Relativity predicts a variable G

F. Lassiaille[1] [1]06000 Nice, France

Abstract. It is shown that relativity predicts a variable G. The proof starts by considering a dimensionless particle in an empty universe. Then two particles, three particles, and an infinite set of particles are studied. This allows to calculate space-time structure for any realistic energy distribution. The proof uses the interchange of limits theorem, and ad hoc sequences of energy distributions. With only one particle, the result is a singularity everywhere if the universe is empty outside of the particle. Those singularities disappear completely with three particles. Then this calculation is done for any realistic energy distribution. An equation of G is given naturally in the process. This equation is a correct approximation in most of the realistic energy distributions. The fundamental principles building Einstein equation are still valid, but now the constant anthropocentric solar system value is shown to be weaker in strong matter density environments, and greater in low matter density environments. It means that the surrounding effect arises, it was introduced by previous works [1,2] And this effect was shown to solve the gravitational mysteries of today in astrophysics and in cosmology. Under a unifying relevant assumption, a solution is also given to the Yang-Mills Millennium problem.

1 Introduction

The purpose of the present document is to show that relativity predicts a variable G. It is also to describe a solution to the Millennium Problem [3,4].

This study follows from previous works. In particular in [2], it was already shown that the relativistic speed of the quarks implies that G is a variable. Here it is shown that it's a theoretical prediction of relativity, independent of the speed of the quarks or any other experimental information about energy distributions.

The proof will start by considering a dimensionless particle in an empty universe. Then space-time structure can be calculated for several particles up to any infinite set of particles. The interchange of limits theorem will be used.

There are many ways to motivate this study. The first one is to searh for a direct link between energy and first metric derivative. Another one is to solve the slight caveats of General Relativity (GR) [2]. Another one is to work on the behaviour of the concept of normal frames, which is a usual key concept in GR. But one of its property has been dismissed in the literature. This work will reveal that Special Relativity (SR) is more than an algebraic rule of GR, it relates to a space-time structure deformation by energy. Another one is to notice that the constancy of G is an anthropocentric assumption which is ruled out by today's experimental data. Another one is to search for the theoretical justification of Newton's law.

It will be proven that relativity predicts a variable equivalent G. This variation will be driven by the surrounding effect [1] which, in its weaker version, is the following. Gravitational force increases in low matter density environments, and decreases in high matter density environments. Then an equation of G will be given naturally in the process. Finally, the Yang-Mills Millennium problem will be addressed.

2 Mathematic reminder: interchange of limits theorems

Norms are equivalent in finite dimension, hence this is true for the four dimensional space-time of General Relativity (GR). By other means, the determination of space-time structure is a function of energy distribution. And this function is a continuous function. Indeed, if any amount of energy at any space-time location is decreased to 0. then the effect of this amount of energy on space-time structure decreases also to 0. This continuity around the 0 value is the result of conservation of energy principle. The restriction to the 0 value is possible because any space-time structure can be seen as flat euclidean almost locally by chosing an ad hoc system of frames.

One formulation of the interchange of limits theorem is the following. If any f_n sequence of energy distributions over space-time tends uniformly to some l limit energy distribution, if S is the function giving space-time structure from an energy distribution, then there is $S(l) = S(lim(f_n)) = lim(S(f_n))$.

The continuity of the determination of space-time structure as a function of energy distribution has another interesting consequence. It is possible to imagine thought experiments in which energy is increased ar decreased progressively. And this can be done without violation of the conservation of energy principle.

3 A privileged frame in relativity

3.1 Definition and properties

In relativity there exists a privileged frame. Moreover, the boost which is associated with the motion of matter in this frame, describes the evolution of this privileged frame. This can be reminded with a thought experiment, avoiding then any complicated and tedious calculation. It is done in appendix 1. The conclusion is that it exists a privileged frame in any space-time event (using the extension of identification which is presented in appendix 1). And for any particle located in a given x event, this privileged frame exists in x and is transformed by the particle, using the boost which is associated with the four-momentum of the particle. Roughly speaking for the understanding, let's write that this boost is calculated in the "old privileged frame", that is, the one "just before the particle", and that it transforms this old privileged frame into the new one, that is, the one "just after the particle". Another definition is that the old one describes space-time locally without the existence of the particle, and the new one does it just after the existence of the particle. Moreover, the identification of the new privileged frame from the old one can be done progressively, using the continuity of the function giving space-time structure from energy distribution.

Also a rescaling of time and space unit occurs after the boost. This will be shown further. But in the present document, it will be written abusively that the local deformation is described by a boost. Indeed, only the context will allow to deduce if a rescaling occurs also, or not. There is more information about that in appendix 3.

This concept of privileged frame is central in GR. It is refered in the literature as the "normal frame". It has been associated here with the rule governing its evolution with respect to matter and with motion of matter. The link between local space-time deformation and matter is given by this concept and this rule. This link is local but implies only the first degree of derivation of the metric. It should be possible to induce from that the second degree of derivation, and then compare the result with Einstein equation. Whatever the result is, a new view of space-time deformation by energy arises.

3.2 Example

A simple example is a P particle in motion along a straight line in a static universe filled with a constant matter density, resulting in a flat Minkowskian universe. The old $R_{0\ system}$ system of privileged frames is represented by the same and constant R_0 frame, a frame for which universe is at rest. This is called the "frame of fixed stars" in old literature. The $R'_{0\ system}$ new system of frames is the real existing one. It results from the existence of P.

This can be refined by assuming a progressive appearance of P in space and time. Then a continuous set of privileged frames is constructed progressively, from $R_{0\ system}$ to $R'_{0\ system}$.

For a gravitational wave (GW), the same definition applies: the "old privileged frame" is the one which would be the privileged one if the GW was not existing, the "new priviled frame" takes this GW existence into account.

4 Fundamental assumption

From now on in the present document, it will be assumed the following assumption.

Assumption (I): a GW propagates at the speed of light.

The relevance of this assumtion (I) is well described in the literature [5].

5 Space-time structure around a uniformly moving relativistic particle

Now the aim is to describe the local space-time deformation of a GW generated by a particle moving at the speed of light. The usual following assumption will be assumed. The P particle is moving at the speed of light along a D straight line in a empty universe structured by a flat Minkowskian space-time.

The local space-time deformation of the GW will be studied at the location where the GW deformation is the greatest. Being only local, the deformation is transforming the time and space axis into new ones. That is, the old privileged frame is transformed into the new one. Therefore this local deformation is described by a linear transform.

Whatever the R inertial frame is chosen, this trajectory will always be a straight line and the speed of the particle will always be the speed of light. Moreover, the property of physics are the same whatever is the chosen inertial frame. Therefore the space-time structure generated by this particle will always be the same whatever the inertial frame is chosen. It means that in R at any given time the locations in which the deformation is the greatest is always the same space cone centered on D.

More precisely, let's study in R the GW generated by P in the E_C event which is a given C space location pertaining to D, and a given t_C time. This GW propagates at the $c/\sqrt{2}$ speed along the CM line where M is the space point of the E_M event reached by the GW at $t_M > t_C$. Let's construct in R the space point H which is the perpendicular projection of M on D. Let's write E_H the event when P is located in H. Then $c/\sqrt{2}$ is the speed from C to M, but c is the speed from C to H and from H to M. Nevertheless the GW propagation is only along the CM line. The GW propagation $c/\sqrt{2}$ speed is the speed of the enveloppe of the GW along its trajectory. This propagation speed is normal to the enveloppe. From E_C and during the same $t_M - t_C$ time duration, the GW has been also propagated, at the c speed from E_C to E_N. E_N is the event spatially located in the N point such as CN is perpendicular to D (and such that N is in the same space plan as D and M). The time of E_N is $t_N = t_M$, the same time as E_M. This is the easier way to understand this GW propagation: perpendicularily to D the GW propagation is done at the speed of light.

The enveloppe is a cone centered on D. This cone is "isosceles". In other words, if M is any point on this cone, if H is the perpendicular projection of M on D, if C is the vertex of the cone, then there is $MH = HC$.

Now let's study the deformation generated by the GW. First of all, in C this deformation is a boost associated with the speed of light in the same direction as P. This is proven in appendix 4.

Along the D line, exactly the same boost propagates itself, at the speed of light. This is proven in appendix 6. Here, since the P trajectory is the same D line, of course this can't be noticed. But it will be noticed as soon as P will deviate from this D trajectory.

