equation of G, now let's assume, also, the following.

Assumption (i): Newton's law is valid. But G may differ from its solar system value.

This is based on experimental data. Newtons' law must be supposed to be valid almost in solar system because this law is validated with high accuracy, at least in solar system [11]. And there are theoretical arguments for this law to stay valid out of solar system, though being used with a different value of G. This will be studied in paragraph 13.

Assumption (ii): the energy of a GW propagating spherically evolves always following the same attenuation function (function of the initial starting energy, and of the propagation distance), regardless of its location and starting energy.

Assumption (iii): in equation (6) the sum of the energy of the contributions generated by P is far weaker than the sum of the energy of the other contributions.

This assumption will be confirmed with surrounding, where a sphere with a 15 kpc ray, which is used for calculating the surrounding value, is fitted to experimental data. The energy which is located in this sphere corresponds to the time component of the rhs of equation (6), that is, the sum of the energy of the contributions of this equation.

Assumption (iiii): the contributions of equation (6) can be replaced by their asymptotic values without modifying consistently the result.

This assumption can be valid if the particles of the universe are isolated enough from each other. This might be realistic because matter is known to be extremely sparse in the universe, whatever the scale is, from particle's physics scale to cosmological scale. Assumptions (i) (ii) (iii) and (iiii) are easier to accept asymptotically.

The calculations are presented in appendix 13. They are based on assumptions (i) (ii) (iii) and (iiii), equations (6), equation (7), and the geodesic equation for Newton's law which is reminded in appendix 13. They result in the following equation of G.

$$G \simeq \frac{c^4}{2\left(\Sigma_{n=0}^{\infty}\sqrt{\frac{C^0(y_n)}{\|x-y_n\|_3}}\right)^2} \tag{13}$$

Equation (13) is valid only for VGWs. In appendix 13 it is shown that this equation is a good approximation under (i) (ii) (iii) and (iiii) assumptions.

The potential divergence of equation (6) might appear worse in equation (13) than in equation (6). Indeed, the $1/\sqrt(r)$ evolution of the contributions of equation (13) shows potentially a quick divergence of the result of this equation. Nevertheless it must be noticed that the resulting gravitational force will obey to the $1/r^2$ rule. Indeed, equation (13) has been formed from that rule. Therefore the final possible divergence is the Olbers's paradox divergence. And it is easy to modify equation (13), inserting a cut-off value of the contributions, for example resulting in the following equations.

$$\begin{aligned} \Phi_{cut}\big(a, b\big) = & \\ b \leqslant R_{cut}: & \quad \sqrt{\frac{a}{b}} \\ b > R_{cut}: & \quad 0 \end{aligned} \tag{14}$$

$$G \simeq \frac{c^4}{2\Bigg(\Sigma_{n=0}^{\infty}\Phi_{cut}\Big(C^0(y_n), \|x-y_n\|_3\Big)\Bigg)^2} \tag{15}$$

Here R_{cut} is the maximum GW propagation distance. The 15 kpc value is suggested by surrounding [1].

This cut-off value does not alter much the qualification of "asymptotic" in the previous reasoning and in the calculations of appendix 13. Indeed in the surrounding gravitational model, the $R_{cut} = 15\ kpc$ value is fitted to experimental data. It is a distance which would require an attracting object like a galaxy in order to contradict this "assymptotic" qualifier. And this equation (15) can still be improved. For example it is possible to replace the simple 0 cut-off value by a slow decrease of the contributions of equation (13).

12 Predicted surrounding effect

The above study shows that relativity predicts a variable G. And this variation given by equation (13) follows the rule untitled "surrounding" in [1]. Of course equation (13) has been constructed under the (i), (ii), (iii) and (iiii) assumptions.

But equation (6) shows already this surrounding effect, without any added assumption. Let's show this by rewriting it, shifting the total energy from left to right, and isolating the resulting speed.

$$\frac{v}{c} = \frac{f(x, y_0)C^0(y_0)}{\Sigma_{n=0}^{\infty} f(x, y_n)C^0(y_n)} \tag{16}$$

The context and notations of paragraph 11.4 are assumed. The universe is still assumed to be at rest and filled with a constant matter density, except for only one P particle at rest with the universe and spatially located in O. y_0 is an event spatially located in O, and $C^\mu(y_0)$ is the four-momentum of the unique VGW generated by P. The $1_w(x, y_0)$ and $1_w(x, y_n)$ terms disappeared because now only VGWs are considered. Indeed, there is always a unique VGW which is propagated by each particle located in y_n and which is received by any given x event. Equation (16) derives directly from equation (6), and shows that the space velocity of the resulting four-velocity is inversely proportional to the denominator, which increases with the energy surrounding the x location.

It can be noticed also that the denominator of the rhs of this equation is a sum of positive scalars calculated in an isotropic manner. It induces naturally to translate this equation (16) into a gravitational model, replacing this value by the energy of the surroundings of the location where the gravitational force is exerted. The result is that the surrounding gravitational model or a gravitational model close to it is predicted by relativity. Therefore the so-called gravitational anomalies of today might be no anomalies at all, but regular predictions of relativity.

The modified gravity theories of today must comply with the MOND [12] model predictions, for the greatest part of experimental data. This has been proven by decades of work in astrophysics. This compliance is naturally acomplished by the surrounding model. Indeed, when acceleration is low, MOND increases it. But when acceleration is low, most of the time it means that the surrounding energy and matter are low, also, and then the surrounding effect increases acceleration too.

13 Revisiting Newton's law

A revisiting of Einstein equation is naturally required by the previous study. This equation is nothing more than the most direct translation of Newton's law from non relativistic physics into relativity. Therefore, the first step is to revisit Newton's law. The Poisson's formulation of Newton's law starts the construction of Einstein equation, not only because it's about Newton's law.

First of all, in vacuum, this formulation expresses the following important non relativistic principle.