Outside of D, on a given M space point, the local GW deformation is probably described by the following linear transform. It might be a b_M boost oriented in the direction of the normal to the cone in M, associated with the $c/\sqrt{2}$ speed and propagated at the $c/\sqrt{2}$ speed. This is not proven in the present document. Hopefully, it is not required for the study. More details are available in appendix

7 and 8.

Outside of the cone, space-time structure is determined by the following equation (the Ricci tensor is null). This is because, as it will be seen further, Einstein equation can still be assumed valid in vacuum.

$$R_{\mu}^{\nu}(g_{\mu\nu}) = 0 \tag{1}$$

$R_{\mu}^{\nu}()$ is the function giving the Ricci tensor from the metric, and $g_{\mu\nu}$ is the metric.

6 Space-time structure around a relativistic particle moving along a circle

Now let's assume that the previous P particle is forced to move along a circle. Then the previous cone transforms into a more complicated geometric figure. But infinitely far from the circle, therefore asymptotically, the enveloppe of the GW propagation at constant time is a sequence of spheres centered in O, the center of the circle. Asymptotically those spheres inflate themselves around O at the speed of light. The rays of these spheres are separated by the same d constant value which is the circle circonference. Now the deformation which is propagated asymptotically is a boost associated with the speed of light and which propagates along the ray of those spheres. This results from the previous study. Indeed, asymptotically, the GW propagates along the direction which is perpendicular to the particle's trajectory, which is also the line starting by the location of the particle. And from the previous study this is done at the c speed.

If the ray of the circle decreases progressively and tends to the 0 value, it means that the circle tends to its limit which is a dimensionless particle located in O. Then the previous cone transforms itself into the previous sequence of spheres, but also d tends to 0. The final result is a sequence of spheres which are infinitely close to each other, all centered on O.

The interchange of limits theorem can be applied to this Dirac distribution of matter, namely here, the dimensionless particle located in O. It can be applied because the circle-like trajectories of microscopic particles are energy distributions which tend to this Dirac distribution. If any doubt exists because the microscopic particle is in a rotating motion around O, then it is possible to add another microscopic particle sharing the same energy and moving along the same circle-like trajectory in the opposite direction. This would cancel the whole rotating motion without changing the final result which will be given further. Finally this distribution tends to the Dirac distribution centered in O, when d tends to 0.

Therefore let's apply the interchange of limit theorem. The space-time structure generated by a 3D Dirac distribution centered in O is the limit of the previously described sequence of spheres, limit when d tends to 0. Those limit deformations are described in any M space point by a boost associated with a speed of light which is oriented along OM. Let's write R, an inertial frame attached to P (in which P is at rest). M belongs to a sphere centered in O which contains all the deformations arriving at the same time in R. Moreover those spheres are now infinitely closed to each other. It means that each space-time event of the universe is modified by this singular boost. This is the space-time structure generated by the 3D Dirac distribution.

Hence the result is radically different from what is told by today's literature. Let's remind that with today's literature the space-time deformation here is exactly the one which corresponds exactly to Newton's law occuring in solar system. But what has been proven here is that this deformation is a singularity everywhere.

This huge difference will explain why G is not a constant but a variable.

7 Partial resolution of the issue of Mach's principle

This resulting space-time structure might be argued to be wrong because it is not realistic. But the correct argument is the opposite. This description appears to be more correct than the one which is given in the literature because the distribution was supposed irrealistic in the first place. Indeed, a Dirac distribution of matter is already by itself, irrealistic since it assumes an empty universe outside of the center of the Dirac distribution. For that reason, only an irrealistic result would be searched for.

Moreover, this GR prediction is in perfect compatibility with Mach's principle [6]. Let's remind briefly the Mach's principle problem in GR. For example the issue appears in the case of a static spherically symmetric universe. A particle is located in the center of this spherical symmetry. If ρ is the matter density filling the universe, then one can distinguish two assumptions. The first one is $\rho > 0$, the second is $\rho = 0$. Close to the particle, ρ appears insignificant in both cases. Therefore, there, the spacetime deformations will be approximately the same for the two assumptions. But in the first assumption it is possible to find an inertial frame, R, at rest with the particle, which is not in rotation with respect to the universe. In R, there are no fictitious forces such as centrifugal forces. But in the second assumption it is not possible to find such a frame. Supposing that R is at rest with respect to the object is not enough. It is not possible to know if R remains inertial or not. One cannot say if in R it will appear fictitious forces or not.

The new GR prediction solves this problem. Now space-time structure becomes singular everywhere for the second case. An answer is given: in each frame at rest with the particle, no fictitious force might ever appear, since those singularities would dissolve them completely. Therefore, with this new, correctly irrealistic prediction, GR becomes more Machian.

8 Two particles

Now let's add another particle, apart from the first one. So there are two particles, at different locations, and out of them the universe is still empty. Everything is still supposed to be static, that is, the two particles are at rest in some given inertial frame.

Outside of the particles, space-time structure is first determined by conservation of GR Lagrangian in vacuum. It is simpler to say that the Ricci tensor is null in vacuum. This is a second degree of derivation differential equation. An integration constant is still required at the end of the calculation. Under today's version of GR, G solar system constant value and the mass of each particle are used for this. Now the determination of this integration constant remains to be proven. Hints and clues for this determination are given in appendix 2. Although this determination would be an improvement, nevertheless it is not mandatory for the study of the present document.

Also, asymptotically the space-time deformation is singular. Indeed, this asymptotic deformation is the same as the previous asymptotic deformation of the distribution with only one particle. And of course each particle still generates locally a space-time singularity. If the masses of the particles are not equal then a tough calculation is required. Let's suppose that they are equal. Then, the calculation is simpler since there is the perpendicular symmetry with respect to the perpendicular plane between the particles. This shows a picture which is still hugely different from what is written in today's literature.

9 Three or more than three particles

Adding a third particle to the scene will disolve those singularities which are located on the straight line containing the particles. As usual it is supposed that the third particle is at rest with respect to the other particles.

With three or more particles, space-time structure is still fully determined by equation (1). And under the assumption of the rest mass of the particles being the same, symmetry considerations might help to calculate simply space-time structure. For the determination of the integration constant, the same arguments apply as in the paragraph 8. They are presented in appendix 11.

10 Space-time structure for any distribution of energy

The studied distribution of energy is a realistic one: an infinite set of dimensionless particles. Indeed modeling reality this way is often a good approximation in astrophysics, from planets to cosmology scales, and in particle physics, because of the sparse nature of matter. Of course a more general distribution of energy might be studied. Notably a uniformly continuous distribution of energy would possibly still allow the use of the interchange of limits theorem. But this is out of the scope of the present document.

Hence let's apply the interchange of limits theorem, to this set of Dirac distributions of matter, namely here, the dimensionless particles of the universe. Let's remind that for each P_i particle, where i is the particle's number, i from 0 to infinity, there is a S_n^i sequence of energy distributions. For each i and n, S_n^i is a distribution made of a circle-like trajectory of a virtual relativistic microscopic particle in an empty universe, this circle being centered on P_i. For each i, i from 0 to infinity, the circle's ray tends to 0 and S_n^i tends to the Dirac distribution of the P_i particle, when n tends to infinity.

$$\lim_{n \to +\infty} (S_n^i) = m_i \delta_i \tag{2}$$

In equation (2), δ_i is the Dirac space distribution centered on the P_i particle, and m_i is the mass of P_i. This simple convergence for each P_i can be transformed into a uniform convergence for the whole set of P_i. It suffices for that to choose, for each n, whatever i is, the same S_n^i circle's ray. For example the value of $1/n$ can be chosen for the circle's rays of S_{n+1}^i, in the system of privileged frames of space-time structure generated by S_n^i.

The final picture is the following. Space-time structure is the result of all the microscopic GWs. Those microscopic GWs are those generated by the virtual microscopic relativistic particles representing the real P_i particles in the above study. From now on in the present document, those GWs generated by those microscopic virtual particles will be called "virtual gravitational waves" (VGW). Considering only the limit distributions ($n \to \infty$), the set of the S_n^i is equal to the set of the $m_i \delta_i$. Then, each space-time event receives a VGW from each real particle in the universe. This gives a clue for calculating space-time structure in a different way.

11 Calculating space-time structure with virtual gravitational waves

11.1 Four-momentum equation

The same distribution of energy is still assumed. In any x space-time event, the four-momentums of all the GWs propagating in x add themselves. The fundamental reason for this is conservation of energy principle. This is shown in appendix 9.

The resulting equation has been described in [2] and is the following.

$$D^\mu(x) = \Sigma_{n=0}^{\infty} 1_w(x, y_n) f(x, y_n) C^\mu(y_n) \tag{3}$$

Equation (3) shows the calculation of the resulting four-momentum in x. For n from 0 to infinity, each y_n event represents a space-time location in which the P_n particle is possibly propagating a GW in x. Considering only the limit distributions ($n \to \infty$), each P_n particle of the universe propagates a VGW in x. The $1_w(x, y_n)$ is equal to 1 if x and y_n events are connected by a null geodesic and if x is located after y_n along this geodesic. It means that the GW generated in y_n is received in x. Considering only the limit distributions, $1_w(x, y_n)$ is allways equal to 1 and can be supressed from the equations. $f(x, y_n)$ is a scalar positive function. It is assumed to be equal to 1 if y_n is equal to x. It expresses the attenuation of the GW energy which is emitted from y_n. $C^\mu(y_n)$ is a four-vector which contains the information of the energy of the GW in y_n. Later on, it will be shown that $C^0(y_n)$ is not the effective energy of the GW, but is proportional to its square root. Nevertheless, in order to avoid a heavy reading, the words "four-momentum" and "energy" will be used for $C^\mu(y_n)$ or $C^0(y_n)$. The context will allow to understand if those are effective energy or contributions to equation (3). And this "contribution" word will mean the $1_w(x, y_n)f(x, y_n)C^\mu(y_n)$ terms which are in the sum of the rhs of equation (3).

There are important remarks about equation (3).

A first remark is that this equation is only valid asymptotically, that is, for great distances between x and each y_n, with respect to the P_n energies. Indeed, $f(x, y_n)C^\mu(y_n)$ is ensured to be a null four-momentum only asymptotically. For example, it has been shown in paragraph 5 that the GW generated by a P_n relativistic particle moving along a straight line in a flat Minkowskian space-time structure would propagate at the $c/\sqrt{2}$ speed and would be described by a boost associated with this speed. It means also that for high speed P_n particles this propagation speed would differ sufficiently from c, and the four-momentum describing locally the deformation would be associated with this speed and would not be a null four-momentum. But in realistic distributions only slow relative motions of particles can be assumed. And considering only the limit distribution and VGWs, the local space-time deformations are propagated at the c speed and are described by boosts associated with this speed, therefore $f(x, y_n)C^\mu(y_n)$ are null four-momentums.

A second remark is the question of wether an infinite number can result from equation (3) or not. For example if an infinite universe is filled with a constant and uniform distribution of particles, then the result is infinite if the f attenuation function decreases less than $1/r^3$. This problem is similar to the Olbers's paradox problem [7]. But a more practical solution can be found here. There is no need to understand the universe expansion and horizon. When translating this equation (3) into a gravitational model such as surrounding [1], a solution is found. Indeed, in surrounding, the fitting of the model with experimental data forces this sum of equation (3) to be translated into a finite value. A practical approach here is simply to ignore the possibility of divergence of this equation, and to fix this issue later on, when working on gravitational models.

11.2 From the four-momentum to the boost

Let's follow the natural calculations. The distribution of energy of paragraph 10 is still assumed. The four-momentum of equation (3) can be generated by a single particle located in x, by the GW of a single particle propagating in x, or by many GWs encountering in x. Let's write this resulting four-momentum of equation (3).

$$D^\mu(x) = \gamma \frac{E}{c} \left(1, \frac{v}{c}, 0, 0\right) \tag{4}$$

E and v are respectively the energy at rest of the four-momentum and its speed in a frame. It has been used $\gamma = 1/\sqrt{1-v^2/c^2}$. This equation (4) has been written in a $R_0(O;ct,x,y,z)$ frame, at rest with the universe, and such that v is along the Ox line. It is possible to find such a frame. Then, from $D^\mu(x)$ is calculated the local space-time deformation which is generated. This is done [2] by using the boost described by the following equation.

$$B^\mu_\nu(x) = \gamma \begin{pmatrix} 1 & -\frac{v}{c} & 0 & 0 \\ -\frac{v}{c} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

This boost is directly deduced from the four-momentum of equation (4).

11.3 From the boost to the metric

Now it is possible to derive the space-time metric from $B^\mu_\nu(x)$.

The distribution of energy of the paragraph 10 is still assumed, but now the universe is filled with a constant matter density. It means that the grid of particles has cells which are small enough for allowing such an approximation.

Let's assume that a P particle is added to the scene, located in O at $t = 0$, at rest in R_0. R_0 can be assumed to be the privileged frame in x, before adding P to the scene. Let's call R'_0 the privileged frame after adding P to the scene. Therefore, R_0 is the "old privileged frame", and R'_0 is the "new privileged frame". Let's write x' the first event when R_0 has been transformed into R'_0, along the time of R_0. R'_0 is obtained by transforming R_0, in x, using the $B^\mu_\nu(x)$ boost.

From R_0 to R'_0 it can also be generated a successive continuous sequence of privileged frames, starting with R_0 and ending with R'_0. For that it suffices to add slowly the P particle energy from 0 to its real value. (Let's remind that avoiding the conservation of energy principle is allowed in a thought experiment).

Then, it is required to rescale the lengths of the "boosted" time and space axis. The boosted time and space axis are the time and space axis which have been modified by the boost, in their states after the boost. The rescaling is done in such a way that the resulting time line described successively by those successive infinitesimal steps is a geodesic. Equivalently this constraint is that the privileged frame must be inertial. This is detailed by the following equations, relating X'^ν the coordinates after the boost, to X^μ the coordinates in R_0, and then relating $X"^\rho$ the final rescaled coordinates in R'_0 to X'^ν.

$$X'^\nu(x') = B^\nu_\mu(x) X^\mu(x) \tag{6}$$

$$X"^\rho(x') = S^\rho_\nu(x') X'^\nu(x) \tag{7}$$

$$g_{\alpha\beta}(x) = B^\rho_\alpha(x) S^\mu_\rho(x') g_{\mu\nu}(x') S^\nu_\kappa(x') B^\kappa_\beta(x) \tag{8}$$

$S^{\mu}_{\rho}(x')$ is a symetric transform which has the ability of being diagonalized in R'_0. Its value is determined by the constraint above (the time line of the set of successive privileged frames must be a geodesic). Equations (6) (7) and (8) show how $g_{\mu\nu}(x')$ the new metric is deduced from $g_{\alpha\beta}(x)$ the old one, due to the action of $B^{\nu}_{\mu}(x)$, which results from the $D^{\mu}(x)$ added energy. Of course equation (8) can be inverted, using the inverse mixed tensors $(B^{-1})^{\beta}_{\kappa}(x)$ and $(S^{-1})^{\rho}_{\mu}(x')$ of, respectively, $B^{\nu}_{\mu}(x)$ and $S^{\mu}_{\rho}(x')$. It results the following equation.

$$g_{\mu\nu}(x') = (S^{-1})^{\rho}_{\mu}(x')(B^{-1})^{\alpha}_{\rho}(x)g_{\alpha\beta}(x)(B^{-1})^{\beta}_{\kappa}(x)(S^{-1})^{\kappa}_{\nu}(x') \qquad (9)$$

Equations (6) (7) and (8) have been obtained by studying the spherically symmetric static case. In the Schwarzschild metric, a P_1 free falling particle, having a negligeable mass, being at rest when located infinitely far from the center of the symmetry, follows a time line which is transformed by those equations [8]. This is proven in appendix 10. Those equations are still valid in the most general case. Indeed, their construction follows the rule of the privileged frame being modified by the boost associated with local matter motion, and this privileged frame is inertial. A more rigorous demonstration might be given. But in the scope of the present document, only the particular spherically symmetric static case is required.

It is already known that $g_{\mu\nu}(x)$ is a diagonal matrix in the R_0 frame and that $g_{\mu\nu}(x')$ is a diagonal matrix in the R'_0 frame. Since the direction of the boost is along the x space axis, only time and x space dimensions are modified by the metric evolution. If the $(S^{-1})^{\rho}_{\nu}(x')$ rescaling is written with an α time rescaling and with a β space rescaling, then, using equation (9) and $\alpha\beta = 1$ usual convention, the resulting metric shows $g_{00}(x') = \alpha^2$ and $g_{11}(x') = -\alpha^{-2}$. This allows to check and understand the involved mechanism of equations (6) (7) and (8).