(i) The flow of the acceleration vector field is constant in vacuum.

It results $div(a) = 0$ in vacuum, where a is the acceleration vector field. This is not something new. But now it has been shown that space-time structure in vacuum can be calculated by considering only the VGWs generated by theoretical relativistic particles. This gives a new insight about this (i) principle: it corresponds to the conservation of flow of GW energy in vacuum. Hence another argument for the Poisson's formulation of Newton's law is given.

Secondly, matter plays the role of a source for this field: matter is a source of any interaction force. And this is not only true for gravitation. Indeed, the force which attracts a given A particle to another given B particle is acting on A in the direction of B. This direction is tangent to the geodesic relying A to B. At the contrary, vacuum does not generate any force. Therefore the F force vector field has a divergence which is a function of ρ, matter density. Let's write Φ this function. It results the following equation.

$$div(F) = \Phi(\rho) \tag{17}$$

Then, let's write the fundamental principle of dynamics.

$$F = ma \tag{18}$$

F is the force generated on A by ρ. m is the mass of A located in a M space location, and a is the acceleration generated in M by ρ.

From equations (17) and (18) the following equation arises.

$$mdiv(a) = \Phi(\rho) \tag{19}$$

Now the reasoning becomes only valid for gravitation. The weak equivalence principle (WEQ) states that a is independent of m for a fixed ρ. It results the following equation.

$$div(a) = f(\rho) \tag{20}$$

f is of course a function which is deduced from Φ. Hence, Newton's law in its Poisson's formulation is almost retrieved by the previous theoretical considerations. The divergence of the acceleration vector field should be a function of energy, such that for a null energy there is a null divergence. This gives also the following equation.

$$f(0) = 0 \tag{21}$$

In equations (20) and (21), nothing is told about neither the sign of $f(\rho)$ nor the exact feature of the f function. And the question of a possible proportionnality of $f(\rho)$ with ρ is related with conservation of energy and the principle of action and reaction. Let's show this. Equation (20) implies for a P particle in an empty universe the following equation.

$$a = \frac{f(\rho)V}{4\pi x^2} \tag{22}$$

In equation (22), a is the acceleration in any given M location, such as $MO = x$, O being the location of a P particle generating this acceleration. ρ and V are respectively the matter density and the volume of P. It was supposed that ρ is constant in P. Then applying the fundamental principle of dynamics, the following equation arises.

$$F = \frac{m' f(\rho) V}{4\pi x^2} \tag{23}$$

Here, F is the force attracting a P' particle located in M, by P. m' is the P' mass. But the principle of action and reaction implies that this equation is invariant by P and P' permutation. The following equation arises, where m is the mass of P.

$$m' f(\rho) V = m f(\rho') V' \tag{24}$$

This being true for V and V' being constant and for any value of m, m', ρ, and ρ', it implies that $f(\rho)V$ is proportional to m . A better demonstration of that would consider the total energy E of P and P', in place of the principle of action and reaction. This energy is the integral of the forces along space distances. Then it is the invariance of E by the P and P' permutation which would be used. This proportionnality would be written the following way.

$$f(\rho) = K\rho \tag{25}$$

Here K is of course the unknown coefficient of this proportionnality. Using equation (25) in equation (23), it yields the following equation.

$$F = \frac{K m m'}{4\pi x^2} \tag{26}$$

But now no theoretical argument can be given here for calculating the $K/(4\pi)$ constant of equation (26). Historically Newton's law has been constructed based on experimental data more than theoretical considerations. To say the least, the G determination was done completely based on experimental data.

But an indirect theoretical argument can be given. Everything was done here under the assumption that a complete vacuum exists outside of P and P'. If the vacuum is not perfect outside of the particles, the reasoning above becomes wrong. The energy surrounding the particles must be taken into account. First of all, matter density of the universe outside of P and P' generates also a divergence by applying equation (20). This added divergence modifies the final result given by equation (23). Secondly, the principle of action and reaction might not be true in its simplest formulation. It is easier to understand that the energy version of the demonstration is wrong. Indeed, rigorously speaking, the total energy of P and P' must be replaced by the total energy of the universe. Indeed, energy exchanges might exist between the particles and their environment. A more practical version would be to approximate the energy of the universe to P and P' energies plus the energy of the surroundings of the particles, to some given extent suitable for a correct approximation.

The whole result of those theoretical arguments is Newton's law. But these arguments tend to prefer a variable G more than a constant G value, this variation depending of the energy surrounding P and P' particles.

14 Revisiting Einstein equation

In the more general relativistic regime the same reasoning might be done, replacing the divergence of equation (20) by Einstein tensor, and matter density by stress-energy tensor. Then the reasoning might give Einstein equation. But this work is above the scope of the present document. Nevertheless, since Einstein equation is the most direct formulation of Newton's law in the context of relativity, the reasoning above done in the non relativistic regime applies indirectly to Einstein equation.

To say the least, what appears still seriously doubtful is the statement that $K/(4\pi)$ is a universal constant, in equation (26). At the contrary, the above discussion shows that one would expect matter density of the universe to play a role in the determination of this constant. A more practical formulation of that would be that the energy of the surroundings of P and P' would play a role in this determination. This was true for Newton's law, and is therefore true for Einstein equation, since it is the most direct translation of Newton's law into relativity.

The $\rho = 0$ particular case implies $div(a) = 0$, from equations (20) and (21), but results also directly from the (i) principle. And the present study shows absolutely no need to modify its relativistic formulation given by equation (2). At the contrary, this equation allows to complete the new construction of space-time structure done in the present document. This equation was used notably, above, in the study about two particles in an empty universe.

By other means, the construction of Einstein equation from Newton's law is extremely simple. It is the simplest way to proceed. Inserting a multiplicative tensor between the stress-energy tensor and Einstein tensor is something natural which were rarely done in the literature. This mutiplicative tensor can be only a function of energy: what else? Now the present document shows that this is exactly the correct translation of Newton's law into relativity. The result would be surrounding, or a gravitational model close to it. A classical way to proceed would be to calculate everything with such a X^μ_ν hypothetic multiplicative tensor and then compare the predictions to experimental data. Very probably, it would show that X^μ_ν must be proportional to the surrounding energy of the location where the force is exerted. Indeed, the surrounding gravitational model indicates strongly that this is exactly what would happen.

Also, today during the construction of the GR Lagrangian for matter, the G anthropocentric solar system constant value is forced without any theoretical argument. This is doubtful. At the contrary, the GR Lagrangian for vacuum is the simple and well sounded scalar curvature. This is another argument for applying it in the present study (it is equivalent to equation (2)).

15 Conclusion about G

Apart from the previous demonstrations, it is interesting to adopt a physics view point and add physics arguments in favour of a variable G following the rule of the surrouding effect. They will show that the previous demonstrations of the present document do not come from nowhere, but are motivated by strong arguments in physics. They are the following.

1) Mach's principle.

2) Correct theoretical construction of Newton's law

3) Sophisticating the construction of Einstein equation.

4) Loss of information in the construction of the stress-energy tensor.

5) Implicit assumption of GR.

Items 1) 2) and 3) h these amount of energyave been described above. Item 4) was used implicitly in the present study. Indeed, there were no such loss of information in the descriptions of the energy distributions of the demonstrations of the present document. This item, as well as item 5), is described in [2]. Item 5) is the experimental argument about the quasi-relativistic speed of the quarks.

Therefore, forgetting one instant the previous demonstrations, from a physics point of view alone the following statement is valid. Outside of solar system, it is much more relevant to use equation (13), or its translation with surrounding, than its solar system value for the determination of G. To say the least, a variable G following the rule of the surrounding effect is much more relevant.

16 Yang-Mills Millennium problem

16.1 General statement

The remark done in [2] about the Yang-Mills Millennium problem is still valid. Moreover, this remark is conspicuously reinforced by the study of the present document. Let's remind briefly this remark. It starts by assuming the following.

Assumption (A): unification of the four forces is driven by gravitation.

It is not indicated how this unification takes place. But it can be imagined that each particle of particle physics is constructed by some given amount of energy in motion, the different trajectories of those amount of energy giving different behavior of the three other forces.

Under this unifying assumption, the Yang-Mills Millennium problem finds a solution. Indeed this assumption awake the full relativity in the context of particle physics: now not only SR, but also GR underlines all the forces. Therefore each of four forces is driven by the surrounding effect. And this effect modifies enormously the interactions between the particles.

16.2 Modified predictions and observations

The first kind of particle physics observations are particles interactions. The present study modifies the physics predictions in the case of triple nuclear collisions [13]. Indeed those collisions would be predicted to behave in a completely different manner, because of the surrounding effect. But triple nuclear collisions are almost impossible to realize. The modifications of these observations might appear when the targetting particle is unchanged but when the target of the interaction is modified. When the target energy decreases, the surrounding effect increases and the scattering is expected to be wider. For example, the Ay puzzle [14, 15] might be enlightened.

The second kind of particle physics observations is cosmic rays. But here the present study will not modify the observed predictions.

The third kind of particle physics observations is static configurations of particles. Let's focus on the most stable group of particles, the atom. The electromagnetic force is now predicted to be much weaker than what is calculated by the simple $1/r^2$ rule, when the electron is part of an atom, than when it is alone. For the nucleus, its most simple form is the hadron. Let's discard hadrons made of two quarks because of their life time. The confinement of quarks is noticeable in hadrons of three quarks. This is very much explained by the present study. The remaining question is why an electron can exist alone, while a quark cannot. The present study gives a simple argument here. Probably an answer is to be found in the difference of magnitude between the electromagnetic force and the strong force, when they are exerted in their confinement state, that is, when such groups are particles are formed. When a hadron is formed, then the strong force is far stronger than the electromagnetic force between an electron and a proton inside of an atom. But when an electron, or a quark is left alone, then the surrounding effect increases the interaction force with respect to its value when the particle were not alone. Therefore, for an electron it is increasing a relatively weak force. But for a quark, it means increasing a force which was already strong. Therefore it can be understood why an electron can exist alone, while a quark cannot. It must be added that in this description the strong force can't be assumed to be a very short range force. In paragraph 16.3 it will be shown that this assumption might be wrong.

For the strong force the interesting modification of the present study involves three body interactions. Any group of three particles closed to each other would experience low values of the strong force between them, because the surrounding effect would be strong, due to their close proximity to one another. But any group of two particles, or any group of three particles having one of them far enough from the two others, would experience stronger values of the force, because the surrounding effect would be weak. This can explain why hadrons with two quarks have a very short life-time, while a hadron with three quarks is very stable. And it explains why an isolated quark is unobserved.

Therefore this allows quark confinement for long duration only when they are close to each other by groups of three. It results a solution of the Millennium problem [3, 4].

16.3 Other observations in static configurations

Nuclear saturation [16, 17] can be understood in a completely different manner. In today's literature this mechanism is understood as suggesting a very short range for the strong force. But another possible explanation is that it is not so much a very short range. Indeed the surrounding effect explains this mechanism in the same usual simple manner. The gravitational potential, exerted on one baryon, by the other baryons of the nucleus, is substracted by its exact value, resulting in a constant gravitational potential. This is the same usual suppressing effect of surrounding [1], which happens for the cosmological version of Einstein equation. It happens also for the bullet cluster. In equation (16), nearly all the contributions of the denominator come from the other baryons of the nucleus. Indeed, a nucleus is extremely isolated. Therefore the sum of the v/c values coming from other baryons is divided by the same sum. A constant value results. This mechanism explains why the volume of the nucleus is proportionnal to the number of baryons: the gravitational potential of the other baryons stays the same whatever is the number of baryons.