As a conclusion, space-time structure is calculated first by calculating a one and only four-momentum in x which contains the information about the local deformation in x. From it the final local space-time deformation is determined. This determination is deducing the speed associated with this momentum, and then the boost associated with this speed. This boost describes the final space-time deformation occuring locally in x. More precisely the final space-time structure in x is described by the boost which is associated with the four-velocity which is the barycentric operation of all the four-velocity of VGWs propagating in x using the total energy of each VGW as its weight. The attenuation of the propagation of the VGWs follows the rule of Ricci tensor being null in vacuum. From this boost is calculated the metric. The question of the stability of this self-induced mechanism arises. But if the universe is static, with this mechanism, space-time structure converges into a stable structure. Indeed if the universe is static, then a thought experiment can be done in which the energy distribution is constructed by adding progressively the particles one after the other. Also each of them can be added progressively, their energy at rest being increased progressively. Each of those successive energy distributions are static. As usual in GR, space-time structure for each of them can be described with the system of the normal frames, system of frames in which space-time appears virtually flat Minkowskian. This system of frames is the system of privileged frames. In this system, in any x space-time event, the sum of the momentum of the VGWs occuring in x is null. This is valid also for the limit of these energy distributions.

11.4 An equation of G

The context and notations of paragraph 11.3 are used. For trying the construction of an equation of G, now let's assume, also, the following.

Assumption (i): Newton's law is valid. But G may differ from its solar system value.

This is based on experimental data. Newtons' law must be supposed to be valid almost in solar system because this law is validated with high accuracy, at least in solar system [9]. And there are theoretical arguments for this law to stay valid out of solar system, though being used with a different value of G. This will be studied further in the present document in the paragraph about a revisiting of Newton's law.

Assumption (ii): the energy of a VGW evolves always following the same attenuation function (function of the initial starting energy, and of the propagation distance), regardless of its location and starting energy.

Assumption (iii): in equation (3) the sum of the energy of the contributions generated by P is far weaker than the sum of the energy of the other contributions.

This assumption will be confirmed with surrounding, where a sphere with a 15 kpc ray, which is used for calculating the surrounding value, is fitted to experimental data. The energy which is located in this sphere corresponds to the time component of the rhs of equation (3), that is, the sum of the energy of the contributions of this equation.

Assumption (iiii): the contributions of equation (3) can be replaced by their asymptotic values without modifying consistently the result.

This assumption can be valid if the particles of the universe are isolated enough from each other. This might be realistic because matter is known to be extremely sparse in the universe, whatever the scale is, from particle's physics scale to cosmological scale. Assumptions (i) (ii) (iii) and (iiii) are easier to accept asymptotically.

The calculations are presented in appendix 11. They are done from assumptions (i) (ii) (iii) and (iiii), equations (3), equation (4), and the geodesic equation for Newton's law which is reminded in the appendix. They result in the following equation of G.

$$G \simeq \frac{c^4}{2\left(\Sigma_{n=0}^{\infty}\sqrt{\frac{C^0(y_n)}{\|x-y_n\|_3}}\right)^2} \tag{10}$$

Equation (10) is used, as usual, with VGWs. It has been shown that this equation is a good approximation under (i) (ii) (iii) and (iiii) assumptions.

The potential divergence of equation (3) might appear worse in equation (10) than in eqution (3). Indeed, the $1/\sqrt(r)$ shows potentially a quick divergence. Nevertheless it must be noticed that the resulting gravitational force will obey to the $1/r^2$ rule. Indeed, equation (10) has been formed from that rule. Therefore the final possible divergence is the Olbers's paradox divergence. And it is easy to modify equation (10), inserting a cut-off value of the contributions, for example resulting in the following equations.

$$\begin{aligned} \Phi_{cut}\big(a, b\big) &= \\ b \leqslant R_{cut}: &\quad \sqrt{\frac{a}{b}} \\ b > R_{cut}: &\quad 0 \end{aligned} \tag{11}$$

$$G \simeq \frac{c^4}{2\bigg(\Sigma_{n=0}^{\infty}\Phi_{cut}\Big(C^0(y_n), \|x - y_n\|_3\Big)\bigg)^2} \tag{12}$$

Here R_{cut} is the maximum GW propagation distance. The $15\ kpc$ value is suggested by surrounding [1].

This cut-off value does not alter much the qualification of "asymptotic" in the previous reasoning and in the calculations of appendix 11. Indeed in the surrounding gravitational model, the $R_{cut} = 15\ kpc$ value is fitted to experimental data. It is a distance which would require an attracting object like a galaxy in order to contradict this "assymptotic" qualifier. And this equation (12) can still be improved. For example it is possible to replace the simple 0 cut-off value by a slow decrease of the contributions of equation (10).

12 Predicted surrounding effect

The above study shows that relativity predicts a variable G. And this variation given by equation (10) follows the rule untitled "surrounding" in [1]. Of course equation (10) has been constructed under the (i), (ii), (iii) and (iiii) assumptions.

But equation (3) shows already this surrounding effect, without any added assumption. Let's show this by rewriting it, shifting the total energy from left to right, and isolating the resulting speed.

$$\frac{v}{c} = \frac{f(x, y_0)C^0(y_0)}{\Sigma_{n=0}^{\infty}f(x, y_n)C^0(y_n)} \tag{13}$$

The $1_w(x, y_0)$ and $1_w(x, y_n)$ terms disappeared because now only VGWs are considered. There is allways a VGW which is propagated by the particle located in y_n and which is received by any given x event. The universe is still assumed to be at rest and filled with a constant matter density, except for only one P particle at rest with the universe. And the VGW of y_0 is still assumed to be the only one emitted by P. Hence the numerator of the rhs of equation (13) is the asymmetric contribution which is also the numerator of equation (28). Equation (13) derives directly from equation (3), and shows that the space velocity of the resulting four-velocity is inversely proportional to the denominator, which increases with the energy surrounding the x location.

It can be noticed also that the denominator of the rhs of this equation is a sum of positive scalars calculated in an isotropic manner. It induces naturally to translate this equation (13) into a gravitational model, replacing this value by the energy of the surroundings of the location where the gravitational force is exerted. The result is that a surrounding gravitational model or a gravitational model close to it is predicted by relativity. Therefore the so-called gravitational anomalies of today might be no anomalies at all, but regular predictions of relativity.

The modified gravity theories of today must comply with the MOND [10] model predictions, for the greatest part of experimental data. This is naturally acomplished by the surrounding model. Indeed, when acceleration is low, MOND increases it. But when acceleration is low, most of the time it means that the surrounding energy and matter are low, also, and then the surrounding effect increases acceleration too.

13 Revisiting Newton's law

A revisiting of Einstein equation is naturally required by the previous study. This equation is nothing more than the most direct translation of Newton's law from non relativistic physics into relativity. Therefore, the first step is to revisit Newton's law. The Poisson's formulation of Newton's law starts the construction of Einstein equation, not only because it's about Newton's law. First of all, in vacuum, this formulation expresses the following important non relativistic principle.

(i) The flow of the acceleration vector field is constant in vacuum.

It results $div(a) = 0$ in vacuum, where a is the acceleration vector field. This is not something new. But now it has been shown that space-time structure in vacuum can be calculated by considering only the VGWs generated by theoretical infinitesimal relativistic particles. This gives a new insight about this (i) principle: it corresponds to the conservation of flow of GW energy in vacuum. Hence another argument for the Poisson's formulation of Newton's law is given.

Secondly, matter plays the role of a source for this field: matter is a source of any interaction force. And this is not only true for gravitation. Indeed, the force which attracts a given A particle to another given B particle is acting on A in the direction of B. This direction is tangent to the geodesic relying A to B. At the contrary, vacuum does not generate any force. Therefore the force vector field has a divergence which is a function of matter density. And it is the same with the acceleration vector field, because of the fundamental principle of dynamics.