It would be interesting to rewrite the strong force as well as the nucleus potentials, taking into account the surrounding effect. Then, comparing the energy distributions with different surrounding effects might yield effective equations which would be more general than today. To say the least, a different understanding of the interactions between hadrons might arise.

16.4 Relevance of the fundamental assumption

Let's try to discuss the validity of assumption (A). It allows to get rid of the GR effect which creates tidal forces, for example in black holes. This effect would mean that space-time deformation would modify energy distribution a second time. Indeed, space-time already modifies energy distribution with gravity. This would appear more as a spontaneous creation of energy. Moreover, assumption (A) is more than an assumption. Indeed, the following argument can be done.

<u>Argument (*)</u>: acceleration generated by gravitation is explained by space-time curvature. It is a simple and elegant rule. It is tempting to apply it to each force.

The only remark which would forbid it is that the ratio σ/m is not constant but depends of the type of particle, where σ is the charge of the particle with respect to the considered force. But this remark can be discussed under assumption (A). Indeed, under this assumption this dependence of the WEP with σ can be explained by the possible mechanism which were described previously: the particular behavior of the gravitational force depends of the particular structure of each particle.

The argument (*) comes from relativity and gravitation. It might appear difficult to find such a convincing hint starting from particle physics. For example it might be difficult to find such a convincing argument with the following assumption.

Assumption (B): unification of the four forces is driven by one of the three forces of particle physics.

Under the (B) assumption, the strong force might be the better candidate. Indeed, for example in figure 1 of [18], it is located on an extreme location as compared to the others. Nevertheless assumptions (A) and (B) appear better than the following one.

Assumption (C): no unification of the four forces exists.

Indeed, there is the apparent energy convergence at Planck scale [19] which contradicts it. But assumptions (A) and (B) must be compared with the following one, which might be done by today's physics.

Assumption (D): unification of the four forces is driven by an effect in which no force plays a leading role.

It might be this (D) assumption which is implicitly assumed todays [20, 21]. But the main arguments for it are symmetry considerations. Their relevance must be compared with the relevance of argument (*).

Also an experimental information in favor of assumption (A) is given by eclipses anomalies. Indeed, under this assumption, strong deviations of the gravitational signal of the sun, by the moon, might be expected during solar eclipses [8, 9].

17 Discussion

A new determination of space-time by energy is presented. It uses the usual GR concept of normal frames, which is called privileged frames in the present document. This new name refers to a property of those frames which has been dismissed in the literature. This property is that the evolution of those frames is driven by energy. And this gives a clue for a new space-time structure calculation from energy. This new calculation is based on the first degree of metric derivation. The second degree of derivation is calculated and then compared to Einstein equation.

The demonstrations done in this document were for the most part, purely mathematics. It results a more complicated relativity, more Machian than before, in which gravitation follows the rule of a surrounding effect. In its weaker formulation this effect is the following. Increasing the energy of the surroundings of the location where the gravitational force is exerted results in decreasing this force with respect to Newton's law. It is proven that G is not a constant, and that its variation is driven by this effect. An equation of G is given, which is a good approximation under four assumptions, which might be valid most of the time in gravitation.

When the complexity of the energy distribution does not allow to use Einstein equation, then a method has been presented, allowing to calculate space-time structure from an energy distribution made of an infinite number of dimensionless particles. Of course this method leads to complicated calculations. But are they more complicated than using Einstein equation without any symmetry rule allowing to simplify?

It still remains to find the general principle allowing to replace completely Einstein equation in any cases. Another Lagrangian might be constructed, based on this new understanding of relativity. But in most of the cases, using Einstein equation with this new equation of G might be a good or very good approximation. Whatever, it remains much better to use a G variation driven by the surrounding effect than its constant anthropocentric solar system value.

Each so-called gravitational anomaly might be simply a prediction of relativity. Of course the work is huge until one can replace "might be" by "is" in the previous sentence. Indeed, the surrounding gravitational model appears to solve all those mysteries in a straightworward way. But it would need not one but several articles to confirm that. And the surrounding gravitational model will have to be tuned. For example, the brutal rectangle window used for calculating the surrounding value must be replaced by a smooth window, soon or later. A need for that appears for conforming surrounding to the wide binaries problem [22–24]. Also, possible regressions might arise, in which this G variation might induce wrong predictions in front of some given experimental data. This work is huge also.

But nevertheless a big step is done in gravitation. In particle physics, the Yang-Mills Millennium problem finds a solution under the relevant unifying assumption that the four forces are different aspects of the same and unique force of gravitation.

Appendix

1 Interchange of limits theorem

This theorem can be seen as the property of continuity of the S function giving space-time structure from energy distribution. S starts from the set of energy distributions which is a linear space of infinite dimension. It reaches another linear space of infinite dimension which is the space of space-time structures. This one can be defined using the matrixes of the metric in some given system of frames. Then, this space is the linear space of the distributions over space-time of 4×4 matrixes. This definition depends of the choice of the system of frames. Those two linear spaces are equipped with the uniform norm. With such a norm, S is continuous. Then, writing this continuity property with the limit of sequences, equation 1 is given.

2 Privileged frame

This thought experiment is simply imagining the energy at rest of a P particle increasing progressively, and at the same time the whole energy of the universe decreasing. At the end of the experiment the universe and the particle have their roles permuted. Now the particle contains the energy of the previous universe, and the universe contains the energy of the previous particle. The first result is that the frame in which time elapses the most is no longer the frame attached to the universe. Now this frame is the frame attached to the particle. It means that the space-time structure is now the symmetrical result of a permutation of those two frames. It means also that during the experiment, the space-time structure has been modified progressively from the first state to the final one. And this operation has allowed to revert the time dilatation. For example, if this was a twin paradox configuration, at the end of his brother's journey, the older twin would become the youngest after the thought experiment. Therefore this space-time modification is simply described by the boost transporting one frame into another. It can be noticed that this reasoning is using the well established supposition that GR is coherent.

Now the need of naming the frames appears. Let's call Ru a frame attached to the universe. It can be supposed that the universe is filled with a constant, homogenous distribution of matter, therefore this matter is supposed to be at rest in Ru. Let's call Rp a frame attached to the particle. The result of the thought experiment is that the particle generates locally a space-time deformation which is described by the boost from Ru to Rp. Of course, this deformation is local to the particle but the more energy at rest of the particle, the more this deformation is valid around the particle. A "more valid deformation" means that the space-time deformation exists significatively over a larger space-time domain.

The space-time deformation appearing in the experiment is described by a boost which allows to transform progressively this privileged frame from R_u to R_p. And it means that this frame remains privileged during the whole process, even though it might be no longer the frame in which time elapses the most. This frame is simply a frame in which the particle is at rest. Its physics relevance is only local to the particle. The result is that it is possible to extend this identification of the privileged frame of relativity to any space-time event in which there exists matter. And this identification can be extended even further to events in which vacuum prevails, by interpolation between those events in which there is matter. This interpolation is done by using space-time structure. The usual system of normal frames is chosen. Let's remind that this system of frames is such that the matrix of the metric is diagonal, its diagonal components being equal to $(-1, 1, 1, 1)$.

This ends what is a reminder about a feature or relativity.

The local space-time deformation of this thought experiment is inflexible. It means that it remains the same whatever is the energy distribution. Notably, whatever are the surroundings of the P particle, the local deformation generated by P remains the same. This is true if the P particle is a dimensionless particle, or at least if it gets a high enough matter density. This is a realistic modelization since matter is known to be extremely condensed. And it is true also for the virtual microscopic particles which are used in the thought experiments of the present document.

3 Integration constant

This appendix is about the integration constant of the space-time structure calculation for the energy distribution of two particles in an empty universe.

This constant is possibly given by the following information. There are singularities along the straight line containing the particles's locations, but only on those points which are not between the particles's locations. They are described by the boost associated with the speed of light in the direction moving away from the particles.

Of course the rigorous way to proceed is to calculate space-time structure using symmetry consideration and equation (2). Then, the asymptotic values of the deformation should allow to calculate the integration constant.

Let's remind that this procedure is already done for the energy distribution with one particle in an empty universe. Symmetry consideration produces the Schwarzschild metric. Equation (2) gives the information that the metric is of the shape $g_{00}(x) = 1 - M/r$, where $g_{00}(x)$ is the time-time component of the metric in the x space-time event, r is the spatial distance from x to the particle, and M is the unknown integration contant. Then the asymptotic value of the local space-time deformation given by paragraph 6 allows to calculate $M = \infty$.

Therefore, it should be possible to calculate this constant for the energy distribution of two particle in an empty universe.

4 Uniformly moving non relativistic particle: local deformation

Let's consider P, a non relativistic particle moving uniformly at the v speed, $v < c$, along the D space line in a flat Minkowskian space-time. The $R(O; ct, x, y, z)$ frame is chosen such as O is contained by D and Ox is in the direction of the P motion. The local deformation around P is first described by the b_v boost associated with the v speed, as shown before.

But equations (9) (10) and (11) show that a rescaling of ct time and x space axis is required. If no rescaling occurs, then it means that the deformation generated by P is wholly described by a boost. Then it is simply the usual change of coordinates obeying to SR rule, and no deformation is noticed. Therefore a rescaling occurs, and this one is coherent with time dilation between R and R'. Therefore from equations (7) (9) (10) and (11), if α and β are respectively the s rescaling of ct and x, there is $\alpha^2 = \beta^{-2} = 1 - v^2/c^2$.

Hence the complete deformation is described by $b_v s$. Of course this deformation results from the assumption that equations (9) (10) and (11) are valid. But those equations themselves result, indirectly, from this particular deformation. Therefore what is required is a check of the coherence of the whole study, at the end of it. And the whole study will be coherent indeed.

In the present document, it will be written abusively that the local deformation is described by a boost. Indeed, only the context will allow to deduce if a rescaling occurs also, or not. Practically speaking, there is always a rescaling between the priviliged frames which are considered in the present document.

5 Uniformly moving relativistic particle: local deformation

Let's prove that the deformation in C is the b_C boost associated with the speed of light along the D line in the direction of the GW propagation.

Starting from appendix 4, taking the limit when v tends to c, the result is obtained. As usual the interchange of limits theorem is used. The simple convergence is enough for it to work since the studied deformation is only local.

6 Relation between the boost and the speed of a gravitational wave

First of all, the space-time deformation generated by a GW is described by a linear map, because it's a local deformation. But it transforms an inertial frame into another inertial frame. Therefore the Minkowskian distance is invariant and the linear map is an isometry. Time inversions, space rotations, and space symetries are dicarded since they are not realistic. It remains only boosts, possibly composed with time and space rescaling (positive rescaling). Let's start by the study of the boost part of this linear map.

It will be assumed that the speed of a GW is a given v positive value with respect to the R frame. What is the relation between v and the V speed which is associated with the boost describing its propagated deformation in R? It might be guessed that $V = v$.

This demonstration is easier to understand in a contradiction way. Let's assume the result is wrong. Therefore, it is assumed that the GW speed is v and that the boost of its deformation is V such as $V \neq v$.