Hence, Newton's law in its Poisson's formulation is almost retrieved by the previous theoretical considerations. The divergence of the acceleration vector field should be a function of energy, such that for a null energy there is a null divergence. This gives the following equations.

$$div(a) = f(\rho) \tag{14}$$

$$f(0) = 0 \tag{15}$$

ρ is matter density. In equations (14) and (15), nothing is told about neither the sign of $f(\rho)$ nor the exact feature of the f function. And the question of a possible proportionnality of $f(\rho)$ with ρ is related with conservation of energy and the principle of action and reaction. Let's show this. Equation (14) implies for a P particle in an empty universe the following equation.

$$a = \frac{f(\rho)V}{4\pi x^2} \tag{16}$$

In equation (16), a is the acceleration in any given M location, such as $MO = x$, O being the location of a P particle generating this acceleration. ρ and V are respectively the matter density and the volume of P. It was supposed that ρ is constant in P. Then applying the fundamental principle of dynamics, the following equation arises.

$$F = \frac{m' f(\rho)V}{4\pi x^2} \tag{17}$$

Here, F is the force attracting a P' particle located in M, by P. m' is the P' mass. But the principle of action and reaction implies that this equation is invariant by P and P' permutation. The following equation arises, where m is the mass of P.

$$m' f(\rho)V = m f(\rho')V' \tag{18}$$

This being true for V and V' being constant and for any value of m, m', ρ, and ρ', it implies that $f(\rho)V$ is proportional to m . A better demonstration of that would consider the total energy E of P and P', in place of the principle of action and reaction. This energy is the integral of the forces along space distances. Then it is the invariance of E by the P and P' permutation which would be used. This proportionnality would be written the following way.

$$f(\rho) = K\rho \tag{19}$$

Here K is of course the unknown coefficient of this proportionnality. Using equation (19) in equation (17), it yields the following equation.

$$F = \frac{Kmm'}{4\pi x^2} \tag{20}$$

But now no theoretical argument can be given here for calculating the $K/(4\pi)$ constant of equation (20). Historically Newton's law has been constructed based on experimental data more than theoretical considerations. To say the least, the G determination was done completely based on experimental data.

But an indirect theoretical argument can be given. Everything was done here under the assumption that a complete vacuum exists outside of P and P'. If the vacuum is not perfect outside of the particles, the reasoning above becomes wrong. The energy surrounding the particles must be taken into account. First of all, matter density of the universe outside of P and P' generates also a divergence by applying equation (14). This added divergence modifies the final result given by equation (17). Secondly, the principle of action and reaction might not be true in its simplest formulation. It is easier to understand that the energy version of the demonstration is wrong. Indeed, rigorously speaking, the total energy of P and P' must be replaced by the total energy of the universe. Indeed, energy exchanges might exist between the particles and their environment. A more practical version would be to approximate the energy of the universe to P and P' energies plus the energy of the surroundings of the particles, to some given extent suitable for a correct approximation.

The whole result of those theoretical arguments is Newton's law. But these arguments tend to prefer a variable G more than a constant G value, this variation depending of the energy surrounding P and P' particles.

14 Revisiting Einstein equation

In the more general relativistic regime the same reasoning might be done, replacing the divergence of equation (14) by Einstein tensor, and matter density by stress-energy tensor. Then the reasoning might give Einstein equation. But this work is above the scope of the present document. Nevertheless, since Einstein equation is the most direct formulation of Newton's law in the context of relativity, the reasoning above done in the non relativistic regime applies indirectly to Einstein equation.

To say the least, what appears still seriously doubtful is the statement that $K/(4\pi)$ is a universal constant, in equation (20). At the contrary, the above discussion shows that one would expect matter density of the universe to play a role in the determination of this constant. A more practical formulation of that would be that the energy of the surroundings of P and P' would play a role in this determination. This was true for Newton's law, and is therefore true for Einstein equation, since it is the most direct translation of Newton's law into relativity.

The $\rho = 0$ particular case implies $div(a) = 0$, from equations (14) and (15), but results also directly from the (i) principle. And the present study shows absolutely no need to modify its relativistic formulation given by equation (1). At the contrary, this equation allows to complete the new construction of space-time structure done in the present document. This equation was used notably, above, in the study about two particles in an empty universe.

By other means, the construction of Einstein equation from Newton's law is extremely simple. It is the simplest way to proceed. Inserting a multiplicative tensor between the stress-energy tensor and Einstein tensor is something natural which were rarely done in the literature. This mutiplicative tensor can be only a function of energy: what else? Now the present document shows that this is exactly the correct translation of Newton's law into relativity. The result would be surrounding, or a gravitational model close to it. A classical way to proceed would be to calculate everything with such a X^{μ}_{ν} hypothetic multiplicative tensor and then compare the predictions to experimental data. Very probably, it would show that X^{μ}_{ν} must be proportional to the surrounding energy of the location where the force is exerted. Indeed, the surrounding gravitational model indicates strongly that this is exactly what would happen.

Also, today during the construction of the GR Lagrangian for matter, the G anthropocentric solar system constant value is forced without any theoretical argument. This is doubtful. At the contrary, the GR Lagrangian for vacuum is the simple and well sounded scalar curvature. This is another argument for applying it in the present study (it is equivalent to equation (1)).

15 Conclusion about G

Apart from the previous demonstrations, there are serious arguments in favour of a variable G following the rule of the surrouding effect. They are the following.

1) Mach's principle.

2) Correct theoretical construction of Newton's law

3) Sophisticating the construction of Einstein equation.

4) Loss of information in the construction of the stress-energy tensor.

5) Implicit assumption of GR.

Items 1) 2) and 3) have been described above. Items 4) 5) and 6) are described in [2]. Item 6) is the experimental argument about the quasi-relativistic speed of the quarks.

Therefore, outside of solar system, it is much more relevant to use equation (10), or its translation with surrounding, than its solar system value for the determination of G.

16 Yang-Mills Millennium problem

The remark done in [2] about the Yang-Mills Millennium problem is still valid. Moreover, this remark is conspicuously reinforced by the study of the present document. Let's remind briefly this remark. It starts by assuming the following.

Assumption (A): unification of the four forces is driven by gravitation.

Under this unifying assumption, the Yang-Mills Millennium problem finds a solution. Indeed this assumption awake the full relativity in the context of particle physics: now not only SR, but also GR underlines all the forces. Therefore each of four forces is driven by the surrounding effect. And this effect modifies enormously the interactions between three particles. This allows quark confinement for long duration only when they are close to each other by groups of three.

Let's discuss the validity of assumption (A). It allows to get rid of the GR effect which creates tidal forces, for example in black holes. This effect would mean that space-time deformation would modify energy distribution a second time. Indeed, space-time already modifies energy distribution with gravity. This would appear more as a spontaneous creation of energy. Moreover, assumption (A) is more than an assumption. Indeed, the following argument can be done.

Argument (*): acceleration generated by gravitation is explained by space-time curvature. It is a simple and elegant rule. It is tempting to apply it to each force. Nothing forbids it.

The argument (*) comes from relativity and gravitation. It might appear difficult to find such a convincing hint starting from particle physics. For example it might be difficult to find such a convincing argument with the following assumption.

Assumption (B): unification of the four forces is driven by one of the three forces of particle physics.

Under the (B) assumption, the strong force might be the better candidate. Indeed, for example in figure 1 of [11], it is located on an extreme location as compared to the others. Nevertheless assumptions (A) and (B) appear better than the following one.

Assumption (C): no unification of the four forces exists.

Indeed, there is the apparent energy convergence at Planck scale which contradicts it. But assumptions (A) and (B) must be compared with the following one, which might be done by todays physics.

Assumption (D): unification of the four forces is driven by an effect in which no force plays a leading role.

It might be this (D) assumption which is implicitly assumed todays [12, 13]. But the main arguments for it are symmetry considerations. Their relevance must be compared with the relevance of argument (*).

A more experimental information in favor of assumption (A) is given by the eclipses anomalies. Indeed, under this assumption, strong deviations of the gravitational signal of the sun, by the moon, might be expected during solar eclipses [14].

the strong scattering of the gravitational force signal might be explained by an interaction between gravitation and matter which would be stronger than

17 Discussion

A new determination of space-time by energy is presented. It uses the usual GR concept of normal frames, which is called privileged frames in the present document. This new name refers to a property of those frames which has been dismissed in the literature. This property is that the evolution of those frames is driven by energy. And this gives a clue for a new space-time structure calculation from energy. This new calculation is based on the first degree of metric derivation. The second degree of derivation is calculated and then compared to Einstein equation.