Then it is considered a P' particle "surfing" on the GW. That is, P' is always located in close vicinity to a moving M point, where the GW deformation is maximum, and which is of course in motion with the GW propagation. Let's write R' a frame which is attached to this M point. The local deformation generated by P' is independent of its energy. Indeed, the determination of the new and old privileged frames before and after the existence of P are given by SR. There are more details about that at the end of appendix 2. For the same reason, the local deformation generated by P' is also independent of the GW energy. And it is the same for the GW: its generated deformation is independent of its energy and of the P' energy. Therefore, in R', P' generates locally a deformation which is noticed because $V \neq v$. If P' energy is low enough, then the P' local deformation transforms more globally into the GW deformation which is a null deformation in R'. In other words, in R', locally it appears a flat Minkowskian space-time, and also a smaller space-time deformation locally to P'. In R', locally to P' a boost is noticed, associated with a w speed which is not null (w is the relativistic substraction of v by W). But the fact that a space-time structure is generated by a GW or a particle does not modify the following rule: with respect to a frame, a particle modifies locally space-time structure with the boost which is associated with its speed. And the reverse is true: with respect to a frame, a particle modifying space-time structure with the boost associated with the w speed is in motion with the w speed. Applying this rule here, it results that P' is in motion in R' at the w speed which is not null. A contradiction arises. This proves the above claim.

Therefore the boost part of the GW local deformation is the one which is associated with its v propagation speed. Now, its rescaling is still unknown. But if $v = c$ which is of course well established, then the associated boost is degenerated and any further rescaling would produce no change on the final result.

7 Uniformly moving relativistic particle: propagation of the deformation along the trajectory

Let's deduce the following, from the result of appendix 5.

The deformation of P in C propagates along the D line in the same direction as the motion of P and is described by the b_C boost which is associated with the speed of light in the same direction.

This can be noticed as soon as P deviates from this D line trajectory. First of all, the GW generated by P propagates along the D line for symmetry reasons. The propagation speed is the speed of light as assumed by assumption (I). And the propagated local deformation is b_C: the demonstration is the same as in appendix 6. Or it is possible to use appendix 6 in the non relativistic case and then taking the limit to the relativistic case.

8 Uniformly moving particle: propagation of the deformation everywhere

Let's argue the following.

Assumption (j): The space-time local deformation which is propagated by the GW is the b_C boost associated with the c speed in the x direction.

This is much more than an assumption. Indeed, the local deformation which generates this GW is described by this b_C boost.

Moreover, let's show in the present appendix that this is coherent with the fact that the enveloppe of the GW propagates at the $c/\sqrt{2}$ speed along the CM direction. This will be an added argument for the (j) assumption.

For this, let's calculate the $\alpha = \widehat{ct, n}$ angle. n is the space axis which is normal to the enveloppe in M. Therefore n is along the CM line, in the direction from C to M. At t_N in R the GW propagation generated by the P particle is located on the Δ half line containing N and C' and starting in C' in the direction of N. M being the middle of N and C' is also part of Δ. At t_N, therefore along Δ, the ct time axis has been rotating around y with a $\pi/4$ angle in the direction of x, resulting in this ct' axis. Indeed this is the effect of the b_C boost along the space points of Δ: the b_C boost transforms R into the $R'(ct', x', y, z)$ frame in such a way. This creates a P' plane containing all these ct' lines starting from the points of Δ. In other words, P' is the new plane of time after the action of the b_C boosts executed along Δ at $t = t_N$ in R. Now, the α angle is the angle between P', with respect to R, and the xy plane. It is also the angle between the n direction and P', therefore the angle between CM and P'. The calculation follows.

$$cos(\alpha) = \frac{\overrightarrow{CM}.\overrightarrow{CM'}}{|CM\|_3|CM'\|_3} = \frac{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2}{\sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2}\sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2 + L^2}} = \frac{1}{\sqrt{3}} \tag{27}$$

L is the $CN = CC'$ distance. M' is the space-time point being the projection of M on P' along the z direction.

By other means, the enveloppe of the GW propagates along the n direction at the $c/\sqrt{2}$ speed. The associated b_M boost modifies the P_M plane directed by the n and ct axis, and containing M. b_M transforms ct along this plane into a ct" axis. The equation of the line associated with this ct" axis is the following.

$$n = (c/\sqrt{2})t \tag{28}$$

n is the coordinate along the n direction starting from M, with respect to R.

Equation (28) can be written $ct = \sqrt{2}n$. Therefore the β angle between ct" and the xy plane is given by the following equation.

$$tan(\beta) = \sqrt{2} \tag{29}$$

$$cos(\beta) = \frac{1}{\sqrt{1+\sqrt{2}^2}} = \frac{1}{\sqrt{3}} \tag{30}$$

Equation (30) is coherent with Equation (27): $\alpha = \beta$.

This was for the new ct" time axis resulting from the b_M boost. Now let's study the new space n" axis resulting from this boost. A direct calculation is simply deducing n" from ct". Since b_M is a boost occuring in M, the resulting ct" and n" axis are transfomed in a coherent way. Indeed, n" is the symmetric result of ct", transformed by the orthogonal symmetry with respect to the symmetry axis defined by the $n = ct$ equation. Therefore the equation of the n" axis starting from M is deduced from equation (28) and is the following.

$$n = \sqrt{2}ct \tag{31}$$

Equation (31) can be written $ct = (1/\sqrt{2})n$. This gives the following value for the γ angle between n" and the xy plane.

$$tan(\gamma) = \frac{1}{\sqrt{2}} \tag{32}$$

$$cos(\gamma) = \frac{1}{\sqrt{1+(1/\sqrt{2})^2}} = \sqrt{\frac{2}{3}} \tag{33}$$

This γ angle is also the δ angle between the Q plane which contains the y and x' axis, and the xy plane, along the n direction. This angle is calculated along the n direction. This means that the P_n plane containing the n and z axis is considered. Then the T line is the intersection of Q with P_n. Also, let's write m the axis associated to T in the direction from C to M. Then, $\delta = \widehat{m,n}$. And δ is calculated the following way.

$$cos(\delta) = \frac{\overrightarrow{CM}.\overrightarrow{CM"}}{|CM\|_3|CM"\|_3} = \frac{\left(\frac{L}{2}\right)^2+\left(\frac{L}{2}\right)^2}{\sqrt{\left(\frac{L}{2}\right)^2+\left(\frac{L}{2}\right)^2}\sqrt{\left(\frac{L}{2}\right)^2+\left(\frac{L}{2}\right)^2+\left(\frac{L}{2}\right)^2}} = \sqrt{\frac{2}{3}} \tag{34}$$

M" is the space-time point being the projection of M on Q along the z direction.

Equation (34) is coherent with Equation (33): $\gamma = \delta$.

9 Final picture for a uniformly moving relativistic particle

This results from the previous appendix. The final picture for a uniformly moving relativistic particle is that in R the deformation occuring in C at the E_C event propagates along the Cn space direction, at the $c/\sqrt{2}$ speed. This deformation is described by the boost associated with the c speed in the x direction. But along the Cn direction, this deformation is coherent with the boost associated with the $c/\sqrt{2}$ speed. This picture is Lorentz invariant.

10 Asymptotic space-time deformation around a relativistic particle moving along a circle

The asymptotic local space-time deformation generated by a relativistic particle moving along a circle is deduced from appendix 9. The GW propagates along the direction which is perpendicular to the particle's trajectory. But asymptotically this direction is also the line starting by the location of the particle. And from appendix 9 this is done at the c speed: that's the propagation of the b_C boost along D, which was described in paragraph 5. It is also the propagation of the enveloppe which is now the sequence of spheres (the sequence of spheres described in paragraph 6, which are centered in O, the center of the circle). Asymptotically those spheres inflate themselves around O at the speed of light. And now this enveloppe propagates at the c speed. Hence the b_C boost describes also this local deformation.

11 The gravitational wave four-momentums add themselves

Let's suppose that n GWs are propagating and encountering themselves in a given x space-time event. Let's write, for i from 0 to n, GW_i the GW having the i indice. For each i, it can be created a P_i particle such that its momentum counteracts exactly the effect of GW_i in x. It means that the P_i speed is the speed of light, and that its total energy in any R frame is the energy of GW_i in R. It means also that the direction of the P_i motion is opposite to the direction of the GW_i motion. The result is that P_i and GW_i together generate no local space-time deformation. Therefore the overall effect of all these P_i particles is such that it cancels the space-time deformation generated by all the GW_i. But the sum of all the P_i particles is a big compound object, let's call it P. So the space-time deformation of all the GW_i is exactly conteracting the space-time deformation generated by P. But conservation of energy principle states that the P four-momentum is the sum of the P_i four-momentums. Therefore the four-momentum of the space-time deformation of the whole set of GW_i is the sum of the GW_i four-momentums.

$$\begin{aligned} F_m\big(\{GW_i, i=0..n\}\big) &= -F_m(P) \\ &= -\Sigma_{i=0}^n F_m(P_i) \\ &= -\Sigma_{i=0}^n \big(-F_m(GW_i)\big) \\ &= \Sigma_{i=0}^n F_m(GW_i) \end{aligned} \tag{35}$$

$F_m()$ is a function giving the four-momentum of a particle or a GW. This is the end of the proof.

12 The free falling particle in the Schwarzschild metric

Let's study, in the Schwarzschild metric, a P_1 free falling particle, being at rest when located infinitely far from the P particle which is located in the center of the symmetry, having an energy weak enough in order not to modify its free fall trajectory.

Let's prove that P_1 follows a time line which is transformed by equations (9) (10) and (11). The same context and notations as in paragraph 11.3 are used. Notably the R_0 frame is attached to P.

The P_1 trajectory is from the start aligned with the time axis of the new and old privileged frames of P (from the start those frames are the same). Since the new privileged frame of P is inertial, the P_1 trajectory is exactly the time line described by the time axis of this new privileged frame. This appears to be the end of the demonstration. Indeed, one can assume that the deformation generated by the motion of P_1 in R_0 is the same as the gravitational deformation generated by P. This would result from a complete coherence of GR. But let's prove that coherence directly here.

For this, in the R_1 frame which is attached to P_1, let's study another P_2 particle which always stays infinitely close to P_1 and which has an energy at rest far weaker than the P_1 rest energy. P_2 follows the same trajectory as P_1. And this can be understood as only the result of the space-time deformation of P_1. Indeed, P_2 energy is far weaker than P_1 energy, and P_2 is always close to P_1. It must be noticed that the space-time gravitational deformation generated by P is without effect locally to P_1, completely replaced by the space-time deformation of P_1. The P_2 free fall allows to ignore it completely.

Therefore the P_1 deformation of space-time is what forces P_2 to follow the same trajectory as P_1. The R_1 frame attached to P_1 is privileged because it's inertial and also because it's privileged from the start, when P_1 is at rest infinitely far from P. But the P_1 deformation is described by the b_1 boost associated with its motion in R_0. Now let's decrease P_1 energy to 0, progressively, without modifying the P_2 energy (it does not mean that the conservation of energy principle is broken). This P_1 energy decrease will not change the P_2 trajectory. This is because P_1 energy (hence P_2 energy, too) has been assumed to be weak enough, in order to give that result. Indeed, since their energies are always assumed to be weak enough, P_1 and P_2 are always in free fall. Hence their trajectories remain the same whatever are their energies, as far as they are weak enough for allowing the free fall. Noteably it is true for P_2.

Therefore, the local space-time deformation generated by P_1 motion in R_0, along its trajectory, is the same as the gravitational space-time deformation generated by P along this trajectory. The result is that b_1 describes the gravitational local deformation generated by P, along the P_1 trajectory. Of course after the execution of b_1, a rescaling is required in order to account for the more global curvature. This ends the demonstration.

This demonstration can be applied to a flat Minkowskian space-time everywhere except for a bounded space-time domain in which a ϵ energy distribution is added. The old and new privileged frames refer now to this ϵ bounded energy. Therefore in the above demonstration, the only modification is that P is replaced by ϵ. Then the demonstration can be re-executed without change for those generalized energy distributions. Once again, the result is the following. The space-time deformation generated by ϵ is described by the successive local deformations of P_1 along its trajectory.