The demonstrations done in this document were for the most part, purely mathematics. It results a more complicated relativity, more Machian than before, in which gravitation follows the rule of a surrounding effect. In its weaker formulation this effect is the following. Increasing the energy of the surroundings of the location where the gravitational force is exerted results in decreasing this force with respect to Newton's law. It is proven that G is not a constant, and that its variation is driven by this effect. An equation of G is given, which is a good approximation under four assumptions, which might be valid most of the time in gravitation.

When the complexity of the energy distribution does not allow to use Einstein equation, then a method has been presented, allowing to calculate space-time structure from an energy distribution made of an infinite number of dimensionless particles. Of course this method leads to complicated calculations. But are they more complicated than using Einstein equation without any symmetry rule allowing to simplify?

It still remains to find the general principle allowing to replace completely Einstein equation in any cases. Another Lagrangian might be constructed, based on this new understanding of relativity. But in most of the cases, using Einstein equation with this new equation of G might be a good or very good approximation. Whatever, it remains much better to use a G variation driven by the surrounding effect than its constant anthropocentric solar system value.

Each so-called gravitational anomaly might be simply a prediction of relativity. Of course the work is huge until one can replace "might be" by "is" in the previous sentence. Indeed, the surrounding gravitational model appears to solve all those mysteries in a straightworward way. But it would need not one but several articles to confirm that. And the surrounding gravitational model will have to be tuned. For example, the brutal rectangle window used for calculating the surrounding value must be replaced by a smooth window, soon or later. For example, the need for that appears for conforming surrounding to the wide binaries problem [15–17]. Also, possible regressions might arise, in which this G variation might induce wrong predictions in front of some given experimental data. This work is huge also.

But nevertheless a big step is done in gravitation. In particle physics, the Yang-Mills Millennium problem finds a solution under the relevant unifying assumption that the four forces are different aspects of the same and unique force of gravitation.

Appendices

1 Privileged frame

This thought experiment is simply imagining the energy at rest of a P particle increasing progressively, and at the same time the whole energy of the universe decreasing. At the end of the experiment the universe and the particle have their roles permuted. Now the particle contains the energy of the previous universe, and the universe contains the energy of the previous particle. The first result is that the frame in which time elapses the most is no longer the frame attached to the universe. Now this frame is the frame attached to the particle. It means that the space-time structure is now the symmetrical result of a permutation of those two frames. It means also that during the experiment, the space-time structure has been modified progressively from the first state to the final one. And this operation has allowed to revert the time dilatation. For example, if this was a twin paradox configuration, at the end of his brother's journey, the older twin would become the youngest after the thought experiment. Therefore this space-time modification is simply described by the boost transporting one frame into another. It can be noticed that this reasoning is using the well established supposition that GR is coherent.

Now the need of naming the frames appears. Let's call Ru a frame attached to the universe. It can be supposed that the universe is filled with a constant, homogenous distribution of matter, therefore this matter is supposed to be at rest in Ru. Let's call Rp a frame attached to the particle. The result of the thought experiment is that the particle generates locally a space-time deformation which is described by the boost from Ru to Rp. Of course, this deformation is local to the particle but the more energy at rest of the particle, the more this deformation is valid around the particle. A "more valid deformation" means that the space-time deformation exists significatively over a larger space-time domain.

The space-time deformation appearing in the experiment is described by a boost which allows to transform progressively this privileged frame from R_u to R_p. And it means that this frame remains privileged during the whole process, even though it might be no longer the frame in which time elapses the most. This frame is simply a frame in which the particle is at rest. Its physics relevance is only local to the particle. The result is that it is possible to extend this identification of the privileged frame of relativity to any space-time event in which there exists matter. And this identification can be extended even further to events in which vacuum prevails, by interpolation between those events in which there is matter. The best way to interpolate is to constrain the interpolated frame to remain inertial.

This ends what is a reminder about a feature or relativity.

The local space-time deformation of this thought experiment is inflexible. It means that it remains the same whatever is the energy distribution. Notably, whatever are the surroundings of the P particle, the local deformation generated by P remains the same. This is true if the P particle is a dimensionless particle, or at least if it gets a high enough matter density. This is a realistic modelization since matter is known to be extremely condensed. And it is true also for the virtual microscopic particles which are used in the thought experiments of the present document.

2 Integration constant

This appendix is about the integration constant of the space-time structure calculation for the energy distribution of two particles in an empty universe.

This constant is possibly given by the following information. There are singularities along the straight line containing the particles's locations, but only on those points which are not between the particles's locations. They are described by the boost associated with the speed of light in the direction moving away from the particles.

Of course the rigorous way to proceed is to calculate space-time structure using symmetry consideration and equation (1). Then, the asymptotic values of the deformation should allow to calculate the integration constant. This procedure is already done for the energy distribution with one particle in an empty universe. Symmetry consideration produces the Schwarzschild metric. Equation (1) gives the information that the metric is of the shape $g_{00}(x) = 1 - M/r$, where $g_{00}(x)$ is the time-time component of the metric in the x space-time event, r is the spatial distance from x to the particle, and M is the unknown integration contant. Then the asymptotic value of the local space-time deformation given by paragraph 6 allows to calculate $M = \infty$. Therefore, it should be possible to calculate this constant for the energy distribution of two particle in an empty universe.

3 Uniformly moving non relativistic particle: local deformation

Let's consider P, a non relativistic particle moving uniformly at the v speed, $v < c$, along the D space line in a flat Minkowskian space-time. The $R(O; ct, x, y, z)$ frame is chosen such as O is contained by D and Ox is in the direction of the P motion. The local deformation around P is first described by the b_v boost associated with the v speed, as shown before.

But equations (6) (7) and (8) show that a rescaling of ct time and x space axis is required. If no rescaling occurs, then it means that the deformation generated by P is wholly described by a boost. Then it is simply the usual change of coordinates obeying to SR rule, and no deformation is noticed. Therefore a rescaling occurs, and this one is coherent with time dilation between R and R'. Therefore from equations (6) (7) and (8), if α and β are respectively the s rescaling of ct and x, there is $\alpha^2 = \beta^{-2} = 1 - v^2/c^2$.

Hence the complete deformation is described by $b_v s$. Of course this deformation results from the assumption that equations (6) (7) and (8) are valid. But those equations themselves result, indirectly, from this particular deformation.

Therefore what is required is a check of the coherence of the whole study, at the end of it. And the whole study will be coherent indeed.

In the present document, it will be written abusively that the local deformation is described by a boost. Indeed, only the context will allow to deduce if a rescaling occurs also, or not.

4 Uniformly moving relativistic particle: local deformation

Let's prove that the deformation in C is the b_C boost associated with the speed of light along the D line in the direction of the GW propagation.

Starting from appendix 3, taking the limit when v tends to c, the result is obtained. As usual the interchange of limits theorem is used. The simple convergence is enough for it to work since the studied deformation is only local.

5 Relation between the boost and the speed of a gravitational wave

It will be assumed that the speed of a GW is a given v positive possible value with respect to the R frame. Generally, what is the relation between v and the V speed which is associated with the boost describing its propagated deformation in R? It might be guessed that $V = v$.

This demonstration is easier to understand in a contradiction way. Let's assume the result is wrong. Therefore, it is assumed that the GW speed is v and that the boost of its deformation is V such as $V \neq v$.

Then it is considered a P' particle "surfing" on the GW. That is, P' is always located in close vicinity to a moving M point, where the GW deformation is maximum, and which is of course in motion with the GW propagation. Let's write R' a frame which is attached to this M point. The local deformation generated by P' is independent of its energy. Indeed, the determination of the new and old privileged frames before and after the existence of P are given by SR. There are more details about that at the end of appendix 1. For the same reason, the local deformation generated by P' is also independent of the GW energy. And it is the same for the GW: its generated deformation is independent of its energy and of the P' energy. Therefore, in R', P' generates locally a deformation which is noticed because $V \neq v$. If P' energy is low enough, then the P' local deformation transforms more globally into the GW deformation which is a null deformation in R'. In other words, in R', locally it appears a flat Minkowskian space-time, and also a smaller space-time deformation locally to P'. In R', locally to P' a boost is noticed, associated with a w speed which is not null (w is the relativistic substraction of v by W). But the fact that a space-time structure is generated by a GW or a particle does not modify the following rule: with respect to a frame, a particle or a GW modifies locally space-time structure with the boost which is associated with its speed. And the reverse is true: with respect to a frame, a particle or a GW modifying space-time structure with the boost associated with the w speed is in motion with the w speed. Applying this rule here, it results that P' is in motion in R' at the w speed which is not null. A contradiction arises. This proves the above claim.