13 An equation of G: calculations

In this appendix an equation of G is tried. Assumptions (i) (ii) (iii) and (iiii) are assumed. The same context and notations as in paragraph 11.3 are used. Let's remind properties of the Schwarzschild metric. The first equation is the following.

$$g_{00}(x') = 1 - \frac{R}{r} \tag{36}$$

Of course $g_{00}(x')$ is the time-time component of the metric in the x' event, R is the Schwarzschild ray of P, r is the distance between P and the x' event. But the following one is another important equation valid in this metric, which relies the metric to the free fall speed.

$$g_{00}(x') = 1 - \frac{v^2}{c^2} \tag{37}$$

v is the speed of the usual P_1 free falling test particle, located in x', which were located initally infinitely far from P. The validity of equation (37) comes from assumption (i). Indeed, equations (36) and (37) are valid together because Newton's law is assumed to be valid. Equation (39), which comes further, can be used to confirm that statement. From equation (7) and (37), using $e = D^1(x)/\big(D^0(x) - D^1(x)\big)$, it results the following equation.

$$g_{00}(x') = \frac{1+2e}{(1+e)^2} \tag{38}$$

By other means, the geodesic equation of P_1 is the following [10].

$$\frac{\partial^2 r}{\partial \tau^2} = -\frac{c^2}{2}\frac{\partial g_{00}}{\partial r} \tag{39}$$

Here τ is the proper time of P_1. Replacing g_{00} by its value given by equation (38), equation (39) becomes the following one.

$$\frac{\partial^2 r}{\partial \tau^2} = c^2 \frac{e}{(1+e)^3}\frac{\partial e}{\partial r} \tag{40}$$

Then, once again, assumption (i) is used, Newton's law is valid. The asymptotic formulation of equation (40) is yielded, relying Newton's law with the asymptotic value of the rhs of equation (40).

$$-\frac{M_0 G}{r^2} \simeq c^2 e \frac{\partial e}{\partial r} \tag{41}$$

Here M_0 is the mass of P. Assumtion (iii) has been used: the contribution of P in equation (6) is far weaker than the sum of the other contributions. Therefore, e is asymptotically equal to 0 in front of 1. The solution of this differential equation (41), gives the following asymptotic value of e.

$$e \simeq \sqrt{\frac{R}{r}} \tag{42}$$

R is the Schwarzschild ray of P. From that is deduced $D^1(x)/D^0(x) = e/(1+e) \simeq \sqrt{R/r}$. It results the following equation.

$$\frac{1_w(x,y_0) f(x,y_0) C^0(y_0)}{\Sigma_{n=0}^{\infty} 1_w(x,y_n) f(x,y_n) C^0(y_n)} \simeq \sqrt{\frac{R}{r}} \tag{43}$$

It has been supposed that the y_0 virtual particle is the only one pertaining to P. It was used $C^0(y_0) = C^1(y_0)$, which reflects the fact that $C^\mu(y_0)$ is a null four-momentum.

Under (iii) assumption, the denominator of equation (43) is constant, that is, independant of the x location. Therefore this equation shows that the $1_w(x,y_0)$ $f(x,y_0)$ $C^0(y_0)$ contribution is proportional to $1/\sqrt{r}$. Under the (ii) assumption, it can be deduced that the $1_w(x,y_n)f(x,y_n)C^0(y_n)$ contributions are proportional to $1/\sqrt{\|x-y_n\|_3}$. $\|x-y_n\|_3$ is the space length calculated in a covariant way along the y_n to x geodesic and observed in x, in R_0.

Also x, and therefore r being constant, from equation (43) the $1_w(x,y_0)$ $f(x,y_0)$ $C^0(y_0)$ contribution is not proportional to $C^0(y_0)$ but proportional to $\sqrt{R}$, therefore to $\sqrt{M_0}$ and to $\sqrt{C^0(y_0)}$. This is in contradiction with the hypothesis at the construction of equation (6). Here it is noticed a proportionnality to the square root of an energy. The answer to this apparent contradiction is that the final picture is coherent. Therefore one can replace each $1_w(x,y_n)$ $f(x,y_n)C^0(y_n)$ contribution of equation (43) by $1_w(x,y_n)\sqrt{C^0(y_n)/\|x-y_n\|_3}$. The result is the following.

$$\frac{1_w(x,y_n)\sqrt{\frac{C^0(y_0)}{r}}}{\Sigma_{n=0}^{\infty}1_w(x,y_n)\sqrt{\frac{C^0(y_n)}{/\|x-y_n\|_3}}} \simeq \sqrt{\frac{R}{r}} \tag{44}$$

It can be used $R = 2M_0G/c^2$ and $C^0(y_0) = M_0c^2$, in order to transform equation (44) into the following one.

$$G \simeq \frac{c^4}{2\left(\Sigma_{n=0}^{\infty}1_w(x,y_n)\sqrt{\frac{C^0(y_n)}{\|x-y_n\|_3}}\right)^2} \tag{45}$$

Now it is possible to use this result with VGWs. It means that the energy distributions of the P_i are replaced by the limits of the S_n^i distributions, as shown by equation (5). The result is equation (13), which is equation (45), without the $1_w(x,y_n)$ terms. Indeed, in any given x and y_n space-time events, it is possible to find a m and a S_m^n distribution centered on y_n, such as the GW, generated by the relativistic virtual particle of S_m^n, propagates in x. Moreover, this S_m^n distribution can be chosen with a circle's ray as weak as we want (remember that those S_m^n distributions are circle like trajectories of relativistic virtual particles). In other words, after the limits of equation (5) are done, each x space-time event is reached by the VGWs of each P_n particle.

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