6 Uniformly moving relativistic particle: propagation of the deformation along the trajectory

Let's deduce the following, from the result of appendix 4.

The deformation of P in C propagates along the D line in the same direction as the motion of P and is described by b_C which is associated with the speed of light in the same direction.

This can be noticed as soon as P deviates from this D line trajectory. First of all, the GW generated by P propagates along the D line for symmetry reasons. The propagation speed is the speed of light as assumed by assumption (I). And the propagated local deformation is b_C: the demonstration is the same as in appendix 5. Or it is possible to use appendix 5 in the non relativistic case and then taking the limit to the relativistic case.

7 Uniformly moving particle: propagation of the deformation everywhere

The same context and notations as in paragraph 5 are used. Let's assume first that P is non relativistic. Therefore P moves with a v speed in the R frame, with $v < c$. In the R' frame attached to P, the GW propagates along a space sphere centered in P, along the $C'l$ half lines, C' being the space location of P in R', and l being any possible space direction in R'. Locally to any M space point on such a sphere, the GW appears to be a plane wave. The calculations in literature shows only that the propagation speed is the speed of light if the GW is weak (when the modification of the metric is far weaker thant the metric itself). Therefore this can't be applied here. Nevertheless, assumption (I) is easier to assume here than in the general case. And in a given R frame in which D is at rest, the GW still propagates at the speed of light. The trajectories of this propagation from C' to M in R' transform into the same trajectories. And the propagation speed is still the speed of light. Let's write E_C a space-time event having the C space location (therefore the P space location) at a given t_C time in R. Let's write E_M an event which is the propagation of the GW generated by this E_C event. In R' the $C'M$ space line is a ray of the propagation sphere. It has been seen that in R', the speed from E_C to E_M is the speed of light. In R, the speed from E_C to E_M is still the speed of light. But there is $\widehat{CM, D} \neq \pi/2$, if the C space point is considered at time t_C. This angle will depend of v. And in R the enveloppe of the propagation is a cone. The CM line is normal to the cone in M, it is the propagation trajectory from the C space point (at t_C in R) to the M space point.

This remains true when v tends to c. Therefore if P is relativistic, then in space with respect to R the propagation speed is $c/\sqrt{2}$ in the Hn direction normal to the cone.

8 Final picture for a uniformly moving relativistic particle

This results from the previous appendix. The final picture for a uniformly moving relativistic particle is that in R the deformation occuring in C at the E_C event propagates along the Hn space direction, at the $c/\sqrt{2}$ speed. This deformation is described by the boost associated with the $c/\sqrt{2}$ speed in the n direction. This picture is Lorentz invariant.

9 The gravitational wave four-momentums add themselves

Let's prove that in any x space-time event, the four-momentums of all the GWs propagating in x add themselves. This will result in the four-momentum which describes locally space-time structure.

The demonstration will first consider the GWs which are generated by the P_i particles, for i from 0 to infinity (using the same context and notations as in paragraph 10). It means that x is in the intersection of only two or three GWs.

Indeed, the set of the space-time locations of the x events such as there exists only one GW propagating is a set of two dimensional manifolds. For only two GWs encountering themselves in x, this set is the one dimensional intersections of those manifolds (two of them). For only three GWs, it is the zero dimensional intersections (isolated events). Four of more than four manifolds propagating in the same event is something extremely rare which can be avoided in physics.

The studied distribution of energy is an infinite set of dimensionless particles. Let's write R the frame which is the privileged frame in x before all the GW_i occur in x. Let's suppose that n GWs are propagating and encountering themselves in a given x space-time event. Let's write, for i from 0 to n, GW_i the GW having the i indice. For each i, it can be created a P_i particle such that its momentum counteracts exactly the effect of GW_i in x. It means that the P_i speed is the speed of light, and that its total energy in R is the energy of GW_i in R. It means also that the direction of the P_i motion is opposite to the motion of GW_i. The result of the existence of P_i in x is that P_i and GW_i together yields the same space-time deformation locally in x as if they were not existing together in x. Therefore the overall effect of all these P_i particles is such that it cancels the space-time deformation generated by all the GW_i. But the sum of all the P_i particles is a big compound object, let's call it P. So the space-time deformation of all the GW_i is exactly conteracting the space-time deformation generated by P. But conservation of energy principle states that the P four-momentum is the sum of the P_i four-momentums. Therefore the four-momentum of the space-time deformation of the whole set of GW_i is the sum of the GW_i four-momentums.

$$
\begin{aligned}
F_m\big(\{GW_i, i = 0..n\}\big) &= -F_m(P) \\
&= -\Sigma_{i=0}^{n} F_m(P_i) \\
&= -\Sigma_{i=0}^{n}\big(-F_m(GW_i)\big) \\
&= \Sigma_{i=0}^{n} F_m(GW_i)
\end{aligned}
\tag{21}
$$

$F_m()$ is a function giving the four-momentum of a particle or a GW. This is the end of the proof.

10 The free falling particle in the Schwarzschild metric

Let's prove that in the Schwarzschild metric, a P_1 free falling particle, being at rest when located infinitely far from the P particle which is located in the center of the symmetry, having an infinitely small energy, (or an energy weak enough in order not to modify its free fall trajectory), follows a time line which is transformed by equations (6) (7) and (8). The same context and notations as in paragraph 11.3 are used. Notably the R_0 frame is attached to P.

The P_1 trajectory is from the start aligned with the time axis of the new and old privileged frames of P (from the start those frames are the same). Since the privileged frame is inertial, the P_1 trajectory is exactly the time line described by the time axis of the new privileged frame. This appears to be the end of the demonstration. Indeed, one can assume that the deformation generated by P_1 is the same as the gravitational deformation generated by P. This would result from a complete coherence of GR. But let's prove that coherence directly here.

For this, in the R_1 frame which is attached to P_1, let's study another P_2 particle which always stays infinitely close to P_1 and which has an energy at rest far weaker than the P_1 rest energy. P_2 follows the same trajectory as P_1. And this can be understood as only the result of the space-time deformation of P_1. Indeed, P_2 energy is far weaker than P_1 energy, and P_2 is always close to P_1. It must be noticed that the space-time gravitational deformation generated by P is without effect locally to P_1, completely replaced by the space-time deformation of P_1. The P_2 free fall allows to ignore it completely.

Therefore the P_1 deformation of space-time is what forces P_2 to follow the same trajectory as P_1. The R_1 frame attached to P_1 is privileged because it's inertial and also because it's privileged from the start, when P_1 is at rest infinitely far from P. But the P_1 deformation is described by the b_1 boost associated with its motion in R_0. Now let's decrease P_1 energy to 0, progressively, without modifying the P_2 energy (it does not mean that the conservation of energy principle is broken). This P_1 energy decrease will not change the P_2 trajectory. This is because P_1 energy (hence P_2 energy, too) has been assumed to be weak enough, in order to give that result.

Since their energies are allways considered infinitelly small, P_1 and P_2 are allways in free fall. Hence their trajectories remain the same whatever is their energies. Noteably it is true for P_2.

Therefore, the local space-time deformation generated by the P_1 motion in R_0, along its trajectory, is the same as the gravitational space-time deformation generated by P along this trajectory.

The result is that b_1 describes the gravitational local deformation generated by P, along the P_1 trajectory. Of course after the execution of b_1, a rescaling is required in order to account for the more global curvature. This ends the demonstration.

This demonstration can be applied to any energy distribution being a flat Minkowskian space-time everywhere except for a bounded space-time domain. Indeed the above demonstration can be re-executed without change for those generalized energy distributions. The behavior and evolution of the privileged frame is the behavior and evolution of the free falling particle, having a negligeable mass, located infinitely far.

11 An equation of G: calculations

In this appendix an equation of G is tried. Assumptions (i) (ii) (iii) and (iiii) are assumed. The same context and notations as in paragraph 11.3 are used.

Let's remind that in the Schwarzschild metric, the fllowing equation relies the metric to the free fall speed.

$$g_{00}(x') = 1 - v^2/c^2 \tag{22}$$

v is the speed of the usual P_1 free falling test particle. The validity of equation (22) comes from assumption (i). Indeed, equations (22) and $g_{00}(x') = 1 - R/r$ where R is the Schwarzschild ray of P, r being the distance between P and the x' event, are valid together because Newton's law is assumed to be valid. Equation (24), which comes further, can be used to confirm that. From equation (4) and (22), using $e = D^1(x)/D^0(x)$, it results the following equation.

$$g_{00}(x') = \frac{1+2e}{(1+e)^2} \tag{23}$$

Then the geodesic equation of P_1 is calculated [8].

$$\frac{\partial^2 r}{\partial \tau^2} = -\frac{c^2}{2}\frac{\partial g_{00}}{\partial r} \tag{24}$$

Here r is the space distance between P_1 and P, and τ is the proper time of P_1. Replacing g_{00} by its value given by equation (23), equation (24) becomes the following one.

$$\frac{\partial^2 r}{\partial \tau^2} = c^2 \frac{e}{(1+e)^3}\frac{\partial e}{\partial r} \tag{25}$$

Then assumption (i) is used, Newton's law is valid. The asymptotic formulation of equation (25) is yielded, relying Newton's law with the asymptotic value of the rhs of equation (25).

$$-\frac{M_0 G}{r^2} \simeq c^2 e \frac{\partial e}{\partial r} \tag{26}$$

Here M_0 is the mass of P. Assumtion (iii) is used. The contribution of P in equation (3) is far weaker than the sum of the other contributions. Therefore, e is asymptotically equal to 0 in front of 1. The solution of this differential equation gives the asymptotic value of e.

$$e \simeq \sqrt{\frac{R}{r}} \tag{27}$$

R is the Schwarzschild ray of P. From that is deduced $D^1(x)/D^0(x) \simeq \sqrt{R/r}$. Now, v being the speed of the free falling particle, it is also the speed which is used in equations (6) and (8). This is shown in appendix 10. Using also equation (3), it results the following equation.

$$\frac{1_w(x,y_0)f(x,y_0)C^0(y_0)}{\Sigma_{n=0}^{\infty}1_w(x,y_n)f(x,y_n)C^0(y_n)} \simeq \sqrt{\frac{R}{r}} \tag{28}$$

It has been supposed that the y_0 virtual particle is the only one pertaining to P. It was used $C^0(y_0) = C^1(y_0)$, which reflects the fact that C^μ is a null four-momentum.

Under (iii) assumption, the denominator of equation (28) is constant, that is, independant of the x location. Therefore this equation shows that the $1_w(x,y_0)$ $f(x,y_0)\,C^0(y_0)$ contribution is proportional to $1/\sqrt{r}$. Under the (ii) assumption, it can be deduced that the $1_w(x,y_n)f(x,y_n)C^0(y_n)$ contributions are proportional to $1/\sqrt{|x-y_n\|_3}$. $\|x-y_n\|_3$ is the space length calculated in a covariant way along the y_n to x geodesic and observed in x, in R_0.

Also x, and therefore r being constant, the $1_w(x,y_0)f(x,y_0)C^0(y_0)$ contribution is not proportional to $C^0(y_n)$ but proportional to $\sqrt{R}$, therefore to $\sqrt{C^0(y_n)}$. This is in contradiction with the hypothesis at the construction of equation (3). Here it is noticed a proportionnality to the square root of an energy. The answer to this contradiction is that the final picture is coherent. For example the effective energy of any GW brought by $C(y_n)$ is calculated from the geodesic equation which derives from the metric. And at the end of it, the $C^0(y_n)$ contribution will be multipied by itself in order to give the effective energy of this GW.

Therefore one can replace each $1_w(x,y_n)f(x,y_n)C^0(y_n)$ contribution of equation (28) by $1_w(x,y_n)\sqrt{C^0(y_n)/\|x-y_n\|_3}$. The result is the following.

$$\frac{1_w(x,y_n)\sqrt{\frac{C^0(y_0)}{r}}}{\Sigma_{n=0}^{\infty}1_w(x,y_n)f(x,y_n)\sqrt{\frac{C^0(y_n)}{/\|x-y_n\|_3}}} \simeq \sqrt{\frac{R}{r}} \tag{29}$$

It can be used $R = 2M_0G/c^2$ and $C^0(y_0) = M_0c^2$, in order to transform equation (29) into the following one.

$$G \simeq \frac{c^4}{2\left(\Sigma_{n=0}^{\infty} 1_w(x, y_n)\sqrt{\frac{C^0(y_n)}{\|x-y_n\|_3}}\right)^2} \tag{30}$$

Now it is possible to use this result with VGWs. It means that the energy distributions of the P_i are replaced by the limits of the S_n^i distributions, as shown by equation (2). The result is equation (10), which is equation (30), without the $1_w(x, y_n)$ terms. Indeed, in any given x and y_n space-time events, it is possible to find a m and a S_m^n distribution centered on y_n, such as the GW, generated by the relativistic virtual particle of S_m^n, propagates in x. Moreover, this S_m^n distribution can be chosen with a circle's ray as weak as we want (remember that those S_m^n distributions are circle like trajectories of relativistic virtual particles). Another argument allowing to erase the $1_w(x, y_n)$ terms is that after the limits of equation (2) are done, each x space-time event is reached by the VGWs of each P_n particle.

References

[1] F. Lassiaille, "Surrounding Matter Theory," *EPJ Web of Conf* **182**, 03006 (2018).

[2] F. Lassiaille, "Relativity in Motion: Short Version", *Nuclear Theory* **Vol. 39** eds. M. Gaidarov, N. Minkov, Heron Press, Sofia (2022).

[3] A. Jaffe and E. Witten, *Quantum Yang-Mills Theory* (Clay Math Institute)

[4] L. D. Faddeev, "Mass in Quantum Yang-Mills Theory (Comment on a Clay Millennium Problem)" In: Benedicks M., Jones P.W., Smirnov S., Winckler B. (eds) *Perspectives in Analysis. Mathematical Physics Studies*, **vol 27** Springer, Berlin, Heidelberg (2005).(arXiv:0911.1013 [math-ph])

[5] BP Abbott, R Abbott, TD Abbott, F Acernese, K Ackley, C Adams, et al., "Gravitational waves and gamma-rays from a binary neutron star merger: GW 170817 and GRB 170817A" *The Astrophysical Journal Letters*, **848** (2): L13. (2017).

[6] D. J. Raine, "Mach's Principle in general relativity", *MNRAS* **171** (3): 507–528 (1975)

[7] A. Unsöld, B. Baschek, "The New Cosmos: An Introduction to Astronomy and Astrophysics", *Physics and astronomy online, Springer* p. 485 (2001).

[8] F. Lassiaille, "Gravitational Model of the Three Elements Theory: Mathematical Explanations," *J. Mod. Phys.* **4** No. 7, pp. 1027-1035 (2013). Vol. 4 No. 7, pp. 1027-1035, 2013.

[9] E. V. Pitjeva, "Determination of the Value of the Heliocentric Gravitational Constant (GM0) from Modern Observations of Planets and Spacecraft" *J. Phys. Chem.* Ref. Data 44 031210 (2015).

[10] M. Milgrom, "A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis", *ApJ* , **270** 365 (1983)

[11] W. Juven, "Unified model beyond grand unification", *Physical Review D.*, **103** (10): 105024 (2021)

[12] H. Georgi and S. L. Glashow, "Unity of All Elementary Particle Forces", *Phys. Rev. Lett.*, **32** 438–441 (1974)

[13] H. Fritzsch and P. Minkowski, ”Unified Interactions of Leptons and Hadrons”, *Annals Phys.*, **93** 193–266 (1975).

[14] A. F. Pugach, D. Olenici, ”Observations of Correlated Behavior of Two Light Torsion Balances and a Paraconical Pendulum in Separate Locations during the Solar Eclipse of January 26th, 2009”, *Advances in Astronomy*, **2012** Article ID 263818, 6 pages, (2012).

[15] I. Banik et al., ”Strong constraints on the gravitational law from Gaia DR3 wide binaries”, *MNRAS*, **527** 4573 (2023)

[16] K.-H. Chae, ”Breakdown of the Newton–Einstein Standard Gravity at Low Acceleration in Internal Dynamics of Wide Binary Stars”, *ApJ*, **952** 128 (2023)

[17] X. Hernandez, ”Internal kinematics of Gaia DR3 wide binaries: anomalous behaviour in the low acceleration regime”, *MNRAS*, **525** 1401 (2023)